機率期終討小考，2020/06/12 14:00~15:00

[1]

a. χ^2 分佈

密度函數：$f_Z(x) = e^{-\frac{1}{2}x^2} \frac{1}{\sqrt{2\pi}}$

b. χ^2 分佈

密度函數：$f_Z(x) = e^{-\frac{1}{2}x^2} \frac{1}{\sqrt{2\pi}}$

c. χ^2 分佈 $(V=2)$

密度函數：$f_Z(x) = e^{-\frac{1}{2}x^2} \frac{1}{\sqrt{2\pi}}$

d. t 分佈 $(V=1)$ $T = \frac{Z}{\sqrt{\frac{1}{V}Q_{\chi^2_1}}} = \frac{Z_0}{Z_1}$

密度函數：$h(t) = \frac{\Gamma}{\Gamma(\frac{1}{2})\sqrt{\pi}} (1+t^2)^{-1}$

e. t 分佈 $\Rightarrow T = \frac{Z}{\sqrt{V/U}} = \frac{Z}{\sqrt{\frac{1}{V}(Z_1^2+\cdots+Z_V^2)}} = \frac{Z_0}{\sqrt{(Z_1^2+Z_2^2)/2}}$，$V=2$.

密度函數：$h(t) = \frac{\Gamma(\frac{3}{2})}{\Gamma\sqrt{2\pi}} (1+\frac{t^2}{2})^{-\frac{3}{2}}$

f. t 分佈，$V=3$.

密度函數：$h(t) = \frac{2\Gamma}{\Gamma(\frac{3}{2})\sqrt{3\pi}} (1+\frac{t^2}{3})^{-2}$

[3]

(a) $\frac{\sigma_A^2}{n} = \frac{9}{25}$ $Z = \frac{\tilde{X}_A - 65}{\frac{3}{5}} = \frac{5(\tilde{X}_A - 65)}{3}$

$P(\tilde{X}_A \le 64) = P((\tilde{X}_A - 65)\frac{5}{3} \le (-\frac{5}{3}))$

$= P(Z \le -\frac{5}{3}) \doteq 0.04476$.

(b) $P(\bar{X}_A \le 64) = P(\bar{X}_A - 65)\frac{5}{3} \le (-\frac{5}{3}))$

$= P(Z \le -\frac{5}{3}) \doteq 0.04476$.

(c) $P(\tilde{X}_A \le x_2) - P(\bar{X}_A > x_1) = 0.9$

$= P(\bar{X}_A - 65)\frac{5}{3} \le (x_2 - 65)\frac{5}{3}) - P(\bar{X}_A - 65)\frac{5}{3} > (x_1 - 65)\frac{5}{3})$

$= P(Z \le (x_2 - 65)\frac{5}{3}) - P(Z \le (x_1 - 65)\frac{5}{3})$.