Problem 1

I assume an initial price at t-1 is 100. Here is my simulation result for three types of price returns:

Classical Brownian Motion:
Expected Price (Mean): 100.00167302366579
Standard Deviation of Price: 0.10123078867662977

Arithmetic Return System:
Expected Price (Mean): 100.1673023665788
Standard Deviation of Price: 10.123078867662972

Log Return or Geometric Brownian Motion:
Expected Price (Mean): 100.68215224722461
Standard Deviation of Price: 10.22349070348781

1. Classical Brownian Motion: This model directly adds the return to the previous price P t-1. Given that Rt has a mean of 0 and a small variance, the expected mean price remains close to P t-1, which is ideal for models assuming price variations due to pure randomness without any trend. The standard deviation is small, reflecting the direct influence of the variance of Rt, indicating minimal price variability which is consistent with the normal distribution's properties.

2. Arithmetic Return System: In this model, the return Rt is scaled by the previous price, effectively treating returns as a percentage change. This scaling translates the zero mean of returns to a slight increase in the expected mean price, likely due to the mathematical behavior of multiplying by 1 + Rt. The variance significantly inflates the standard deviation, showing that percentage changes introduce more pronounced price variability. This model is well-suited for financial markets where returns are considered relative to the asset's previous value.

3. Log Return or Geometric Brownian Motion: This approach models returns as continuously compounded, using the exponential function, which is common in financial mathematics for stock price simulations. The exponential treatment of returns creates a log-normal distribution of prices, which typically skews higher due to the nature of exponential growth. The mean price shows a notable increase over P t-1, and the standard deviation is slightly higher than in the Arithmetic model, reflecting increased volatility due to the exponential growth factor.

Each model provides insights into how different assumptions about the nature of returns impact the expected price and its variability. The Classical Brownian Motion model is straightforward, showing minimal variability around the initial price, making it suitable for scenarios with random, non-directional price changes. The Arithmetic Return System offers a more realistic portrayal for typical financial assets, where returns are often viewed as a percentage change, leading to greater variability. Finally, the Log Return or Geometric

Brownian Motion is ideal for modeling prices over longer periods or under continuous compounding conditions, like stock indices or interest rates, where the exponential nature of returns can significantly influence the trajectory and variability of prices.

Problem 2

Value at Risk (VaR) Estimates:
Normal: -0.03817295454890714
EW Normal: -0.031944547727144924
T distribution: -0.03242585900409048
AR(1): -0.0384353
Historical: -0.028843489039740173

The Value at Risk (VaR) estimates for the `META` stock calculated using different models display a spectrum of potential losses based on distinct statistical assumptions. The Normal Distribution VaR and the Exponentially Weighted Normal Distribution VaR project losses at 3.82% and 3.19%, respectively. These figures show the impact of weighting schemes: the Exponentially Weighted model, which emphasizes recent data, predicts a less severe loss compared to the Normal model that treats all data points equally. This highlights how perceptions of risk can vary significantly depending on whether recent trends or long-term patterns are prioritized.

The MLE Fitted T Distribution VaR and the AR(1) Model VaR also offer insightful contrasts, estimating potential losses at 3.24% and 3.84%, respectively. The T distribution accounts for the fat tails of the distribution, suggesting a higher probability of extreme losses than the Normal distribution, and thus a slightly less severe outlook than the Normal model but more conservative than the EW model. On the other hand, the AR(1) model, which factors in the serial correlation in returns, provides the most conservative estimate. This model assumes that past performance is a strong indicator of future risk, which might be particularly relevant for stocks with high volatility or those in unstable markets.

Finally, the Historical Simulation VaR, which comes in at 2.88%, offers the most optimistic assessment by using actual historical return data to simulate future risk. This method eschews theoretical distributions in favor of empirical evidence, potentially providing a more realistic outlook but possibly underestimating risk if past conditions differ significantly from future volatility. Each VaR model thus serves different strategic needs: statistical models might be preferable for their predictive powers under typical market conditions, while historical models could be favored for their straightforward, data-driven insights into asset behavior under observed conditions.

Problem 3

Two different models were used to calculate the Value at Risk (VaR) for three portfolios, utilizing both an exponentially weighted covariance matrix and a simple covariance matrix which I choose. The VaR estimates from each method differ due to the underlying assumptions and computations specific to each model.

Exponentially weighted covariance matrix method uses a decay factor λ (set to 0.97), which places a higher weight on more recent data points. This approach assumes that more recent observations are more indicative of future risks and market conditions. The results from this model showed higher VaR values for all portfolios (A: $0.0138, B: $0.0152, C: $0.0141, Total: $0.0430). The higher VaR values suggest a more conservative estimation, reflecting a sensitivity to recent market volatilities that may have increased perceived risk levels.

The simple covariance matrix treats all data points equally, irrespective of when they occurred. This model assumes a stable historical volatility and correlation between stock returns, providing a uniform view over the entire dataset. Using this model, the VaR values were lower (A: $0.0119, B: $0.0127, C: $0.0129, Total: $0.0375). These lower values suggest a less conservative approach, indicating that the perceived risk, when averaged out over a long period without giving additional weight to recent data, is lower.

Reason for Choice:
The Simple Covariance Matrix model was chosen for comparison to provide a baseline that evaluates risk under the assumption of uniform data significance across the entire time horizon. This model serves as a contrast to the exponentially weighted model, which might overemphasize recent events, thereby showing the effect of different weighting strategies on risk assessment.

Impact on Results:
The choice of the simple covariance matrix model significantly affected the results, yielding lower VaR estimates compared to the exponentially weighted covariance matrix. This occurs because the simple model dilutes any recent spikes in volatility by averaging them with periods of lower volatility, potentially underestimating the risk if recent trends are more volatile than the historical average. This contrast highlights the potential for underestimation of risk when using simpler models in periods following market turbulence, and conversely, the potential for overestimation when using models that heavily weight recent events during calm periods.