長庚大學期中、期末考試答案用紙

科目 機率

____學年度 第____學期____考 復I 系 姓名 王妤霈 學號 B0729025

[1]

(1) $f_X(x) = \binom{10}{x}\left(\frac{1}{10}\right)^x\left(\frac{9}{10}\right)^{10-x}$, $x = 0.1.2.\dots.10$.

x	0	1	2	3	4	5	6	7	8	9	10
$f_X(x)$	$\left(\frac{9}{10}\right)^{10}$	$C_1^{10}\left(\frac{9}{10}\right)^9\left(\frac{1}{10}\right)$	$C_2^{10}\left(\frac{9}{10}\right)^8\left(\frac{1}{10}\right)^2$	$C_3^{10}\left(\frac{9}{10}\right)^7\left(\frac{1}{10}\right)^3$	$C_4^{10}\left(\frac{9}{10}\right)^6\left(\frac{1}{10}\right)^4$	$C_5^{10}\left(\frac{9}{10}\right)^5\left(\frac{1}{10}\right)^5$	$C_6^{10}\left(\frac{9}{10}\right)^4\left(\frac{1}{10}\right)^6$	$C_7^{10}\left(\frac{9}{10}\right)^3\left(\frac{1}{10}\right)^7$	$C_8^{10}\left(\frac{9}{10}\right)^2\left(\frac{1}{10}\right)^8$	$C_9^{10}\left(\frac{9}{10}\right)\left(\frac{1}{10}\right)^9$	$C_{10}^{10}\left(\frac{1}{10}\right)^{10}$

(2) $\mu = np = 10 \times \frac{1}{10} = 1$ #

(3) $\sqrt{\sigma^2} = \sqrt{npq} = \sqrt{\frac{1}{10} \times \frac{9}{10} \times 10} = \sqrt{\frac{9}{10}} = 0.9487$ #

(4) $f_Y(y) = \frac{C_y^{10} C_{10-y}^{90}}{C_{10}^{100}}$, $y = 0.1.2\dots10$.

y	0	1	2	3	4	5	6	7	8	9	10
$f_Y(y)$	$\frac{C_{10}^{90}C_0^{10}}{C_{10}^{100}}$	$\frac{C_9^{90}C_1^{10}}{C_{10}^{100}}$	$\frac{C_8^{90}C_2^{10}}{C_{10}^{100}}$	$\frac{C_7^{90}C_3^{10}}{C_{10}^{100}}$	$\frac{C_6^{90}C_4^{10}}{C_{10}^{100}}$	$\frac{C_5^{90}C_5^{10}}{C_{10}^{100}}$	$\frac{C_4^{90}C_6^{10}}{C_{10}^{100}}$	$\frac{C_3^{90}C_7^{10}}{C_{10}^{100}}$	$\frac{C_2^{90}C_8^{10}}{C_{10}^{100}}$	$\frac{C_1^{90}C_9^{10}}{C_{10}^{100}}$	$\frac{C_0^{90}C_{10}^{10}}{C_{10}^{100}}$

(5) $E(Y) + std[Y] = 1.94868$

(6) $f_Z(z) = b^*(z; 5, \frac{1}{10}) = C_4^{z-1} \times \left(\frac{1}{10}\right)^5\left(\frac{9}{10}\right)^{z-5}$

[2]

(1) $f_W(w) = P(w; 100) = \frac{e^{-100} \times 100^w}{w!}$

(2) $E[W] + std(W) = 100 + \sqrt{100} = 110$ #

(3) $P(|W-100| \leq 2 \cdot 10) = P(80 \leq w \leq 120) = \sum_{w=0}^{120} P(w;100) - \sum_{w=0}^{79}(w;100) = 0.9599$ #

(4) $P(w > 120) = 1 - P(w \leq 120) = 1 - \sum_{w=100}^{120} P(120, 100) = 0.0227$ #

(5) 不能接受，因為100天平均應為100件，而 $w>120$ 經常發生，

代表可接受機率為2%，因此不可接受.

[3]

(1) $P(x = 10 \mid p = 0.05)$.

$\Rightarrow \binom{100}{x}(0.05)^{10}(0.95)^{90} = 0.0282$.

(2) 若約定 $0.0282 < 0.05$ 故視為很小的機率，因此，在 $p=0.05$ 的假設下，$x=10$，$n=100$ 應該不會發生，但證據 $x=10$，$n=100$ 確實發生了，所以假設不為真 #

[4.] $n \to \infty$，$p \to 0$，且 $np = \lambda$.

$$\lim_{n\to\infty} C_x^n p^x (1-p)^{n-x} = \lim_{n\to\infty} \frac{n!}{x!(n-x)!}\left(\frac{\lambda}{n}\right)^x\left(1-\frac{\lambda}{n}\right)^{n-x}$$

$$= \lim_{n\to\infty} \frac{\lambda^x}{x!} \frac{n \times (n-1) \times \dots \times (n-x+1)}{n^x}\left(1-\frac{\lambda}{n}\right)^n\left(1-\frac{\lambda}{n}\right)^{-x}$$

$$\frac{\lambda^x}{x!} e^{-\lambda}$$ #

（請翻面繼續作答）