DS-GA 3001.001 Special Topics in Data Science: Probabilistic Time Series Analysis
Homework 1

Due date: September 28, by 6 pm

Problem 1. Consider a time series constructed as:

$$x_t = 2\cos\left(2\pi\frac{t+15}{50}\right) + w_t, \tag{1}$$

with w_t independent white noise with zero mean and variance σ^2.

- What is the ACF of this process?
- Simulate a series of n=500 observations from this process for $\sigma^2 = 1$. Compute the sample ACF of the data you generated up to 100 and comment.

Problem 2. Given the following ARMA(p, q) models:

$$\begin{aligned} x_t &= 0.8x_{t-1} - 0.2x_{t-2} + w_t - 1.1w_{t-1} \\ x_t &= 0.7x_{t-1} - 0.1x_{t-2} + w_t - 0.2w_{t-1} \end{aligned}$$

- Rewrite the above expressions using the backward operator.
- Determine the parameters p and q for the resulting model. Identify and eliminate potential parameter redundancy.
- Are these models causal and/or invertible?

Problem 3. Prove that if random variables U and V can be written as linear combinations of the form $U = \sum_i a_i X_i$, $V = \sum_j b_j Y_j$, then the following relation holds:

$$\text{cov}(U, V) = \sum_{ij} a_i b_j \text{cov}(X_i, Y_j).$$

Problem 4. Consider the series of the form:

$$y_t = a_2 t^2 + a_1 t + a_0 + x_t$$

where a_i are nonzero parameters and x_t is a stationary series.

- Repeatedly apply difference operator ∇ to reduce y to a stationary series.
- Rewrite the original expressions using the backward operator.
- Comment on the generalization of this idea to a m-th order polynomial dependence in t.