CS311 Homework 5

160 points, 20 points per problem

Problem 1

(a) Give an algorithm that multiplies two degree-1 polynomials with only three multiply operations. That is, given coefficients a, b, c, and d, the algorithm should compute the values of the coefficients in the expanded form of $(ax + b) * (cx + d)$. Hint: One multiplication will be (a + b)(c + d).

(b) Give a divide-and-conquer algorithm for multiplying two polynomials of degree n, and prove using the master theorem that your algorithm runs in $\Theta(n^{log_2(3)})$ time. You may assume that $n + 1$ is a power of 2.

Problem 2

Give an $O(log(n))$ time algorithm that computes the following function:

MEDIAN-OF-TWO(l_1, l_2)
Input: l_1 and l_2 are two sorted lists of integers. Each list has n elements - so there are $2n$ elements in total - and the value of each element in the lists is unique.
Output: the value of the n^{th} smallest integer in the set of $2n$ integers in l_1 and l_2.

Problem 3

Give an $O(n)$ average case running time algorithm that computes the following function:

Kth-SMALLEST($list$, k)
Input: an unsorted list $list$ of unique integers and an integer k
Output: the value of the k^{th} smallest integer from $list$

Problem 4

Let T be a tree with n vertices. We say that a vertex v is a *minimal separator of T* if its removal splits T into two or more subtrees, each with at most n/2 nodes.

(a) Show that every finite tree has at least one minimal separator.

(b) Give an $O(|V|)$ algorithm for identifying a minimal separator in a given tree.

Problem 5

Give an algorithm that computes the following function:

BST-RECONSTRUCTION($traversal$)
Input: An array of elements $traversal$ generated by a pre-order traversal of some binary search tree T.
Output: An binary search tree identical to the original T.

Problem 6

Let $G = (V, E)$ be a connected, undirected graph. Prove or disprove:

$\exists v \in V \mid \Big[G' = (V \setminus \{v\}, E) \text{ is connected } \Big]$

Problem 7

Do problem 5-26 from the text.

Problem 8

Do problem 6-6 from the text.

Problem 9

Do problem 6-7 from the text.