NC STATE UNIVERSITY

Iterative Quantum Amplitude Estimation

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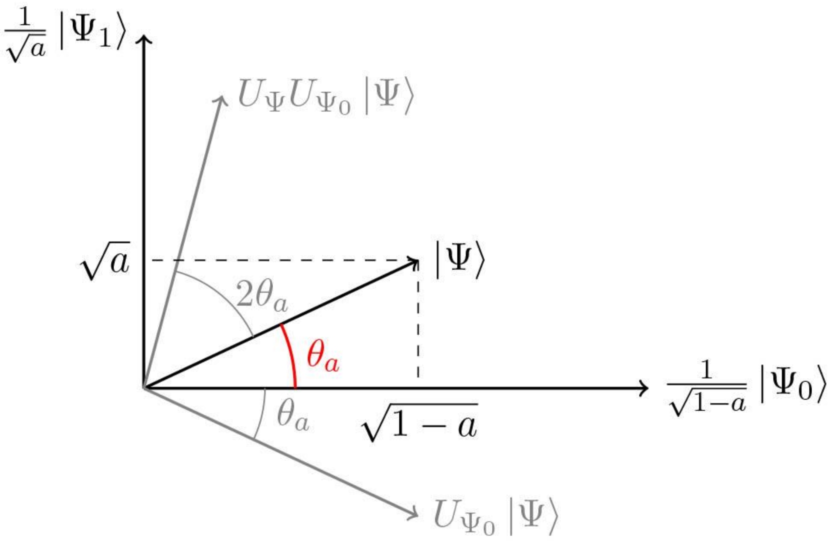
Amplitude estimation

- Amplitude Amplification use Q operators (same with Grover's Algorithm)



$$|\psi\rangle_n = \sum_{i=0}^{N-1} \sqrt{p_i}\,|i\rangle_n$$

Let F: $|i\rangle_n|0\rangle \rightarrow |i\rangle_n(\sqrt{1-f(i)}|0\rangle + \sqrt{f(i)}|1\rangle)$

Applying F to $|\psi\rangle_n|0\rangle$

$$\sum_{i=0}^{N-1} \sqrt{1-f(i)}\sqrt{p_i}\,|i\rangle_n|0\rangle + \sum_{i=0}^{N-1} \sqrt{f(i)}\sqrt{p_i}\,|i\rangle_n|1\rangle$$

$$\cos(\theta_a)|i\rangle_n|0\rangle + \sin(\theta_a)|i\rangle_n|1\rangle$$

$$\cos(3\theta_a)|i\rangle_n|0\rangle + \sin(3\theta_a)|i\rangle_n|1\rangle$$

Iterative Quantum Amplitude Estimation

- Amplitude estimation by using Q operators
  - By using $\sin^2\left((2k+1)\theta_a\right) \approx \cos((4k+2)\theta_a)$, we can get θ_a, if we know the argument is restricted to either $[0,\pi]$ or $[\pi, 2\pi]$
  - Hence, we want to find the largest k such that the scaled interval $[(4k+2)\theta_l, (4k+2)\theta_u]_{mod\ 2\pi}$ is fully contained either in $[0,\pi]$ or $[\pi, 2\pi]$
  - Then, we can improve our estimate for θ_a with high confidence.

- Sketch of the algorithm
  - STEP 1: Set $[\theta_l, \theta_u]$ = $[0,\pi/2]$, $k_i = 0$, $i = 0$
  - STEP 2: Set $K_i = 4k_i + 2$

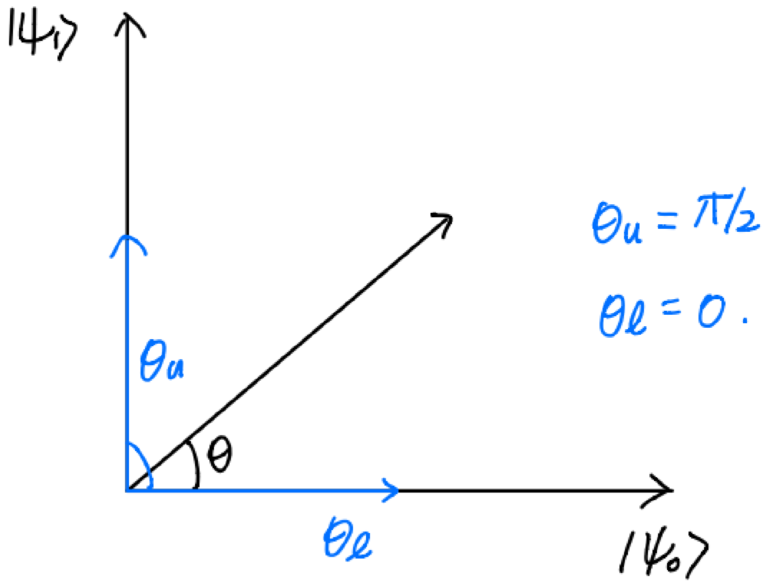
    Until get certain level of confidence interval $\left(\frac{(K_i\theta_l - K_i\theta_u)}{2} < L\right)$, measure the last qubit (# of shots).
  - STEP 3: Find the largest k such that the scaled interval $[K_i\theta_l, K_i\theta_u]_{mod\ 2\pi}$ is fully contained

    either in $[0,\pi]$ or $[\pi, 2\pi]$ by using subroutine (FindNextK).
  - STEP 4: If $\theta_l - \theta_u > 2\epsilon$, $i = i + 1$ and go to STEP 2,

    Otherwise, return $[a_l, a_u]$ = $[\sin^2\theta_l, \sin^2\theta_u]$

Iterative Quantum Amplitude Estimation

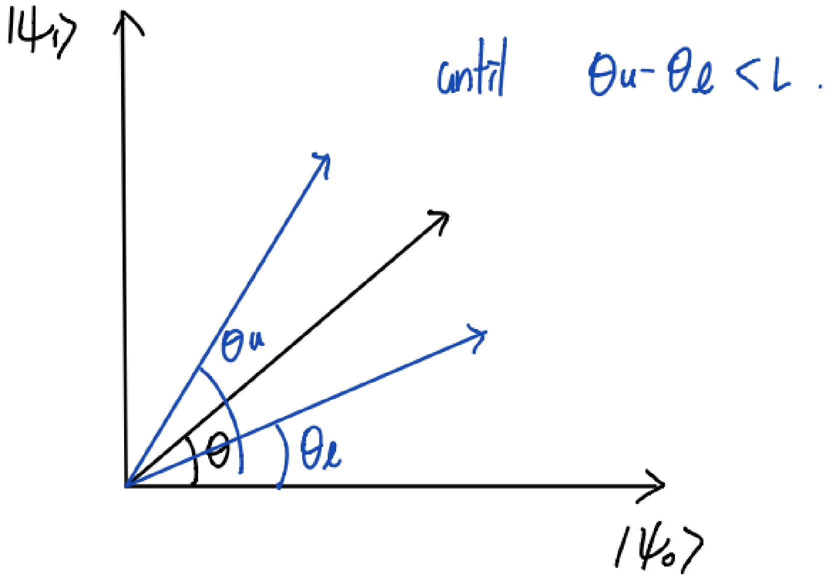
- STEP 1: Set $[\theta_l, \theta_u] = [0, \pi/2]$, $k_i = 0$, $i = 0$



Iterative Quantum Amplitude Estimation

- STEP 2: Set $K_i = 4k_i + 2$

  Until get certain level of confidence interval $\left(\frac{(K_i\theta_u - K_i\theta_l)}{2} < L\right)$, measure the last qubit (# of shots).



Iterative Quantum Amplitude Estimation

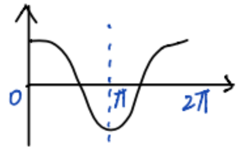
- STEP 3: Find the largest k such that the scaled interval $[K_i\theta_l, K_i\theta_u]_{mod\ 2\pi}$ is fully contained either in $[0,\pi]$ or $[\pi, 2\pi]$ by using subroutine (FindNextK).
  $\sin^2\big((2k+1)\theta_a\big) \approx \cos((4k+2)\theta_a)$

How many Q operators can we apply?

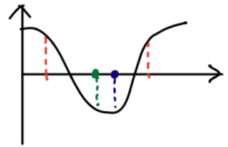
Since we use 4k+2 instead of 2k+1.

we want the scaled interval $[(4k+2)\theta_l, (4k+2)\theta_u]_{mod\ 2\pi}$

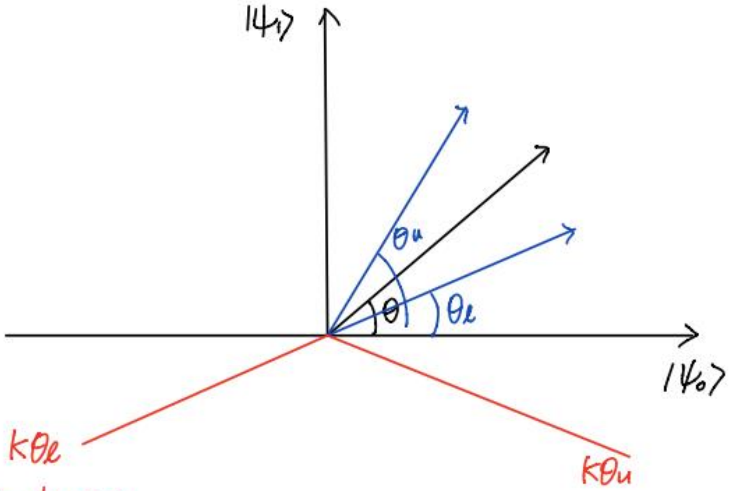
is fully contained either in $[0,\pi]$ or $[\pi,2\pi]$:



if the interval is not fully contained either in $[0,\pi]$ or $[\pi,2\pi]$:



we cannot get to know θ_a is green or blue point.

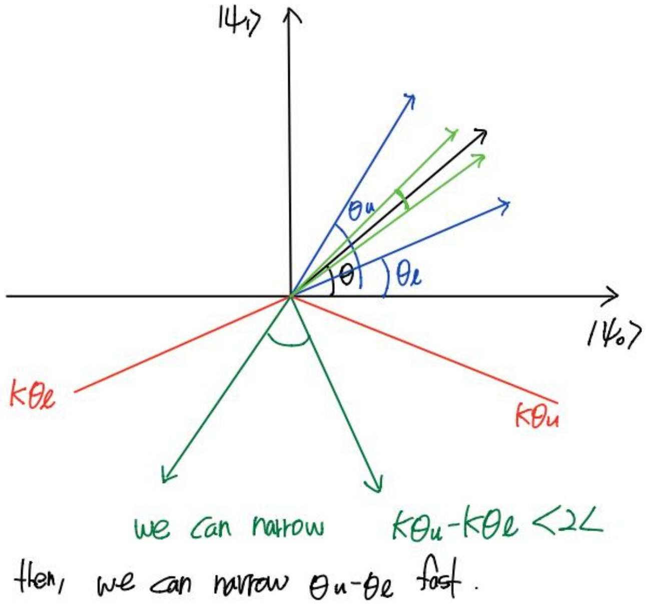


$K = 4k+2$
$\Updownarrow$
the number of Q operator.

Iterative Quantum Amplitude Estimation

- STEP 2: Set $K_i = 4k_i + 2$

Until get certain level of confidence interval $\left(\frac{(K_i\theta_u - K_i\theta_l)}{2} < L\right)$, measure the last qubit (# of shots).



Amplitude Estimation without phase estimation

- When we have just one qubit $|\psi\rangle = a|1\rangle + b|0\rangle$, can we rotate it $\theta/4$ degree?
- There are some papers
- "Zalka, Christof. "Grover's quantum searching algorithm is optimal." Physical Review A 60.4 (1999): 2746."
- Bennett, Charles H., et al. "Strengths and weaknesses of quantum computing." SIAM journal on Computing 26.5 (1997): 1510-1523.
- Above paper had shown that no quantum algorithm can solve the search problem in fewer than $O(\sqrt{N})$ queries.