2.10 WRITING AND SOLVING INEQUALITIES IN ONE VARIABLE

2.10.0 Lesson Overview

In this lesson, you will write and solve inequalities to answer questions about a situation. You will also practice finding the solution set to an inequality by reasoning about its composition and parts.

When you finish this lesson, you will be able to:

- Analyze the structure of an inequality in one variable to help determine if the solution is greater or less than the solution to the related equation.
- Write and solve inequalities to answer questions about a situation.

Here are the **activities** that will help you reach those goals:

- 2.10.1: Writing an Inequality to Represent a Constraint
- 2.10.2: Using Inequalities to Solve a Problem
  - 2.10.2: Self Check
  - 2.10.2: Additional Resources
- 2.10.3: Different Ways of Solving an Inequality
  - 2.10.3: Self Check
  - 2.10.3: Additional Resources
- 2.10.4: Matching Inequalities and Solutions
- 2.10.5: Solving Inequalities
  - 2.10.5: Self Check
  - 2.10.5: Additional Resources
- 2.10.6: Writing Inequalities to Represent a Situation

After that, you'll **practice and review**.

- 2.10.7: Practice
- 2.10.8: Lesson Summary

2.10.1 Writing an Inequality to Represent a Constraint

Warm Up

Kiran is getting dinner for his drama club on the evening of their final rehearsal. He plans to buy some prepared dishes from a supermarket.

- The budget for the dinner is $60.
- The prepared dishes are sold by the pound, at $5.29 per pound.
- He also plans to buy two large bottles of sparkling water at $2.49 each.

1. Represent the constraints in the situation mathematically. If you use variables, specify what each one means.

2. How many pounds of prepared dishes can Kiran buy? Be prepared to show your reasoning.

2.10.2 Using Inequalities to Solve a Problem

Activity

Use the following information to answer questions 1 – 2.

> Han is about to mow some lawns in his neighborhood. His lawn mower has a 5-gallon fuel tank, but Han is not sure how much gasoline is in the tank.
> He knows, however, that the lawn mower uses 0.4 gallon of gasoline per hour of mowing.

1. What are all the possible values for x, the number of hours Han can mow without refilling the lawn mower?
2. Write one or more inequalities to represent your response. Be prepared to show your reasoning.

Video: Using Inequalities to Solve a Problem

Watch the following video to learn more about how to solve a problem using inequalities.

Access multimedia content (http://openstax.org/books/algebra-1/pages/2-10-2-using-inequalities-to-solve-a-problem)

Self Check

Samantha is feeding the horses at her barn. She has 6 buckets of oats. If each horse eats $\frac{3}{4}$ bucket of oats, how many horses can she feed at most?

a. $x \leq 10$, Samantha can feed a maximum of 10 horses.
b. $x \leq 8$, Samantha can feed a maximum of 8 horses.
c. $x \leq 4$, Samantha can feed a maximum of 4 horses.
d. $x \leq 6$, Samantha can feed a maximum of 6 horses.

Additional Resources

Using Inequalities to Solve a Problem

Let's look at an example where we might be able to solve a problem using inequalities.

A school is repainting its classroom walls. The school has at most 30 gallons of paint. The painter needs $\frac{5}{2}$ gallons of paint for each classroom. How many classrooms can the school paint at most?

We know that the answer is greater than or equal to 0 classrooms, so $x \geq 0$.

We also know that each classroom requires $\frac{5}{2}$ or 2.5 gallons of paint.

Let's take a guess at how many classrooms and check if we are close to the solution. How many gallons of paint would we need to paint 10 classrooms?

$10 \times 2.5 = 25$, and since $25 < 30$, it could be possible for the school to paint 10 classrooms.

We can try 11 classrooms.

$11 \times 2.5 = 27.5$, and since $27.5 < 30$, it could be possible for the school to paint 11 classrooms.

We can try 12 classrooms.

$12 \times 2.5 = 30$, and since $30 = 30$, the most classrooms the school could paint is 12 classrooms.

TRY IT **Using Inequalities to Solve a Problem**

Use the following information for questions 1 – 2.

While working in a car factory, a worker can use a drill for as long as the batteries last. The worker has at most 9 fully charged batteries. Each car the worker helps assemble uses $\frac{3}{4}$ of a full battery.

1. What are all of the possible values for c, the number of cars the work can help assemble? Write one or more inequalities to represent your response.
2. Explain the meaning of the inequalities.

2.10.3 Different Ways of Solving an Inequality

Activity

Use the following scenario to answer questions 1 – 2.

Andre and Priya used different strategies to solve the following inequality, but they reached the same solution.
$2(2x + 1.5) \leq 18 - x$
Make sense of each strategy until you can explain what each student has done.

Andre	Priya
$2(2x + 1.5) = 18 - x$ $4x + 3 = 18 - x$ $4x - 15 = -x$ $-15 = -5x$ $3 = x$ Testing to see if $x = 4$ is a solution: $2(2 \cdot 4 + 1.5) < 18 - 4$ $2(9.5) < 14$ $19 < 14$ The inequality is false, so 4 is not a solution. If a number greater than 3 is not a solution, the solution must be less than 3, or $x < 3$.	$2(2x + 1.5) = 18 - x$ $4x + 3 = 18 - x$ $5x + 3 = 18$ $5x = 15$ $x = 3$ In $4x + 3 = 18 - x$, there is $4x$ on the left and $-x$ on the right. If x is a negative number, $4x + 3$ could be positive or negative, but $18 - x$ will always be positive. For $4x + 3 < 18 - x$ to be true, x must include negative numbers or x must be less than 3.

1. Summarize the strategy Andre used to solve the inequality.
2. Summarize the strategy Priya used to solve the inequality.
3. Examine four inequalities:
   Inequality A: $\frac{1}{5}p > -10$
   Inequality B: $4(x + 7) \leq 4(2x + 8)$
   Inequality C: $-9n < 36$
   Inequality D: $\frac{c}{3} < -2(c - 7)$
   Work with a partner to decide on at least two inequalities to solve. Solve one inequality using Andre's strategy (by testing values on either side of the given solution), while your partner uses Priya's strategy (by reasoning about the parts of the inequality). Switch strategies for the other inequality.

Are you ready for more?

Extending Your Thinking

1. Using positive integers between 1 and 9 and each positive integer at most once, fill in values to get two constraints so that $x = 7$ is the only integer that will satisfy both constraints at the same time.
$\square x + \square < \square x + \square$

2. Using positive integers between 1 and 9 and each positive integer at most once, fill in values to get two constraints so that $x = 7$ is the only integer that will satisfy both constraints at the same time.
$\square x + \square > \square x + \square$

Self Check

Solve the following inequality by solving the related equation and reasoning what the answer may be given the situation.

$\frac{1}{3}p < 4$

a. $p < \frac{4}{3}$; The related equation is $p = \frac{4}{3}$. Since the solution set must contain negative numbers, p must be less than $\frac{4}{3}$.
b. $p < \frac{3}{4}$; The related equation is $p = frac34$. Since the solution set must contain negative numbers, p must be less than $\frac{3}{4}$.
c. $p < 12$; The solution to the related equation is $p = 12$. Since $\frac{1}{3}p$ must be less than 4, the solution cannot include large positive numbers, so numbers larger than 12 cannot be included in the solution.
d. $p > 12$; The solution to the related equation is $p = 12$. For $\frac{1}{3}p$ to be less than 4, the solution must contain large positive numbers.

Additional Resources

Reasoning about Solution Sets to Linear Inequalities

You can derive a lot of information about the solution set to an inequality by studying the inequality itself.

Let's look at the inequality shown.

$12x > -3(x - 1)$

The left side of the inequality contains $12x$. The right side of the inequality contains $-3x$.

We can think to ourselves: "What values of x would make $12x$ greater than $-3x$?"

If x is a large positive number, then $12x$ will still be positive. If x is a large positive number, then $-3x$ will be a negative number. This makes the inequality true, so the solution set must include large positive numbers.

Let's solve the related equation.

$$\begin{aligned} 12x &> -3(x-1) \\ 12x &= -3(x-1) \\ 12x &= -3x+3 \\ 15x &= 3 \\ x &= \frac{3}{15} \end{aligned}$$

Since we know that the solution set must include large positive numbers, then $x > \frac{3}{15}$ must be the solution to the inequality.

TRY IT **Reasoning about Solution Sets to Linear Inequalities**

For questions 1 – 2, solve the related equation for each inequality. Use what you know about the inequality to determine the solution set.

1. $-5y \geq 30$
2. $4p < -4$

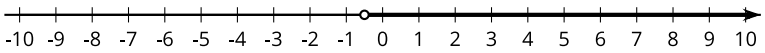
2.10.4 Matching Inequalities and Solutions

Activity

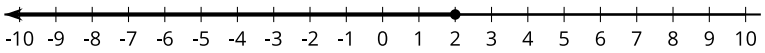
Match each inequality to a graph that represents its solutions. Be prepared to show your reasoning.

- $6x \leq 3x$
- $\frac{1}{4}x > -\frac{1}{2}$
- $5x + 4 \geq 7x$
- $8x - 2 < -4(x - 1)$
- $\frac{4x-1}{3} > -1$
- $\frac{12}{5} - \frac{x}{5} \leq x$

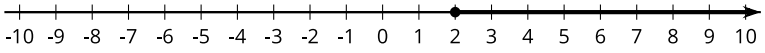
Graph A



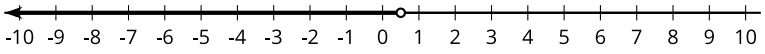
Graph B



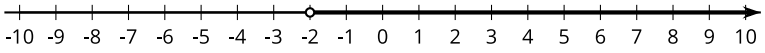
Graph C



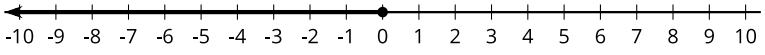
Graph D



Graph E



Graph F



Use this inequality for questions 1 – 2:

$6x \leq 3x$

1. Match the graph to the equation.

2. Explain or show your reasoning.

Use this inequality for questions 3 and 4:

$\frac{1}{4}x > -\frac{1}{2}$

3. Match the graph to the equation.

4. Explain or show your reasoning.

Use this inequality for questions 5 – 6:

$5x + 4 \geq 7x$

5. Match the graph to the equation.

6. Explain or show your reasoning.

Use this inequality for questions 7 and 8:

$8x - 2 < -4(x - 1)$

7. Match the graph to the equation.

8. Explain or show your reasoning.

Use this inequality for questions 9 – 10:

$\frac{4x-1}{3} > -1$

9. Match the graph to the equation.

10. Explain or show your reasoning.

Use this inequality for questions 11 and 12:

$\frac{12}{5} - \frac{x}{5} \leq x$

11. Match the graph to the equation.

12. Explain or show your reasoning.

2.10.5 Solving Inequalities

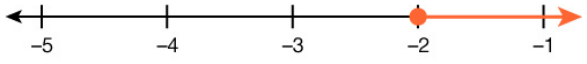
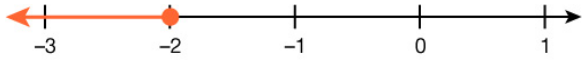
Activity

In this activity, we're going to solve inequalities. Work with a partner to solve each inequality given. Then match it with the number line representing its solution.

As you complete the activity, discuss with your partner how you know each number line is the solution to its inequality.

When you think you've found the correct number line for an inequality, substitute a test point that is on the highlighted part of the number line into the inequality. Is it true?

Try a test point that is not highlighted. Is the inequality false?

1. $x - 4 \leq -6$	Graph A 
2. $x + 6 \geq 8$	Graph B 
3. $4x \leq 8$	Graph C 
4. $-\frac{1}{2}x \leq 1$	Graph D 

Use the table to choose the matching graph to the inequality.

1. $x - 4 \leq -6$

2. $x + 6 \geq 8$

3. $4x \leq 8$

4. $-\frac{1}{2}x \leq 1$

For each of the inequalities above, match it to the property you used to solve.

5. $x - 4 \leq -6$	A. Multiplication Property of Inequality
6. $x + 6 \geq 8$	B. Addition Property of Inequality

7. $4x \leq 8$	C. Division Property of Inequality
8. $-\frac{1}{2}x \leq 1$	D. Subtraction Property of Inequality

5. $x - 4 \leq -6$

6. $x + 6 \geq 8$

7. $4x \leq 8$
Select **two** properties that could be used to solve the inequality

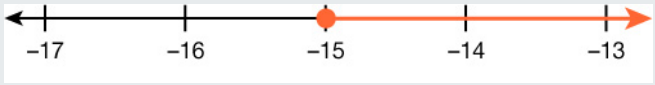
8. $-\frac{1}{2}x \leq 1$
Select **two** properties that could be used to solve the inequality

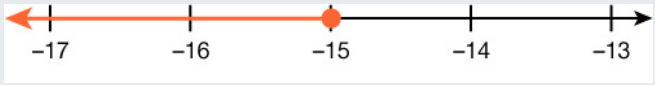
9. Is there anything unique about the method used to solve $-\frac{1}{2}x \leq 1$? Explain.

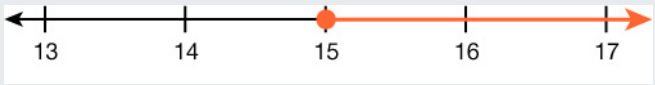
Self Check

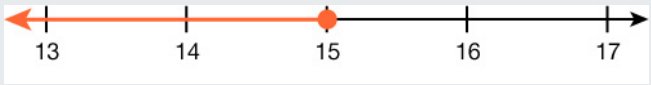
Solve the inequality. Which number line represents the solution?

$-\frac{1}{3}m \leq 5$

a.


b.


c.


d.


Additional Resources

Solving Linear Inequalities

A linear inequality is much like a linear equation—but the equal sign is replaced with an inequality sign. So, when we solve linear equations, we are able to use the properties of equality to add, subtract, multiply, or divide both sides and still keep the equality. Similar properties hold true for inequalities.

We can add or subtract the same quantity from both sides of an inequality and still keep the inequality. For example:

$$\begin{array}{ll} -4 < 2 & -4 < 2 \\ -4 - 5 < 2 - 5 & -4 + 7 < 2 + 7 \\ -9 < -3 \text{ True} & 3 < 9 \text{ True} \end{array}$$

Notice that the inequality sign stayed the same.

This leads us to the Addition and Subtraction Properties of Inequality.

Addition and Subtraction Properties of Inequality

For any numbers a, b, and c, if $a < b$, then

$a + c < b + c$

$a - c < b - c$

For any numbers a, b, and c, if $a > b$, then

$a + c > b + c$

$a - c > b - c$

We can add or subtract the same quantity from both sides of an inequality and still keep the inequality.

What happens to an inequality when we divide or multiply both sides by a constant?

Let's first multiply and divide both sides by a positive number.

$10 < 15$ $\qquad$ $10 < 15$

$10(5) < 15(5)$ $\qquad$ $\frac{10}{5} < \frac{15}{5}$

$50 < 75$ True $\qquad$ $2 < 3$ True

The inequality signs stayed the same.

Does the inequality stay the same when we divide or multiply by a negative number?

$10 < 15$ $\qquad$ $10 < 15$

$10(-5) \;?\; 15(-5)$ $\qquad$ $\frac{10}{-5} \;?\; \frac{15}{-5}$

$-50 \;?\; -75$ $\qquad$ $-2 \;?\; -3$

$-50 > -75$ $\qquad$ $-2 > -3$

Notice that when we filled in the inequality signs, the inequality signs reversed their direction.

When we divide or multiply an inequality by a positive number, the inequality sign stays the same. When we divide or multiply an inequality by a negative number, the inequality sign reverses.

This gives us the Multiplication and Division Properties of Inequality.

Multiplication and Division Properties of Inequality

For any numbers a, b, and c,

multiply or divide by a positive

if $a < b$ and $c > 0$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

if $a > b$ and $c > 0$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

Multiply or divide by a negative

if $a < b$ and $c < 0$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

if $a > b$ and $c < 0$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

When we divide or multiply an inequality by a:

- positive number, the inequality stays the same.
- negative number, the inequality reverses.

Sometimes when solving an inequality, as in the next example, the variable ends up on the right. We can rewrite the inequality in reverse to get the variable to the left.

$x > a$ has the same meaning as $a < x$.

Think about it as "If Xander is taller than Andy, then Andy is shorter than Xander."

In Examples 1 and 2: solve the inequality, graph the solution on the number line, and write the solution in interval notation.

EXAMPLE 1

$x - \frac{3}{8} \leq \frac{3}{4}$

Solution

Step 1 - Add $\frac{3}{8}$ to both sides of the inequality.

$x - \frac{3}{8} + \frac{3}{8} \leq \frac{3}{4} + \frac{3}{8}$

Step 2 - Simplify.

$x \leq \frac{9}{8}$

Step 3 - Graph the solution on the number line.

Step 4 - Write the solution in interval notation.

$(-\infty, \frac{9}{8}]$

EXAMPLE 2

$-15 < \frac{3}{5}z$

Solution

Step 1 - Multiply both sides of the inequality by $\frac{5}{3}$. Since $\frac{5}{3}$ is positive, the inequality stays the same.

$(\frac{5}{3})(-15) < (\frac{5}{3})(\frac{3}{5}z)$

Step 2 - Simplify.

$-25 < z$

Step 3 - Rewrite with the variable on the left.

$z > -25$

Step 4 - Graph the solution on the number line.

Step 5 - Write the solution in interval notation.

$(-25, \infty)$

TRY IT **Solving Linear Inequalities**

Solve the following linear inequalities:

1. $9y < 54$
2. $-13m \geq 65$

2.10.6 Writing Inequalities to Represent a Situation

Cool Down

Use the following scenerio for questions 1 – 2.

> Lin's job pays $8.25 an hour plus $10 of transportation allowance each week. She has to work at least 5 hours a week to keep the job, and she can earn up to $175 per week, including $10 to cover transportation per week.

1. Represent this situation mathematically. If you use variables, specify what each one means.
2. How many hours per week can Lin work? Be prepared to show your reasoning.

2.10.7 Practice

Complete the following questions to practice the skills you have learned in this lesson.

1. Xander is loading a pallet with boxes that each weighs 45 pounds. The pallet can safely support no more than 900 pounds. How many boxes can he safely load onto the pallet?
   ___ boxes or less

2. The elevator in Yehire's apartment building has a sign that says the maximum weight is 2100 pounds. If the average weight of one person is 150 pounds, how many people can safely ride the elevator?
   a maximum of ___ people

3. Jaden is looking at apartments with three of his friends. They want the monthly rent to be no more than $2360. If the roommates split the rent evenly among the four of them, what is the maximum rent each will pay?
   $___

4. Melody got a $20 gift card for the coffee shop. Her favorite iced drink costs $3.79. What is the maximum number of drinks she can buy with the gift card?
   ___ drinks

5. Teegan likes to play golf. He has budgeted $60 next month for the driving range. It costs him $10.55 for a bucket of balls each time he goes. What is the maximum number of times he can go to the driving range next month?
   ___ times

6. Ryan charges his neighbors $17.50 to wash their car. How many cars must he wash next summer if his goal is to earn at least $1500?
   ___ cars

7. Alejandro's water bill is $24.80 per month plus $2.20 per ccf (hundred cubic feet) of water. What is the maximum number of ccf Alejandro can use if he wants his bill to be no more than $60?
   ___ ccf

8. Look at the inequality $5m + 6 < 14 - 2m$. Which of the following is NOT true?

   a. If m is a negative number, then $14 - 2m$ will always be negative.
   b. If m is a positive number, then $5m + 6$ will always be positive.
   c. If m is a negative number, then $14 - 2m$ will always be positive.
   d. If m is a negative number, then $5m + 6$ could be a positive or negative number.

9. Solve the inequality: $x + 8 > -12$

   a. $x > 4$
   b. $x > -4$
   c. $x > 20$
   d. $x > -20$

10. Solve the inequality: $x - 15 < -3$

    a. $x < -18$
    b. $x < 12$
    c. $x < -12$
    d. $x < 18$

11. Solve the inequality: $x + 19 \leq 23$

    a. $x \leq 4$
    b. $x \leq 42$
    c. $x \leq -4$
    d. $x \geq 4$

12. Solve the inequality: $7x < -56$

a. $x < -49$
b. $x < -63$
c. $x < -8$
d. $x > -8$

13. Solve the inequality: $\frac{x}{2} \leq \frac{5}{2}$

a. $x \geq 5$
b. $x \leq 2$
c. $x \leq 5$
d. $x \leq \frac{5}{2}$

14. Solve the inequality: $-3x > 21$

a. $x < -7$
b. $x > -7$
c. $x < -24$
d. $x < 18$

2.10.8 Lesson Summary

In this lesson, you learned how to:

- Analyze the structure of an inequality in one variable to help determine if the solution is greater or less than the solution to the related equation.
- Write and solve inequalities to answer questions about a situation.

Here are the **activities** that helped you reach those goals:

- 2.10.1: Writing an Inequality to Represent a Constraint
  - In this activity, you practiced writing an inequality to represent a constraint, reasoning about its solutions, and interpreting the solutions.
- 2.10.2: Using Inequalities to Solve a Problem
  - In this activity, you used inequalities to solve a problem about a situation whose constraints might be unfamiliar and in which multiple quantities are unknown.
  - 2.10.2: Self Check
  - 2.10.2: Additional Resources
- 2.10.3: Different Ways of Solving an Inequality
  - In this activity, you explored some ways to decide whether the solution set to an inequality is greater than or less than a particular boundary value (identified by solving a related equation).
  - 2.10.3: Self Check
  - 2.10.3: Additional Resources
- 2.10.4: Matching Inequalities and Solutions
  - In this activity, you received additional practice in reasoning about the solutions to inequalities without any context.
- 2.10.5: Solving Inequalities
  - In this activity, you used the Addition and Subtraction Properties of Inequality to solve different inequalities. You also used the Multiplication and Division Properties of Inequality to solve inequalities. You learned that when using the Multiplication and Division Properties of Inequality, you must reverse the symbol when you multiply or divide by a negative number.
  - 2.10.5: Self Check
  - 2.10.5: Additional Resources
- 2.10.6: Writing Inequalities to Represent a Situation
  - In this activity, you wrote inequalities to represent a situation.

After these activities, you completed the following **practice**:

- 2.10.7: Practice

Checking In

On a scale of 1 to 5, how confident do you feel about the learning goals of this lesson?