1 Background

2 Model

2.1 Data Generating Process

In this model, pictures were modeled as a function (light strength) on R^2, particularly in the primary model, pictures were modeled as square with light strength 1 and centered at its known position on unit square, x_i and y_i with a common width d defined as distance from center to one edge. The picture i was a function:

$$f_i(x, y) = \begin{cases} 1 & |x - x_i| \leq d \text{ and } |y - y_i| \leq d \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Perception center was modeled as a Gaussian kernel using 3 parameters, 1) position on x axis, $x_c \in [0, 1]$, 2) position on y axis $y_c \in [0, 1]$ and 3) concentration bandwidth $\gamma > 0$. Note this kernel as $p_\theta(x, y)$ in which $\theta = (x_c, y_c, \gamma)$:

$$p_\theta(x, y) = \frac{1}{2\pi\gamma^2} exp(-\frac{(x - x_c)^2 + (y - y_c)^2}{2\gamma^2}) \tag{2}$$

The expected response of the neuron network Y_i was model as expectation of picture f_i times some amplitude a and add some Gaussian noise:

$$Y_i = a \int f_i(x, y)p_\theta(x, y)dxdy + \epsilon_i \tag{3}$$

in which $\epsilon_i \sim N(0, \sigma^2)$ i.i.d.

Amplitude a corresponds to the expected response when picture is infinitely large. Note that we can scale a and f_i at the same time without changing EY_i, thus we need to specify some convention on f_i, e.g. $\max_{x,y} f_i(x, y) = 1$.

As a summary, there are 5 parameters in this primary model, 1)-2) $(x_c, y_c) \in [0, 1] \times [0, 1]$: the center of perception 3) γ: bandwidth of perception kernel, 4) $a > 0$: amplitude of signal and 5) $\sigma > 0$ standard deviation of noise.

Data has three parts 1)-2) $(x_i, y_i) \in [0, 1] \times [0, 1]$: center of picture 3) $d > 0$ common width of the picture.

2.2 Reconstruction

We take a Bayesian approach. Reconstruction means sample from the posterior of the parameters. (x_c, y_c) had a uniform prior on unit square, γ had an improper uniform prior on $[0, \infty)$, a had an improper uniform prior on $[0, \infty)$, σ had a conjugate prior of inverse Gamma with parameter $\alpha = 1$, $\beta = 1$.

Markov chain Monte Carlo method was used to sample from the posterior, all parameters except σ was sampled using MH algorithm with a normal random walk proposal, while σ was updated using conjugation.

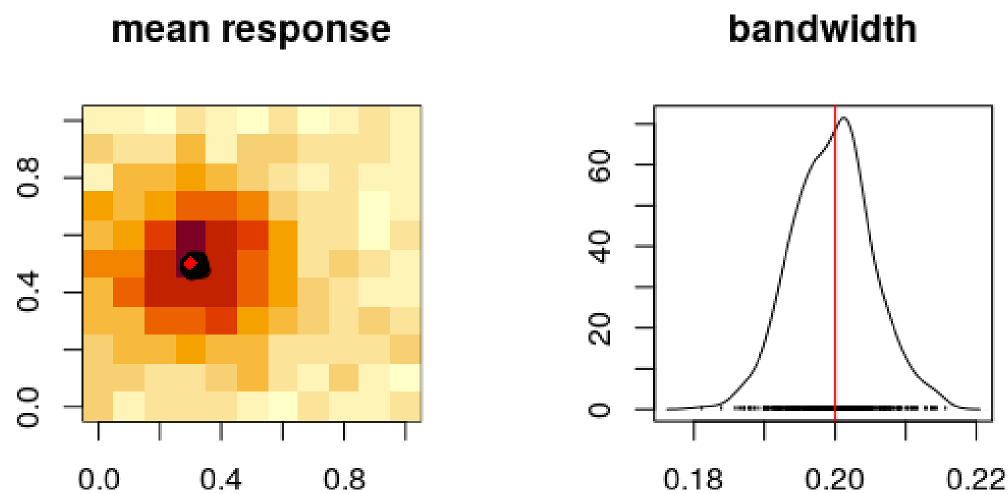

Figure 1: Image width 0.1

3 Primary result

Below simulation was done using $(x_c, y_c) = (0.3, 0.5)$ (red square), bandwidth $\gamma = 0.2$, amplitude $a = 10$, noise standard deviation $\sigma = 0.3$. Picture was put in grids on $0, 1$ depends on the width of pictures. Each experiment had 5 repeats (i.e. pictures were put on each grid 5 times independently).

Black dots showed 500 posterior sample on the center, density plot showed the posterior distribution estimated for bandwidth γ. Background map showed the mean response of 5 repeats.

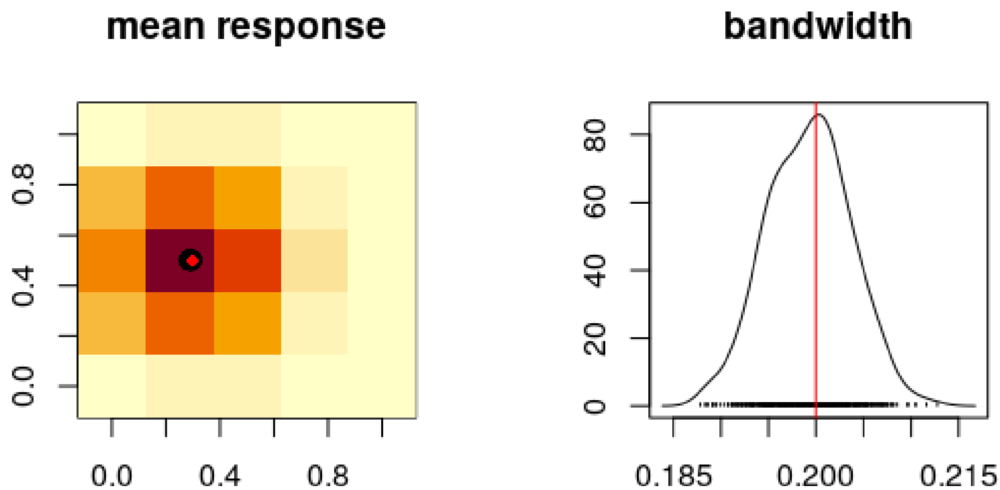

Figure 2: Image width 0.25

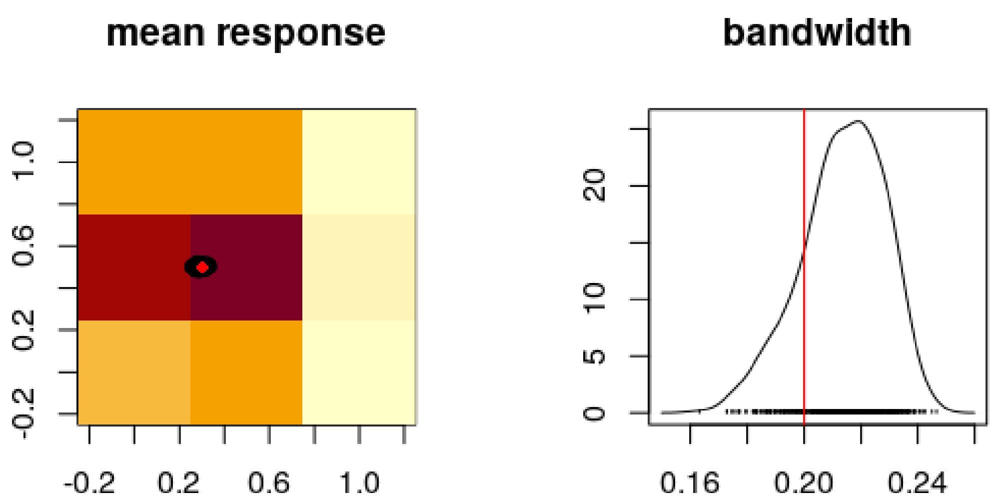

Figure 3: Image width 0.5

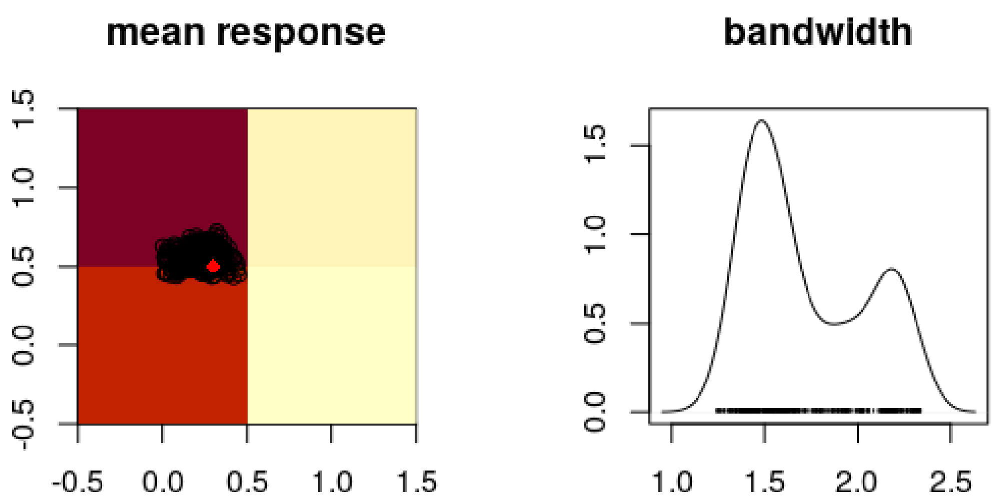

Figure 4: Image width 1