A binomial pricing method for Snowball autocallable

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ABSTRACT

The paper is going to price snowball autocallable by Binomial Model, which is unconventional as the product is usually priced by Monte Carlo Simulation and Partial Differential Equations (PDE) Method. Using Snowball VWO issued by Barclays Bank as an example, we set up three datasets of the underlying ETF prices and obtain three volatilities. For each dataset, we replicated three portfolios based on the payoffs on three call valuation days and computed the sums of these three portfolios to be the predicted prices. The pricing results shows a strong correlation between volatility and the price of snowball autocallable. However, by comparing these theoretical prices with the price stated in the pricing supplement, we find that the snowball autocallable is overvalued in reality. The results in this paper benefit pricing researchers who attempt to use Binomial Model for pricing snowball derivatives and investors who are interested in snowball autocallable.

Keywords-Snowball Autocallable; Option pricing; Binomial Model

1. INTRODUCTION

Pricing analysis has always been one of the hot issues in the field of financial derivatives since a reasonable price is a prerequisite for transactions in the financial market. As is elaborated in the Efficient Markets Hypothesis (EMH) [1]., the price of an asset reflects its real value, and there ought to be no arbitrage opportunities. Therefore, how to price derivatives reasonably and effectively is of great significance, which plays an important role in ensuring the orderly operation of the financial market.

In recent years, the growth of global economy has been slowing down and the financial market has been experiencing changes and fluctuations. With the decline of interest rates and the occurrence of a series of credit risk events, traditional fixed income products are losing their appeal, while a variety of new over-the-counter derivatives, including snowball autocallable, are enjoying great fondness by investors and securities companies, due to their high possibilities of gaining profits especially in volatile or low-yield markets and their function of hedging against a fall in the underlying asset price effectively.

However, for snowball autocallable have just sprung up in recent decades, the theoretical and empirical study of pricing them have not been complete and mature, which arouses our interest in investigating their pricing mechanism.

At present, the most well-known and far-reaching pricing models are Black-Scholes Model (B-S Model) [2] proposed by Black and Scholes in 1973 and Binomial Model proposed by Cox, Ross and Rubinstein in 1979 [3], neither of which can be directly applied to snowball autocallable due to its unique and complicated structure, unfortunately. As a consequence, academics have been examining and exploring different pricing method based on diverse variants of snowball autocallable. Lederman valued an multi underlying autocallable product using the Least Squares Monte Carlo approach [4]. Snowball structured products can also be linked to assets other than stocks. Shim examined a Korean snowball currency option and derived a pricing formula utilizing integration, providing a more effortless method to hedge compared to the Monte Carlo method [5]. Empirically speaking, the most commonly used pricing methods for snowball structured products are B-S Model and partial differential equations (PDE) method. Pan derived a pricing formula using the Black Scholes partial differentiation equation through listing out all payoff situations for the double barrier option [6]. However, PDE method is effective only when the product is linked to one single underlying asset [7]. Zhu priced a snowball product using Monte Carlo Simulation and compared performance of the B-S model and variance gamma model, the latter of which produces a better fit for the snowball product [8]. New pricing formulas were devised to increase the accuracy and simplify the model at the same time. Tong presented a recursive pricing method based on multivariate subordinate Orstein-Uhlenbeck

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processes for autocallable [9]. Nevertheless, the proposed model needs to adjust for more complicated structures. Huang and Guo also proposed a quadrature-based method based on B-S model for exotic options with early-exercised features [10].

Nonetheless, while many intricate models proposed have difficulty to implement and comprehend, simpler and more cost-effective models, such as Binomial Model, are favored by most financial institutions [10]. The utilization of Binomial Model, one of the fundamental models for option pricing, has been examined in depth in previous literatures. For example, Song, Wang & Sun suggested that the theoretical value of a SSE500 ETF option generated from the Binomial Model produces a smaller deviation compared to that from the B-S Model [11]. Previous research has also shown the applicableness of Binomial Model in pricing autocallable. Zhu analyzed the Binomial and Trinomial Model for autocallable by separating each branch based on the knock-in events and found that the deeper the tree is, the closer the theoretical value is to the real value [8].

Binomial Model is widely used and welcomed because it functions as an effective means to price various options with diverse structures in a comparatively accurate (by increasing the depth of the tree), comprehensible and easily computable way. Nevertheless, there are few existing literatures employing the Binomial Model by the means of replicating payoffs to price snowball autocallable.

Therefore, to provide instructive experience and suggestions for pricing snowball autocallable, the paper prices Snowball Autocall VWO issued by Barclays Bank PLC in 2015 by employing the Binomial Model through the replicating portfolio in all payoff situations. With empirical data, the paper then compares the difference between theoretical value and the real value of snowball autocallable and accordingly provides a thorough analysis on the reason behind the overpricing phenomenon of snowball structured products.

2. DATA

Due to the path-dependent nature of the snowball autocallable product, the paper intentionally chooses to price Snowball Autocall VWO that started in 2015 and matured in 2018 to guarantee the time series data are sufficient for the operation of Binomial Model. By pricing such a liquidated product, the essay can better focus on the accuracy and precision of the Binomial Model.

In order to price Snowball Autocall VWO, data and information that are relevant to the pricing process need gathering. The paragraphs below demonstrate the information of the priced product, the source of and the reason for the selected datasets, and the used risk-free rates for the pricing.

The paper focuses on an annual Snowball Autocall VWO note issued by Barclays Bank PLC in 2015 and due on June 1, 2018, with three call valuation days during the period. The initial price, call price and barrier price are the main parameters for the snowball autocallable product, because the Automatic Call function (explained in TABLE 1.), the main feature of the product, is fully based on the end price of the product on each call valuation day. The estimated value of note on the Initial Valuation Day is $959.9 per note. TABLE 1. lists out the main parameters and relevant clauses of the Snowball Autocall VWO according to the pricing supplement [12].

Table 1. Clauses for Snowball Autocall VWO [12]

Name of the product	**Snowball Autocall VWO**
Denominations	\$1,000 and above (integral multiple of \$1,000)
Value per note	\$959.9
Initial Valuation Date	May 29, 2015
Call Valuation Dates	*Number* — *Call Valuation Dates* 1 — June 6, 2016 2 — May 30, 2017 3 — June 1, 2018
Final Valuation Date	May 29, 2018
Reference Asset	Vanguard FTSE Emerging Markets ETF (Bloomberg Ticker Symbol: VWO UP <Equity>)
Initial Price (S_0)	\$42.34
Barrier Price (B)	$S_0 \times 80\%$=\$33.87
Call Price (K)	$S_0 \times 100\%$=\$42.34
ETF Price on Call Valuation Date (number i)	$S_i\ (i = 1,2,3)$
Final Price	S_3
Principle	Pr
Payoff	Pa
Knock-out situation	$S_i \gg K\ (i = 1,2,3)$
Knock-in situation	$S_i < K\ (i = 1,2,3)$
Automatic Call	Any knock-out situation happens $i.e.\quad \exists i \in \{1,2,3\}\ s.t.\ S_i \gg K$
No-knock end	No knock-out situation happens on the 3 days and no knock-in situation happens on the Final Valuation Day.
Knock-in end	No knock-out situation happens on the 3 days and knock-in situation happens on the Final Valuation Day[a].

a. For detailed payoff situation. please refer to Table 2: Payoff situations for the autocallable notes based on valuation dates.

The payoffs in various cases are elaborated in TABLE 2. , according to the Snowball Autocall VWO pricing supplement [12].

Table 2. Payoff Situations of Snowball Autocall VWO on Call Valuation Days [12]

Cases of $S_i(i = 1,2,3)$	**Call Valuation Day 1**	**Call Valuation Day 2**	**Call Valuation Day 3**
Automatic Call: $S_1 \geq K$	Product matures with $Pa = 110\% \times Pr$	------	------
Automatic Call: $S_1 < K$ $S_2 \geq K$	0 (wait until Call Valuation Day 2)	Product matures with $Pa = 120\% \times Pr$	------
Automatic Call: $S_1 < K$ $S_2 < K$ $S_3 \geq K$	0 (wait until Call Valuation Day 2)	0 (wait until Call Valuation Day 3)	Product matures with $Pa = 130\% \times Pr$
No-knock end $S_1 < K$ $S_2 < K$ $B \leq S_3 < K$	0 (wait until Call Valuation Day 2)	0 (wait until Call Valuation Day 3)	$Pa = 0$
Knock-in end $S_1 < K$ $S_2 < K$ $S_3 < B$	0 (wait until Call Valuation Day 2)	0 (wait until Call Valuation Day 3)	$Pa = Pr \times (1 - \frac{S_0 - S_3}{S_0})$

We gathered the daily Closing Prices for Vanguard FTSE Emerging Markets ETF (the reference asset of Snowball Autocall VWO) from the Yahoo Finance Website [13] and then separated them into 3 datasets for further calculation, as listed below.

Dataset 1 comprises daily Closing Prices of ETF from June 3, 2015 to May 31, 2018, with the same time span, 3 years, as that of the Snowball Autocall VWO. Based on the volatility of the actual stock price, the calculated price P_1 (solved in TABLE 8.) will be more accurate to the real payoff of the product, which leads to a more rational pricing result according to the non-arbitrage pricing principle.

Dataset 2 comprises daily Closing Prices of ETF from January 3, 2013 to April 29, 2015. They are the actual historical data that Barclays had when pricing for the Snowball Autocall VWO before its issuance. Investors could also access them in the product pricing supplement to have an understanding and judgement about the overall trend of the reference asset. Dataset 2 contributes the historical volatility to the calculated price P_2 (solved in TABLE 8.).

Dataset 3 comprises the daily Closing Prices of ETF from January 2, 2014 to December 31, 2014. As volatility closely positively correlates with the length of time span [14], we aim to lower the volatility by shortening the time span in order to enhance the accuracy of pricing. While the previous two datasets have a time span of 3 years, Dataset 3 only uses the ETF data from the year 2014 to rule out factors such as inflation. Using volatility from Dataset 3, we get the calculated price P_3 (solved in TABLE 8.).

Risk-free interest rates serve for calculating the opportunity cost of the investment and the net present value (NPV). They are collected from the Duff & Phelps recommended risk free rates, which are based on the normalized 20-year U.S. treasury yield [15]. Pricing calculations employ the average risk-free rates, which is shown in TABLE 3.

Table 3. Duff & Phelps recommended risk free rates [15]

Time span (mm/dd/yyyy)	Average Risk-free rate r(%)
1/31/2015--12/31/2016	$r_1 = 4.00$
1/31/2015--12/31/2017	$r_2 = 3.83$
1/31/2015--12/30/2018	$r_3 = 3.75$

3. METHODS

First and foremost, we work out the natural logarithm of ETF Closed Price data from Dataset 1,2 and 3 and then figure out their daily difference, as is shown below:

$$Y(i) = LN(S_{i+1}) - LN(S_i) \tag{1}$$

Then, we can figure out the sample standard deviation s using $\bar{Y}$ (the mean of the dataset) and n (the number of days in the dataset), and the annual standard deviation σ, which is also called volatility:

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2} \tag{2}$$

$$\sigma = s \times \sqrt{252} \tag{3}$$

Noticeably, the number 252 is the number of trading days in a year. So far, we get the respective volatility of each Dataset, as is shown in TABLE 4.

Next, we employ the Binomial Model to predict the future stock price, thereby pricing the Snowball Autocall VWO product.

Binomial Model, a method extensively used in the field of option pricing, focuses on the prices of the reference asset at certain discrete time between the initial date to the maturity date of the option. The time span in years is divided evenly into intervals, each of which is Δt in length. In every interval, reference asset price S_0 can either go up to $S_u = S_0 \times u$ (u is the up factor, $u > 1$) at probability p or go down to $S_d = S_0 \times d$ (d is the down factor, $d < 1$) at probability $1 - p$.

$$S_u = S_0 \times u \tag{4}$$

$$S_d = S_0 \times d \tag{5}$$

Option Prices P_0, P_u and P_d are respectively equal to the payoffs brought by reference asset price S_0, S_u and S_d. Given that volatility σ of the asset and continuous risk-free interest rate r are fixed in the time span, the formulas of u, d and the probability p (based on the non-arbitrage pricing principle) are displayed as (6), (7) and (8), where e is the Euler's number.

Table 4. Time span and volatility σ in the 3 Datasets

Datasets	**Time span (mm/dd/yyyy)**	**Volatility σ**
Dataset 1	6/3/2015--5/31/2018	0.1908
Dataset 2	1/3/2013--4/29/2015	0.1760
Dataset 3	1/2/2014--12/31/2014	0.1611

$$u = e^{\sigma\sqrt{\Delta t}} \tag{6}$$

$$d = e^{-\sigma\sqrt{\Delta t}} \tag{7}$$

$$p = \frac{e^{r\sqrt{\Delta t}} - d}{u - d} \tag{8}$$

Hence, option price P_0 ought to be the present value of the expectation of its future price. In mathematical language,

$$P_0 = e^{-r\Delta t}[P_u p + P_d(1-p)] \tag{9}$$

Since there are many intervals within the time span, the Binomial Model is composed of many layers, as is shown in Fig 1. The option price at the front node can be worked out using prices at its back nodes, using (9). Therefore, by iterating option prices from back to the front (root point) over and over again, we can get the calculated option price P at the initial date, which is used in trading the option according to the Binomial Model.

Given all the knowledge and methods above, we return to the problem of using Binomial Model to price VWO Snowball.

By referring to the 3 Datasets, we can obtain the ETF price respectively on the initial valuation day, 1st call valuation day and 2nd call valuation day, as is shown in TABLE 5.

Then we can accordingly figure out the predicted up price S_{ui} and down price S_{di} by plugging S_{i-1} $(i = 1,2,3)$ into (4) and (5), as is elaborated in TABLE 6. This is because, for example, when calculating the two possible values S_u and S_d on the 2nd call valuation day, the daily Closing Price on the 1st call valuation date should be used as the initial stock price.

Table 5. ETF price on the 3 valuation days

Day	**ETF price ($)**
Initial Valuation Day	$S_0 = 42.34$
1st Call Valuation Day	$S_1 = 34.86$
2nd Call Valuation Day	$S_2 = 41.04$

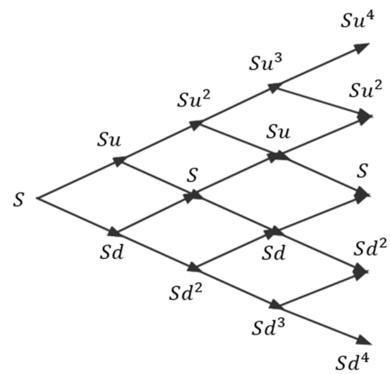

Figure 1. Schematic diagram of Binomial Model

Table 6. Mathematical expressions of S_u and S_d on the 3 call valuation days

Call Valuation Days	**Up(S_u)**	**Down(S_d)**
1st Call Valuation Day	$S_{u1} = S_0 e^{\sigma\sqrt{\frac{254}{365}}}$	$S_{d1} = S_0 e^{-\sigma\sqrt{\frac{254}{365}}}$
2nd Call Valuation Day	$S_{u2} = S_1 e^{\sigma\sqrt{\frac{247}{365}}}$	$S_{d2} = S_1 e^{-\sigma\sqrt{\frac{247}{365}}}$
3rd Call Valuation Day	$S_{u3} = S_2 e^{\sigma\sqrt{\frac{247}{365}}}$	$S_{d3} = S_2 e^{-\sigma\sqrt{\frac{247}{365}}}$

Table 7. predicted up price and down price through each Dataset

Datasets	**σ**	**S_{u1}(\$)**	**S_{d1}(\$)**	**S_{u2}(\$)**	**S_{d2}(\$)**	**S_{u3}(\$)**	**S_{d3}(\$)**
Dataset 1	0.1908	49.64	36.11	40.78	29.79	48.11	35.01
Dataset 2	0.1760	49.04	36.56	40.29	30.16	47.52	35.45
Dataset 3	0.1611	48.43	37.01	39.80	28.86	42.91	35.89

Therefore, we can work out predicted up price S_{ui} and down price S_{di} through each Dataset, by using σ in TABLE 4. and formulas in TABLE 6. The results are shown below in TABLE 7.

We construct the replicating portfolio in which holding the VWO Snowball make the same profits as depositing α_i amount of money into bank and simultaneously investing in δ_i shares of stock. Both α_i and δ_i can be integer or fraction, as the replicating portfolio assumes it is feasible to purchase shares of stocks in fractions. Since the Snowball may be called or not on three call valuation days, we construct 3 replicating portfolios respectively on each day, considering every payoff situation. For the sake of simplicity, the replicating portfolio is based on the payoff of minimal denomination, which is \$1,000.

Let r be the risk-free interest rate, and we illustrate the 3 replicating portfolios in order, as is displayed below.

For the 1st Call Valuation Day, according to TABLE 1. and TABLE 7. , for all the 3 Datasets, there exists a relationship:

$$S_{d1} < K \ll S_{u1} \ ,$$

which means if the ETF price mounts to S_{u1}, VWO Snowball will knock out and bring \$100 as profits, while if the ETF price decreases to S_{d1}, VWO Snowball will still be held with 0 paid. In addition, since investor only holds the portfolio for one year, the risk-free interest rate would be r_1 (displayed in TABLE 3.). In mathematical language,

$$\begin{cases} (1+r_1)\alpha_1 + S_{u1}\delta_1 = 100 \\ (1+r_1)\alpha_1 + S_{d1}\delta_1 = 0 \end{cases} \quad (10)$$

For the 2nd Call Valuation Day, noticeably, the fact that the product didn't knock out on the 1st Call Valuation Day indicates that deposit time in bank should also be 2 years. Therefore, the risk-free interest rate would be r_2 (displayed in TABLE 3.). According to TABLE 1. and TABLE 6. , for all the 3 Datasets, there exists a relationship:

$$S_{d2} < S_{u2} < K \ ,$$

which means VWO Snowball will still be held with 0 paid. In mathematical language,

$$\begin{cases}(1+r_2)^2\alpha_2 + S_{u2}\delta_2 = 0 \\ (1+r_2)^2\alpha_2 + S_{d2}\delta_2 = 0\end{cases} \tag{11}$$

For the 3rd Call Valuation Day, similarly, deposit time in bank should be 3 years. Therefore, the risk-free interest rate would be r_3 (displayed in TABLE 3.). According to TABLE 1. and TABLE 6. , for all the 3 Datasets, there exists a relationship:

$$B < S_{d3} < K \ll S_{u3} \ ,$$

which means if the ETF price mounts to S_{u3}, VWO Snowball will knock out and bring \$300 as profits, while if the ETF price decreases to S_{d1}, VWO Snowball will not knock in and therefore bring 0 profit.

$$\begin{cases}(1+r_3)^3\alpha_3 + S_{u3}\delta_3 = 300 \\ (1+r_3)^3\alpha_3 + S_{d3}\delta_3 = 0\end{cases} \tag{12}$$

Provided the value of α_i and δ_i, the total price of the Snowball VWO can be solved by adding up the present value of the 3 replicating portfolios. In mathematical language,

$$P = \sum_{i=1}^{3} \alpha_i + S_{i-1}\delta_i \tag{13}$$

4. RESULTS AND DISCUSSION

4.1. Results

According to formulas and procedures in Section *B*, we can finally work out the calculated price of Snowball VWO. The system of equations is solved by using the matrix function in excel. P_1, P_2, P_3 are respectively figured out from Dataset 1, Dataset 2 and Dataset 3, as is shown in TABLE 8.

Table 8. Pricing Results for Snowball VWO

Part Present Value (\$)	1st Call Valuation Day	2nd Call Valuation Day	3rd Call Valuation Day	Calculated Price of the product (\$)
Dataset 1	48.07	0	172.2493	P_1=220.31
Dataset 2	56.92	0	231.1263	P_2=288.05
Dataset 3	58.36	0	241.8892	P_3=300.25

In addition, from Dataset 1 to Dataset 3, we base the selection of the dataset on simulating the actual situation the issuer (Barclays Bank PLC) facing when pricing the Snowball Autocall VWO, particularly in terms of the range of time and calculated volatility. Therefore, we successfully obtain higher calculated price, which is closer to the real price.

Moreover, by using different volatility computed from different period, we find that the higher the volatility is, the lower the price of Snowball VWO is. This correlation results from the fact that higher volatility implies higher risk, which leads to more uncertainty and suggests a comparatively low payoff by Snowball VWO.

4.2. Discussion

While Binomial Model is welcomed for its clearness and simplicity in pricing, there are by now few literatures linking snowball autocallable to it. The paper creatively employs the Binomial Model to price the snowball autocallable, a relatively new product and a big hit in the financial market, and generates the according pricing results in TABLE 8.

While the real price set by Barclays Bank PLC is \$959.90, the calculated prices by this paper are around \$300. This may results from the difference between Barclays's internal pricing models and the Binomial Model that we used. For better numerical approximation, we could go on to take other models such as Monte Carlo method. Furthermore, the Binomial Model utilizes the risk-free interest rate and a constant volatility, while the two parameters keep changing through time in reality. Finally, other literatures also found that snowball autocallable are sometimes overpriced in reality [16], as in general pricing models in market fail to take commissions and credit risk of the issuer into account when pricing.

In addition, the replicated portfolio equation in the paper is highly dependent on the value of volatility, as it determines the value of S_u and S_d on each call valuation day, which in turn determines the respective payoff. Nevertheless, comparatively insufficient and constant volatilities are used in this model. Therefore, for pricing future snowball autocallable, methods such as autoregressive integrated moving average model or recursion method need to be employed to predict the future value of the stock.

The results produced in this paper suggests that pricing snowball product is indeed a work of art and science. Thus, the financial markets should be cautious about the pricing and the supervision of such a structured financial product.

5. CONCLUSION

Taking the Snowball VWO issued by Barclays Bank as an example, the paper utilized the Binomial Model to price a snowball autocallable, which is often priced using Monte Carlo Simulation and PDE method. Due to the path-dependent nature of the snowball autocallable, the employment of the Binomial Model explicitly presents the respective payoffs on different valuation days. We set up three datasets of the underlying ETF prices and obtain three volatilities. For each dataset, we then replicate three portfolios based on the three call valuation days of the priced product Snowball VWO and calculate the sum of three portfolios to price the Snowball VWO. The predicted prices are \$220.31, \$288.05, and \$300.25, displaying a strong correlation between volatility and the price of snowball autocallable--higher volatility leads to lower pricing results. Besides, by comparing the real price and the three pricing results, we find that the product is overvalued in reality, which may be attributed to the discrepancy between models used by the paper and the issuer, the use of constant volatility, and the assumption of disregarding other factors such as commissions and credit risk of the issuer. The results may benefit the pricing researchers who attempt to use the Binomial Model for pricing snowball derivatives and investors who are interested in snowball autocallable.

REFERENCES

[1] Fama, E. F. (1995). Random walks in stock market prices. Financial analysts journal, 51(1), 75-80.

[2] Black, F., & Scholes, M. (2019). The pricing of options and corporate liabilities. In World Scientific Reference on Contingent Claims Analysis in Corporate Finance: Volume 1: Foundations of CCA and Equity Valuation, 3-21. doi: 10.1142/9857

[3] Cox, J. C., Ross, S. A., & Rubinstein, M. (1979). Option pricing: A simplified approach. Journal of financial Economics, 7(3), 229-263.

[4] Lederman, R. (2017). The valuation and hedging of multi-underlying autocallable structured products (master's thesis). Université Paris-Dauphine. Retrieved on November 18, 2021 from https://raphaellederman.github.io/assets/images/autocallable.pdf.

[5] Shim, G. (2011). A Snowball Currency Option . Journal of the Korean Society for Industrial and Applied Mathematics, 15(1), 31–41. doi: 10.12941/jksiam.2011.15.1.031

[6] Pan, Q. (2019). Pricing research on automatic knock-out option structure linked to one underlying asset. Modern trade and Industry, (34),108-109. doi: 10.19311/j.cnki.1672-3198.2019.34.052.

[7] Hansson, F. (2012). A pricing and performance study on auto-callable structured products. Retrieved on December 24, 2021 from https://www.diva-portal.org/smash/get/diva2:555900/FULLTEXT01.pdf

[8] Zhu, Y. (2020). Income receipts of pricing strategy studies (master's thesis). University of Nanjing. doi: 10.27235/d.cnki.gnjiu.2020.000467

[9] Tong, K. Z. (2019). A recursive pricing method for autocallable under multivariate subordination. Quantitative Finance and Economics, 3(3), 440–455. doi: 10.3934/qfe.2019.3.440

[10] Huang, M., & Luo, G. (2019). A simple and efficient numerical method for pricing discretely monitored early-exercise options. Cornell University. Retrieved on November 18, 2021, from https://arxiv.org/pdf/1905.13407.pdf.

[11] Song, H., Wang, H. & Sun, Y. (2015). Study on SSE50ETF option pricing method. Securities Association of China. (eds.) Innovation and Development: Proceedings of China securities Industry 2015.1029-1034.

[12] Barclays. (2015). Pricing supplement - snowball Autocall Vwo. SEC.gov. Retrieved on December 24, 2021 from https://www.sec.gov/Archives/edgar/data/312070/000119312515210341/d936678d424b2.htm.
[13] Yahoo Finance. (2021). Vanguard Ftse Emerging Markets (VWO) stock historical prices & data. Retrieved on October 31, 2021 from https://finance.yahoo.com/quote/VWO/history?period1=1356998400&period2=1430352000&interval=1d&filter=history&frequency=1d&includeAdjustedClose=true.
[14] Chen, J. (2021, October 6). What is time-varying volatility? Investopedia. Retrieved on December 4, 2021 from https://www.investopedia.com/terms/t/time-varying-volatility.asp.
[15] Duff & Phelps. (2021). Table: Equity Risk Premium & Risk-free Rates. Duff & Phelps. Retrieved on October 31, 2021 from https://www.duffandphelps.com/-/media/assets/pdfs/publications/articles/dp-erp-rf-table-2021.pdf.
[16] Albuquerque, R., Gaspar, R. M., & Michel, A. (2015). Investment analysis of autocallable contingent income securities. Financial Analysts Journal, 71(3), 61–83. doi: 10.2469/faj.v71.n3.4