NC STATE UNIVERSITY

# Amplitude Amplification

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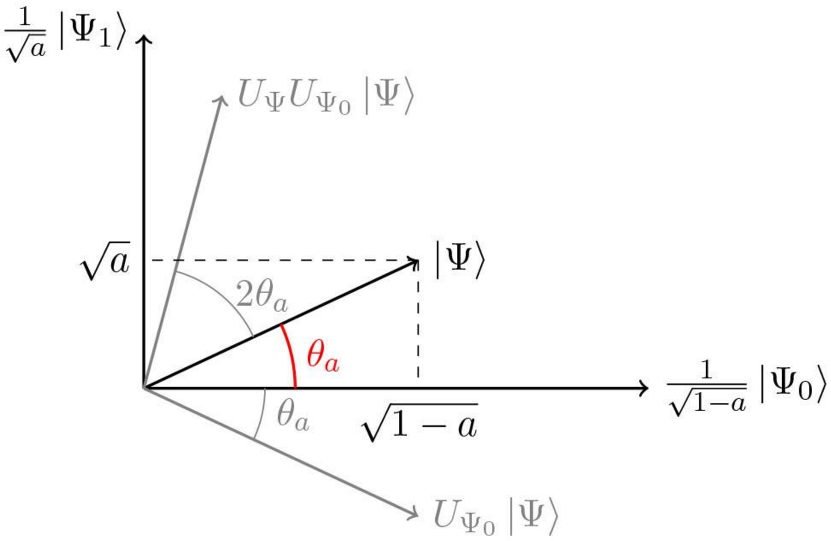
# Amplitude estimation

- Amplitude Amplification use Q operators (same with Grover's Algorithm)




$$|\psi\rangle_n = \sum_{i=0}^{N-1} \sqrt{p_i}\,|i\rangle_n$$

Let $F$: $|i\rangle_n|0\rangle \rightarrow |i\rangle_n(\sqrt{1-f(i)}|0\rangle + \sqrt{f(i)}|1\rangle)$

Applying F to $|\psi\rangle_n|0\rangle$

$$\sum_{i=0}^{N-1} \sqrt{1-f(i)}\sqrt{p_i}\,|i\rangle_n|0\rangle + \sum_{i=0}^{N-1} \sqrt{f(i)}\sqrt{p_i}\,|i\rangle_n|1\rangle$$

$$\cos(\theta_a)|i\rangle_n|0\rangle + \sin(\theta_a)|i\rangle_n|1\rangle$$

$$\cos(3\theta_a)|i\rangle_n|0\rangle + \sin(3\theta_a)|i\rangle_n|1\rangle$$

# Amplitude Amplification

- Let $\mathcal{H}$ denote the Hilbert space representing the state space of a quantum system. Every Boolean function $\mathcal{X}: \mathbb{Z} \rightarrow \{0,1\}$ induces a partition of $\mathcal{H}$ into a direct sum of two subspaces, a good subspace and a bad subspace. The good subspace is the subspace spanned by the set of basis $|x\rangle$ for which $\mathcal{X}(x) = 1$, and the bad subspace is its orthogonal complement in $\mathcal{H}$.

- Every pure state $|\gamma\rangle$ in $\mathcal{H}$ has a unique decomposition as $|\gamma\rangle = |\gamma_1\rangle + |\gamma_0\rangle$, where $|\gamma_1\rangle$ denotes the projection onto the good subspace, and $|\gamma_0\rangle$ denotes the projection onto the bad subspace.

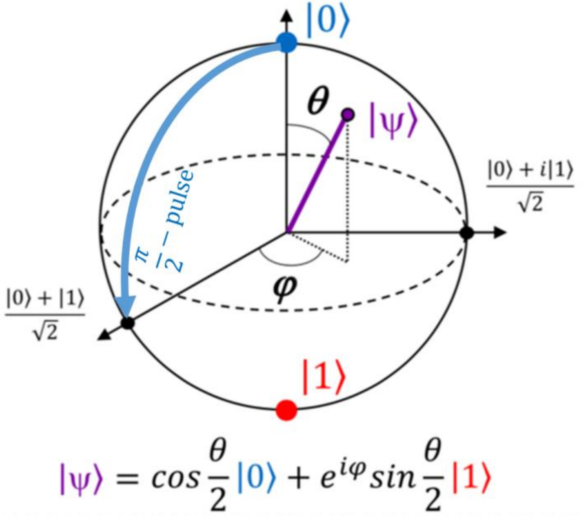
- Let $\mathrm{a}_\gamma = \langle\gamma_1|\gamma_1\rangle$ denote the probability that measuring $|\gamma\rangle$ produces a good state, and similarly, let $\mathrm{b}_\gamma = \langle\gamma_0|\gamma_0\rangle$. Since $|\gamma_1\rangle$ and $|\gamma_0\rangle$ are orthogonal, we have $\mathrm{a}_\gamma + b_r = 1$.

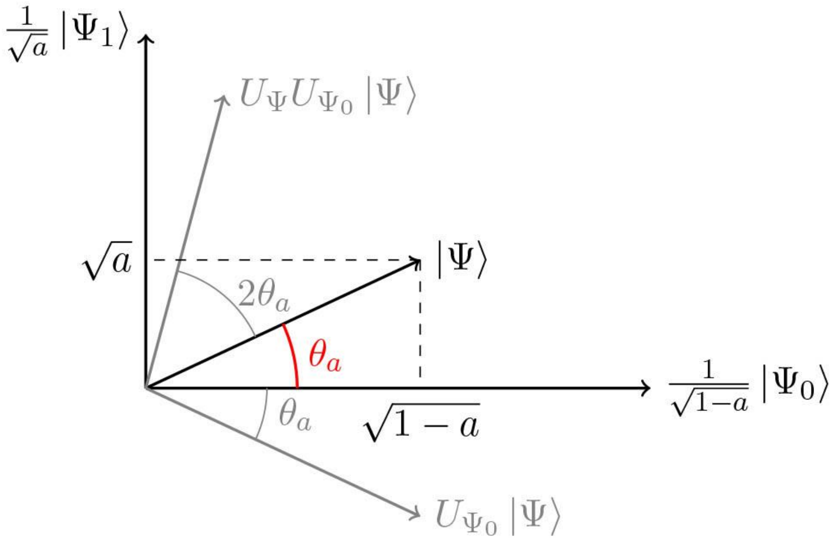
- Hence, given normalized state vector $|\gamma\rangle$ in $\mathcal{H}$, we can uniquely decompose it as

$$|\gamma\rangle = \sin(\theta)\,|\gamma_1\rangle + \cos(\theta)\,|\gamma_0\rangle$$

# Amplitude Estimation without phase estimation

- When we have just one qubit $|\psi\rangle = a|1\rangle + b|0\rangle$, can we rotate it $\theta/4$ degree?



# Some other recent papers

- “Aaronson, Scott, and Patrick Rall. "Quantum approximate counting, simplified." Symposium on Simplicity in Algorithms. Society for Industrial and Applied Mathematics, (2020).
- Nakaji, Kouhei. "Faster amplitude estimation." arXiv preprint arXiv:2003.02417 (2020).
- ✓ Above two papers have very similar concept with Iterative Quantum Amplitude Estimation

- Brown, Eric G., Oktay Goktas, and W. K. Tham. "Quantum Amplitude Estimation in the Presence of Noise." arXiv preprint arXiv:2006.14145 (2020).
- ✓ Analyzed QPE-free QAE under common noise models that may afflict NISQ devices and report on the optimality of various query schedules in the noisy regime.

- Rao, Pooja, et al. "Quantum amplitude estimation algorithms on IBM quantum devices." Quantum Communications and Quantum Imaging XVIII. Vol. 11507. International Society for Optics and Photonics, (2020).
- ✓ They implemented just two algorithm (MLE version, Iterative version) for very simple integral problem.

# Iterative Quantum Amplitude Estimation

- Amplitude estimation by using Q operators
  - By using $\sin^2\left((2k+1)\theta_a\right) \approx \cos((4k+2)\theta_a)$, we can get $\theta_a$, if we know the argument is restricted to either $[0,\pi]$ or $[\pi, 2\pi]$
  - Hence, we want to find the largest $k$ such that the scaled interval $[(4k+2)\theta_l, (4k+2)\theta_u]_{mod\ 2\pi}$ is fully contained either in $[0,\pi]$ or $[\pi, 2\pi]$
  - Then, we can improve our estimate for $\theta_a$ with high confidence.

- Sketch of the algorithm
  - STEP 1: Set $[\theta_l, \theta_u]$ = $[0,\pi/2]$, $k_i = 0$, $i = 0$
  - STEP 2: Set $K_i = 4k_i + 2$

    Until get certain level of confidence interval $\left(\frac{(K_i\theta_l - K_i\theta_u)}{2} < L\right)$, measure the last qubit (# of shots).
  - STEP 3: Find the largest $k$ such that the scaled interval $[K_i\theta_l, K_i\theta_u]_{mod\ 2\pi}$ is fully contained

    either in $[0,\pi]$ or $[\pi, 2\pi]$ by using subroutine (FindNextK).
  - STEP 4: If $\theta_l - \theta_u > 2\epsilon$, $i = i + 1$ and go to STEP 2,

    Otherwise, return $[a_l, a_u]$ = $[\sin^2\theta_l, \sin^2\theta_u]$

# Reference


- Bernhardt, Chris. Quantum Computing for Everyone. Mit Press, 2019.
- Rieffel, Eleanor G., and Wolfgang H. Polak. Quantum computing: A gentle introduction. MIT Press, 2011.
- Stamatopoulos, Nikitas, et al. "Option Pricing using Quantum Computers." arXiv preprint arXiv:1905.02666 (2019).
- Woerner, Stefan, and Daniel J. Egger. "Quantum risk analysis." npj Quantum Information 5.1 (2019).
- Suzuki, Yohichi, et al. "Amplitude Estimation without Phase Estimation." arXiv preprint arXiv:1904.10246 (2019).
- Grinko, Dmitry, et al. "Iterative Quantum Amplitude Estimation." arXiv preprint arXiv:1912.05559 (2019).
- Zoufal, Christa, Aurélien Lucchi, and Stefan Woerner. "Quantum generative adversarial networks for learning and loading random distributions." npj Quantum Information 5.1 (2019): 1-9.
- https://quantum-computing.ibm.com/support/guides/introduction-to-quantum-circuits?section=5cae613866c1694be21df8cc