1 Explicit Feedback

For explicit feedback, we assume that a pair of purchase and click $< w, l >$ is the result that (1) w is superior than l on the most pertinent aspect $g_k = 1$ and (2) w at least ties with l on other aspects.

We have the following equation:

$$p(< w, l > |\Theta, g) = \prod_{k=1}^{K} \frac{w_k}{w_k + \theta l_k}^{g_k} \frac{\theta w_k}{l_k + \theta w_k}^{1-g_k}, \tag{1}$$

where g the hidden variable is sampled from u, $\Theta = \{\theta, w_k, l_k, u\}$ is the parameter space.

The likelihood over all sessions is defined by

$$p(D|\Theta) = \prod_u \prod_{d \in D(u)} \Sigma_k (\prod_{w \in W(d)} \prod_{l \in L(d)} p(< w, l > |\Theta, g)p(g|u)) \tag{2}$$

Due to the summation part, the log-likelihood can not be analytically optimized. To maximize the log-likelihood, we follow the EM framework.

E-step

$$\begin{aligned} p(g|d, \Theta^t) &\propto p(g|u, \Theta^t)p(d|g, \Theta^t) \\ p(g_k = 1|d, \Theta^t) &\propto u_k^t \tfrac{w_k^t}{w_k^t + \theta^t l_k^t} \textstyle\prod_{k' \neq k} [\tfrac{\theta^t w_{k'}^t}{l_{k'}^t + \theta^t w_{k'}^t}] \end{aligned} \tag{3}$$

M-step

For any positive variables x, y, we have

$$\ln \frac{y}{x} \geq 1 - \frac{x}{y}$$

Therefore we can derive a lower bound for the log-likelihood over the complete data, given the parameters learnt from previous round.

$$\begin{aligned} l(\Theta, g) &= \textstyle\sum_u \sum_{d \in D(u)} \log\{u_k \tfrac{w_k}{w_k + \theta l_k} \prod_{k' \neq k} [\tfrac{\theta^t w_{k'}}{l_{k'} + \theta^t w_{k'}}]\} \\ &\geq Q^t(\Theta, g) \\ \textstyle\sum_u \sum_{d \in D(u)} \{\log u_k + [1 - \tfrac{w_k + \theta l_k}{w_k^t + \theta^t l_k^t} + \log \tfrac{w_k}{w_k^t + \theta^t l_k^t}] \sum_{k' \neq k} [1 - \tfrac{l_{k'} + \theta w_{k'}}{l_{k'}^t + \theta^t w_{k'}^t} + \log \tfrac{\theta w_{k'}}{l_{k'}^t + \theta^t w_{k'}^t}]\} \end{aligned} \tag{4}$$

where l_k^t, w_k^t, θ^t are the parameters learnt from $t - th$ round.

The objective in M-step is $Q^t = \sum_u \sum_{d \in D(u)} p(g_k = 1|d, \Theta^t) Q^t(\Theta, g)$. To maximize Q^t we adopt coordinate descent.

Fix v for all w and l and θ, we update u by. (See the corresponding derivation in our paper)

Fix u, θ, we update w and l by.

Fix u, v, we update θ by.

Stochastic S-step In stochastic EM, we simply add an S-step after the E-step. We draw the value of g for each session d by

$$k \sim u_k^t \frac{w_k^t}{w_k^t + \theta^t l_k^t} \prod_{k' \neq k} [\frac{\theta^t w_{k'}^t}{l_{k'}^t + \theta^t w_{k'}^t}]. \tag{5}$$

Next replace the M-step by

$$\sum_u \sum_{d \in D(u)} \{[1 - \frac{w_k + \theta l_k}{w_k^t + \theta^t l_k^t} + \log \frac{w_k}{w_k^t + \theta^t l_k^t}] \sum_{k' \neq k} [1 - \frac{l_{k'} + \theta w_{k'}}{l_{k'}^t + \theta^t w_{k'}^t} + \log \frac{\theta w_{k'}}{l_{k'}^t + \theta^t w_{k'}^t}]\} \tag{6}$$

2 Implicit Feedback