Differential Privacy

CHAPTER

ONE

INTRODUCTION

This is a book about differential privacy, for programmers. It is intended to give you an introduction to the challenges of data privacy, introduce you to the techniques that have been developed for addressing those challenges, and help you understand how to implement some of those techniques.

The book contains numerous examples B T Q S P, including implementations of many concepts. Each chapter is generated from a self-contained Jupyter Notebook. You can click on the "download" button at the top-right of the chapter, and then select ".ipynb" to download the notebook for that chapter, and you'll be able to execute the examples yourself. Many of the examples are generated by code that is hidden (for readability) in the chapters you'll see here. You can show this code by clicking the "Click to show" labels adjacent to these cells.

This book assumes a working knowledge of Python, as well as basic knowledge of the pandas and NumPy libraries. You will also benefit from some background in discrete mathematics and probability - a basic undergraduate course in these topics should be more than sufficient.

This book is open source, and the latest version will always be available online here. The source code is available on GitHub. If you would like to fix a typo, suggest an improvement, or report a bug, please open an issue on GitHub.

The techniques described in this book have developed out of the study of E B U B Q S J W B D Z. For our purposes, we will define data privacy this way:

Definition

% B U B Q S J W B D Z techniques have the goal of allowing analysts to learn about U S F O E T information specific to J O E J W J E V B M T.

This is a broad definition, and many different techniques fall under it. But it's important to note what this definition F Y D M V E E T. Techniques for ensuring T F D, like encryption. Encrypted data doesn't reveal B O Z U I J O G I I to meet the first requirement of our definition. The distinction between security and privacy is an important one: privacy techniques involve an J O U F G F S F O D F B O M information, and attempt to control X I B U D B O C F M F B S O F E from that release, security techniques usually Q S F W F O U release of information, and control X I P D B O B D E T T. This book covers privacy techniques, and we will only discuss security when it has important implications for privacy.

This book is primarily focused on differential privacy. The first couple of chapters outline some of the reasons why: differential privacy (and its variants) is the only formal approach we know about that seems to provide robust privacy protection. Commonly-used approaches that have been used for decades (like de-identification and aggregation) have more recently been shown to break down under sophisticated privacy attacks, and even more modern techniques (like k-Anonymity) are susceptible to certain attacks. For this reason, differential privacy is fast becoming the gold standard in privacy protection, and thus it is the primary focus of this book.

CHAPTER

TWO

DE-IDENTIFICATION

% F J E F O U J G J D B U J P O is the process of removing J E F O U J G Z J O H J G P S N B U J P O from a dataset. The term E F J E O J E F O U J G J D B U J P O is sometimes used synonymously with the terms B O P O Z N J [B U J P O and [BQUJPVCE P O Z N J [B U J P O

Learning Objectives

After reading this chapter, you be able to:

- Define the following concepts:
  - De-identification
  - Re-identification
  - Identifying information / personally identifying information
  - Linkage attacks
  - Aggregation and aggregate statistics
  - Differencing attacks
- Perform a linkage attack
- Perform a differencing attack
- Explain the limitations of de-identification techniques
- Explain the limitations of aggregate statistics

Identifying information has no formal definition. It is usually understood to be information which would be used to identify us uniquely in the course of daily life - name, address, phone number, e-mail address, etc. As we will see later, it's J N Q P T T J C M F to formalize the concept of identifying information, because B M M information is identifying. The term Q F S T P O B M M Z J E F O U J G J B C M F J O G P S N B U J P O is often used synonymously with identifying information.

How do we de-identify information? Easy - we just remove the columns that contain identifying information!

```
adult_data = adult.copy().drop(columns=['Name', 'SSN'])
adult_pii = adult[['Name', 'SSN', 'DOB', 'Zip']]
adult_data.head(1)
```

```
       DOB    Zip  Age  Workclass  fnlwgt  Education  Education-Num  \
0  9/7/1967  64152   39  State-gov   77516  Bachelors             13

  Marital Status    Occupation   Relationship   Race   Sex  Capital Gain  \
0  Never-married  Adm-clerical  Not-in-family  White  Male          2174
```

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```
   Capital Loss  Hours per week        Country Target
0             0              40  United-States  <=50K
```

We'll save some of the identifying information for later, when we'll use it as B V Y J M J to perform a U F J E F O U J £ D B U J P attack.

2.1 Linkage Attacks

Imagine we want to determine the income of a friend from our de-identified data. Names have been removed, but we happen to know some auxiliary information about our friend. Our friend's name is Karrie Trusslove, and we know Karrie's date of birth and zip code.

To perform a simple M J O L B H F B U U B D we look at the overlapping columns between the dataset we're trying to attack, and the auxiliary data we know. In this case, both datasets have dates of birth and zip codes. We look for rows in the dataset we're attacking with dates of birth and zip codes that match Karrie's date of birth and zip code. In databases, this is called a K P J O of two tables, and we can do it in Pandas using `merge`. If there is only one such row, we've found Karrie's row in the dataset we're attacking.

```
karries_row = adult_pii[adult_pii['Name'] == 'Karrie Trusslove']
pd.merge(karries_row, adult_data, left_on=['DOB', 'Zip'], right_on=['DOB', 'Zip'])
```

```
               Name          SSN       DOB    Zip  Age  Workclass  fnlwgt  \
0  Karrie Trusslove  732-14-6110  9/7/1967  64152   39  State-gov   77516

   Education  Education-Num Marital Status    Occupation   Relationship  \
0  Bachelors             13  Never-married  Adm-clerical  Not-in-family

    Race   Sex  Capital Gain  Capital Loss  Hours per week        Country  \
0  White  Male          2174             0              40  United-States

  Target
0  <=50K
```

Indeed, there is only one row that matches. We have used auxiliary data to re-identify an individual in a de-identified dataset, and we're able to infer that Karrie's income is less than $50k.

2.1.1 How Hard is it to Re-Identify Karrie?

This scenario is made up, but linkage attacks are surprisingly easy to perform in practice. How easy? It turns out that in many cases, just one data point is sufficient to pinpoint a row!

```
pd.merge(karries_row, adult_data, left_on=['Zip'], right_on=['Zip'])
```

```
               Name          SSN     DOB_x    Zip     DOB_y  Age  Workclass  \
0  Karrie Trusslove  732-14-6110  9/7/1967  64152  9/7/1967   39  State-gov

   fnlwgt  Education  Education-Num Marital Status    Occupation  \
0   77516  Bachelors             13  Never-married  Adm-clerical

    Relationship   Race   Sex  Capital Gain  Capital Loss  Hours per week  \
0  Not-in-family  White  Male          2174             0              40

```

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```
         Country Target
0  United-States  <=50K
```

So ZIP code is sufficient **by itself** to allow us to re-identify Karrie. What about date of birth?

```
pd.merge(karries_row, adult_data, left_on=['DOB'], right_on=['DOB'])
```

```
               Name          SSN       DOB  Zip_x  Zip_y  Age  \
0  Karrie Trusslove  732-14-6110  9/7/1967  64152  64152   39
1  Karrie Trusslove  732-14-6110  9/7/1967  64152  67306   64
2  Karrie Trusslove  732-14-6110  9/7/1967  64152  62254   46

          Workclass  fnlwgt  Education  Education-Num      Marital Status  \
0         State-gov   77516  Bachelors             13       Never-married
1           Private  171373       11th              7             Widowed
2  Self-emp-not-inc  119944    Masters             14  Married-civ-spouse

        Occupation   Relationship   Race     Sex  Capital Gain  Capital Loss  \
0     Adm-clerical  Not-in-family  White    Male          2174             0
1  Farming-fishing      Unmarried  White  Female             0             0
2  Exec-managerial        Husband  White    Male             0             0

   Hours per week        Country Target
0              40  United-States  <=50K
1              40  United-States  <=50K
2              50  United-States   >50K
```

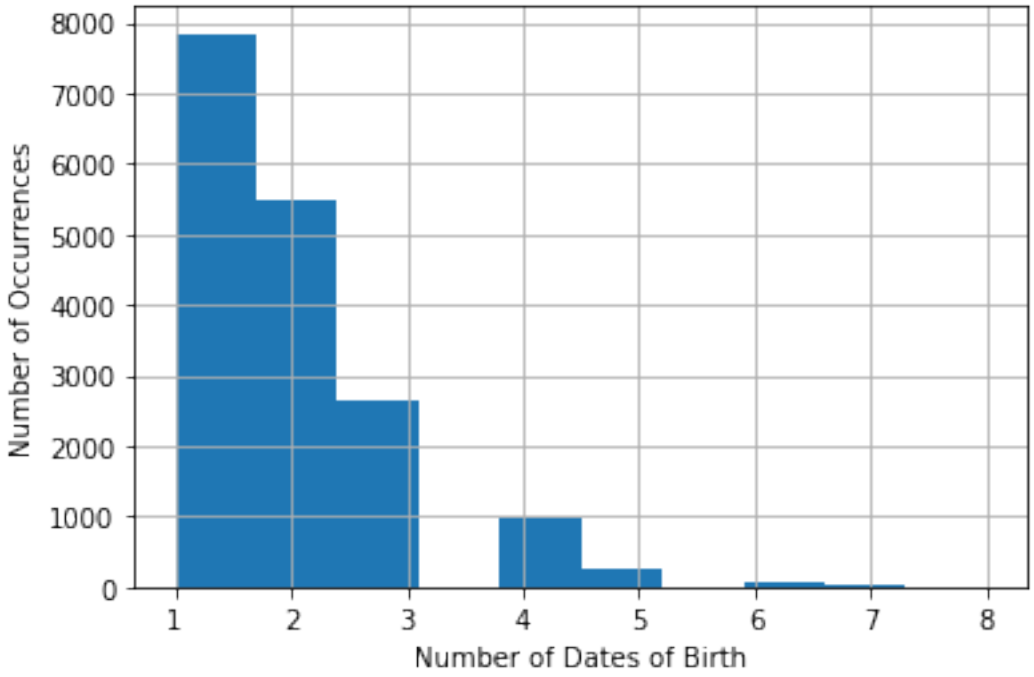
This time, there are three rows returned - and we don't know which one is the real Karrie. But we've still learned a lot!

- We know that there's a 2/3 chance that Karrie's income is less than $50k
- We can look at the differences between the rows to determine what additional auxiliary information would I F M Q to distinguish them (e.g. sex, occupation, marital status)

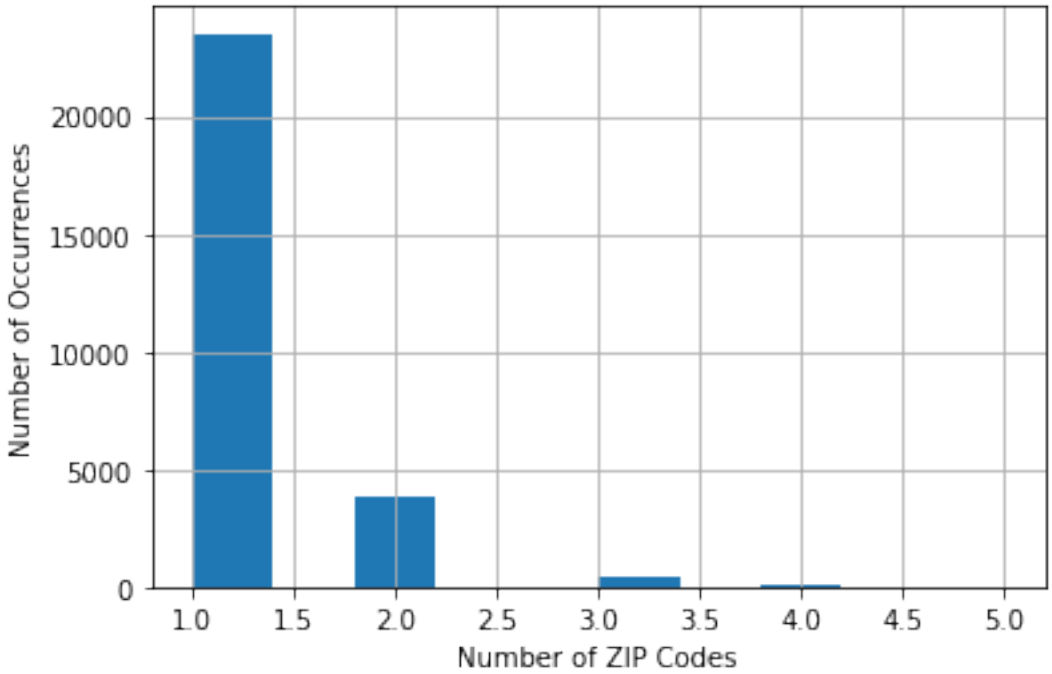
2.1.2 Is Karrie Special?

How hard is it to re-identify others in the dataset? Is Karrie especially easy or especially difficult to re-identify? A good way to gauge the effectiveness of this type of attack is to look at how "selective" certain pieces of data are - how good they are at narrowing down the set of potential rows which may belong to the target individual. For example, is it common for birthdates to occur more than once?

We'd like to get an idea of how many dates of birth are likely to be useful in performing an attack, which we can do by looking at how common "unique" dates of birth are in the dataset. The histogram below shows that U I F W B T U of N B K P S J U Z dates of birth occur 1, 2, or 3 times in the dataset, and O P E B U F occur more than 8 times. This means that date of birth is fairly T F M F D U J W F in narrowing down the possible records for an individual.

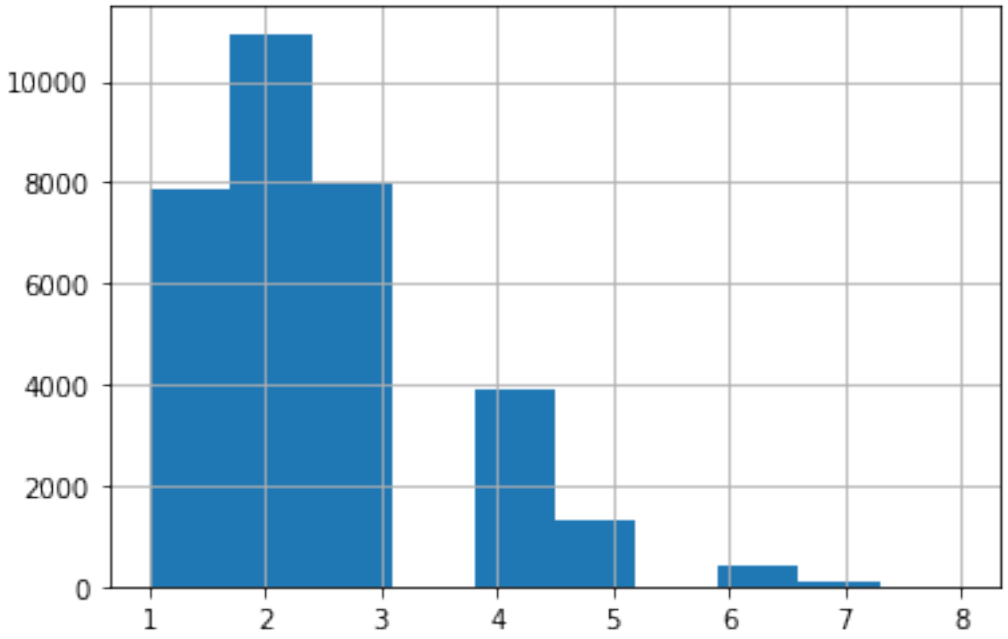


We can do the same thing with ZIP codes, and the results are even worse - ZIP code happens to be very selective in this dataset. Nearly all the ZIP codes occur only once.

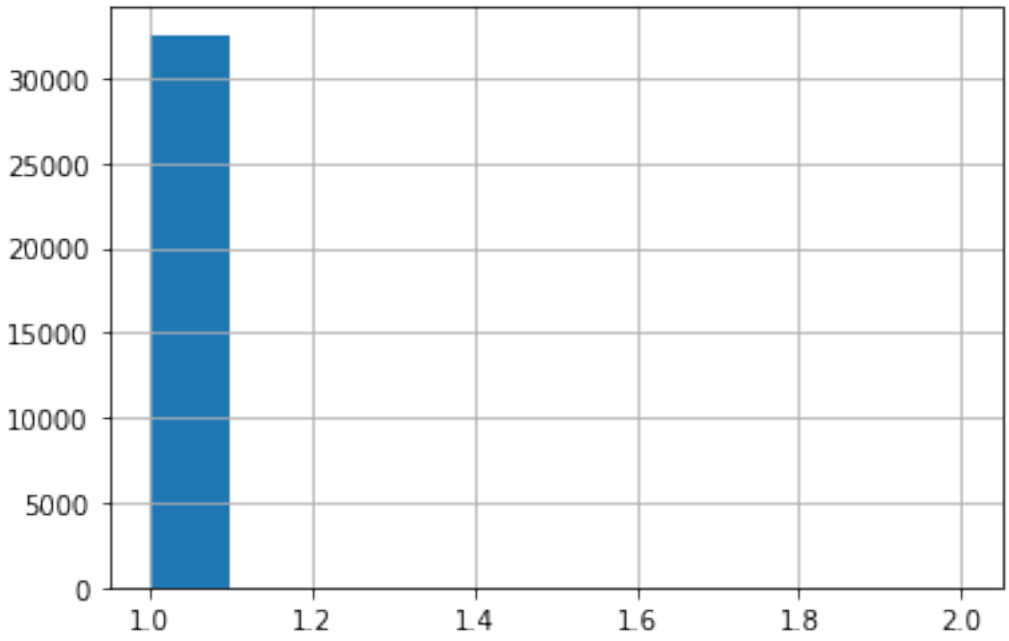


2.1.3 How Many People can we Re-Identify?

In this dataset, how many people can we re-identify uniquely? We can use our auxiliary information to find out! First, let's see what happens with just dates of birth. We want to know how many Q P T T J C M are returned for each data record in the dataset. The following histogram shows the number of records with each number of possible identities. The results show that we can uniquely identify almost 7,000 of the data records (out of about 32,000), and an additional 10,000 data records are narrowed down to two possible identities.



So it's not possible to re-identify a majority of individuals using K V of birth. What if we collect more information, to narrow things down further? If we use both date of birth and ZIP, we're able to do much better. In fact, we're able to uniquely re-identify basically the whole dataset.



When we use both pieces of information, we can re-identify **essentially everyone**. This is a surprising result, since we generally assume that many people share the same birthday, and many people live in the same ZIP code. It turns out that the combination of these factors is **extremely** selective. According to Latanya Sweeney's work [1], 87% of people in the US can be uniquely re-identified by the combination of date of birth, gender, and ZIP code.

Let's just check that we've actually re-identified Barnabe Haime, by printing out the number of possible data records for each identity:

```
Barnabe Haime           2
Antonin Chittem         2
Bambi Cooney            1
Cherida Degli Antoni    1
Letta Fellos            1
Name: Name, dtype: int64
```

Looks like we missed two people! In other words, in this dataset, only **two people** share a combination of ZIP code and date of birth.

2.2 Aggregation

Another way to prevent the release of private information is to release only aggregate data.

```
adult['Age'].mean()
```

```
38.58164675532078
```

2.2.1 Problem of Small Groups

In many cases, aggregate statistics are broken down into smaller groups. For example, we might want to know the average age of people with a particular education level.

```
adult[['Education-Num', 'Age']].groupby('Education-Num').mean().head(3)
```

```
                    Age
Education-Num
1             42.764706
2             46.142857
3             42.885886
```

Aggregation is supposed to improve privacy because it's hard to identify the contribution of a particular individual to the aggregate statistic. But what if we aggregate over a group with just one person in it? In that case, the aggregate statistic reveals one person's age exactly, and provides no privacy protection at all! In our dataset, most individuals have a unique ZIP code - so if we compute the average age by ZIP code, then most of the "averages" actually reveal an individual's exact age.

```
adult[['Zip', 'Age']].groupby('Zip').mean().head()
```

```
      Age
Zip
4    55.0
12   24.0
16   59.0
```

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```
17    42.0
18    24.0
```

The US Census Bureau, for example, releases aggregate statistics at the C M P D L M F W F M. Some census blocks have large populations, but some have a population of zero! The situation above, where small groups prevent aggregation from hiding information about individuals, turns out to be quite common.

How big a group is "big enough" for aggregate statistics to help? It's hard to say - it depends on the data and on the attack - so it's challenging to build confidence that aggregate statistics are really privacy-preserving. However, even very large groups do not make aggregation completely robust against attacks, as we will see next.

2.2.2 Differencing Attacks

The problems with aggregation get even worse when you release multiple aggregate statistics over the same data. For example, consider the following two summation queries over large groups in our dataset (the first over the whole dataset, and the second over all records except one):

```
adult['Age'].sum()
```

```
1256257
```

```
adult[adult['Name'] != 'Karrie Trusslove']['Age'].sum()
```

```
1256218
```

If we know both answers, we can simply take the difference and determine Karrie's age completely! This kind of attack can proceed even if the aggregate statistics are over W F S Z M B S H F H S P V Q T

```
adult['Age'].sum() - adult[adult['Name'] != 'Karrie Trusslove']['Age'].sum()
```

```
39
```

This is a recurring theme.

- Releasing E B U B that is useful makes ensuring Q S J W B D Z very difficult
- Distinguishing between N B M J D J P V T and O P O N B M J D J P V T queries is not possible

2.3 Summary

- A M J O L B H F B U U B D L involves combining B V Y J M J B S Z E B U B with E F J E F O U J G J F E E B U B to S F J E F O U J G Z individuals.
- In the simplest case, a linkage attack can be performed via a K P J O of two tables containing these datasets.
- Simple linking attacks are surprisingly effective:
  - Just a single data point is sufficient to narrow things down to a few records
  - The narrowed-down set of records helps suggest additional auxiliary data which might be helpful
  - Two data points are often good enough to re-identify a huge fraction of the population in a particular dataset
  - Three data points (gender, ZIP code, date of birth) uniquely identify 87% of people in the US

CHAPTER

THREE

K-ANONYMITY

k-Anonymity [2] is a G P S N B M Q S J W The definition of k-Anonymity is designed to formalize our intuition that a piece of auxiliary information should not narrow down the set of possible records for an individual "too much." Stated another way, k-Anonymity is designed to ensure that each individual can "blend into the crowd."

Learning Objectives

After reading this chapter, you will understand:

- The definition of k-Anonymity
- How to check for k-Anonymity
- How to generalize data to enforce k-Anonymity
- The limitations of k-Anonymity

Informally, we say that a dataset is "k-Anonymized" for a particular k if each individual in the dataset is a member of a group of size at least k, such that each member of the group shares the same R V B T J J E F Q U J F S T (a selected subset of all the dataset's columns) with all other members of the group. Thus, the individuals in each group "blend into" their group - it's possible to narrow down an individual to membership in a particular group, but not to determine which group member is the target.

Definition

Formally, we say that a dataset D satisfies k-Anonymity for a value of k if:

- For each row $r_1 \in D$, there exist at least $k-1$ other rows $r_2 \ldots r_k \in D$ such that $\Pi_{qi(D)} r_1 = \Pi_{qi(D)} r_2, \ldots, \Pi_{qi(D)} r_1 = \Pi_{qi(D)} r_k$

where $qi(D)$ is the quasi-identifiers of D, and $\Pi_{qi(D)} r$ represents the columns of r containing quasi-identifiers (i.e. the projection of the quasi-identifiers).

3.1 Checking for k-Anonymity

We'll start with a small dataset, so that we can immediately see by looking at the data whether it satisfies k-Anonymity or not. This dataset contains age plus two test scores; it clearly doesn't satisfy k-Anonymity for $k > 1$. Any dataset trivially satisfies k-Anonymity for $k = 1$, since each row can form its own group of size 1.

```
   age  preTestScore  postTestScore
0   42             4             25
1   52            24             94
2   36            31             57
3   24             2             62
4   73             3             70
```

To implement a function to check whether a dataframe satisfies k-Anonymity, we loop over the rows; for each row, we query the dataframe to see how many rows match its values for the quasi-identifiers. If the number of rows in any group is less than k, the dataframe does not satisfy k-Anonymity for that value of k, and we return False. Note that in this simple definition, we consider B M columns to contain quasi-identifiers; to limit our check to a subset of all columns, we would need to replace the `df.columns` expression with something else.

```
def isKAnonymized(df, k):
    for index, row in df.iterrows():
        query = ' & '.join([f'{col} == {row[col]}' for col in df.columns])
        rows = df.query(query)
        if rows.shape[0] < k:
            return False
    return True
```

As expected, our example dataframe does O R satisfy k-Anonymity for $k = 2$, but it does satisfy the property for $k = 1$.

```
isKAnonymized(df, 1)
```

```
True
```

```
isKAnonymized(df, 2)
```

```
False
```

3.2 Generalizing Data to Satisfy k-Anonymity

The process of modifying a dataset so that it satisfies k-Anonymity for a desired k is generally accomplished by H F O F S B M J [J O H the data - modifying values to be less specific, and therefore more likely to match the values of other individuals in the dataset. For example, an age which is accurate to a year may be generalized by rounding to the nearest 10 years, or a ZIP code might have its rightmost digits replaced by zeros. For numeric values, this is easy to implement. We'll use the `apply` method of dataframes, and pass in a dictionary named `depths` which specifies how many digits to replace by zeros for each column. This gives us the flexibility to experiment with different levels of generalization for different columns.

```
def generalize(df, depths):
    return df.apply(lambda x: x.apply(lambda y: int(int(y/(10**depths[x.
↪name]))*(10**depths[x.name]))))
```

Now, we can generalize our example dataframe. First, we'll try generalizing each column by one "level" - i.e. rounding to the nearest 10.

```
depths = {
    'age': 1,
    'preTestScore': 1,
    'postTestScore': 1
}
df2 = generalize(df, depths)
df2
```

```
   age  preTestScore  postTestScore
0   40             0             20
1   50            20             90
2   30            30             50
3   20             0             60
4   70             0             70
```

Notice that even after generalization, our example data [illegible] satisfy k-Anonymity for $k = 2$.

```
isKAnonymized(df2, 2)
```

```
False
```

We can try generalizing more - but then we'll end up removing [illegible] data!

```
depths = {
    'age': 2,
    'preTestScore': 2,
    'postTestScore': 2
}
generalize(df, depths)
```

```
   age  preTestScore  postTestScore
0    0             0              0
1    0             0              0
2    0             0              0
3    0             0              0
4    0             0              0
```

This example illustrates one of the key challenges of achieving k-Anonymity:

Challenge

Achieving k-Anonymity for meaningful values of k often requires removing quite a lot of information from the data

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3.3 Does More Data Improve Generalization?

Our example dataset is too small for k-Anonymity to work well. Because there are only 5 individuals in the dataset, building groups of 2 or more individuals who share the same properties is difficult. The solution to this problem is more data: in a dataset with more individuals, less generalization will typically be needed to satisfy k-Anonymity for a desired k.

Let's try the same census data we examined for de-identification. This dataset contains more than 32,000 rows, so it should be easier to achieve k-Anonymity.

We'll consider the ZIP code, age, and educational achievement of each individual to be the quasi-identifiers. We'll project just those columns, and try to achieve k-Anonymity for $k = 2$. The data is already k-Anonymous for $k = 1$.

Notice that we take just the first 100 rows from the dataset for this check - try running `isKAnonymized` on a larger subset of the data, and you'll find that it takes a very long time (for example, running the $k = 1$ check on 5000 rows takes about 20 seconds on my laptop). For $k = 2$, our algorithm finds a failing row quickly and finishes fast.

```
df = adult_data[['Age', 'Education-Num']]
df.columns = ['age', 'edu']
isKAnonymized(df.head(100), 1)
```

```
True
```

```
isKAnonymized(df.head(100), 2)
```

```
False
```

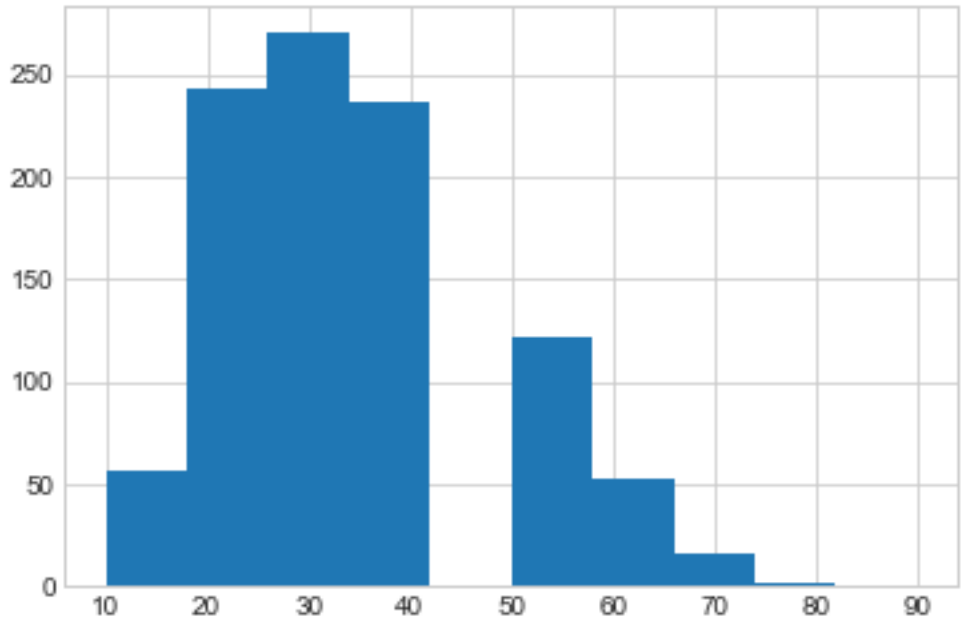
Now, we'll try to generalize to achieve k-Anonymity for $k = 2$. We'll start with generalizing both age and educational attainment to the nearest 10.

```
# outliers are a real problem!
depths = {
    'age': 1,
    'edu': 1
}
df2 = generalize(df.head(1000), depths)
isKAnonymized(df2, 2)
```

```
False
```

The generalized result still does not satisfy k-Anonymity for $k = 2$! In fact, we can perform this generalization on all 32,000 rows and still fail to satisfy k-Anonymity for $k = 2$ - so adding more data does not necessarily help as much as we expected.

The reason is that the dataset contains individuals who are very different from the rest of the population. These individuals do not fit easily into any group, even after generalization. Even considering ages, we can see that adding more data is not likely to help, since very low and very high ages are poorly represented in the dataset.



Achieving the optimal generalization for k-Anonymity is very challenging in cases like this. Generalizing each row N P S F would be overkill for the well-represented individuals with ages in the 20-40 range, and would hurt utility. However, more generalization is clearly needed for individuals at the upper and lower ends of the age range. This is the kind of challenge that occurs regularly in practice, and is difficult to solve automatically. In fact, optimal generalization for k-Anonymity has been shown to be NP-hard.

Challenge

Outliers make achieving k-Anonymity very challenging, even for large datasets. Optimal generalization for k-Anonymity is NP-hard.

3.4 Removing Outliers

One solution to this problem is simply to clip the age of each individual in the dataset to lie within a specific range, eliminating outliers entirely. This can also hurt utility, since it replaces real ages with fake ones, but it can be better than generalizing each row more. We can use Numpy's `clip` method to perform this clipping. We clip ages to be 60 or below, and leave educational levels alone (by clipping them to a very large value).

```
# clipping away outliers
depths = {
    'age': 1,
    'edu': 1
}
dfp = df.clip(upper=np.array([60, 10000000000000]), axis='columns')
df2 = generalize(dfp.head(500), depths)
isKAnonymized(df2, 7)
```

```
True
```

Now, the generalized dataset satisfies k-Anonymity for $k = 7$! In other words, our level of generalization was appropriate, but outliers prevented us from achieving k-Anonymity before, even for $k = 2$.

3.5 Summary

- k-Anonymity is a property of data, which ensures that each individual "blends in" with a group of at least k individuals.
- k-Anonymity is computationally expensive even to check: the naive algorithm is $O(n^2)$, and faster algorithms take considerable space.
- k-Anonymity can be achieved by modifying a dataset by [illegible], so that particular values become more common and groups are easier to form.
- Optimal generalization is extremely difficult, and outliers can make it even more challenging. Solving this problem automatically is NP-hard.

CHAPTER

FOUR

DIFFERENTIAL PRIVACY

Learning Objectives

After reading this chapter, you will be able to:

- Define differential privacy
- Explain the importance of the privacy parameter ϵ
- Use the Laplace mechanism to enforce differential privacy for counting queries

Like k-Anonymity, differential privacy is a property of algorithms (i.e. it's possible to prove that a data release has the property). Unlike k-Anonymity, however, differential privacy is a property of algorithms, and not a property of data. That is, we can prove that an algorithm satisfies differential privacy; to show that a data release satisfies differential privacy, we must show that the algorithm which produced it satisfies differential privacy.

Definition

A function which satisfies differential privacy is often called a mechanism. We say that a mechanism F satisfies differential privacy if for all inputs x and x' that differ in the data of a single individual, and all possible outputs S,

$$\frac{\Pr[F(x) = S]}{\Pr[F(x') = S]} \leq e^{\epsilon} \tag{4.1}$$

Two datasets are considered neighbors if they differ in the data of a single individual. Note that F is typically a randomized function, which has many possible outputs under the same input. Therefore, the probability distribution describing its outputs is not just a point distribution.

The important implication of this definition is that F's output will be pretty much the same, whether it includes the data of any specific individual. In other words, the randomness built into F should be "enough" so that an observed output from F will not reveal which of x or x' was the input. Imagine that my data is present in x but not in x'. If an adversary can't determine which of x or x' was the input to F, then the adversary can't tell whether or not my data was even in the input - let alone the contents of that data.

The ϵ parameter in the definition is called the privacy parameter or the privacy loss. It provides a knob to tune the "amount of privacy" the definition provides. Small values of ϵ require F to provide very similar outputs when given similar inputs, and therefore provide higher levels of privacy; large values of ϵ allow less similarity in the outputs, and therefore provide less privacy.

How should we set ϵ to prevent bad outcomes in practice? Nobody knows. The general consensus is that ϵ should be around 1 or smaller, and values of ϵ above 10 probably don't do much to protect privacy - but this rule of thumb could turn out to be very conservative. We will have more to say on this subject later on.

4.1 The Laplace Mechanism

Differential privacy is typically used to answer specific queries. Let's consider a query on the census data, X J U IRTV-U ential privacy.

"How many individuals in the dataset are 40 years old or older?"

```
adult[adult['Age'] >= 40].shape[0]
```

```
14237
```

The easiest way to achieve differential privacy for this query is to add random noise to its answer. The key challenge is to add enough noise to satisfy the definition of differential privacy, but not so much that the answer becomes too noisy to be useful. To make this process easier, some basic N F D I B O a v e b e n developed in the field of differential privacy, which describe exactly what kind of - and how much - noise to use. One of these is called the - B Q M B D F N [4] D I B O J T N

Definition

According to the Laplace mechanism, for a function $f(x)$ which returns a number, the following definition of $F(x)$ satisfies ϵ-differential privacy:

$$F(x) = f(x) + \text{Lap}\left(\frac{s}{\epsilon}\right) \tag{4.2}$$

where s is the T F O T a f U J a W d u Z $\text{Lap}(S)$ denotes sampling from the Laplace distribution with center 0 and scale S.

The T F O T a f U J a W d u Z is the amount f's output changes when its input changes by 1. Sensitivity is a complex topic, and an integral part of designing differentially private algorithms; we will have much more to say about it later. For now, we will just point out that D P V O U J O a l w a y s h a v e a s e n s i t i v i t y of 1: if a query counts the number of rows in the dataset with a particular property, and then we modify exactly one row of the dataset, then the query's output can change by at most 1.

Thus we can achieve differential privacy for our example query by using the Laplace mechanism with sensitivity 1 and an ϵ of our choosing. For now, let's pick $\epsilon = 0.1$. We can sample from the Laplace distribution using Numpy's `random.laplace`.

```
sensitivity = 1
epsilon = 0.1

adult[adult['Age'] >= 40].shape[0] + np.random.laplace(loc=0, scale=sensitivity/
↪epsilon)
```

```
14254.574490023344
```

You can see the effect of the noise by running this code multiple times. Each time, the output changes, but most of the time, the answer is close enough to the true answer (14,235) to be useful.

4.2 How Much Noise is Enough?

How do we know that the Laplace mechanism adds enough noise to prevent the re-identification of individuals in the dataset? For one thing, we can try to break it! Let's write down a malicious counting query, which is specifically designed to determine whether Karrie Trusslove has an income greater than $50k.

```
karries_row = adult[adult['Name'] == 'Karrie Trusslove']
karries_row[karries_row['Target'] == '<=50K'].shape[0]
```

```
1
```

This result definitely violates Karrie's privacy, since it reveals the value of the income column for Karrie's row. Since we know how to ensure differential privacy for counting queries with the Laplace mechanism, we can do so for this query:

```
sensitivity = 1
epsilon = 0.1

karries_row = adult[adult['Name'] == 'Karrie Trusslove']
karries_row[karries_row['Target'] == '<=50K'].shape[0] + \
  np.random.laplace(loc=0, scale=sensitivity/epsilon)
```

```
-0.43344754770407024
```

Is the true answer 0 or 1? There's too much noise to be able to reliably tell. This is how differential privacy is J O U R N E Y work - the approach does not S F K queries which are determined to be malicious; instead, it adds enough noise that the results of a malicious query will be useless to the adversary.

CHAPTER

FIVE

PROPERTIES OF DIFFERENTIAL PRIVACY

Learning Objectives

After reading this chapter, you will be able to:

- Explain the concepts of sequential composition, parallel composition, and post processing
- Calculate the cumulative privacy cost of multiple applications of a differential privacy mechanism
- Determine when the use of parallel composition is allowed

This chapter describes three important properties of differentially private mechanisms that arise from the definition of differential privacy. These properties will help us to design useful algorithms that satisfy differential privacy, and ensure that those algorithms provide accurate answers. The three properties are:

- Sequential composition
- Parallel composition
- Post processing

5.1 Sequential Composition

The first major property of differential privacy is T F R V F O U J B M [illegible] which bounds the total privacy cost of releasing multiple results of differentially private mechanisms on the same input data. Formally, the sequential composition theorem for differential privacy says that:

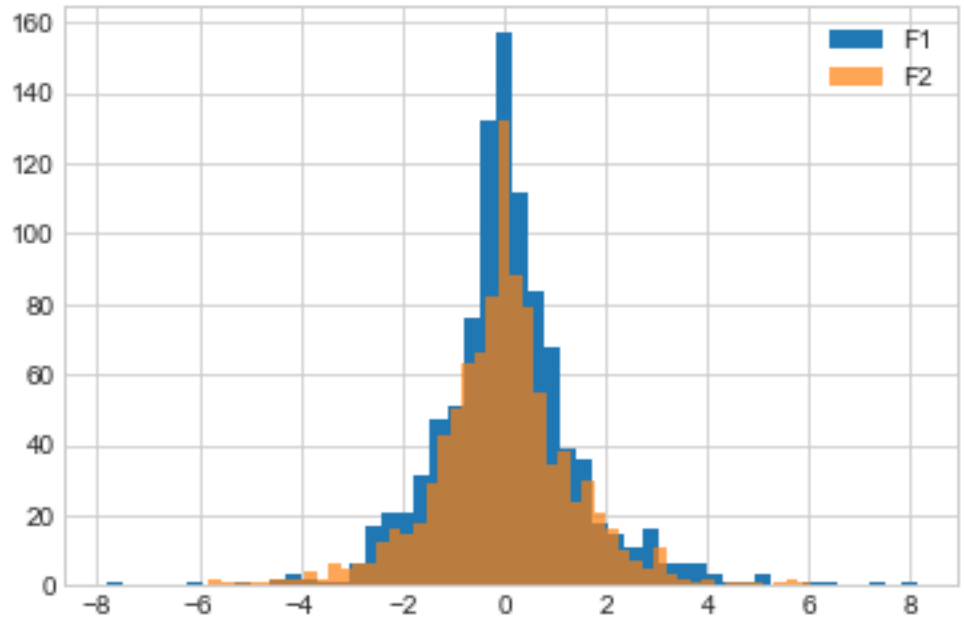
- If $F_1(x)$ satisfies ϵ_1-differential privacy
- And $F_2(x)$ satisfies ϵ_2-differential privacy
- Then the mechanism $G(x) = (F_1(x), F_2(x))$ which releases both results satisfies $\epsilon_1 + \epsilon_2$-differential privacy

Sequential composition is a vital property of differential privacy because it enables the design of algorithms that consult the data more than once. Sequential composition is also important when multiple separate analyses are performed on a single dataset, since it allows individuals to bound the U P privacy cost they incur by participating in all of these analyses. The bound on privacy cost given by sequential composition is an V Q bound - the actual privacy cost of two particular differentially private releases may be smaller than this, but never larger.

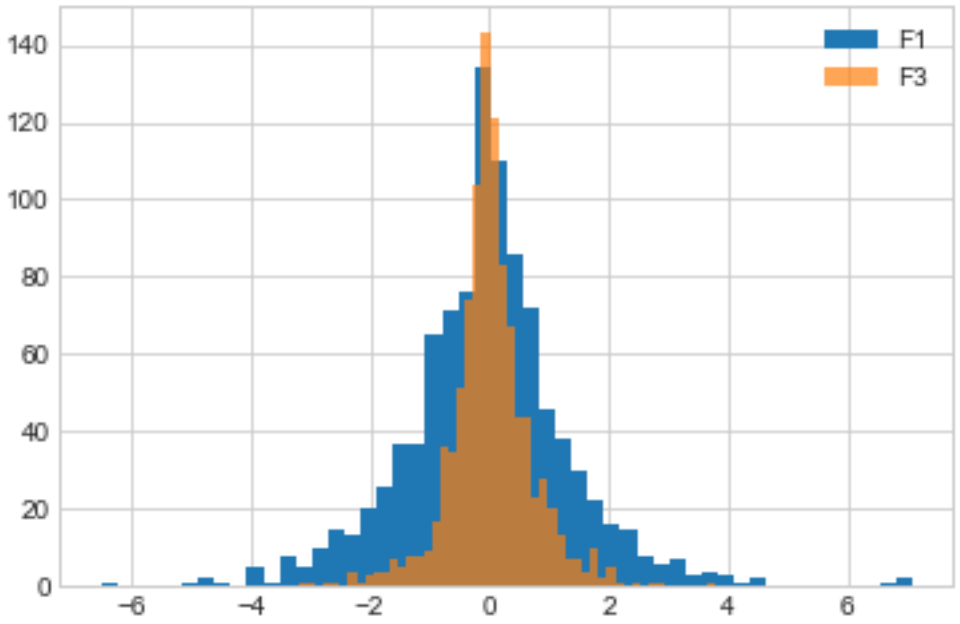
The principle that the ϵs "add up" makes sense if we examine the distribution of outputs from a mechanism which averages two differentially private results together. Let's look at some examples.

```
epsilon1 = 1
epsilon2 = 1
epsilon_total = 2

# satisfies 1-differential privacy
def F1():
    return np.random.laplace(loc=0, scale=1/epsilon1)

# satisfies 1-differential privacy
def F2():
    return np.random.laplace(loc=0, scale=1/epsilon2)

# satisfies 2-differential privacy
def F3():
    return np.random.laplace(loc=0, scale=1/epsilon_total)

# satisfies 2-differential privacy, by sequential composition
def F_combined():
    return (F1() + F2()) / 2
```

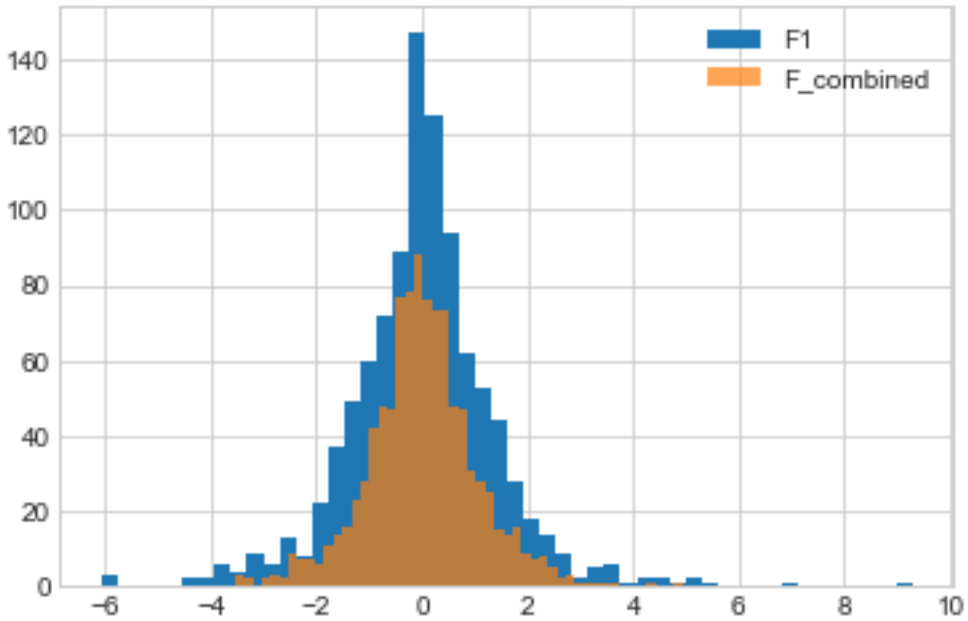
If we graph `F1` and `F2`, we see that the distributions of their outputs look pretty similar.



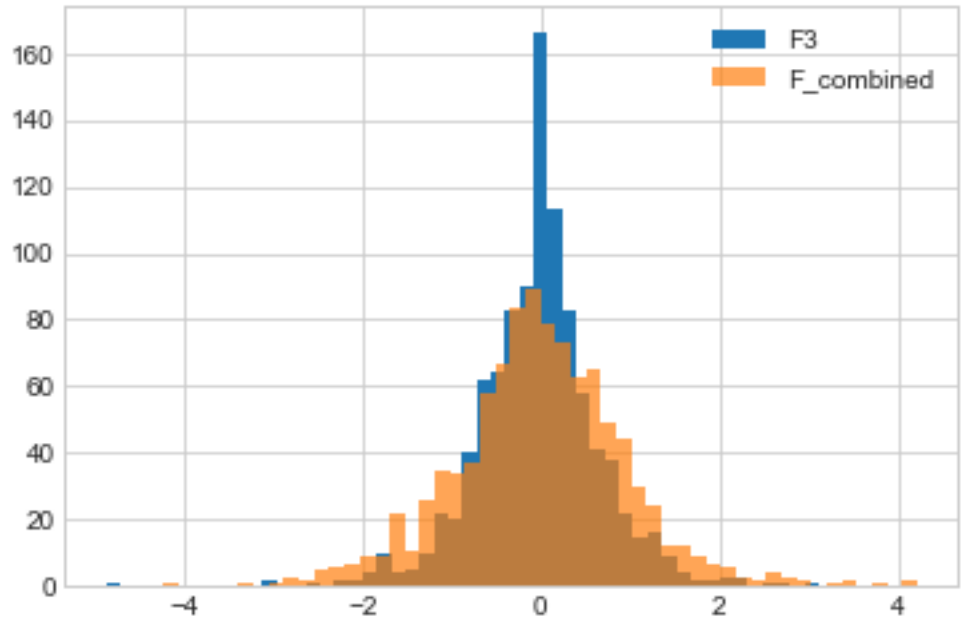
If we graph `F1` and `F3`, we see that the distribution of outputs from `F3` looks "pointier" than that of `F1`, because its higher ϵ implies less privacy, and therefore a smaller liklihood of getting results far from the true answer.



If we graph `F1` and `F_combined`, we see that the distribution of outputs from `F_combined` is pointier. This means its answers are more accurate than those of `F1`, so it makes sense that its ϵ must be higher (i.e. it yields less privacy than `F1`).



What about `F3` and `F_combined`? Recall that the ϵ values for these two mechanisms are the same - both have an ϵ of 2. Their output distributions should look the same.



In fact, `F3` looks "pointier"! Why does this happen? Remember that sequential composition yields an V Q Q on the total ϵ of several releases, the actual cumulative impact on privacy might be lower. That's the case here - the actual privacy loss in this case appears to be somewhat lower than the upper bound ϵ determined by sequential composition. Sequential composition is an extremely useful way to control total privacy cost, and we will see it used in many different ways, but keep in mind that it is not necessarily an exact bound.

5.2 Parallel Composition

The second important property of differential privacy is called Q B S B M M F M [6]. Parallel composition can be seen as an alternative to sequential composition - a second way to calculate a bound on the total privacy cost of multiple data releases. Parallel composition is based on the idea of splitting your dataset into disjoint chunks and running a differentially private mechanism on each chunk separately. Since the chunks are disjoint, each individual's data appears in F Y B D U M Z one chunk - so even if there are k chunks in total (and therefore k runs of the mechanism), the mechanism runs exactly once on the data of each J O E J W F M Y B M

- If $F(x)$ satisfies ϵ-differential privacy
- And we split a dataset X into k disjoint chunks such that $x_1 \cup ... \cup x_k = X$
- Then the mechanism which releases all of the results $F(x_1), ..., F(x_k)$ satisfies ϵ-differential privacy

Note that this is a much better bound than sequential composition would give. Since we run F k times, sequential composition would say that this procedure satisfies $k\epsilon$-differential privacy. Parallel composition allows us to say that the total privacy cost is just ϵ.

The formal definition matches up with our intuition - if each participant in the dataset contributes one row to X, then this row will appear in F Y B D U M Z one of the chunks $x_1, ..., x_k$. That means F will only "see" this participant's data P O F , U J N F meaning a privacy cost of ϵ is appropriate for that individual. Since this property holds for all individuals, the privacy cost is ϵ for everyone.

5.2.1 Histograms

In our context, a *histogram* is an analysis of a dataset which splits the dataset into "bins" based on the value of one of the data attributes, and counts the number of rows in each bin. For example, a histogram might count the number of people in the dataset who achieved a particular educational level.

```
               Education
HS-grad            10501
Some-college        7291
Bachelors           5355
Masters             1723
Assoc-voc           1382
```

Histograms are particularly interesting for differential privacy because they automatically satisfy parallel composition. Each "bin" in a histogram is defined by a possible value for a data attribute (for example, `'Education' == 'HS-grad'`). It's impossible for a single row to have *two* values for an attribute simultaneously, so defining the bins this way *guarantees* that they will be disjoint. Thus we have satisfied the requirements for parallel composition, and we can use a differentially private mechanism to release *all* of the bin counts with a total privacy cost of just ϵ.

```
epsilon = 1

# This analysis has a total privacy cost of epsilon = 1, even though we release many␣
↪results!
f = lambda x: x + np.random.laplace(loc=0, scale=1/epsilon)
s = adult['Education'].value_counts().apply(f)
s.to_frame().head(5)
```

```
                  Education
HS-grad        10500.079011
Some-college    7291.326638
Bachelors       5355.571888
Masters         1721.755188
Assoc-voc       1382.600081
```

5.2.2 Contingency Tables

A *contingency table* or *cross tabulation* (often shortened to *crosstab*) is like a multi-dimensional histogram. It counts the frequency of rows in the dataset with particular values for more than one attribute at a time. Contingency tables are frequently used to show the relationship between two variables when analyzing data. For example, we might want to see counts based on both education level and gender:

```
pd.crosstab(adult['Education'], adult['Sex']).head(5)
```

```
Sex        Female  Male
Education
10th          295   638
11th          432   743
12th          144   289
1st-4th        46   122
5th-6th        84   249
```

Like the histogram we saw earlier, each individual in the dataset participates in exactly *one* count appearing in this table. It's impossible for any single row to have multiple values simultaneously, for any set of data attributes considered in building the contingency table. As a result, it's safe to use parallel composition here, too.

```
ct = pd.crosstab(adult['Education'], adult['Sex'])
f = lambda x: x + np.random.laplace(loc=0, scale=1/epsilon)
ct.applymap(f).head(5)
```

```
Sex             Female        Male
Education
10th        295.863380  641.726775
11th        431.457752  743.818341
12th        142.007554  289.007022
1st-4th      46.453134  121.412048
5th-6th      84.192272  249.134223
```

It's also possible to generate contingency tables of more than 2 variables. Consider what happens each time we add a variable, though: each of the counts tends to get smaller. Intuitively, as we split the dataset into more chunks, each chunk has fewer rows in it, so all of the counts get smaller.

These shrinking counts can have a significant affect on the accuracy of the differentially private results we calculate from them. If we think of things in terms of signal and noise, a large count represents a strong T J H-Q's M likely to be disrupted too much by relatively weak noise (like the noise we add above), and therefore the results are likely to be useful even after the noise is added. However, a small count represents a weak T J H-potentially B T X as the noise itself - and after we add the noise, we won't be able to infer anything useful from the results.

So while it may seem that parallel composition gives us something "for free" (more results for the same privacy cost), that's not really the case. Parallel composition simply moves the tradeoff between accuracy and privacy along a different axis - as we split the dataset into more chunks and release more results, each result contains a weaker signal, and so it's less accurate.

5.3 Post-processing

The third property of differential privacy we will discuss here is called Q P T U Q S The idea is simple: it's impossible to reverse the privacy protection provided by differential privacy by post-processing the data in some way. Formally:

- If $F(X)$ satisfies ϵ-differential privacy
- Then for any (deterministic or randomized) function g, $g(F(X))$ satisfies ϵ-differential privacy

The post-processing property means that it's always safe to perform arbitrary computations on the output of a differentially private mechanism - there's no danger of reversing the privacy protection the mechanism has provided. In particular, it's fine to perform post-processing that might reduce the noise or improve the signal in the mechanism's output (e.g. replacing negative results with zeros, for queries that shouldn't return negative results). In fact, many sophisticated differentially private algorithms make use of post-processing to reduce noise and improve the accuracy of their results.

The other implication of the post-processing property is that differential privacy provides resistance against privacy attacks based on auxiliary information. For example, the function g might contain auxiliary information about elements of the dataset, and attempt to perform a linkage attack using this information. The post-processing property says that such an attack is limited in its effectiveness by the privacy parameter ϵ, regardless of the auxiliary information contained in g.

CHAPTER 1

Introduction

1.1 MOTIVATION

We have witnessed the rapid growth of machine learning (ML) technologies in empowering diverse artificial intelligence (AI) applications, such as computer vision, automatic speech recognition, natural language processing, and recommender systems [Pouyanfar et al., 2019, Hatcher and Yu, 2018, Goodfellow et al., 2016]. The success of these ML technologies, in particular deep learning (DL), has been fueled by the availability of vast amounts of data (a.k.a. the big data) [Trask, 2019, Pouyanfar et al., 2019, Hatcher and Yu, 2018]. Using these data, DL systems can perform a variety of tasks that can sometimes exceed human performance; for example, DL empowered face-recognition systems can achieve commercially acceptable levels of performance given millions of training images. These systems typically require a huge amount of data to reach a satisfying level of performance. For example, the object detection system from Facebook has been reported to be trained on 3.5 billion images from Instagram [Hartmann, 2019].

In general, the big data required to empower AI applications is often large in size. However, in many application domains, people have found that big data are hard to come by. What we have most of the time are "small data," where either the data are of small sizes only, or they lack certain important information, such as missing values or missing labels. To provide sufficient labels for data often requires much effort from domain experts. For example, in medical image analysis, doctors are often employed to provide diagnosis based on scan images of patient organs, which is tedious and time consuming. As a result, high-quality and large-volume training data often cannot be obtained. Instead, we face silos of data that cannot be easily bridged.

The modern society is increasingly made aware of issues regarding the data ownership: who has the right to use the data for building AI technologies? In an AI-driven product recommendation service, the service owner claims ownership over the data about the products and purchase transactions, but the ownership over the data about user purchasing behaviors and payment habits is unclear. Since data are generated and owned by different parties and organizations, a traditional and naive approach is to collect and transfer the data to one central location where powerful computers can train and build ML models. Today, this methodology is no longer valid.

While AI is spreading into ever-widening application sectors, concerns regarding user privacy and data confidentiality expand. Users are increasingly concerned that their private information is being used (or even abused) by commercial and political purposes without their permission. Recently, several large Internet corporations have been fined heavily due to their

leakage of users' private data to commercial companies. Spammers and under-the-table data exchanges are often punished in court cases.

In the legal front, law makers and regulatory bodies are coming up with new laws ruling how data should be managed and used. One prominent example is the adoption of the General Data Protection Regulation (GDPR) by the European Union (EU) in 2018 [GDPR website, 2018]. In the U.S., the California Consumer Privacy Act (CCPA) will be enacted in 2020 in the state of California [DLA Piper, 2019]. China's Cyber Security Law and the General Provisions of Civil Law, implemented in 2017, also imposed strict controls on data collection and transactions. Appendix A provides more information about these new data protection laws and regulations.

Under this new legislative landscape, collecting and sharing data among different organizations is becoming increasingly difficult, if not outright impossible, as time goes by. In addition, the sensitive nature of certain data (e.g., financial transactions and medical records) prohibits free data circulation and forces the data to exist in isolated data silos maintained by the data owners [Yang et al., 2019]. Due to industry competition, user privacy, data security, and complicated administrative procedures, even data integration between different departments of the *same company faces heavy resistance*. The prohibitively high cost makes it almost impossible to integrate data scattered in different institutions [WeBank AI, 2019]. Now that the old privacy-intrusive way of collecting and sharing data is outlawed, data consolidation involving different data owners is extremely challenging going forward.

How to solve the problem of data fragmentation and isolation while complying with the new stricter privacy-protection laws is a major challenge for AI researchers and practitioners. Failure to adequately address this problem will likely lead to a new AI winter [Yang et al., 2019].

Another reason why the AI industry is facing a data plight is that the benefit of collaborating over the sharing of the big data is not clear. Suppose that two organizations wish to collaborate on medical data in order to train a joint ML model. The traditional method of transferring the data from one organization to another will often mean that the original data owner will lose control over the data that they owned in the first place. The value of the data decreases as soon as the data leaves the door. Furthermore, when the better model as a result of integrating the data sources gained benefit, it is not clear how the benefit is fairly distributed among the participants. This fear of losing control and lack of transparency in determining the distribution of values is causing the so-called data fragmentation to intensify.

With edge computing over the Internet of Things, the big data is often not a single monolithic entity but rather distributed among many parties. For example, satellites taking images of the Earth cannot expect to transmit all data to data centers on the ground, as the amount of transmission required will be too large. Likewise, with autonomous cars, each car must be able to process much information locally with ML models while collaborate globally with other cars and computing centers. How to enable the updating and sharing of models among the multiple sites in a secure and yet efficient way is a new challenge to the current computing methodologies.

1.2 FEDERATED LEARNING AS A SOLUTION

As mentioned previously, multiple reasons make the problem of data silos become impediment to the big data needed to train ML models. It is thus natural to seek solutions to build ML models that do not rely on collecting all data to a centralized storage where model training can happen. An idea is to train a model at each location where a data source resides, and then let the sites communicate their respective models in order to reach a consensus for a global model. In order to ensure user privacy and data confidentiality, the communication process is carefully engineered so that no site can second-guess the private data of any other sites. At the same time, the model is built as if the data sources were combined. This is the idea behind "federated machine learning" or "federated learning" for short.

Federated learning was first practiced in an edge-server architecture by McMahan et al. in the context of updating language models on mobile phones [McMahan et al., 2016a,b, Konecný et al., 2016a,b]. There are many mobile edge devices each holding private data. To update the prediction models in the Gboard system, which is the Google's keyboard system for auto-completion of words, researchers at Google developed a federated learning system to update a collective model periodically. Users of the Gboard system gets a suggested query and whether the users clicked the suggested words. The word-prediction model in Gboard keeps improving based on not just a single mobile phone's accumulated data but all phones via a technique known as federated averaging (`FedAvg`). Federated averaging does not require moving data from any edge device to one central location. Instead, with federated learning, the model on each mobile device, which can be a smartphones or a tablet, gets encrypted and shipped to the cloud. All encrypted models are integrated into a global model under encryption, so that the server at the cloud does not know the data on each device [Yang et al., 2019, McMahan et al., 2016a,b, Konecný et al., 2016a,b, Hartmann, 2018, Liu et al., 2019]. The updated model, which is under encryption, is then downloaded to all individual devices on the edge of the cloud system [Konecný et al., 2016b, Hartmann, 2018, Yang et al., 2018, Hard et al., 2018]. In the process, users' individual data on each device is not revealed to others, nor to the servers in the cloud.

Google's federated learning system shows a good example of B2C (business-to-consumer), in designing a secure distributed learning environment for B2C applications. In the B2C setting, federated learning can ensure privacy protection as well as increased performance due to a speedup in transmitting the information between the edge devices and the central server.

Besides the B2C model, federated learning can also support the B2B (business-to-business) model. In federated learning, a fundamental change in algorithmic design methodology is, instead of transferring data from sites to sites, we transfer model parameters in a secure way, so that other parties cannot "second guess" the content of others' data. Below, we give a formal categorization of the federated learning in terms of how the data is distributed among the different parties.

1.2.1 THE DEFINITION OF FEDERATED LEARNING

Federated learning aims to build a joint ML model based on the data located at multiple sites. There are two processes in federated learning: model training and model inference. In the process of model training, information can be exchanged between parties but not the data. The exchange does not reveal any protected private portions of the data at each site. The trained model can reside at one party or shared among multiple parties.

At inference time, the model is applied to a new data instance. For example, in a B2B setting, a federated medical-imaging system may receive a new patient who's diagnosis come from different hospitals. In this case, the parties collaborate in making a prediction. Finally, there should be a fair value-distribution mechanism to share the profit gained by the collaborative model. Mechanism design should done in such a way to make the federation sustainable.

In broad terms, federated learning is an algorithmic framework for building ML models that can be characterized by the following features, where a model is a function mapping a data instance at some party to an outcome.

- There are two or more parties interested in jointly building an ML model. Each party holds some data that it wishes to contribute to training the model.
- In the model-training process, the data held by each party does not leave that party.
- The model can be transferred in part from one party to another under an encryption scheme, such that other parties cannot re-engineer the data at any given party.
- The performance of the resulting model is a good approximation of ideal model built with all data transferred to a single party.

More formally, consider N data owners $\{\mathcal{F}_i\}_{i=1}^N$ who wish to train a ML model by using their respective datasets $\{\mathcal{D}_i\}_{i=1}^N$. A conventional approach is to collect all data $\{\mathcal{D}_i\}_{i=1}^N$ together at one data server and train a ML model $\mathcal{M}_{SUM}$ on the server using the centralized dataset. In the conventional approach, any data owner $\{\mathcal{F}_i$ will expose its data $\{\mathcal{D}_i$ to the server and even other data owners.

Federated learning is a ML process in which the data owners collaboratively train a model $\mathcal{M}_{FED}$ without collecting all data $\{\mathcal{D}_i\}_{i=1}^N$. Denote $\mathcal{V}_{SUM}$ and $\mathcal{V}_{FED}$ as the performance measure (e.g., accuracy, recall, and F1-score) of the centralized model $\mathcal{M}_{SUM}$ and the federated model $\mathcal{M}_{FED}$, respectively.

We can capture what we mean by performance guarantee more precisely. Let δ be a non-negative real number. We say that the federated learning model $\mathcal{M}_{FED}$ has δ-performance loss if

$$|\mathcal{V}_{SUM} - \mathcal{V}_{FED}| < \delta. \tag{1.1}$$

The previous equation expresses the following intuition: if we use secure federated learning to build a ML model on distributed data sources, this model's performance on future data is approximately the same as the model that is built on joining all data sources together.

We allow the federated learning system to perform a little less than a joint model because in federated learning data owners do not expose their data to a central server or any other data owners. This additional security and privacy guarantee can be worth a lot more than the loss in accuracy, which is the δ value.

A federated learning system may or may not involve a central coordinating computer depending on the application. An example involving a coordinator in a federated learning architecture is shown in Figure 1.1. In this setting, the coordinator is a central aggregation server (a.k.a. the parameter server), which sends an initial model to the local data owners A–C (a.k.a. clients or participants). The local data owners A–C each train a model using their respective dataset, and send the model weight updates to the aggregation server. The aggregation sever then combines the model updates received from the data owners (e.g., using federated averaging [McMahan et al., 2016a]), and sends the combined model updates back to the local data owners. This procedure is repeated until the model converges or until the maximum number of iterations is reached. Under this architecture, the raw data of the local data owners never leaves the local data owners. This approach not only ensures user privacy and data security, but also saves communication overhead needed to send raw data. The communication between the central aggregation server and the local data owners can be encrypted (e.g., using homomorphic encryption [Yang et al., 2019, Liu et al., 2019]) to guard against information leakage.

The federated learning architecture can also be designed in a peer to peer manner, which does not require a coordinator. This ensures further security guarantee in which the parties communicate directly without the help of a third party, as illustrated in Figure 1.2. The advantage of this architecture is increased security, but a drawback is potentially more computation to encrypt and decrypt messages.

Federated learning brings several benefits. It preserves user privacy and data security by design since no data transfer is required. Federated learning also enables several parties to collaboratively train a ML model so that each of the parties can enjoy a better model than what it can achieve alone. For example, federated learning can be used by private commercial banks to detect multi-party borrowing, which has always been a headache in the banking industry, especially in the Internet finance industry [WeBank AI, 2019]. With federated learning, there is no need to establish a central database, and any financial institution participating in federated learning can initiate new user queries to other agencies within the federation. Other agencies only need to answer questions about local lending without knowing specific information of the user. This not only protects user privacy and data integrity, but also achieves an important business objective of identifying multi-party lending.

While federated learning has great potential, it also faces several challenges. The communication link between the local data owner and the aggregation server may be slow and un-

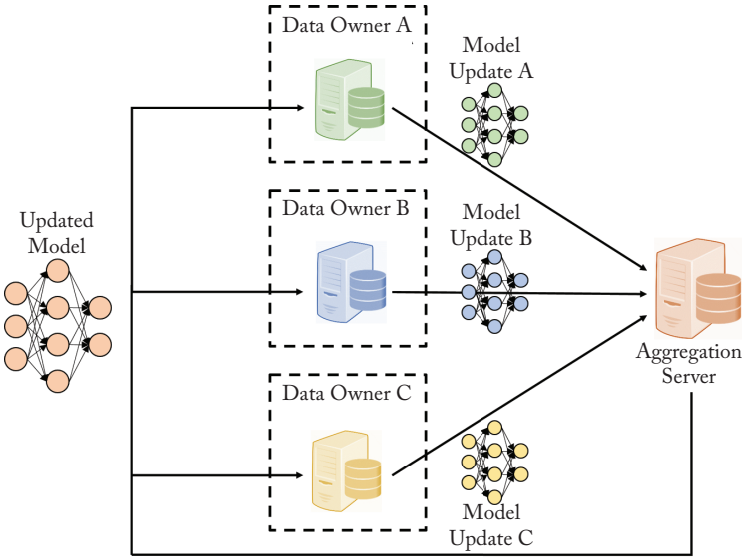


Figure 1.1: An example federated learning architecture: client-server model.

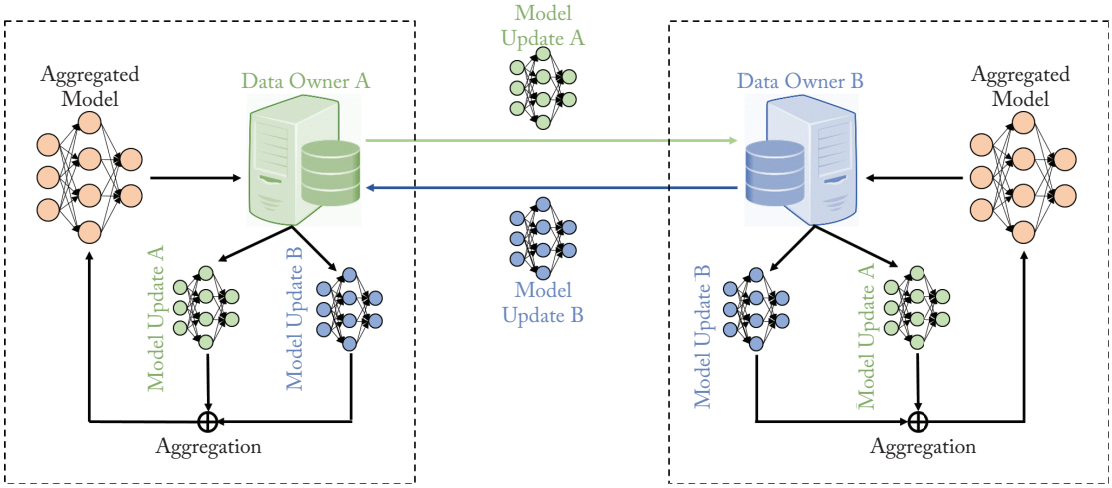


Figure 1.2: An example federated learning architecture: peer-to-peer model.

stable [Hartmann, 2018]. There may be a very large number of local data owners (e.g., mobile users). In theory, every mobile user can participate in federated learning, making the system unstable and unpredictable. Data from different participants in federated learning may follow non-identical distributions [Zhao et al., 2018, Sattler et al., 2019, van Lier, 2018], and different participants may have unbalanced numbers of data samples, which may result in a biased model

or even failure of training a model. As the participants are distributed and difficult to authenticate, federated learning model poisoning attacks [Bhagoji et al., 2019, Han, 2019], in which one or more malicious participants send ruinous model updates to make the federated model useless, can take place and confound the whole operation.

1.2.2 CATEGORIES OF FEDERATED LEARNING

Let matrix $\mathcal{D}_i$ denote the data held by the i th data owner. Suppose that each row of the matrix $\mathcal{D}_i$ represents a data sample, and each column represents a specific feature. At the same time, some datasets may also contain label data. We denote the feature space as $\mathcal{X}$, the label space as $\mathcal{Y}$, and we use $\mathcal{I}$ to denote the sample ID space. For example, in the financial field, labels may be users' credit. In the marketing field labels may be the user's purchasing desire. In the education field, $\mathcal{Y}$ may be the students' scores. The feature $\mathcal{X}$, label $\mathcal{Y}$, and sample IDs $\mathcal{I}$ constitute the complete training dataset $(\mathcal{I}, \mathcal{X}, \mathcal{Y})$. The feature and sample spaces of the datasets of the participants may not be identical. We classify federated learning into horizontal federated learning (HFL), vertical federated learning (VFL), and federated transfer learning (FTL), according to how data is partitioned among various parties in the feature and sample spaces. Figures 1.3–1.5 show the three federated learning categories for a two-party scenario [Yang et al., 2019].

HFL refers to the case where the participants in federated learning share overlapping data features, i.e., the data features are aligned across the participants, but they differ in data samples. It resembles the situation that the data is horizontally partitioned inside a tabular view. Hence, we also call HFL as sample-partitioned federated learning, or example-partitioned federated learning [Kairouz et al., 2019]. Different from HFL, VFL applies to the scenario where the participants in federated learning share overlapping data samples, i.e., the data samples are aligned amongst the participants, but they differ in data features. It resembles the situation that data is vertically partitioned inside a tabular view. Thus, we also name VFL as feature-partitioned federated learning. FTL is applicable for the case when there is neither overlapping in data samples nor in features.

For example, when the two parties are two banks that serve two different regional markets, they may share only a handful of users but their data may have very similar feature spaces due to similar business models. That is, with limited overlap in users but large overlap in data features, the two banks can collaborate in building ML models through horizontal federated learning [Yang et al., 2019, Liu et al., 2019].

When two parties providing different services but sharing a large amount of users (e.g., a bank and an e-commerce company), they can collaborate on the different feature spaces that they own, leading to a better ML model for both. That is, with large overlap in users but little overlap in data features, the two companies can collaborate in building ML models through vertical federated learning [Yang et al., 2019, Liu et al., 2019]. Split learning, recently proposed by Gupta and Raskar [2018] and Vepakomma et al. [2019, 2018], is regarded here as a special case of vertical federated learning, which enables vertically federated training of deep neural

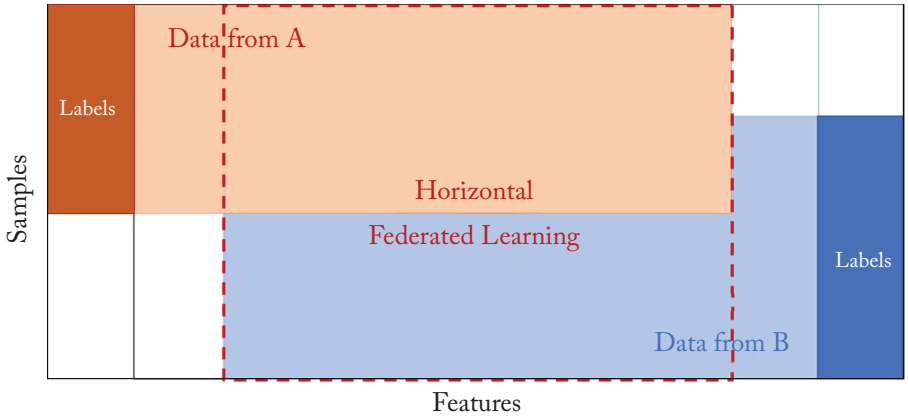


Figure 1.3: Illustration of HFL, a.k.a. sample-partitioned federated learning where the overlapping features from data samples held by different participants are taken to jointly train a model [Yang et al., 2019].

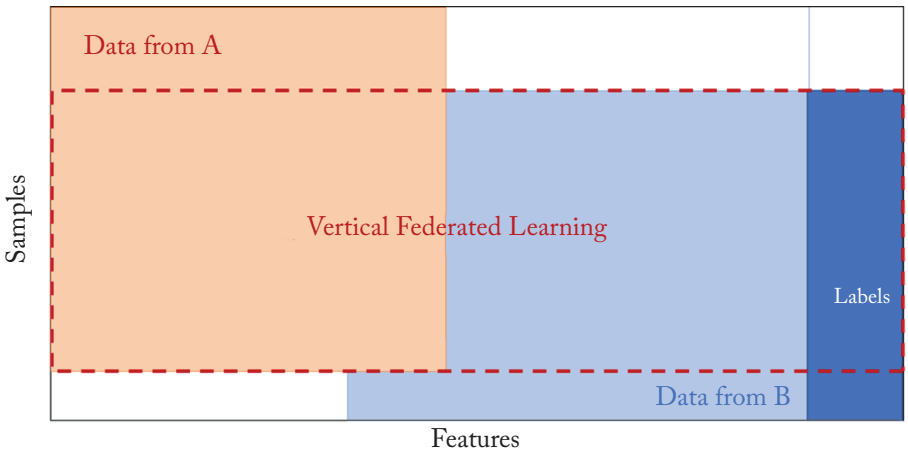


Figure 1.4: Illustration of VFL, a.k.a feature-partitioned federated learning where the overlapping data samples that have non-overlapping or partially overlapping features held by multiple participants are taken to jointly train a model [Yang et al., 2019].

networks (DNNs). That is, split learning facilitates training DNNs in federated learning settings over vertically partitioned data [Vepakomma et al., 2019].

In scenarios where participating parties have highly heterogeneous data (e.g., distribution mismatch, domain shift, limited overlapping samples, and scarce labels), HFL and VFL may not be able to build effective ML models. In those scenarios, we can leverage transfer learning

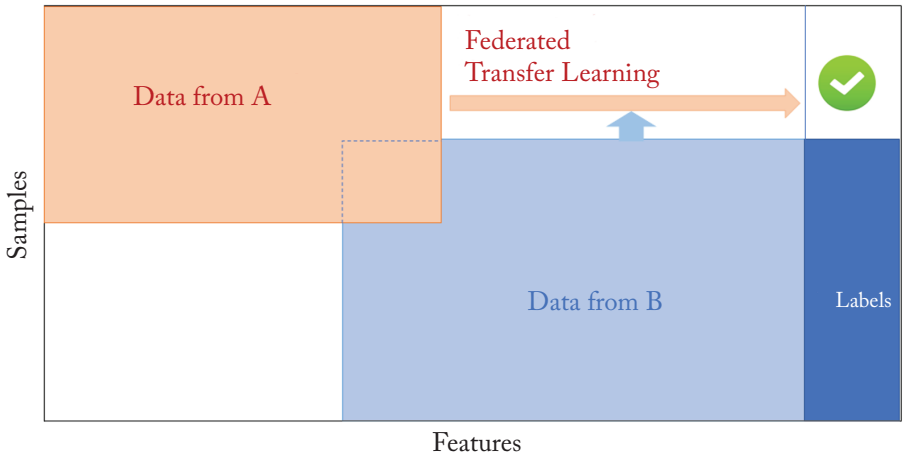


Figure 1.5: Federated transfer learning (FTL) [Yang et al., 2019]. A predictive model learned from feature representations of aligned samples belonging to party A and party B is utilized to predict labels for unlabeled samples of party A.

techniques to bridge the gap between heterogeneous data owned by different parties. We refer to federated learning leveraging transfer learning techniques as FTL.

Transfer learning aims to build effective ML models in a resource-scarce target domain by exploiting or transferring knowledge learned from a resource-rich source domain, which naturally fits the federated learning setting where parties are typically from different domains. Pan and Yang [2010] divides transfer learning into mainly three categories: (i) instance-based transfer, (ii) feature-based transfer, and (iii) model-based transfer. Here, we provide brief descriptions on how these three categories of transfer learning techniques can be applied to federated settings.

- **Instance-based FTL.** Participating parties selectively pick or re-weight their training data samples such that the distance among domain distributions can be minimized, thereby minimizing the objective loss function.
- **Feature-based FTL.** Participating parties collaboratively learn a common feature representation space, in which the distribution and semantic difference among feature representations transformed from raw data can be relieved and such that knowledge can be transferable across different domains to build more robust and accurate shared ML models.

  Figure 1.5 illustrates an FTL scenario where a predictive model learned from feature representations of aligned samples belonging to party A and party B is utilized to predict labels for unlabeled samples of party A. We will elaborate on how this FTL is performed in Chapter 6.

- **Model-based FTL.** Participating parties collaboratively learn shared models that can benefit for transfer learning. Alternatively, participating parties can utilize pre-trained models as the whole or part of the initial models for a federated learning task.

We will further explain in detail the HFL and VFL in Chapter 4 and Chapter 5, respectively. In Chapter 6, we will elaborate on a feature-based FTL framework proposed by Liu et al. [2019].

1.3 CURRENT DEVELOPMENT IN FEDERATED LEARNING

The idea of federated learning has appeared in different forms throughout the history of computer science, such as privacy-preserving ML [Fang and Yang, 2008, Mohassel and Zhang, 2017, Vaidya and Clifton, 2004, Xu et al., 2015], privacy-preserving DL [Liu et al., 2016, Phong, 2017, Phong et al., 2018], collaborative ML [Melis et al., 2018], collaborative DL [Zhang et al., 2018, Hitaj et al., 2017], distributed ML [Li et al., 2014, Wang, 2016], distributed DL [Vepakomma et al., 2018, Dean et al., 2012, Ben-Nun and Hoefler, 2018], and federated optimization [Li et al., 2019, Xie et al., 2019], as well as privacy-preserving data analytics [Mangasarian et al., 2008, Mendes and Vilela, 2017, Wild and Mangasarian, 2007, Bogdanov et al., 2014]. Chapters 2 and 3 will present some examples.

1.3.1 RESEARCH ISSUES IN FEDERATED LEARNING

Federated learning was studied by Google in a research paper published in 2016 on arXiv.[1] Since then, it has been an area of active research in the AI community as evidenced by the fast-growing volume of preprints appearing on arXiv. Yang et al. [2019] provide a comprehensive survey of recent advances of federated learning.

Recent research work on federated learning are mainly focused on improving security and statistical challenges [Yang et al., 2019, Mancuso et al., 2019]. Cheng et al. [2019] proposed SecureBoost in the setting of vertical federated learning, which is a novel lossless privacy-preserving tree-boosting system. SecureBoost provides the same level of accuracy as the non-privacy-preserving approach. It is theoretically proven that the SecureBoost framework is as accurate as other non-federated gradient tree-boosting algorithms that rely on centralized datasets [Cheng et al., 2019].

Liu et al. [2019] presents a flexible federated transfer learning framework that can be effectively adapted to various secure multi-party ML tasks. In this framework, the federation allows knowledge to be shared without compromising user privacy, and enables complimentary knowledge to be transferred in the network via transfer learning. As a result, a target-domain party can build more flexible and powerful models by leveraging rich labels from a source-domain party.

[1] arXiv is a repository of electronic preprints (e-prints) hosted by Cornell University. For more information, visit arXiv website `https://arxiv.org/`.

In a federated learning system, we can assume that participating parties are honest, semi-honest, or malicious. When a party is malicious, it is possible for a model to taint its data in training. The possibility of model poisoning attacks on federated learning initiated by a single non-colluding malicious agent is discussed in Bhagoji et al. [2019]. A number of strategies to carry out model poisoning attack were investigated. It was shown that even a highly constrained adversary can carry out model poisoning attacks while simultaneously maintaining stealth. The work of Bhagoji et al. [2019] reveals the vulnerability of the federated learning settings and advocates the need to develop effective defense strategies.

Re-examining the existing ML models under the federated learning settings has become a new research direction. For example, combining federated learning with reinforcement learning has been studied in Zhuo et al. [2019], where Gaussian differentials on the information shared among agents when updating their local models were applied to protect the privacy of data and models. It has been shown that the proposed federated reinforcement learning model performs close to the baselines that directly take all joint information as input [Zhuo et al., 2019].

Another study in Smith et al. [2017] showed that multi-task learning is naturally suited to handle the statistical challenges of federated learning, where separate but related models are learned simultaneously at each node. The practical issues, such as communication cost, stragglers, and fault tolerance in distributed multi-task learning and federated learning, were considered. A novel systems-aware optimization method was put forward, which achieves significant improved efficiency compared to the alternatives.

Federated learning has also been applied in the fields of computer vision (CV), e.g., medical image analysis [Sheller et al., 2018, Liu et al., 2018, Huang and Liu, 2019], natural language processing (NLP) (see, e.g., Chen et al. [2019]), and recommender systems (RS) (see, e.g., Ammad-ud-din et al. [2019]). This will be further reviewed in Chapter 8.

Regarding applications of federated learning, the researchers at Google have applied federated learning in mobile keyboard prediction [Bonawitz and Eichner et al., 2019, Yang et al., 2018, Hard et al., 2018], which has achieved significant improvement in prediction accuracy without exposing mobile user data. Researchers at Firefox have used federated learning for search word prediction [Hartmann, 2018]. There is also new research effort to make federated learning more personalizable [Smith et al., 2017, Chen et al., 2018].

1.3.2 OPEN-SOURCE PROJECTS

Interest in federated learning is not only limited to theoretical work. Research on the development and deployment of federated learning algorithms and systems is also flourishing. There are several fast-growing open-source projects of federated learning.

• **Federated AI Technology Enabler (FATE)** [WeBank FATE, 2019] is an open-source project initiated by the AI department of WeBank[2] to provide a secure computing framework to support the federated AI ecosystem [WeBank FedAI, 2019]. It implements secure computation protocols based on homomorphic encryption (HE) and secure multi-party computation (MPC). It supports a range of federated learning architectures and secure computation algorithms, including logistic regression, tree-based algorithms, DL (artificial neural networks), and transfer learning. For more information on FATE, readers can refer to the GitHub FATE website [WeBank FATE, 2019] and the FedAI website [WeBank FedAI, 2019].

• **TensorFlow**[3] **Federated** project [Han, 2019, TFF, 2019, Ingerman and Ostrowski, 2019, Tensorflow-federated, 2019] (TFF) is an open-source framework for experimenting with federated ML and other computations on decentralized datasets. TFF enables developers to simulate existing federated learning algorithms on their models and data, as well as to experiment with novel algorithms. The building blocks provided by TFF can also be used to implement non-learning computations, such as aggregated analytics over decentralized data. The interfaces of TFF are organized in two layers: (1) the federated learning (FL) application programming interface (API) and (2) federated Core (FC) API. TFF enables developers to declaratively express federated computations, so that they can be deployed in diverse runtime environments. Included in TFF is a single-machine simulation run-time for experimentation.

• **TensorFlow-Encrypted** [TensorFlow-encrypted, 2019] is a Python library built on top of TensorFlow for researchers and practitioners to experiment with privacy-preserving ML. It provides an interface similar to that of TensorFlow, and aims to make the technology readily available without requiring user to be experts in ML, cryptography, distributed systems, and high-performance computing.

• **coMind** [coMind.org, 2018, coMindOrg, 2019] is an open-source project for training privacy-preserving federated DL models. The key component of coMind is the implementation of the federated averaging algorithm [McMahan et al., 2016a, Yu et al., 2018], which is training ML models in a collaborative way while preserving user privacy and data security. coMind is built on top of TensorFlow and provides high-level APIs for implementing federated learning.

• **Horovod** [Sergeev and Balso, 2018, Horovod, 2019], developed by Uber, is an open-source distributed training framework for DL. It is based on the open message passing interface

[2]WeBank, opened in December 2014 upon receiving its banking license in China. It is the first digital-only bank in China. WeBank is devoted to offering individuals and SMEs under-served by the current banking system with a variety of convenient and high-quality financial services. For more information on WeBank, please visit `https://www.webank.com/en/`.

[3]TensorFlow is an open-source DL framework, developed and maintained by Google Inc. TensorFlow is widely used in research and implementation of DL. For more information on TensorFlow, readers can refer to its project website `https://www.tensorflow.org/` and its GitHub website `https://github.com/tensorflow`.

(MPI) and works on top of popular DL frameworks, such as TensorFlow and PyTorch.[4] The goal of Horovod is to make distributed DL fast and easy to use. Horovod supports federated learning via open MPI and currently, encryption is not yet supported.

• **OpenMined/PySyft** [Han, 2019, OpenMined, 2019, Ryffel et al., 2018, PySyft, 2019, Ryffel, 2019] provides two methods for privacy preservation: (1) federated learning and (2) differential privacy. OpenMined further supports two methods of secure computation through multi-party computation and homomorphic encryption. OpenMined has made available the PySyft library [PySyft, 2019], which is the first open-source federated learning framework for building secure and scalable ML models [Ryffel, 2019]. PySyft is simply a hooked extension of PyTorch. For users who are familiar with PyTorch, it is very easy to implement federated learning systems with PySyft. Federated learning extension based on the TensorFlow framework is currently being developed within OpenMined.

• **LEAF Beanchmark** [LEAF, 2019, Caldas et al., 2019], maintained by Carnegie Mellon University and Google AI, is a modular benchmarking framework for ML in federated settings, with applications in federated learning, multi-task learning, meta-learning, and on-device learning. LEAF includes a suite of open-source federated datasets (e.g., FEMNIST, Sentiment140, and Shakespeare), a rigorous evaluation framework, and a set of reference implementations, aiming to capture the reality, obstacles, and intricacies of practical federated learning environments. LEAF enables researchers and practitioners in these domains to investigate new proposed solutions under more realistic assumptions and settings. LEAF will include additional tasks and datasets in its future releases.

1.3.3 STANDARDIZATION EFFORTS

As more developments are made in the legal front on the secure and responsible use of users' data, technical standard needs to be developed to ensure that organizations use the same language and follow a standard guideline in developing future federated learning systems. Moreover, there is increasing need for the technical community to communicate with the regulatory and legal communities over the use of the technology. As a result, it is important to develop international standards that can be adopted by multiple disciplines.

For example, companies striving to satisfy the GDPR requirements need to know what technical developments are needed in order to satisfy the legal requirements. Standards can provide a bridge between regulators and technical developers.

One of the early standards is initiated by the AI Department at WeBank with the Institute of Electrical and Electronics Engineers (IEEE) P3652.1 Federated Machine Learning Working Group (known as Federated Machine Learning (C/LT/FML)) was established in December

[4]PyTorch is a popular DL framework and is widely used in research and implementation. For more information, visit the official PyTorch website `https://pytorch.org/` and the GitHub PyTorch website `https://github.com/pytorch/pytorch`.

2018 [IEEE P3652.1, 2019]. The objective of this working group is to provide guidelines for building the architectural framework and applications of federated ML. The working group will define the architectural framework and application guidelines for federated ML, including:

1. The description and definition of federated learning;
2. The types of federated learning and the application scenarios to which each type applies;
3. Performance evaluation of federated learning; and
4. The associated regulatory requirements.

The purpose of this standard is to provide a feasible solution for the industrial application of AI without exchanging data directly. This standard is expected to promote and facilitate collaborations in an environment where privacy and data protection issues have become increasingly important. It will promote and enable to the use of distributed data sources for the purpose of developing AI without violating regulations or ethical concerns.

1.3.4 THE FEDERATED AI ECOSYSTEM

The Federated AI (FedAI) ecosystem project was initiated by the AI Department of WeBank [WeBank FedAI, 2019]. The primary goal of the project is to develop and promote advanced AI technologies that preserve user privacy, data security, and data confidentiality. The federated AI ecosystem features four main themes.

- Open-source technologies: FedAI aims to accelerate open-source development of federated ML and its applications. The FATE project [WeBank FATE, 2019] is a flagship project under FedAI.
- Standards and guidelines: FedAI, together with partners, are drawing up standardization to formulate the architectural framework and application guidelines of federated learning, and facilitate industry collaboration. One representative work is the IEEE P3652.1 federated ML working group [IEEE P3652.1, 2019].
- Multi-party consensus mechanisms: FedAI is studying incentive and reward mechanisms to encourage more institutions to participate in federated learning research and development in a sustainable way. For example, FedAI is undertaking work to establish a multi-party consensus mechanism based on technologies like blockchain.
- Applications in various verticals: To open up the potential of federated learning, FedAI endeavors to showcase more vertical field applications and scenarios, and to build new business models.

1.4 ORGANIZATION OF THIS BOOK

The organization of this book is as follows. Chapter 2 provides background information on privacy-preserving ML, covering well-known techniques for data security. Chapter 3 describes distributed ML, highlighting the difference between federated learning and distributed ML. Horizontal federated learning, vertical federated learning, and federated transfer learning are elaborated in detail in Chapter 4, Chapter 5, and Chapter 6, respectively. Incentive mechanism design for motivating the participation in federated learning is discussed in Chapter 7. Recent work on extending federated learning to the fields of computer vision, natural language processing, and recommender systems are reviewed in Chapter 8. Chapter 9 presents federated reinforcement learning. The prospect of applying federated learning into various industrial sectors is summarized in Chapter 10. Finally, we provide a summary of this book and looking ahead in Chapter 11. Appendix A provides an overview of recent data protection laws and regulations in the European Union, the United States, and China.

CHAPTER 2

Background

In this chapter, we introduce the background knowledge related to federated learning, covering privacy-preserving machine learning techniques and data analytics.

2.1 PRIVACY-PRESERVING MACHINE LEARNING

Data leakage and privacy violation incidents have brought about heightened public awareness of the need for AI systems to be able to preserve user privacy and data confidentiality. Researchers are interested in developing techniques for privacy-preserving properties to be built inside machine learning (ML) systems. The resulting systems are known as privacy-preserving machine learning systems (PPML). In fact, 2018 was considered a breakout year for PPML [Mancuso et al., 2019]. PPML is a broad term that generally refers to ML equipped with defense measures for protecting user privacy and data security. The system security and cryptography community has also proposed various secure frameworks for ML.

In Westin [1968], Westin defined information privacy as follows: "the claim of individuals, groups, or institutions to determine for themselves when, how, and to what extent information about them is communicated to others." This essentially defines the right to control the access and handling of one's information. The main idea of information privacy is to have control over the collection and handling of one's personal data [Mendes and Vilela, 2017].

In this chapter, we will introduce several popular approaches used in PPML including secure multi-party computation (MPC), homomorphic encryption (HE) for privacy-preserving model training and inference, as well as differential privacy (DP) for preventing unwanted data disclosure. Privacy-preserving gradient descent methods will also be discussed.

2.2 PPML AND SECURE ML

Before going into the details of PPML, we first clarify the difference between PPML and secure ML. PPML and secure ML differ mainly in the types of security violations that they are designed to deal with [Barreno et al., 2006]. In secure ML, the adversary (i.e., attacker) is assumed to violate the *integrity and availability* of a data-analytic system, while in PPML, the adversary is assumed to violate the *privacy and confidentiality* of an ML system.

Most of the time, compromise in security is caused by the intentional attack by a third party. We are concerned with three major types of attacks in ML.

- **Integrity attack.** An attack on *integrity* may result in intrusion points being classified as normal (i.e., false negatives) by the ML system.
- **Availability attack.** An attack on *availability* may lead to classification errors (both false negatives and false positives) such that the ML system becomes unusable. This is a broader type of integrity attacks.
- **Confidentiality attack.** An attack on *confidentiality* may result in sensitive information (e.g., training data or model) of an ML system being leaked.

Table 2.1 gives a comparison between PPML and secure ML in terms of security violations, adversary attacks, and defense techniques.

Table 2.1: Comparison between PPML and secure ML

	Security Violations	Adversary Attacks	Defence Techniques
PPML	Privacy Confidentiality	Reconstruction attack Inversion attack Membership-inference attack	Secure multi-party computation Homomorphic encryption Differential privacy
Secure ML	Integrity Availability	Poisoning attack Adversarial attack Oracle attack	Defensive distillation Adversarial training Regularization

In this chapter, we mainly focus on PPML and defense techniques against privacy and confidentiality violations in ML. Interested readers can refer to Barreno et al. [2006] for a more detailed explanation of secure ML.

2.3 THREAT AND SECURITY MODELS

2.3.1 PRIVACY THREAT MODELS

In order to preserve privacy and confidentiality in ML, it is important to understand the possible threat models. In ML tasks, the participants usually take up three different roles: (1) as the input party, e.g., the data owner; (2) as the computation party (e.g., the model builder and inference service provider); and (3) as the result party (e.g., the model querier and user) [Bogdanov et al., 2014].

Attacks on ML may happen in any stage, including data publishing, model training, and model inference. *Attribute-inference attacks* can happen in the data publishing stage, where adversaries may attempt to de-anonymize or target data-record owners for malevolent purposes. The attacks during ML model training are called *reconstruction attacks*, where the computation

party aims to reconstruct the raw data of the data providers or to learn more information about the data providers than what the model builders intend to reveal.

For federated learning, reconstruction attacks are the major privacy concerns. In the inference phase of ML models, an adversarial result party may conduct *reconstruction attack*, *model inversion attacks*, or *membership-inference attacks*, using reverse engineering techniques to gain extra information about the model or raw training data.

Reconstruction Attacks. In reconstruction attacks, the adversary's goal is to extract the training data or feature vectors of the training data during ML model training or model inference. In centralized learning, raw data from different data parties are uploaded to the computation party, which makes the data vulnerable to adversaries, such as a malicious computation party. Large companies may collect raw data from users to train an ML model. However, the collected data may be used for other purposes or sent to a third-party without informed consent from the users. In federated learning, each participating party carries out ML model training using their local data. Only the model weight updates or gradient information are shared with other parties. However, the gradient information may also be leveraged to reveal extra information about the training data [Aono et al., 2018]. Plain-text gradient updating may also violate privacy in some application scenarios. To resist reconstruction attacks, ML models that store explicit feature values such as support vector machine (SVM) and k-nearest neighbors (kNN) should be avoided. During model training, secure multi-party computation (MPC) [Yao, 1982] and homomorphic encryption (HE) [Rivest et al., 1978] can be used to defend against such attacks by keeping the intermediate values private. During model inference, the party computing the inference result should only be granted black-box access to the model. MPC and HE can be leveraged to protect the privacy of the user query during model inference. MPC, HE, and their corresponding applications in PPML will be introduced in Sections 2.4.1 and 2.4.2, respectively.

Model Inversion Attacks. In model inversion attacks, the adversary is assumed to have either white-box access or black-box access to the model. For the case of white-box access, the adversary knows the clear-text model without stored feature vectors. For the case of black-box access, the adversary can only query the model with data and collect the responses. The adversary's target is to extract the training data or feature vectors of the training data from the model. The black-box access adversary may also reconstruct the clear-text model from the response by conducting an *equation solving attack*. Theoretically, for an N-dimensional linear model, an adversary can steal it with $N + 1$ queries. Such a problem can be formalized as solving θ from $(x, h_\theta(x))$. The adversary can also learn a similar model using the query-response pairs to simulate the original model. To resist model inversion attacks, less knowledge of the model should be exposed to the adversary. The access to model should be limited to black-box access, and the output should be limited as well. There are several strategies proposed to reduce the success rate of model inversion attack. Fredrikson et al. [2015] choose to report only rounded

confidence values. Al-Rubaie and Chang [2016] take the predicted class labels as response, and the aggregated prediction results of multiple testing instances are returned to further enhance model protection. Bayesian neural networks combined with homomorphic encryption have been developed [Xie et al., 2019], to resist such attacks during secure neural network inference.

Membership-Inference Attacks. In membership-inference attacks, the adversary has black-box access to a model, as well as a certain sample, as its knowledge. The adversary's target is to learn if the sample is inside the training set of the model. The adversary infers whether a sample belongs to the training set or not based on the ML model output. The adversary conducts such attacks by finding and leveraging the differences in the model predictions on the samples belonging to the training set vs. other samples. Defense techniques that are proposed to resist model inversion attacks, such as result generalization by reporting rounded prediction results are shown to be effective to thwart such attacks [Shokri et al., 2017]. Differential privacy (DP) [Dwork et al., 2006] is a major approach to resist membership inference attacks, which will be introduced in Section 2.4.3.

Attribute-Inference Attacks. In attribute-inference attacks, the adversary tries to de-anonymize or target record owners for malevolent purpose. Anonymization by removing personally identifiable information (PII) (also known as sensitive features), such as user IDs and names, before data publishing appears to be a natural approach for protecting user privacy. However, it has been shown to be ineffective. For example, Netflix, the world's largest online movie rental service provider, released a movie rating dataset, which contains anonymous movie ratings from 500,000 subscribers. Despite anonymization, Narayanan and Shmatikov [2008] managed to leverage this dataset along with the Internet Movie Database (IMDB) as background knowledge to re-identify the Netflix users in the records, and further managed to deduce the user's apparent political preferences. This incident shows that anonymization fails in the face of strong adversaries with access to alternative background knowledge. To deal with attribute-inference attacks, group anonymization privacy approaches have been proposed in Mendes and Vilela [2017]. Privacy preservation in group anonymization privacy is achieved via generalization and suppression mechanisms.

Model Poisoning Attacks. It has been shown that federated learning may be vulnerable to model poisoning attacks [Bhagoji et al., 2019], also known as backdoor attacks [Bagdasaryan et al., 2019]. For example, a malicious participant in federated learning may inject a hidden backdoor functionality into the trained federated model, e.g., to cause a trained word predictor to complete certain sentences with an attacker-chosen word [Bagdasaryan et al., 2019]. Bhagoji et al. [2019] proposed a number of strategies to carry out model poisoning attacks, such as boosting of the malicious participant's model update, an alternating minimization strategy that alternately optimizes for the legit training loss and the adversarial backdoor objective, and using parameter estimation for the benign updates to improve attack success. Bagdasaryan et al. [2019] developed a new model-poisoning methodology using model replacement, where a constrain-

and-scale technique is proposed to evade anomaly detection-based defenses by incorporating the evasion into the attacker's loss function during model training. Possible solutions against model poisoning attacks include blockchain-based approaches [Preuveneers et al., 2018] and trusted execution environment (TEE) based approaches [Mo and Haddadi, 2019].

2.3.2 ADVERSARY AND SECURITY MODELS

For cryptographic PPML techniques, including MPC and HE, two types of adversaries are concerned in the literature.

- **Semi-honest adversaries.** In the semi-honest (a.k.a honest-but-curious, and passive) adversary model, the adversaries abide by the protocol honestly, but also attempt to learn more information beyond the output from the received information.

- **Malicious adversaries.** In the malicious (a.k.a. active) adversary model, the adversaries deviate from the protocol and can behave arbitrarily.

The semi-honest adversary model is widely considered in most PPML studies. The main reason is that, in federated learning, it is beneficial to each party to honestly follow the ML protocol, since malicious behaviors also break the benefits of the adversaries themselves. The other reason is that, in cryptography, it is a standard method to build a protocol secure against semi-honest adversaries first, then modify it to be secure against malicious adversaries via zero-knowledge proof.

For both security models, the adversaries corrupt a fraction of the parties, and the corrupted parties may collude with each other. The corruption of parties can be static or adaptive. The complexity of an adversary can be either polynomial-time or computational unbounded, corresponding to information-theoretic secure and computational secure, respectively. The security in cryptography is based on the notion of indistinguishability. Interested readers can refer to Lindell [2005] and Lindell and Pinkas [2009] for detailed analysis of adversary and security models.

2.4 PRIVACY PRESERVATION TECHNIQUES

In this section, we discuss privacy preservation techniques. We cover three types of such approaches, namely (1) MPC, (2) HE, and (3) DP.

2.4.1 SECURE MULTI-PARTY COMPUTATION

Secure Multi-Party Computation (MPC), a.k.a. secure function evaluation (SFE), was initially introduced as a secure two-party computation problem (the famous Millionaire's Problem), and generalized in 1986 by Andrew Yao [1986]. In MPC, the objective is to jointly compute a function from the private input by each party, without revealing such inputs to the other parties.

MPC tells us that for any functionality, it is possible to compute it without revealing anything other than the output.

Definition

MPC allows us to compute functions of private input values so that each party learns only the corresponding function output value, but not input values from other parties. For example, given a secret value x that is split into n shares so that a party P_i only knows x_i, all parties can collaboratively compute

$$y_1, \ldots, y_n = f(x_1, \ldots, x_n)$$

so that party P_i learns nothing beyond the output value y_i corresponding to its own input x_i.

The standard approach to prove that an MPC protocol is secure is the *simulation paradigm* [Lindell, 2017]. To prove an MPC protocol is secure against adversaries that corrupt t parties under the simulation paradigm, we build a simulator that, when given inputs and outputs of t colluding parties, generates t transcripts, so that the generated transcripts are *indistinguishable* to that generated in the actual protocol.

In general, MPC can be implemented through three different frameworks, namely: (1) Oblivious Transfer (OT) [Keller et al., 2016, Goldreich et al., 1987]; (2) Secret Sharing (SS) [Shamir, 1979, Rabin and Ben-Or, 1989]; and (3) Threshold Homomorphic Encryption (THE) [Cramer et al., 2001, Damgård and Nielsen, 2003]. From a certain point of view, both oblivious transfer protocols and threshold homomorphic encryption schemes use the idea of secret sharing. This might be the reason why secret sharing is widely regarded as the core of MPC. In the rest of this section, we will introduce oblivious transfer and secret sharing.

Oblivious Transfer

OT is a two-party computation protocol proposed by Rabin in 1981 [Rabin, 2005]. In OT, the sender owns a database of message-index pairs $(M_1, 1), \ldots, (M_N, N)$. At each transfer, the receiver chooses an index i for some $1 \leq i \leq N$, and receives M_i. The receiver does not learn any other information about the database, and the sender does not learn anything about the receiver's selection i. Here, we give the definition of 1-out-of-n OT.

Definition 2.1 1-out-of-n OT: Suppose Party A has a list $(x_1, \ldots, x_n)$ as the input, Party B has $i \in 1, \ldots, n$ as the input. 1-out-of-n OT is an MPC protocol where A learns nothing about i and B learns nothing else but x_i.

When $n = 2$, we get 1-out-of-2 OT which has the following property: 1-out-of-2 OT is universal for two-party MPC [Ishai et al., 2008]. That is, given a 1-out-of-2 OT protocol, one can conduct any secure two-party computation.

Many Constructions of OT has been proposed such as Bellare–Micali's [Bellare and Micali, 1990], Naor–Pinka's [Naor and Pinkas, 2001], and Hazay–Lindell's [Hazay and Lindell,

2010] approaches. Here, we demonstrate the Bellare-Micali's construction of OT, which utilizes Diffie–Hellman key exchange and is based on the computational Diffie–Hellman (CDH) assumption [Diffie and Hellman, 1976]. The Bellare–Micali's construction works as follows: the receiver sends two public keys to the sender. The receiver only holds one private key corresponding to one of the two public keys, and the sender does not know which public key it is. Then, the sender encrypts the two massages with their corresponding public keys, and sends the ciphertexts to the receiver. Finally, the receiver decrypts the target ciphertext with the private key.

Bellare–Micali Construction. In a discrete logarithm setting $(\mathbb{G}, g, p)$, where $\mathbb{G}$ is a group of prime order p, $g \in \mathbb{G}$ is a generator, and $H : G \to \{0,1\}^n$ is a hash function. Suppose the sender A has $x_0, x_1 \in \{0,1\}^n$, and the receiver B has $b \in \{0,1\}$.

1. A chooses a random element $c \leftarrow G$ and sends it to B.

2. B chooses $k \leftarrow \mathbb{Z}_p$ and sets $PK_b = g^k$, $PK_{1-b} = c/PK_b$, then sends PK_0 to A. A sets $PK_1 = c/PK_0$.

3. A encrypts x_0 with ElGamal scheme using PK_0 (i.e., setting $C_0 = [g^{r_0}, HASH(PK_0^{r_0}) * x_0]$ and encrypting x_1 using PK_1). Then, A sends (C_0, C_1) to B.

4. B decrypts C_b using private key k to obtain $x_b = PK_b^{r_b} * x_b / g^{r_b k}$.

Yao's Garbled Circuit (GC). [Yao, 1986] is a well-known OT-based secure two-party computation protocol that can evaluate any function. The key idea of Yao's GC is to decompose the computational circuits into generation and evaluation stages. The circuits consisting of gates like AND, OR, and NOT can be used to compute any arithmetic operation. Each party is in charge of one stage and the circuit is garbled in each stage, so that any of them cannot get information from the other one, but they can still achieve the result according to the circuit. GC consists of an OT protocol and a block cipher. The complexity of the circuit grows at least linearly with the input size. Soon after GC was proposed, GMW [Goldreich et al., 1987] extended GC to the multi-party setting against malicious adversaries. For more detailed survey of GC, readers can refer to Yakoubov [2017].

OT Extension. Impagliazzo and Rudich [1989] showed that OT provably requires "public-key" type of assumptions (such as factoring, discrete log, etc.). However, Beaver [1996] pointed out that OT can be "extended" in the sense that it is enough to generate a few "seed" OTs based on public-key cryptography, which can then be extended to any number of OTs using symmetric-key cryptosystems only. OT extension is now widely applied in MPC protocols [Keller et al., 2016, Mohassel and Zhang, 2017, Demmler et al., 2015] to improve efficiency.

Secret Sharing

Secret sharing is a concept of hiding a secret value by splitting it into random parts and distributing these parts (a.k.a. shares) to different parties, so that each party has only one share and thus only one piece of the secret [Shamir, 1979, Beimel, 2011]. Depending on the specific secret sharing schemes used, all or a known threshold of shares are needed to reconstruct the original secret value [Shamir, 1979, Tutdere and Uzunko, 2015]. For example, Shamir's Secret Sharing is constructed based on polynomial equations and provides information-theoretic security, and it is also efficient using matrix calculation speedup [Shamir, 1979]. There are several types of secret sharing, mainly including arithmetic secret sharing [Damård et al., 2011], Shamir's secret sharing [Shamir, 1979], and binary secret sharing [Wang et al., 2007]. As arithmetic secret sharing is mostly adopted by existing SMPC-based PPML approaches and binary secret sharing are closely related to OT which is discussed in Section 2.4.1, here we focus on arithmetic secret sharing.

Consider that a party P_i wants to share a secret S among n parties $\{P_i\}_{i=1}^n$ in a finite field F_q. To share S, the party P_i randomly samples $n-1$ values $\{s_i\}_{i=1}^{n-1}$ from $\mathbb{Z}_q$ and set $s_n = S - \sum_{i=1}^{n-1} s_i \bmod q$. Then, P_i distributes s_k to party P_k, for $k \neq i$. We denote the shared S as $\langle S \rangle = \{s_i\}_{i=1}^n$.

The arithmetic addition operation is carried out locally at each party. The secure multiplication is performed by using Beaver triples [Beaver, 1991]. The Beaver triples can be generated in an offline phase. The offline phase (i.e., preprocessing) serves as a *trusted dealer* who generates Beaver triples $\{(\langle a \rangle, \langle b \rangle, \langle c \rangle) | ab = c\}$ and distributes the shares among the n parties.

To compute $\langle z \rangle = \langle x \rangle \cdot \langle y \rangle = \langle x * y \rangle$, $P_i{}_{i=1}^n$ first computes $\langle e \rangle = \langle x \rangle - \langle a \rangle$, $\langle f \rangle = \langle y \rangle - \langle b \rangle$. Then, e and f are reconstructed. Finally, P_i computes $\langle z \rangle = \langle c \rangle + e\langle x \rangle + f\langle y \rangle$ locally, and a random party P_j adds its share into ef. We denote element-wise multiplication of vectors as $\langle \cdot \rangle \odot \langle \cdot \rangle$.

Secure multiplication can also be performed by leveraging the Gilboa's protocol [Gilboa, 1999], in which n-bit arithmetic multiplication can be conducted via n 1-out-of-2 OTs. Suppose that Party A holds x and Party B holds y. Now we show Gilboa's protocol, which results in A holding $\langle z \rangle_A$ and B holding $\langle z \rangle_B$ such that $z = x \cdot y$. Let l be the maximum length of the binary representation of the numbers involved in our protocol. Denote the $m\times$ 1-out-of-2 OT for l-bit strings as OT_l^m. Denote the ith bit of x as $x[i]$. The secure 2-party multiplication via OT can be conducted as follows.

1. A represents x in binary format.
2. B builds OT_l^l. For the ith OT, randomly pick $a_{i,0}$ and compute $a_{i,1} = 2^i y - a_{i,0}$. Use $(-a_{i,0}, a_{i,1})$ as the input for the ith OT.
3. A inputs X[i] as the choice bit in the ith OT and obtains x[i] $\times 2^i y - a_{i,0}$.
4. A computes $\langle z \rangle_A = \sum_{i=1}^l (x[i] \times 2^i y - a_{i,0})$
   B computes $\langle z \rangle_B = \sum_{i=1}^l a_{i,0}$.

The offline phase can be carried out efficiently with the help of a *semi-honest dealer* who generates Beaver triples and distributes them among all the parties. To perform such a preprocessing step without a *semi-honest dealer*, there are several protocols available, such as SPDZ [Damård et al., 2011], SPDZ-2 [Damård et al., 2012], MASCOT [Keller et al., 2016], and HighGear [Keller et al., 2018].

- SPDZ is an offline protocol in the preprocessing model based on somewhat homomorphic encryption (SHE) in the form of BGV, first described in Damård et al. [2011].
- SPDZ-2 [Damård et al., 2012] is a protocol based on threshold SHE cryptography (with a shared decryption key).
- MASCOT is an oblivious-transfer-based protocol, proposed in Keller et al. [2016]. It is far more computationally efficient than SPDZ and SPDZ-2.
- In 2018, Keller et al. [2018] developed a BGV-based SHE protocol, called the HighGear protocol, which achieves better performance than the MASCOT protocol.

Application in PPML

Various MPC-based approaches have been designed and implemented for PPML in the past. Most MPC-based PPML approaches leverage a two-phase architecture, comprising of an offline phase and an online phase. The majority of cryptographic operations are conducted in the offline phase, where multiplication triples are generated. The ML model is then trained in the online phase using the multiplication triples generated in the offline phase. The DeepSecure [Rouhani et al., 2017] is a GC-based framework for secure neural network inference, where the inference function has to be represented as a Boolean circuit. The computation and communication cost in GC only depend on the total number of AND gates in the circuit.

SecureML [Mohassel and Zhang, 2017] is another two-party framework for PPML employing two-phase architecture. Parties in federated learning distributes arithmetic shared of their data among two non-colluding servers, who run secure two-party model training protocols. Both Linearly HE (LHE)-based and OT-based protocols are proposed for multiplication triples generation in offline phase. The online phase is based on arithmetic secret sharing and division GC. Therefore, only linear operations are allowed in model training, and various approximations are done to nonlinear functions.

The Chameleon framework is another hybrid MPC framework based on ABY for neural network model inference [Demmler et al., 2015]. Arithmetic secret sharing is used to conduct linear operations, and GC as well as GMW [Goldreich et al., 1987] are used for nonlinear operations. Conversion protocols are also implemented to convert data representations among different protocols.

Privacy-preserving ID3 learning based on OT is addressed in Lindell and Pinkas [2002]. Shamir's threshold secret sharing is used for secure model aggregation for PPML with security against both honest-but-curious and malicious adversaries [Bonawitz et al., 2017], where a

group of clients do MPC to evaluate the average of their private input models, and disclose the average to the parameter server for model update. Recently, MPC-based approaches pursuing security against malicious corrupted majority has been studied. For example, linear regression and logistic regression training and evaluation with SPDZ is studied in Chen et al. [2019]. The authors in Damgård et al. [2019] embraces SPDZ_{2^k} [Cramer et al., 2018] for actively secure private ML against a dishonest majority. It implements decision tree and SVM evaluation algorithms.

2.4.2 HOMOMORPHIC ENCRYPTION

HE is generally considered as an alternative approach to MPC in PPML. HE can also be used to achieve MPC as discussed in Section 2.4.1. The concept of HE was proposed in 1978 by Rivest et al. [1978] as a solution to perform computation over ciphertext without decrypting the ciphertext. Since then, numerous attempts have been made by researchers all over the world to design such homomorphic schemes.

The encryption system proposed by Goldwasser and Micali [1982] was a provably secure encryption scheme that reached a remarkable level of safety. It allows an additive operation over ciphertext, but is able to encrypt only a single bit. Paillier [1999] proposed a provable security encryption system that also allows an additive operation over ciphertext in 1999. It has been widely used in various applications. A few years later, in 2005, Boneh et al. [2005] invented a system of provable security encryption, which allows unlimited number of additive operations and one multiplicative operation. Gentry made a breakthrough in 2009 and proposed the first HE scheme that supports both additive and multiplicative operations for unlimited number of times [Gentry, 2009].

Definition

An HE scheme $\mathcal{H}$ is an encryption scheme that allows certain algebraic operations to be carried out on the encrypted content, by applying an efficient operation to the corresponding ciphertext (without knowing the decryption key). An HE scheme $\mathcal{H}$ consists of a set of four functions:

$$\mathcal{H} = \{KeyGen, Enc, Dec, Eval\}, \tag{2.1}$$

where

- *KeyGen*: Key generation. A cryptographic generator g is taken as the input. For asymmetric HE, a pair of keys $\{pk, sk\} = KeyGen(g)$ are generated, where pk is the public key for encryption of the plaintext and sk is the secret (private) key for decryption of the ciphertext. For symmetric HE, only a secret key $sk = KeyGen(g)$ is generated.
- *Enc*: Encryption. For asymmetric HE, an encryption scheme takes the public key pk and the plaintext m as the input, and generates the ciphertext $c = Enc_{pk}(m)$ as the output. For symmetric HE, an HE scheme takes the secret key sk and the plaintext m, and generates ciphertext $c = Enc_{sk}(m)$.

- *Dec*: Decryption. For both symmetric and asymmetric HE, the secret key sk and the ciphertext c are taken as the input to produce the corresponding plaintext $m = Dec_{sk}(c)$.
- *Eval*: Evaluation. The function *Eval* takes the ciphertext c and the public key pk (for asymmetric HE) as the input, and outputs a ciphertext corresponding to a functioned plaintext.

Let $Enc_{enk}(\cdot)$ denote the encryption function with *enk* as the encryption key. Let $\mathcal{M}$ denote the plaintext space and $\mathcal{C}$ denote the ciphertext space. A secure cryptosystem is called *homomorphic* if it satisfies the following condition:

$$\forall m_1, m_2 \in \mathcal{M}, \quad Enc_{enk}(m_1 \odot_{\mathcal{M}} m_2) \leftarrow Enc_{enk}(m_1) \odot_{\mathcal{C}} Enc_{enk}(m_2) \tag{2.2}$$

for some operators $\odot_{\mathcal{M}}$ in $\mathcal{M}$ and $\odot_{\mathcal{C}}$ in $\mathcal{C}$, where $\leftarrow$ indicates the left-hand side term is equal to or can be directly computed from the right-hand side term without any intermediate decryption. In this book we denote homomorphic encryption operator as $[[\cdot]]$, and we overload the addition and multiplication operators over ciphertexts as follows.

- **Addition:** $Dec_{sk}([[u]] \odot_{\mathcal{C}} [[v]]) = Dec_{sk}([[u + v]])$, where "$\odot_{\mathcal{C}}$" may represent multiplication of the ciphertexts (see, e.g., Paillier [1999]).
- **Scalar multiplication:** $Dec_{sk}([[u]] \odot_{\mathcal{C}} n) = Dec_{sk}([[u \cdot n]])$, where "$\odot_{\mathcal{C}}$" may represent taking the power of n of the ciphertext (see, e.g., Paillier [1999]).

Categorization of HE Schemes

HE schemes can be categorized into three classes: Partially Homomorphic Encryption (PHE), Somewhat Homomorphic Encryption (SHE), and Fully Homomorphic Encryption (FHE). In general, for HE schemes, the computational complexity increases as the functionality grows. Here, we provide a brief introduction to different types of HE schemes. Interested readers can refer to Armknecht et al. [2015] and Acar et al. [2018] for more details regarding different classes of HE schemes.

Partially Homomorphic Encryption (PHE). For PHE, both $(\mathcal{M}, \odot_{\mathcal{M}})$ and $(\mathcal{C}, \odot_{\mathcal{C}})$ are groups. The operator $\odot_{\mathcal{C}}$ can be applied on ciphertexts for a unlimited number of times. PHE is a *group homomorphism* technique. Specifically, if $\odot_{\mathcal{M}}$ is addition operator, the scheme is *additively homomorphic*, and if $\odot_{\mathcal{M}}$ is a multiplication operator, we say that the scheme is *multiplicative homomorphic*. The references Rivest et al. [1978] and ElGamal [1985] represent two typical multiplicative HE schemes. Examples of additive HE schemes can be found in Goldwasser and Micali [1982] and Paillier [1999].

Somewhat Homomorphic Encryption (SHE). An HE scheme is called SHE if some operations (e.g., addition and multiplication) can be applied for only a limited number of times. Some literature also refer to the schemes supporting arbitrary operations while only some limited

circuits (e.g., the branching programs [Ishai and Paskin, 2007], garbled circuit [Yao, 1982]) as SHE. Examples are BV [Brakerski and Vaikuntanathan, 2011], BGN [Boneh et al., 2005], and IP [Ishai and Paskin, 2007]. SHE schemes introduce *noise* for security. Each operation on the ciphertext increases the noise of the output ciphertext, and multiplication is the main technique for increasing noise. When the noise exceeds an upper bound, decryption cannot be conducted correctly. This is the reason why most SHE schemes require a limited number of times of applying the operations.

Fully Homomorphic Encryption (FHE). FHE schemes allow both additive and multiplicative operations with unlimited number of times over ciphertexts. It is worth noticing that *additive* and *multiplicative* operations are the only two operations needed to compute arbitrary functions. Consider $A, B \in \mathbb{F}_2$. The *NAND* gate can be constructed by $1 + A * B$. Thanks to its functional completeness, the NAND gate can be used to construct any gate. Therefore, any functionality can be evaluated by FHE. There are four main families of FHE [Acar et al., 2018]: (1) Ideal Lattice-based FHE (see, e.g., Gentry [2009]); (2) Approximate-GCD based FHE (see, e.g., Dijk et al. [2010]); (3) (R)LWE-based FHE (e.g., Lyubashevsky et al. [2010] and Brakerski et al. [2011]); and (4) NTRU-like FHE (see, e.g., López-Alt et al. [2012]).

The existing FHE schemes are built on SHE, by assuming circular security and implementing an expensive *bootstrap* operation. The bootstrap operation re-encrypts the ciphertexts, by evaluating the decryption and encryption functions over the ciphertexts and the encrypted secret key, in order to reduce the noise of ciphertext for further computation. As a result of the costly bootstrap operation, FHE schemes are very slow and not competitive against general MPC approaches in practice. Researchers are now focusing on finding more efficient SHE schemes that satisfy certain requirements, instead of trying to develop an FHE scheme. In addition, FHE schemes assume circular security (a.k.a. key dependent message (KDM) security), which keeps the secret key secure by encrypting it with the public key. However, no FHE is proven to be semantically secure with respect to any function and is IND-CCA1 secure [Acar et al., 2018].

Application in PPML

Many research efforts based on HE have been devoted to PPML in the past. For example, Hardy et al. [2017] proposed algorithms for privacy-preserving two-party logistic regression for vertically partitioned data. Paillier's scheme is leveraged in secure gradient descent to train the logistic regression model, where constant-multiplication and addition operations are conducted via a mask encrypted by Paillier's scheme and the intermediate data computed by each party. The encrypted masked intermediate results are exchanged between the two parties in the secure gradient descent algorithm. Finally, the encrypted gradient is sent to a coordinator for decryption and model update.

CryptoNets [Gilad-Bachrach et al., 2016] is an HE-based methodology announced by Microsoft Research that allows secure evaluation (inference) of encrypted queries over already

trained neural networks on cloud servers: queries from the clients can be classified securely by the trained neural network model on a cloud server without inferring any information about the query or the result. The CryptoDL [Hesamifard et al., 2017] framework is a leveled HE-based approach for secure neural network inference. In CryptoDL, several activation functions are approximated using low-degree polynomials and mean-pooling is used as a replacement for max-pooling. The GAZELLE [Juvekar et al., 2018] framework is proposed as a scalable and low-latency system for secure neural network inference. In GAZELLE, to conduct secure nonlinear function evaluation in neural network inference, HE and traditional secure two-party computation techniques (such as GC) are combined in an intricate way. The packed additive homomorphic encryption (PAHE) embraced in GAZELLE allows single instruction multiple data (SIMD) arithmetic homomorphic operations over encrypted data.

FedMF [Chai et al., 2019] uses Paillier's HE for secure federated matrix factorization assuming honest-but-curious server and honest clients. Secure federated transfer learning is studied via Paillier's HE scheme in Liu et al. [2019], where the semi-honest third party is into the discard by mixing HE with additive secret sharing in decryption process.

2.4.3 DIFFERENTIAL PRIVACY

DP was originally developed to facilitate secure analysis over sensitive data. With the rise of ML, DP has become an active research field again in the ML community. This is motivated by the fact that many exciting results from DP can be applied to PPML [Dwork et al., 2016, 2006]. The key idea of DP is to confuse the adversaries when they are trying to query individual information from the database so that adversaries cannot distinguish individual-level sensitivity from the query result.

Definition

DP is a privacy definition initially proposed by Dwork et al. [2006], developed in the context of statistical disclosure control. It provides an information-theoretic security guarantee that the outcome of a function to be insensitive to any particular record in the dataset. Therefore, DP can be used to resist the membership inference attack. The (ϵ, δ)-differential privacy is defined as follows.

Definition 2.2 (ϵ, δ)-differential privacy. A randomized mechanism $\mathcal{M}$ preserves (ϵ, δ)-differential privacy if given any two datasets D and D' differing by only one record, and for all $S \subset Range(\mathcal{M})$,

$$\Pr[\mathcal{M}(d) \in S] \leq \Pr[\mathcal{M}(D') \in S] \times e^{\epsilon} + \delta, \tag{2.3}$$

where ϵ is the privacy budget and δ is the failure probability.

The quantity $\ln \frac{\Pr[\mathcal{M}(D) \in S]}{\Pr[\mathcal{M}(D') \in S]}$ is called the *privacy loss*, with $\ln$ denoting natural logarithm operation. When $\delta = 0$, the stronger notion of ϵ-differential privacy is achieved.

DP has utility-privacy trade-offs as it introduces noise to data. Jayaraman and Evans [2019] found out that current mechanisms for differential privacy for ML rarely offer acceptable utility-privacy trade-offs: settings that provide limited accuracy loss provide little effective privacy, and settings that provide strong privacy result in large accuracy loss.

Categorization of DP Schemes

Typically, there are mainly two ways to achieve DP by adding noise to the data. One is the addition of noise according to the sensitivity of a function [Dwork et al., 2006]. The other is choosing noise according to an exponential distribution among discrete values [McSherry and Talwar, 2007].

The sensitivity of a real-valued function expresses the maximum possible change in its value due to the addition or removal of a single sample.

Definition 2.3 Sensitivity. For two datasets D and D' differing by only one record, and a function $\mathcal{M} : \mathcal{D} \rightarrow \mathcal{R}^d$ over an arbitrary domain, the sensitivity of $\mathcal{M}$ is the maximum change in the output of $\mathcal{M}$ over all possible inputs:

$$\Delta\mathcal{M} = \max_{D,D'} \|\mathcal{M}(D) - \mathcal{M}(D')\|, \tag{2.4}$$

where $\|\cdot\|$ is a norm of the vector. The l_1-sensitivity or the l_2-sensitivity is defined when the l_1-norm or l_2-norm is applied, respectively.

We denote the Laplace distribution with parameter b as $Lap(b)$. $Lap(b)$ has a probability density function $P(z|b) = \frac{1}{2b}\exp(-|z|/b)$. Given a function $\mathcal{M}$ with sensitivity $\Delta\mathcal{M}$, the addition of noise drawn from a calibrated Laplace distribution $Lap(\Delta\mathcal{M}/\epsilon)$ maintains ϵ-differential privacy [Dwork et al., 2006].

Theorem 2.4 *Given a function $\mathcal{M} : \mathcal{D} \rightarrow \mathcal{R}^d$ over an arbitrary domain D, for any input X, the function:*

$$\mathcal{M}(X) + Lap\left(\frac{\Delta\mathcal{M}}{\epsilon}\right)^d \tag{2.5}$$

provides ϵ-differential privacy. The ϵ-differential privacy can also be achieved by adding independently generated Laplace noise from distribution $Lap(\Delta\mathcal{M}/\epsilon)$ to each of the d output terms.

The addition of Gaussian or binomial noise, scaled to the l_2-sensitivity of the function, sometimes yields better accuracy, while only ensuring the weaker (ϵ, δ)-differential privacy [Dwork et al., 2006, Dwork and Nissim, 2004].

The *exponential mechanism* [McSherry and Talwar, 2007] is another way to obtain DP. The exponential mechanism is given a quality function q that scores outcomes of a calculation, where higher scores are better. For a given database and ϵ parameter, the quality function induces a

probability distribution over the output domain, from which the exponential mechanism samples the outcome. This probability distribution favors high-scoring outcomes, while ensuring ϵ-differential privacy.

Definition 2.5 Let $q : (\mathcal{D}^n \times \mathcal{R}) \rightarrow \mathbb{R}$ be a quality function, which given a dataset $d \in \mathcal{D}^n$, assigns a score to each outcome $r \in \mathcal{R}$. For any two datasets D and D' differing by only one record, let $S(q) = \max_{r,D,D'} \|q(D,r) - q(D',r)\|_1$. Let $\mathcal{M}$ be a mechanism for choosing an outcome $r \in \mathcal{R}$ given a dataset instance $d \in D^n$. Then, the mechanism $\mathcal{M}$, defined as

$$\mathcal{M}(d,q) = \left\{ \text{return } r \text{ with probability } \propto \exp\left(\frac{\epsilon q(d,r)}{2S(q)}\right) \right\} \tag{2.6}$$

provides ϵ-differential privacy.

The DP algorithms can be categorized according to how and where the perturbation is applied.

1. **Input perturbation**: The noise is added to the training data.
2. **Objective perturbation**: The noise is added to the objective function of the learning algorithms.
3. **Algorithm perturbation**: The noise is added to the intermediate values such as gradients in iterative algorithms.
4. **Output perturbation**: The noise is added to the output parameters after training.

DP still exposes the statistics of a party, which are sensitive in some cases, such as financial data, medical data and other commercial and health applications. Readers who are interested in DP and willing to learn more about it can refer to the tutorial given by Dwork and Roth [2014].

Application in PPML

In federated learning, to enable model training on distributed datasets held by multiple parties, *local differential privacy* (LDP) can be used. With local differential privacy, each input party would perturb their data, then release the obfuscated data to the un-trusted server. The main idea behind local differential privacy is *randomized response* (RR).

Papernot et al. [2016] utilized the teacher ensemble framework to first learn a teacher model ensemble from the distributed datasets among all the parties. Then, the teacher model ensemble is used to make noisy predictions on a public dataset. Finally, the labeled public dataset is used to train a student model. The privacy loss is precisely controlled by the number of public data samples inferred by the teacher ensemble. Generative adversarial network (GAN) is further applied in Papernot et al. [2018] to generate synthetic training data for the training of the student

model. Although this approach is not limited to a single ML algorithm, it requires adequate data quantity at each location.

Moments accountant is proposed for differentially private stochastic gradient descent (SGD), which computes the overall privacy cost in neural networks model training by taking into account the particular noise distribution under consideration [Abadi et al., 2016]. It proves less privacy loss for appropriately chosen settings of the noise scale and the clipping threshold.

The differentially private Long Short Term Memory (LSTM) language model is built with user-level differential privacy guarantees with only a negligible cost in predictive accuracy [McMahan et al., 2017]. Phan et al. [2017] proposed a private convolutional deep belief network (pCDBN) by leveraging the functional mechanism to perturb the energy-based objective functions of traditional convolutional deep belief networks. Generating differentially private datasets using GANs is explored in Triastcyn and Faltings [2018], where a Gaussian noise layer is added to the discriminator of a GAN to make the output and the gradients differentially private with respect to the training data. Finally, the privacy-preserving artificial dataset is synthesized by the generator. In addition to the DP dataset publishing, differentially private model publishing for deep learning is also addressed in Yu et al. [2019], where concentrated DP and a dynamic privacy budget allocator are embraced to improve the model accuracy.

Geyer et al. [2018] studied differentially private federated learning and proposed an algorithm for client-level DP preserving federated optimization. It was shown that DP on a client level is feasible and high model accuracy can be reached when sufficiently many participants are involved in federated learning.