5.7 ANALYZING GRAPHS

5.7.0 Lesson Overview

In this lesson, you will determine whether situations are characterized by exponential growth or by exponential decay given real-world scenarios of descriptions and graphs.

When you finish this lesson, you will be able to:

- Use graphs to compare and contrast situations that involve exponential decay.
- Use information from a graph to write an equation that represents exponential decay.

Here are the **activities** that will help you reach those goals:

- 5.7.1: Patterns in Real Numbers
- 5.7.2: Comparing Graphs
  - 5.7.2: Self Check
  - 5.7.2: Additional Resources
- 5.7.3: Describing Graphs
  - 5.7.3: Self Check
  - 5.7.3: Additional Resources
- 5.7.4: Quantities that Change Exponentially

After that, you'll **practice and review**.

- 5.7.5: Practice
- 5.7.6: Lesson Summary

5.7.1 Patterns in Real Numbers

Warm Up

In the table, find as many patterns as you can. Use one or more patterns to help you complete the table. Be prepared to explain your reasoning.

fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$
decimal	0.5	0.25	0.125	a.____	b.____

1. What is the value for a?

2. What is the value for b?

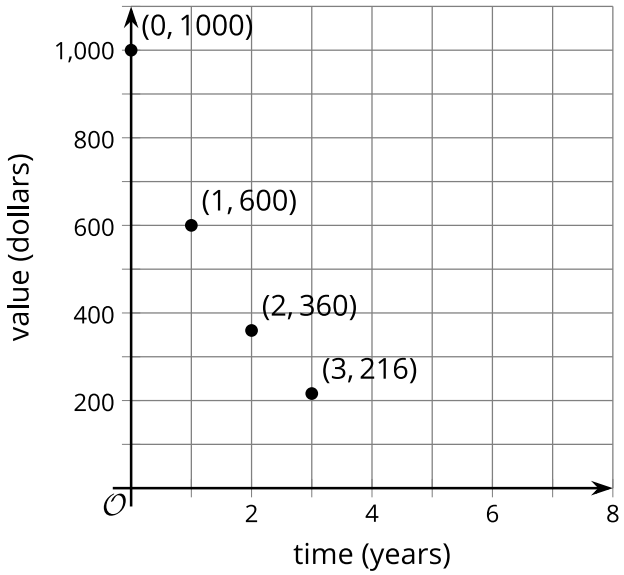
3. Describe the pattern you used to find the values that completed the table.

5.7.2 Comparing Graphs

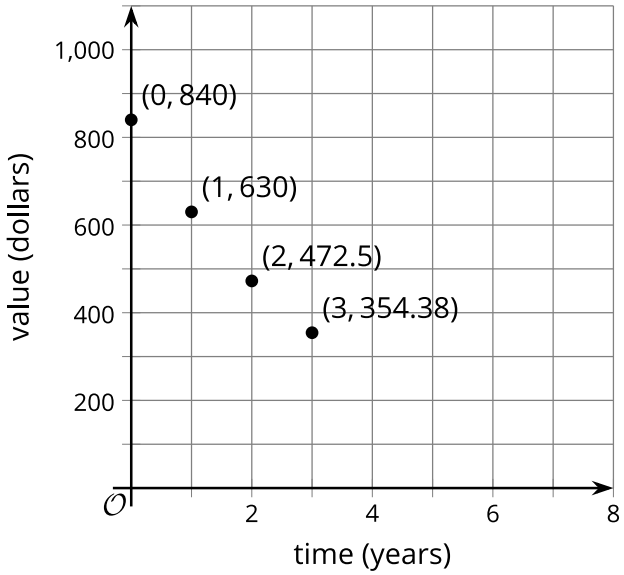
Activity

The value of some cell phones changes exponentially after initial release. Here are graphs showing the depreciation of two phones 1, 2, and 3 years after they were released.

Phone A



Phone B



1. Which phone is more expensive to buy when it is first released?

2. Which graph depicts the value of the cell phone falling in value more quickly? Explain how you know.

3. How does the value of phone A change with every passing year?

4. How does the value of phone B change with every passing year?

5. If the phones continue to depreciate by the same factor each year, what will the value of each phone be 4 years after its initial release?

a. Enter your answer for Phone A.

b. Enter your answer for Phone B.

6. For each cell phone, write an equation that relates the value of the phone in dollars to the years since release, t. Use v for the value of Phone A and w for the value of Phone B.

a. Enter your answer for Phone A.

b. Enter your answer for Phone B.

Are you ready for more?

Extending Your Thinking

When given data, it is not always clear how to best model it. In this case, we were told the value of the cell phones was changing exponentially. Suppose, however, we were instead just given the initial values of the cell phones when released and the values after each of the first three years.

1. Use technology to compute the best fit line for each cell phone. Round any numbers to the nearest dollar.

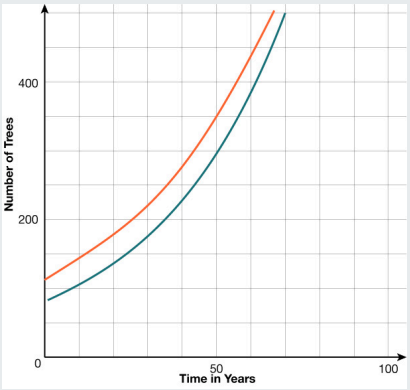
a. Enter your answer for Phone A.

b. Enter your answer for Phone B.

2. Explain why, in this situation, an exponential model might be more appropriate than the linear model you just created.

Self Check

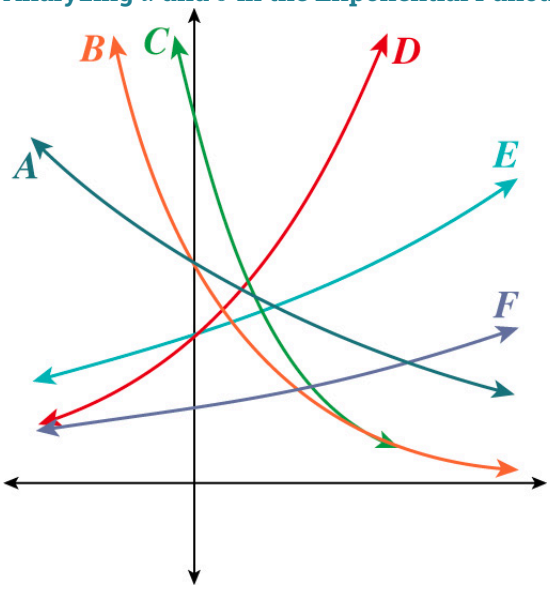
For the following exercise, consider this scenario: For each year t, the population of trees in Forest A is represented by the function $A(t) = 115(1.025)^t$, (red curve). In a neighboring forest, Forest B, the population of the same type of tree is represented by the function $B(t) = 82(1.029)^t$, (blue curve). Looking at the graph provided, which forest had a greater number of trees initially?



a. Forest B, because it has a greater growth rate.
b. Forest A, because it has a greater growth rate.
c. Forest A, because it starts higher on the y-axis.
d. Both forests started with the same number of trees.

Additional Resources

Analyzing a and b in the Exponential Function $y = ab^x$



1. Which graph has the largest value for b?
2. Which graph has the smallest value for b?
3. Which graph has the largest value for a?
4. Which graph has the smallest value for a?

When using the form $y = ab^x$, it is important to remember that a is the initial value, or the y-intercept, and b is the growth factor. The growth factor reflects an exponential growth function when the value is greater than one. The growth factor reflects an exponential decay function when the value is between zero and one.

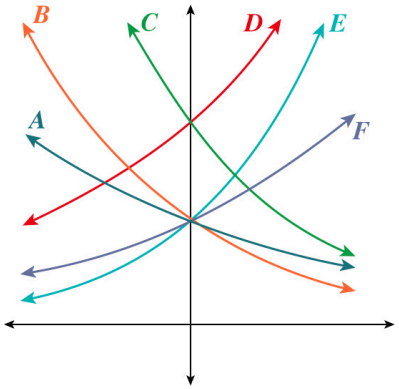
For question 1, you are looking for the largest growth factor, which would mean an exponential growth function that is increasing the most rapidly. First, identify the graphs that reflect exponential growth, which would be D, E, and F, and then decide which is the steepest. Graph D increases much faster than both graphs E and F.

For question 2, you are looking for the steepest rate of exponential decay, so you need the graph that has the smallest b-value. The graphs that reflect exponential decay are graphs A, B, and C. The graph that has a b-value that is closest to 0 is graph C. Graph C has the steepest exponential decay because its b-value is the closest to 0 and, therefore, the smallest.

For question 3, you are looking for the largest initial value or y-intercept. This could be with a growth or decay function; it just needs to cross the y-axis higher than the other graphs. Graph C has the highest a value.

For question 4, you are looking for the smallest initial value or y-intercept. This could be a growth or decay function; it just needs to cross the y-axis lower than the other graphs. Graph F has the lowest a value.

TRY IT **Analyzing a and b in the Exponential Function $y = ab^x$**



1. Which graphs represent exponential growth?
2. Which graphs represent exponential decay?
3. Which graph has the highest value for b?
4. Which graph has the highest value for a?

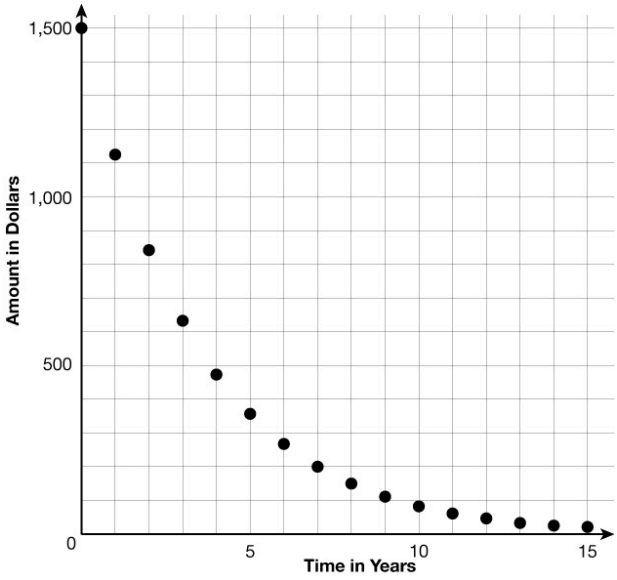
5.7.3 Describing Graphs

Activity

Your teacher will give you a set of cards (https://openstax.org/r/set-cards-5-7-3) containing descriptions of situations and graphs. Match each situation with a graph that represents it. Record your matches and be prepared to explain your reasoning.

Use the cards to answer questions 1 – 4. Match each graph representation to the description of the situation characterized by exponential change. Then choose the relationship between the number of years since purchase and the value.

1. Choose the description of exponential change shown in the graph.



a. The value of a stock doubles approximately every 4 years.

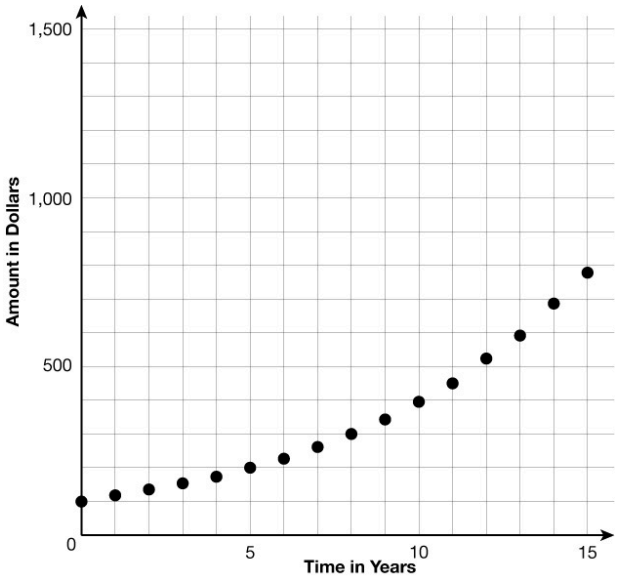
b. A car loses $\frac{1}{4}$ of its value every year after purchase.

c. The value of a stock triples roughly every 8 years.

d. A laptop loses $\frac{2}{5}$ of its value every year after purchase.

2. What is the equation for this graph?

3. Choose the description of exponential change shown in the graph.



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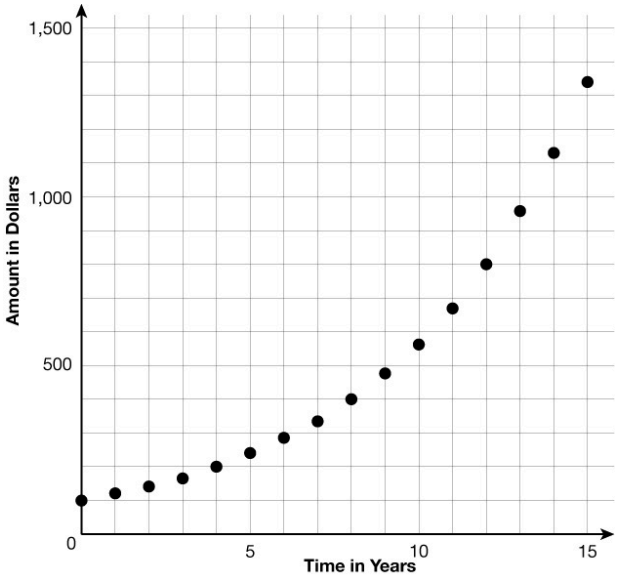
d. A laptop loses $\frac{2}{5}$ of its value every year after purchase.

4. What is the equation for this graph?

5. Choose the description of exponential change shown in the graph.



a. The value of a stock doubles approximately every 4 years.

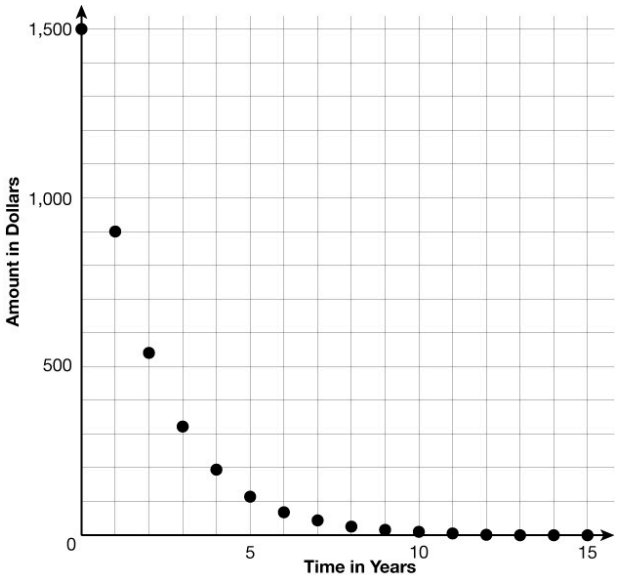
b. A car loses $\frac{1}{4}$ of its value every year after purchase.

c. The value of a stock triples roughly every 8 years.

d. A laptop loses $\frac{2}{5}$ of its value every year after purchase.

6. What is the equation for this graph?

7. Choose the description of exponential change shown in the graph.



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b. A car loses $\frac{1}{4}$ of its value every year after purchase.

c. The value of a stock triples roughly every 8 years.

d. A laptop loses $\frac{2}{5}$ of its value every year after purchase.

8. What is the equation for this graph?

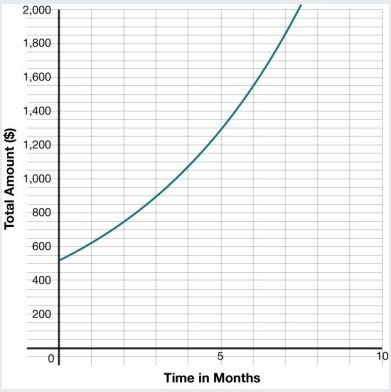
Video: Analyzing Graphs of Exponential Scenarios

Watch the following video to learn more about how to analyze a graph to determine which exponential scenario it represents.

Access multimedia content (http://openstax.org/books/algebra-1/pages/5-7-3-describing-graphs)

Self Check

Which scenario could be represented by the following graph?



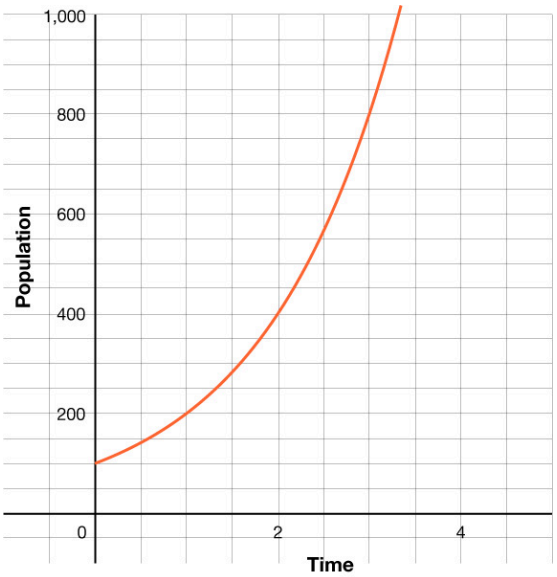
a. A recent graduate's credit card balance started around $500 and decreases at a rate of approximately 20% each month.
b. The population of sea birds started at 518 and is increasing exponentially each month.
c. The population of sea birds started at 518 and is decreasing exponentially each month.
d. A recent graduate's credit card balance started around $500 and increases at a rate of approximately 20% each month.

Additional Resources

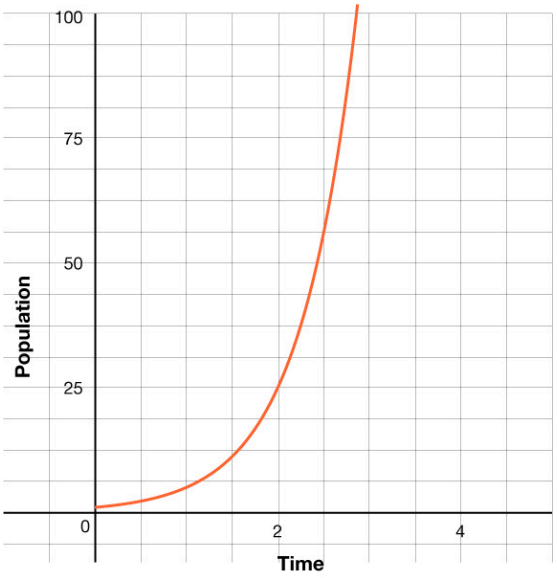
Connecting Exponential Situations and Graphs

Match each graph representation to the description of the situation characterized by exponential change.

Graph 1



Graph 2

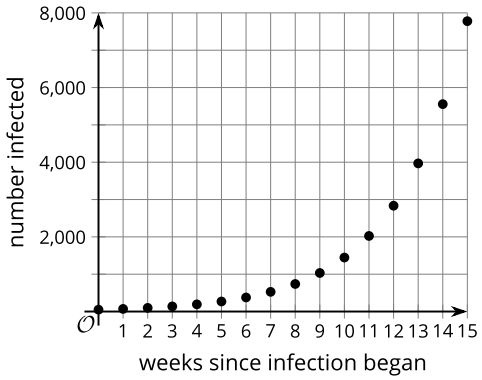


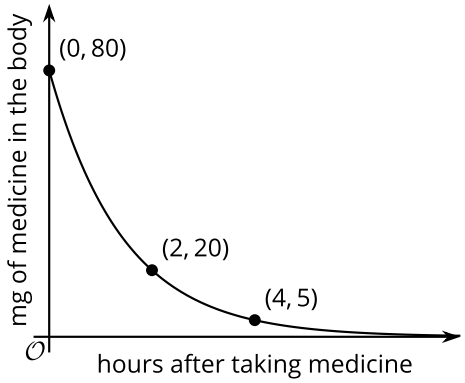
Situation 1: A dangerous bacterial compound forms in a closed environment but is immediately detected. This bacteria is known to double in concentration in a closed environment every hour and can be modeled by the function $P(t) = 100 \cdot 2^t$, where t is measured in hours.

Situation 2: Loggerhead turtles reproduce every 2 to 4 years, laying approximately 120 eggs in a clutch. Using this information, we can derive an approximate equation to model the turtle population. As is often the case in biological studies, we will count only the female turtles. If we start with a population of one female turtle in a protected area and assume that all turtles survive, we can roughly approximate the population of female turtles by $P(t) = 5^t$.

Since each graph represents exponential growth, and they both have the same labels, you have to look more closely at each. The biggest difference is the y-intercept or initial value. Graph 1 has an initial value of 100, and Graph 2 has a very small initial value. When you reread the situations, you can see that Situation 1 starts with 100 bacteria, and Situation 2 starts with 1 female turtle. This helps you see that Situation 1 belongs to Graph 1 and Situation 2 belongs to Graph 2.

TRY IT **Connecting Exponential Situations and Graphs**

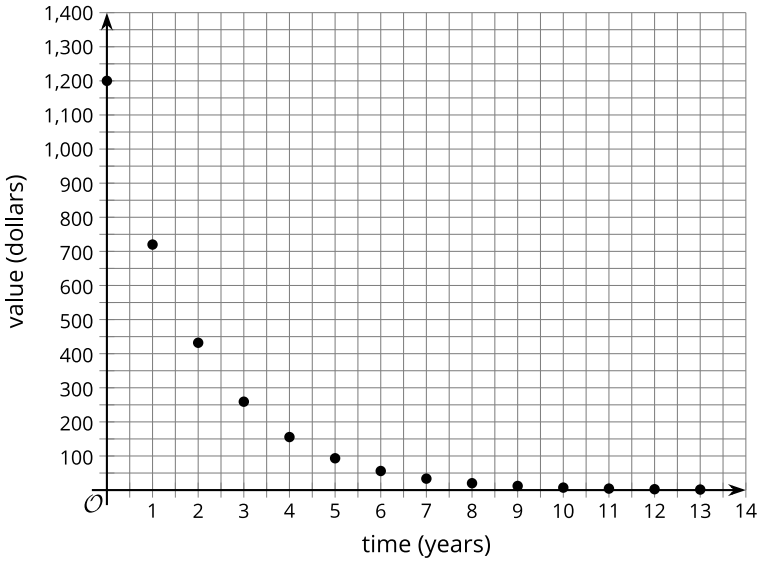




Write a possible situation for each graph. Make sure to include vocabulary about exponential functions.

5.7.4 Quantities that Change Exponentially

Cool Down



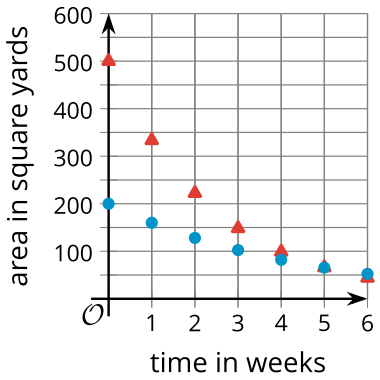
1. This graph represents one of the following descriptions. Which one?
   a. A phone loses $\frac{4}{5}$ of its value every year after purchase; the relationship between the number of years since purchasing the phone and the value of the phone.
   b. The number of stores a company has tripled approximately every 5 years; the relationship between the number of years and the number of stores.
   c. A camera loses $\frac{2}{5}$ of its value every year after purchase; the relationship between the number of years since purchasing the camera and the value of the camera.
2. Explain how you know the graph represents the description you chose.

5.7.5 Practice

Complete the following questions to practice the skills you have learned in this lesson.

For questions 1 – 2, use the following scenario:

The two graphs show models characterized by exponential decay representing the area covered by two different algae blooms, in square yards, w weeks after different chemicals were applied.



1. Which algae bloom covered a larger area when the chemicals were applied?

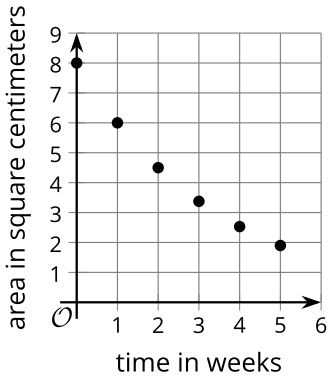
a. The algae bloom represented by the circles because the curve is always below the other one on the graph.
b. Both algae blooms covered the same area after 5 weeks.
c. The algae bloom represented by the circles because the curve has a lower y-intercept.
d. The algae bloom represented by the triangles because the curve has a higher y-intercept.

2. Which algae population is decreasing more rapidly? Explain how you know.

a. The algae bloom represented by the triangles because its graph is steeper.
b. The algae bloom represented by the circles because the curve is always below the other one on the graph.
c. Both algae blooms are decreasing at the same rate.
d. The algae bloom represented by the triangles because the curve has a higher y-intercept.

For questions 3 – 5, use the following scenario:

A medicine is applied to a burn on a patient's arm. The area of the burn in square centimeters decreases exponentially and is shown in the graph.



3. What fraction of the burn area remains each week?

a. $\frac{2}{3}$ or 0.66
b. $\frac{1}{4}$ or 0.25
c. $\frac{3}{4}$ or 0.75

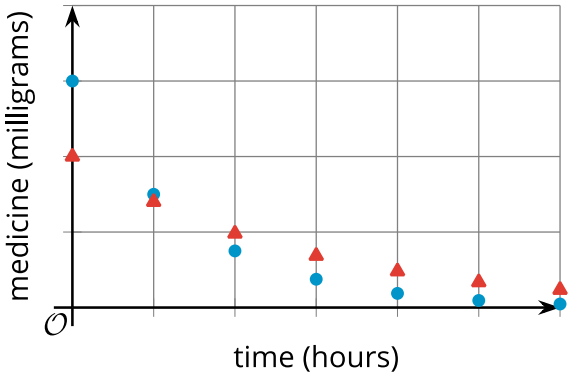
d. $\frac{1}{2}$ or 0.5

4. Which of the following is an equation representing the area of the burn, a, after t weeks?

a. $a = \frac{-6}{5}t + 8$
b. $a = -2t + 8$
c. $a = 8(\frac{3}{4})^t$
d. $a = 8(\frac{1}{4})^t$

5. What is the area of the burn after 7 weeks? Round to three decimal places.

6. The graphs show the amounts of medicine in two patients after receiving injections. The circles show the medicine in patient A, and the triangles show the medicine in patient B.

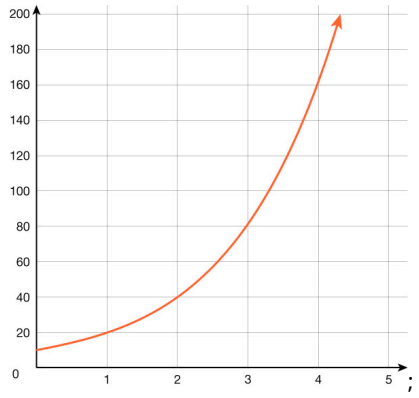


One equation that gives the amount of medicine in milligrams, m, in patient A, h hours after an injection, is $m = 300(\frac{1}{2})^h$.

What could be an equation for the amount of medicine in patient B?

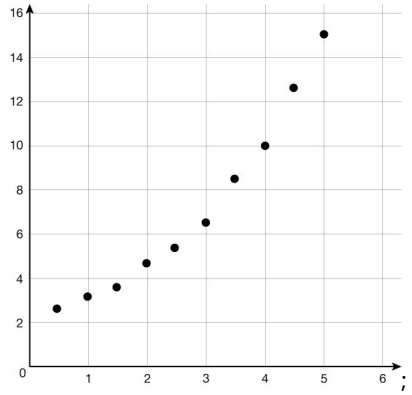
a. $m = 200(\frac{7}{10})^h$
b. $m = 200(\frac{3}{10})^h$
c. $m = 500(\frac{7}{10})^h$
d. $m = 500(\frac{3}{10})^h$

7. Which scenario could describe the graph?



a. A population of bacteria starts with 200 and decays by a factor of $\frac{1}{2}$ every hour.
b. A population of bacteria increases by a factor of 10 every hour and starts with 2 bacteria.
c. A savings account starts with $200 and decays by a factor of $\frac{1}{2}$ each month.
d. A population of bacteria doubles every hour and starts with 10 bacteria.

8. Which equation could represent the graph?



a. $y = 0.479(2.104)^x$
b. $y = 2.104(0.479)^x$
c. $y = 2.5x + 2$
d. $y = 2.104(1.479)^x$

9. Given a formula for an exponential function, is it possible to determine whether the function grows or decays exponentially just by looking at the formula? Explain.

a. No, it is not possible to determine growth and decay from the formula.
b. Yes, if the growth rate is greater than one it's growth; if the growth rate is, less than zero it's decay.
c. Yes, if the growth rate is between zero and one it's decay; if the growth rate is greater than one it is growth.
d. Yes, if the initial value is greater than one it's growth; if the initial value is less than one it's decay.

5.7.6 Lesson Summary

In this lesson, you learned how to:

- Use graphs to compare and contrast situations that involve exponential decay.
- Use information from a graph to write an equation that represents exponential decay.

Here are the **activities** that helped you reach those goals:

- 5.7.1: Patterns in Real Numbers
  - In this activity, you analyzed all the possible patterns connecting a set of numbers in a table. The numbers were written in both decimal and fraction form, and you may have noticed different patterns in each format.
- 5.7.2: Comparing Graphs
  - In this activity, you looked at the graphs of the values of different cell phones over time. You were able to analyze each cell phone and compare them to one another by looking at key features of the graphs.
  - 5.7.2: Self Check
  - 5.7.2: Additional Resources
- 5.7.3: Describing Graphs
  - In this activity, you connected the verbal description of a graph to the matching graph. Each description was a real-life scenario, and all of the graphs were exponential. You had to consider words that represent different growth rates and whether the situation was growth or decay.
  - 5.7.3: Self Check
  - 5.7.3: Additional Resources
- 5.7.4: Quantities that Change Exponentially
  - In this activity, you analyzed a graph and determined which of the scenarios provided it matches. It was a wrap up of all of the other skills from this lesson.

After these activities, you completed the following **practice**:

- 5.7.5: Practice

Checking In

On a scale of 1 to 5, how confident do you feel about the learning goals of this lesson?