CS311 Homework 2

200 points, 20 points per problem

Problem 1

Using a loop invariant, prove that the following property holds for MAX:

$\forall\, 0 \le i < n,\ list[i] \le \textsc{max}(list)$

MAX(list)

$i = 0$
$maxValue = list[i]$
while $i < n$ **do**
    if $list[i] > maxValue$ **then**
        $maxValue = list[i]$
    end if
    $i = i + 1$
end while
return $maxValue$

Problem 2

Using mathematical induction, prove that a tree with n vertices has precisely $n - 1$ edges.

Problem 3

Prove using the definition of Θ that $6n^4 + 2n^3 + n + 12 \in \Theta(n^4)$.

Problem 4

Prove using the definition of Θ that any polynomial of degree k (that is, any function of the form $a_k * n^k + a_{k-1} * n^{k-1} + \ldots + a_1 * n^1 + a_0 * n^0$) is a member of $\Theta(n^k)$, where $a_k, a_{k-1}, \ldots, a_1$, and a_0 are nonnegative constants.

Problem 5

Do problem 2-12 from the text.

Problem 6

Do problem 2-20 from the text.

Problem 7

Do problem 2-13 from the text.

Problem 8

Suppose $f_1 \in \Theta(g_1)$ and $f_2 \in \Theta(g_2)$. Prove using the definition of Θ that $f_1/f_2 \in \Theta(g_1/g_2)$.

Problem 9

Suppose $f \in O(g)$.

(a) Prove using the definition of O that $f + g \in O(g)$.

(b) Prove using the definition of O that $O(f + g) = O(g)$.

Problem 10

Prove or disprove: Big Oh defines an equivalence relation on the set of all functions $f : N \to R$.

Problem 11

An adversary challenges you to a game. He gives you a board upon which is drawn a grid of squares with 2^n rows and 2^n columns (So there are therefore $(2^n)^2$ squares in the grid). He also gives you a bag containing a large number of L-shaped pieces like the one shown below. The challenge is this: the adversary will first select a single square on the grid and mark it as unusable. If you can then place the L-shaped tiles in such a way that they cover the entire grid - with no overlaps, and leaving only the unusable square uncovered - you win. Otherwise, the adversary wins. Prove using mathematical induction that you can win the game for any n and any adversary-selected unusable square.

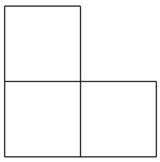

Figure 1: An L-shaped piece. Each square is the size of one grid location.