MACM 316 – Computing Assignment 3

Due Date: February 14 at 11:00pm.

Submission Instructions: You must upload one .pdf file in Crowdmark that consists of two pages: page 1 is your report which should fit all discussions, data and figures into a single page; and page 2 is a listing of your code. The deadline is **11:00pm** on the due date. The actual due time is set to 11:05pm and if Crowdmark indicates that you submitted late, you will be assigned a grade of 0 on this assignment. Your TA has emailed you a Crowdmark link that you should save since it will allow you to upload your completed assignments.

- Please review the **Guidelines for Assignments** carefully.
- Acknowledge any collaborations or assistance from colleagues/TAs/instructor.
- If you have any questions about Matlab or aspects of this assignment, then you are strongly encouraged to attend tutorials and drop-in workshops.

Computing Assignment – Modified Newton's Method

In this assignment, you will investigate a variation of Newton's Method called the Modified Newton's Method. You will compare the performance of both methods using the following nonlinear function

$$f(x) = (x-1)^2 e^x$$

which has a double root at $x = 1$ (that is, a root of multiplicity $m = 2$).

(a) Apply the Newton iteration formula to $f(x)$ using the Matlab code `newton.m` (posted on Canvas under Lecture 2a). Use the default convergence tolerance of 10^{-6} and record the iterations starting from an initial guess of $x_0 = 2$.

(b) We know from class that a convergent iteration generates a sequence x_k for $k = 0, 1, 2, \ldots$ that satisfies

$$E_{k+1} \leqslant \alpha E_k^p \qquad \text{where} \quad E_k = |x_k - x^*|,$$

for constants $p > 0$ and α. Then we say that x_k converges to x^* (as $k \to \infty$) with order of convergence p and rate of convergence α. Use your Newton iterations x_k from part (a) and the exact root $x^* = 1$ to plot the absolute errors E_{k+1} versus E_k on a log-log scale. Estimate the convergence order p and rate α from your plot, and clearly explain how you calculated their values.

(c) Modify the `newton.m` code to implement the following iteration

$$x_{k+1} = x_k - \frac{f(x_k) f'(x_k)}{[f'(x_k)]^2 - f(x_k) f''(x_k)}$$

which is known as the Modified Newton's Method. Repeat your calculation from part (a) starting with the same initial point $x_0 = 2$. Again plot your errors on a log-log scale and estimate the order of convergence p. Explain any differences between the two iterations. List several advantages and disadvantages of Modified Newton's Method relative to Newton's Method.

(d) How easy would it be to apply the Bisection Method to the function $f(x)$? Explain.