Evaluate Temporal Overlap using Absolute Encounter Intensity

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Model Daily Activity Pattern using Time Inhomogeneous Poisson Process with Periodic Intensity Function

Detection of camera trapping can be viewed as time-to-event data. One common selection to model this type of process is Poisson point process. Two main assumptions for Poisson point process are:

- Independency: Number of events in any disjoint time intervals are independent
- Ordinariness:

$$P(N(t+\delta t) - N(t) = 1) = \lambda(t)\delta t + o(\delta t)$$
$$P(N(t+\delta t) - N(t) > 1) = o(\delta t)$$

From assumption 1, we can derive the waiting time ($T - T_0$, how long did it take from time T_0 to next event)'s distribution to be:

$$P(T - T_0 > t) = exp(\int_{T_0}^{T} \lambda(u)du)$$

Denote $\int_0^t \lambda(u)du = \Lambda(t)$, then $\int_{T_0}^T \lambda(u)du = \Lambda(T) - \Lambda(T_0)$. The $\lambda(t) \geq 0$ is called *intensity function* and $\Lambda(t)$ is called *leading function* of the process. The *intensity function* can be understand as in a short time period, how many events (e.g. detections) should be expect.

From assumption 1 and 2, we derive the likelihood function for time-at-event. Denote time to the *ith* event as T_i, then $X_i = T_i - T_{i-1}$ and $X_0 = T_0$

$$f_{T_1,T_2 \ldots T_n} = exp(-\Lambda(T_n)) \prod_{i=0}^{n} \lambda(T_i)$$

To model daily activity pattern from time-to-event data, we can consider a family of periodic intensity function with period of $P = 1\ day = 86400s$. To construct this family of positive periodic functions, we could use an exponential of Fourier series with amplitude parameters A_i and phase parameters ϕ_i and N terms, i.e.

$$\lambda(t) = exp(\frac{T_0}{2} + \sum_{i=1}^{N} A_i cos(\frac{2\pi it}{P} - \phi_i))$$

Then plug into likelihood function, we can have a MLE for parameters of the Fourier series and also an AIC to help us choose N.

Result from the intensity function

From the intensity function of different species, we can derive different probabilities. Denote $\lambda_i(t)$ as the detection intensity of species i.

Then

$$\int_0^P \lambda(u)du$$

is the detection probability for a day.

Also we have:

$$P(\text{it is spp i}|\text{detect spp i or j at t}) = \frac{\lambda_i(t)}{\lambda_i(t) + \lambda_j(t)}$$

Estimation

Since likelihood is tractable, we can do MLE on amplitude and phase parameters of the Fourier series. AIC can be calculated based on N, the number of terms (smoothness) of the Fourier series using the fact that number of parameters is $2N + 1$.

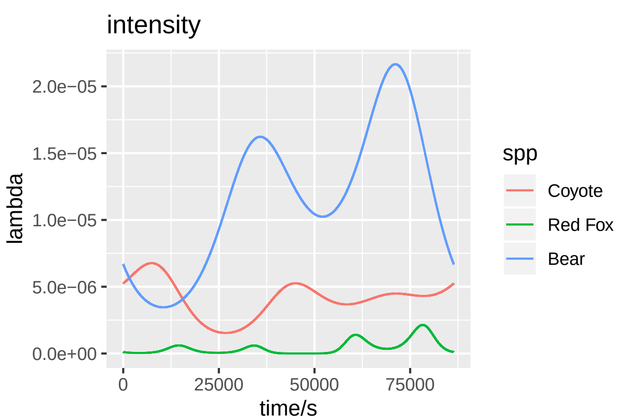
Some results with APIS, Stockton data:

AIC table for number of Fourier terms:

Table 1: Sample AIC table

	Bear_black	Coyote	Fox_red
N=3	9375.146	3642.153	351.5552
N=4	9377.779	3645.593	348.8128

Absolute intensity of fox and coyote:



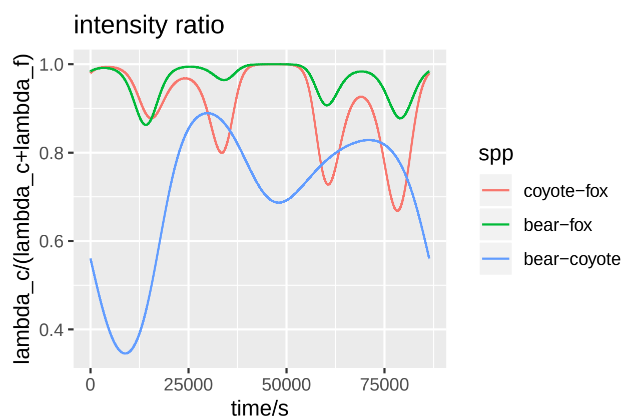


Table 2: Daily Detection Probability

	P
Bear_black	0.6324003
Coyote	0.3001481

	P
Fox_red	0.0374846

For fox, the intensity did not changed so much compare with coyote, and in most of the time, we should expect to see a coyote rather than a red fox.