2.11 QR via householder transform

Algorithm: Input: $A \in \mathbb{C}^{m \times n}$ $\quad m \geq n$

for $k = 1, \cdots m$ do

$x = A(k:m, k)$

$v = \frac{x_1}{|x_1|} \|x\|_2 e_1 + x$

$v = \frac{v}{\|v\|}$

$A(k:m, k:n) = A(k:m, k:n) - 2v\left[v^* A(k:m, k:n)\right]$

end for

Explanation: $k=1$. $\quad x = A(1:m, 1) = a_1$

$A(k:m, k:n) = A$

$H = I_m - 2\frac{vv^*}{\|v\|_2^2}$ H is unitary and Hermitian.

By lemma: $Hx = -\frac{x_1}{|x_1|}\|x\|_2 e_1$

$A \leftarrow HA = (Ha_1, Ha_2, \cdots Ha_n)$

$$= \begin{pmatrix} * & * & \cdots & * \\ 0 & * & & \vdots \\ \vdots & \vdots & & \\ 0 & * & & * \end{pmatrix}$$

$Q_1 = H$

$A \leftarrow Q_1 A$

$k=2$. $\quad x = A(2:m, 2)$

$H = I_{(m-1)} - \frac{2vv^*}{\|v\|_2^2}$

$HA(2:m, 2:n)$ $\quad Q_2 = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & & H & \\ 0 & & & \end{pmatrix}$

original

$Q_2 A = Q_2 Q_1 A$

$$= \begin{pmatrix} * & * & * & \cdots & * \\ 0 & * & * & & \vdots \\ \vdots & 0 & * & & \\ 0 & 0 & * & & * \end{pmatrix}$$

$Q_n \cdots Q_1 A = \begin{pmatrix} * & \cdots & * \\ & \ddots & \vdots \\ 0 & & * \end{pmatrix} = R$

p74 ~ p75 operation count $\sim 2mn^2 - \frac{2n^3}{3}$ (cost of algorithm)