

CS311 Homework 2

200 points, 20 points per problem

Problem 1

Using a loop invariant, prove that the following property holds for MAX:

$$\forall 0 \leq i < n, \text{list}[i] \leq \text{MAX}(\text{list})$$

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MAX(list)
1:  $i = 0$ 
2:  $\text{maxValue} = \text{list}[i]$ 
3: while  $i < n$  do
4:   if  $\text{list}[i] > \text{maxValue}$  then
5:      $\text{maxValue} = \text{list}[i]$ 
6:   end if
7: end while
8:  $i = i + 1$ 
9: return  $\text{maxValue}$ 
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Problem 2

Using mathematical induction, prove that a tree with n vertices has precisely $n - 1$ edges.

Problem 3

Prove using the definition of Θ that $6n^4 + 2n^3 + n + 12 \in \Theta(n^4)$.

Problem 4

Prove using the definition of Θ that any polynomial of degree k (that is, any function of the form $a_k * n^k + a_{k-1} * n^{k-1} + \dots + a_1 * n^1 + a_0 * n^0$) is a member of $\Theta(n^k)$.

Problem 5

Do problem 2-12 from the text.

Problem 6

Do problem 2-20 from the text.

Problem 7

Do problem 2-13 from the text.

Problem 8

Suppose $f_1 \in \Theta(g_1)$ and $f_2 \in \Theta(g_2)$. Prove using the definition of Θ that $f_1/f_2 \in \Theta(g_1/g_2)$.

Problem 9

Suppose $f \in O(g)$.

- (a) Prove using the definition of O that $f + g \in O(g)$.
- (b) Prove using the definition of O that $O(f + g) = O(g)$.

Problem 10

Prove or disprove: Big Oh defines an equivalence relation on the set of all functions $f : N \rightarrow R$.

Problem 11

An adversary challenges you to a game. He gives you a board upon which is drawn a grid of squares with 2^n rows and 2^n columns (So there are therefore $(2^n)^2$ squares in the grid). He also gives you a bag containing a large number of L-shaped pieces like the one shown below. The challenge is this: the adversary will first select a single square on the grid and mark it as unusable. If you can then place the L-shaped tiles in such a way that they cover the entire grid - with no overlaps, and leaving only the unusable square uncovered - you win. Otherwise, the adversary wins. Prove using mathematical induction that you can win the game for any n and any adversary-selected unusable square.

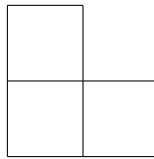


Figure 1: An L-shaped piece. Each square is the size of one grid location.