Problem 2

VaR (Normal): 0.00047847878001674874

ES (Normal): -3.157020323009356 e-05 ≈ -0.00003

VaR (MLE-fitted T-distribution): -0.07647592805615931

ES (MLE-fitted T-distribution): 2.0976158187056058

VaR (Historical Simulation): -0.07598069069686242

ES (Historical Simulation): -0.11677669788562187

The differences between VaR and ES under the three probabilistic distributions reveal important aspects of risk modeling. Using a normal distribution with exponentially weighted variance results in the lowest VaR and ES values, which suggests that this method underestimates the risk of extreme events. This is because the normal distribution assumes returns are symmetrically distributed with thinner tails, meaning it is less sensitive to large, unexpected losses. The ES, which measures the average loss beyond the VaR threshold, is especially small in the normal distribution, indicating that it assumes losses beyond the VaR level won't increase dramatically.

In contrast, both the MLE-fitted T-distribution and historical simulation show significantly higher VaR and ES values. The T-distribution, which accounts for "fat tails," indicates much higher potential losses, reflecting a more conservative risk estimate. The higher ES suggests that, in the tail of the distribution, extreme losses are expected to be more severe. Historical simulation, which uses actual past returns, also produces higher VaR and ES than the normal distribution, but its results are closer to the T-distribution. This indicates that past data captures extreme events better than a normal distribution, but the T-distribution anticipates even more severe outliers. These differences highlight the importance of choosing the right method for risk modeling, as the normal distribution may underestimate risks in volatile markets, while methods like the T-distribution or historical simulation provide a more realistic view of potential losses.

Problem 3

Week 05 Results (Copula-Based VaR and ES)

- Portfolio A:
  - VaR: 0.0121
  - ES: 0.0166
- Portfolio B:
  - VaR: 0.0100
  - ES: 0.0137
- Portfolio C:
  - VaR: 0.0146
  - ES: -0.0200
- Total VaR: 0.0368
- Total ES: 0.0102

Week 04 Results (Exponential and Simple Covariance)

Exponentially Weighted Covariance (λ = 0.97)

- Portfolio A: VaR: 0.0137
- Portfolio B: VaR: 0.0151
- Portfolio C: VaR: 0.0141
- Total VaR: 0.0430

Simple Covariance Model

- Portfolio A: VaR: 0.0121
- Portfolio B: VaR: 0.0129
- Portfolio C: VaR: 0.0131
- Total VaR: 0.0381

The comparison between the copula-based VaR and ES results from Week 05 and the traditional covariance models from Week 04 reveals key insights about risk modeling. For Portfolio A, the copula-based VaR (0.0121) is almost identical to the simple covariance model, but lower than the exponentially weighted model. This suggests that while the copula captures the portfolio's dependencies, it yields similar risk estimates to simpler models in some cases. For Portfolio B, however, the copula VaR (0.0100) is significantly lower than both traditional approaches, indicating that the copula reduces perceived risk by modeling non-linear relationships between assets, which are missed by traditional covariance approaches.

When looking at the total VaR, the copula-based method (0.0368) produces a noticeably lower overall risk estimate compared to the exponentially weighted covariance (0.0430) and the simple covariance model (0.0381). This implies that the copula approach better captures diversification effects by more accurately modeling the interdependencies between portfolios. By reducing the overall risk, the copula demonstrates its ability to account for complex relationships that traditional covariance models tend to oversimplify. In this context, the copula-based approach provides a more nuanced understanding of portfolio risk, especially when considering multi-asset portfolios.