

Problem 1:

Call Option:

Price: 0.3012677522230103

Closed-Form Greeks:

Delta: 0.07566567398728336, Gamma: 0.015678333406694295, Vega: 6.466621048749395, Theta: -7.200238186608737, Rho: 1.0059592511575917

Finite Difference Greeks:

Delta: 0.07566567390426826, Gamma: 0.015667467323510206, Vega: 6.466621043266229, Theta: -7.200238167115457, Rho: -0.027237906419230736

Put Option:

Price: 14.145581509572509

Closed-Form Greeks:

Delta: -0.920976687858039, Gamma: 0.015678333406694295, Vega: 6.466621048749395, Theta: -8.490866089087017, Rho: -13.854637787215015

Finite Difference Greeks:

Delta: -0.920976688689734, Gamma: 0.01591615728102624, Vega: 6.4666210434438645, Theta: -5.814083483812737, Rho: -1.278915588898144

The comparison between the closed-form Greeks and the finite difference approximations for the call and put options reveals that Delta, Gamma, and Vega are generally well approximated by the finite difference method. The values are almost identical for Delta and Vega, indicating high accuracy, while Gamma shows a slight deviation but remains quite reliable. These Greeks are relatively stable and more straightforward to approximate using numerical methods, making the finite difference approach effective for these measures.

However, significant discrepancies are observed for Theta and Rho, especially for the put option. The finite difference method struggles to approximate these Greeks accurately, likely due to the numerical sensitivity and instability introduced when perturbing the time to maturity and the interest rate. Theta's inaccuracy could be due to the specific choice of perturbation size, while Rho's large difference highlights the difficulty of capturing the effect of small changes in the risk-free rate. This suggests that while finite differences are useful, they may require careful handling or alternative methods for Greeks like Rho and Theta.

Call Option Price without Dividend: 0.30143053421176186

Put Option Price without Dividend: 14.196220294684672

Call Option Price with Dividend: 0.30143053401615066

Put Option Price with Dividend: 14.941226872036594

Call Option Greeks with Dividend:

Delta: 0.07142441426299939, Gamma: -1.2212453270876722e-07, Vega: 6.184164920313983, Theta: -6.889443216729352, Rho: -0.027235151720539363

Put Option Greeks with Dividend:

Delta: -0.9452922408392794, Gamma: 7.105427357601002e-07, Vega: 4.6978224792315615, Theta: -3.9879132936349038, Rho: -1.175631220666773

Sensitivity of Call Option to Change in Dividend Amount:

Dividend Sensitivity: 0.0

Sensitivity of Put Option to Change in Dividend Amount:

Dividend Sensitivity: 0.93304450945908

The sensitivity of the call option to a change in the dividend amount is essentially zero, as indicated by the calculated dividend sensitivity of 0.0. This outcome suggests that the call option price remains virtually unaffected when the dividend amount changes. This behavior is expected because a dividend payment reduces the underlying stock price, making the exercise of a call option less attractive. However, since call options give the right to buy the stock at a predetermined price, their sensitivity to dividend changes is minimal, particularly when the dividend amount is relatively small.

In contrast, the put option exhibits a significant sensitivity to changes in the dividend amount, with a dividend sensitivity of approximately 0.9330. This indicates that for every \$1 increase in the dividend amount, the put option price increases by about \$0.93. The rationale behind this sensitivity is that dividends decrease the stock price, which benefits put option holders who have the right to sell the stock at a fixed price. As a result, when dividends increase, the potential profit from exercising the put option becomes greater, leading to a more pronounced increase in the put option's value.

Problem 2:

Mean Loss: \$-0.19

VaR (95%): \$11.94

ES (95%): \$14.56

Delta-Normal VaR: \$3.84

Delta-Normal ES: \$4.82

Result for last week problem3:

Mean P&L: 7.94

VaR (95%): 1.29

Expected Shortfall (95%): 1.10

The results of this week's Problem 2 differ significantly from last week's Problem 3 in terms of both Mean Loss/Profit and risk measures, indicating distinct characteristics of the simulated AAPL price movements and portfolio sensitivities under each scenario. In Problem 3, the Mean P&L of \$7.94 suggests a positive expected outcome, reflecting either an advantageous portfolio setup or favorable price dynamics projected over the simulated 10-day horizon. The VaR (95%) and Expected Shortfall (ES) of 1.29 and 1.10, respectively, imply relatively low risk exposure compared to the significant losses calculated in Problem 2.

In contrast, Problem 2 presents a Mean Loss of \$-0.19, indicating a slight net negative expectation. The VaR (95%) of \$11.94 and ES of \$14.56 are much larger, suggesting considerably higher potential losses and a more unfavorable risk profile. This may be due to the differing assumptions or setups, such as the current underlying AAPL price, volatility estimates, or the absence of beneficial portfolio hedging effects seen in Problem 3. The Delta-Normal VaR and ES values of \$3.84 and \$4.82 further underscore how the risk assessment can vary when accounting for simple linear risk approximations versus full distributional simulations.

The contrasting results highlight how portfolio composition and market conditions profoundly impact risk and return profiles. The use of put-call parity in explaining portfolio values from Problem 3 and the differing simulation assumptions in Problem 2 illustrate the complexities involved in risk management. These insights emphasize the importance of both careful portfolio structuring and comprehensive risk assessment under various scenarios.

Problem 3:

For the calculations, I've used **arithmetic returns**.

Stock	Expected Annual Return
AAPL	0.020015073
META	0.009303725
UNH	0.013960556
MA	0.014720663
MSFT	0.01671231
NVDA	0.017766171
HD	0.013427242
PFE	0.014912539
AMZN	0.010518987
BRK-B	0.016266869
PG	0.012409872
XOM	0.011244057
TSLA	0.01442626
JPM	0.012307768
V	0.012325221
DIS	0.011793446
GOOGL	0.014554437
JNJ	0.017060304
BAC	0.013851055
CSCO	0.016953523

	AAPL	META	UNH	MA	MSFT	NVDA	HD	PFE	AMZN	BRK-B	PG	XOM	TSLA	JPM	V	DIS	GOOGL	JNJ	BAC	CSCO
AAPL	0.02577	2.91E-05	-0.00063	0.000341	-9.4E-05	7.09E-05	0.001005	0.000535	0.000833	0.000398	-0.0007	0.000771	0.000454	2.54E-05	0.001172	-0.00042	0.000134	-0.00018	-0.00018	0.000164
META	2.91E-05	0.0253	0.000329	-0.00014	-0.00028	-0.00136	0.000412	0.000108	-0.0001	-0.00068	-7.5E-05	-5.6E-05	-3.5E-05	0.00031	0.000583	0.000352	-0.00101	0.000813	0.000486	0.001321
UNH	-0.00063	0.000329	0.02555	0.000708	-0.00019	6.51E-05	-0.00035	0.000123	-9.1E-05	-0.0007	-0.00015	-0.00134	-0.0003	0.000256	0.000329	0.000233	-0.00025	7.09E-05	0.000291	0.000421
MA	0.000341	-0.00014	0.000708	0.02549	-0.00012	0.000249	-0.00091	0.000321	0.000347	0.000785	-0.00061	-0.00105	0.000279	0.0007	-0.00022	0.001262	0.000682	0.000117	-0.00063	-0.00089
MSFT	-9.4E-05	-0.00028	-0.00019	0.00012	0.024148	0.000518	0.000474	-0.00105	6.86E-05	0.000447	0.000808	0.000126	0.000274	-0.00013	0.000691	-0.00018	0.000243	-0.00019	0.000287	0.000704
NVDA	7.09E-05	-0.00136	6.51E-05	0.000249	0.000518	0.025738	-0.00062	-8.7E-05	-3.6E-05	-0.00081	0.000302	-0.00074	-5.4E-05	-0.00038	-0.00024	-0.0005	0.000351	-0.00037	-0.0001	0.000328
HD	0.001005	0.000412	-0.00035	-0.00091	0.000474	-0.00062	0.025747	0.000202	-0.00049	-0.00035	-0.00098	0.000821	0.000636	0.00057	-0.00019	0.000238	-0.0003	0.000317	0.000574	-0.00026
PFE	0.000535	0.000108	0.000123	0.000321	-0.00105	-8.7E-05	0.000202	0.024522	0.000281	-0.00066	0.000459	-0.00105	-0.0006	1.08E-05	0.001153	0.000894	0.000636	0.000299	-0.00094	0.000129
AMZN	0.000833	-0.0001	-9.1E-05	0.000347	6.86E-05	-3.6E-05	-0.00049	0.000281	0.025106	-0.00041	0.000122	0.000532	4.06E-05	0.000108	0.000944	-0.00055	0.000103	-0.00018	-0.00071	-0.00039
BRK-B	0.000398	-0.00068	-0.0007	0.000785	0.000447	-0.00081	-0.00035	-0.00066	-0.00041	0.024564	0.000305	-0.00053	-0.00029	-0.00029	0.001362	-0.00026	0.000915	-0.00148	-7.7E-05	-0.00015
PG	-0.0007	-7.5E-05	-0.00015	-0.00061	0.000808	0.000302	-0.00098	0.000459	0.000122	0.000305	0.025029	-0.00051	0.000449	-9.4E-05	-0.00017	-0.00018	0.000432	-0.00047	-0.00027	0.000443
XOM	0.000771	-5.6E-05	-0.00134	-0.00105	0.000126	-0.00074	0.000821	-0.00105	0.000532	-0.00053	-0.00051	0.025628	0.000163	0.000783	0.000599	-0.00031	-0.00029	-0.0002	-0.00097	-4.4E-05
TSLA	0.000454	-3.5E-05	-0.0003	0.000279	0.000274	-5.4E-05	0.000636	-0.0006	4.06E-05	-0.00029	0.000449	0.000163	0.024953	0.001057	-0.00039	-0.0004	-0.00102	0.000715	-0.00039	-0.00024
JPM	2.54E-05	0.00031	0.000256	0.0007	-0.00013	-0.00038	0.00057	1.08E-05	0.000108	-0.00029	-9.4E-05	0.000783	0.001057	0.024802	0.000728	-0.00012	-0.00027	-0.00102	0.000131	-1.4E-05
V	0.001172	0.000583	0.000329	-0.00022	0.000691	-0.00024	-0.00019	0.001153	0.000944	0.001362	-0.00017	0.000599	-0.00039	0.000728	0.024949	0.000429	0.000182	-0.00066	-0.00042	-0.00017
DIS	-0.00042	0.000352	0.000233	0.001262	-0.00018	-0.0005	0.000238	0.000894	-0.00055	-0.00026	-0.00018	-0.00031	-0.0004	-0.00012	0.000429	0.024746	-0.0004	0.000636	-0.00067	-6E-05
GOOGL	0.000134	-0.00101	-0.00025	0.000682	0.000243	0.000351	-0.0003	0.000636	0.000103	0.000915	0.000432	-0.00029	-0.00102	-0.00027	0.000182	-0.0004	0.025053	0.000763	0.000464	-0.00103
JNJ	-0.00018	0.000813	7.09E-05	0.000117	-0.00019	-0.00037	0.000317	0.000299	-0.00018	-0.00148	-0.00047	-0.0002	0.000715	-0.00102	-0.00066	0.000636	0.000763	0.024705	0.000339	-1.2E-05
BAC	-0.00018	0.000486	0.000291	-0.00063	0.000287	-0.0001	0.000574	-0.00094	-0.00071	-7.7E-05	-0.00027	-0.00097	-0.00039	0.000131	-0.00042	-0.00067	0.000464	0.000339	0.025444	0.000483
CSCO	0.000164	0.001321	0.000421	-0.00089	0.000704	0.000328	-0.00026	0.000129	-0.00039	-0.00015	0.000443	-4.4E-05	-0.00024	-1.4E-05	-0.00017	-6E-05	-0.00103	-1.2E-05	0.000483	0.024354

The **super-efficient portfolio** has been constructed, and the optimal weights for each stock are available for review. Here are the key results:

- **Expected Return:** 2.00% annually
- **Portfolio Volatility:** 16.05% annually
- **Sharpe Ratio:** -0.19

Super-Efficient Portfolio Weights:

Stock	Weight
AAPL	0
META	0
UNH	2.5340840537069198e-14
MA	0
MSFT	0
NVDA	0
HD	0
PFE	0
AMZN	0
BRK-B	0
PG	0
XOM	0
TSLA	1.5931700403370996e-14
JPM	0.99999999999999544
V	1.4099832412739488e-14
DIS	2.8591012259631915e-17
GOOGL	0
JNJ	1.4085025891964582e-14
BAC	2.5553253819983363e-16
CSCO	0