CSC 591 Project B4 HW 4
Quantum Enhanced Simulation Optimization

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1 Introduction

The general optimization problem we consider it to find a setting of controllable parameters that minimizes a given objective function, i.e.,

$$\min_{\theta \in \Theta} J(\theta)$$

where $\theta \in \Theta$ represents the (vector of) input variables, $J(\theta)$ is the (scalar) objective function, and Θ is the constraint set, which may be either explicitly given or implicitly defined.

The assumption in the simulation optimization (SO) setting is that $J(\theta)$ is not available directly, but must be estimated through simulation, e.g., the simulation output provides $\hat{J}(\theta)$, a noisy estimate of $J(\theta)$. The most common form for J is an expectation, e.g.,

$$J(\theta) = E[L(\theta, w)],$$

where w represents a sample path (simulation replication), L is the sample performance measure.

However, simulation is used to evaluate a function that is computationally too expensive to be evaluated analytically, e.g., because a system of interest is too complex or because it involves uncertainty. Especially in the latter case, Monte Carlo simulation is popular since the estimation error scales as $\mathcal{O}(M^{-1/2})$ from the central limit theorem, where M denotes the number of function evaluations, and is independent of the dimension of the system.

Quantum Amplitude Estimation (QAE) is a quantum algorithm that provides a quadratic speedup over classical Monte Carlo simulation, i.e., its estimation error scales as $\mathcal{O}(M^{-1})$. Like Monte Carlo simulation, QAE can be applied to a large variety of problems. Recent research has already investigated the applicability of this algorithm to several tasks, for instance, risk analysis and option pricing.

The canonical version of QAE is a combination of Quantum Phase Estimation (QPE) and Grover's Algorithm. However, since QPE consists of many controlled amplification operations followed by a quantum Fourier transform, the whole procedure is hard to implement with current and near-term quantum computers. Hence, removing the QPE-dependency would help to reduce the resource requirements of QAE in terms of qubits and circuit depth and lower the bar for practial applications of QAE.

However, since simulation optimization is composed of lots of things such as simulation and optimization, we need not only QAE for expectation value, but also additional quantum circuits for quantum-enhanced simulation optimization.

1. How we can load the probability p_x into the amplitudes of n qubits by operator $\mathcal{P}_X$:

$$\mathcal{P}_X \left|0\right\rangle_n = \sum_{x=0}^{N-1} \sqrt{p_\phi(x)} \left|x\right\rangle_n$$

Although we can make the circuit for each probability distribution, the paper suggests quantum generative adversarial networks for learning and loading random distributions.

2. How we can make the circuit for the following operator F:

$$F \left|x\right\rangle \left|0\right\rangle = \sqrt{1 - f(x)} \left|x\right\rangle \left|0\right\rangle + \sqrt{f(x)} \left|x\right\rangle \left|1\right\rangle$$

Most recent paper used the approach which is approximates f using a Taylor expansion and uses controlled Pauli-rotations to rotate the function values onto the qubit amplitudes.

3. Can we run not only the simulation, but also optimization by using quantum computer?

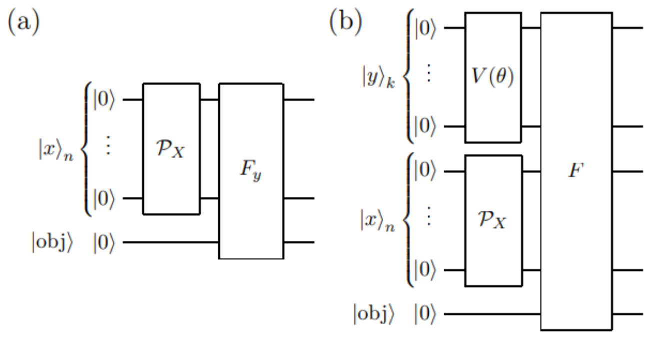

Figure 1: two circuits for simulation optimization

By using figure 1(a), we can get $E[f(x)]$ and thus, make the surrogate model for the optimization. It means that we used the quantum computer for simulation part and the classical computer for optimization part. However, when the decison variables are not continuous, optimization problem is NP-hard problem and thus, we can get quantum advantage again. Hence, the very recent paper suggested figure 1(b) circuit, which includes both simulation and optimization part. Unlike the existing quantum optimization algorithm which can be applicable to QUBO, this is new type of quantum optimization algorithm, which can be applied to other classes of optimization problems. In this project, I want to implement and improve these algorithms in various perspectives.

2 Time line

Date	Description
2020/10/15	Implement various QAE without QPE and try to improve it
2020/10/22	Implement the operator $\mathcal{P}_X$ and do the research related to it
2020/10/29	Make the circuit for the operator F (for any function $f(x)$)
2020/11/05	Implement the Figure 1(b) circuit for entire quantum SO
2020/11/12	Use entire algorithm for an application problem
2020/11/19	Prepare the presentation

Reference

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URL: https://github.ncsu.edu/yha3/CSC591