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Fundamental Algorithms Problem Set 7

Q1 LCS of 10010101 and 010110110

Solution: $X_8 = 1\ 0\ 0\ 1\ 0\ 1\ 0\ 1$

$Y_9 = 0\ 1\ 0\ 1\ 1\ 0\ 1\ 1\ 0$

$\because X_1 \neq Y_1$

$\therefore$ Analyze $X_1 = Y_2$ and $X_2 = Y_1$

① $X_1 = Y_2$

X: 1 | 0 0 1 0 1 0 1 ⇒ a) 0 1 | 0 1 0 1 ⇒ max: 0110

Y: 1 | 0 1 1 0 1 1 0   0 1 | 1 0

b). 1 | 0 1 0 1 ⇒ max: 1101

1 | 1 0 1 1 0   1010

1011

Thus, common string set under ① is:

100110

101101

101010

101011

② $X_2 = Y_1$

X: 0 | 0 1 0 1 0 1 ⇒ a) 0 1 | 0 1 0 1 ⇒ max: 01010

Y: 0 | 1 0 1 1 0 1 1 0   0 1 | 1 0 1 1 0   01011

01101

b) 1 | 0 1 | 0 1

1 | 0 1 | 1 0 1 1 0 ⇒ max: 10101

Thus, common string set under ② is:

001010

001011

001101

010101

⇒ Because the longest length of common strings is 6 Thus, LCS of 10010101 and 010110110 is:

100110	001010
101101	001011
101010	001101
101011	010101

Q2. Write all parenthesizations of ABCDE

Solution: Splitting 1-4 gives $P(1)P(4) = 5$ parenthesizations

A(B(C(DE))), A(B((CD)E)), A((BC)(DE)),

A((B(CD))E), A(((BC)D)E)

Splitting 2-3 gives $P(2)P(3) = 2$ parenthesizations

(AB)((CD)E), (AB)(C(DE))

Splitting 3-2 gives $P(3)P(2) = 2$ parenthesizations

((AB)C)(DE), (A(BC))(DE)

Splitting 4-1 gives $P(4)P(1) = 5$ parenthesizations

(A(B(CD)))E, (A((BC)D))E, ((AB)(CD))E,

(((AB)C)D)E, ((A(BC))D)E

Q3

Solution: a) $x_1, \dots x_j x_i \dots x_m$

$1 \le i \le n$

$1 \le j < i$, $j = i-1$

```
for i : 1 to n {
    if (xi > xj)
        INC[i] = INC[j]+1;
    else
        INC[i] = INC[j];
}
return INC[i];
```

b) LIS:
```
for i to m. {
    if (xi+1 > xi)
        LIS = INC[i] + 1 ;
    else
        LIS = INC[i]
}
```

DIS: for i to m {

```
    if (x_{i+1} < x_i)
        DIS = DEC[i]+1;
    else
        DIS = DEC[i];
}
```

c) Suppose $i < j$. Prove it's impossible to have $INC[i] = INC[j]$ and $DEC[i] = DEC[j]$

Proof: Given $i < j$, suppose $x_i < x_j$

Then $INC[j] \geq INC[i]+1$;

Thus, $INC[i] = INC[j]$ is impossible.

Suppose $x_i > x_j$.

Then $DEC[j] \geq DEC[i]+1$;

Thus, $DEC[i] = DEC[j]$ is impossible

d) Deduce: Let $m = ab+1$, then any sequence $x_1, \cdots x_m$ of distinct real numbers either $LIS > a$ or $DIS > b$

The length of $x_1 \cdots x_m$ is $(ab+1)$

① Base case: $m=1$ $(ab=0)$

when the length of the sequence is 1

Then $LIS = DIS = 1 > 0$, $LIS > a$ or $DIS > b$ works

② Assume $LIS > a-1$ or $DIS > b-1$ works on $m = (a-1)(b-1)+1$

Then when test x_{ab+1},

If $x_{ab+1} > x_{a-1b+1}$ then $LIS = \underset{LIS > a-1}{LIS} + 1$ } => LIS is certainly $> a$

If $x_{ab+1} < x_{a-1b+1}$ then $DIS = \underset{DIS > b-1}{DIS} + 1$ } => DIS is certaily $> b$

Thus, Any sequence $x_1 \cdots x_m$ of distinct real numbers either $LIS > a$ or $DIS > b$

Q4. Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is 5, 10, 3, 12, 5, 50, 6

Solution:

$A_1 = 5, 10$

$A_2 = 10, 3$

$A_3 = 3, 12$

$A_4 = 12, 5$

$A_5 = 5, 50$

$A_6 = 50, 6$ $\quad p_0 = 5 \quad p_1 = 10 \quad p_2 = 3 \quad p_3 = 12 \quad p_4 = 5 \quad p_5 = 50 \quad p_6 = 6$

$m[i, j] = 0$ if $i = j$

$m[i, j] = \{\min\{m[i, k] + m[k+1, j] + p_{i-1} p_k p_j\}\}$, if $i < j$

$m[1,1] = m[2,2] = m[3,3] = m[4,4] = m[5,5] = m[6,6] = 0$

$m[1,2] = 5 \times 10 \times 3 = 150$

$m[2,3] = 10 \times 3 \times 12 = 360$

$m[3,4] = 3 \times 12 \times 5 = 180$

$m[4,5] = 12 \times 5 \times 50 = 3000$

$m[5,6] = 5 \times 50 \times 6 = 1500$

m	1	2	3	4	5	6
1	0	150(1)	330(2)	405(2)	1655(4)	2010(2)
2		0	360(2)	330(2)	2430(2)	1950(2)
3			0	180(3)	930(4)	1770(4)
4				0	3000(4)	1860(4)
5					0	1500
6						0

Min of $m[1,3] = 330$

$k=1$: $m[1,3] = \min\{m[1,1] + m[2,3] + p_0 p_1 p_3\} = 0 + 360 + 5 \times 10 \times 12 = 960$

$k=2$: $- - - - = - - m[1,2] + m[3,3] + p_0 p_2 p_3] = 150 + 0 + 5 \times 3 \times 12 = 330$

Min of $m[1,4] = 405$

$k=1$: $\min(m[1,1] + m[2,4] + p_0 p_1 p_4) = 0 + 330 + 5 \times 10 \times 5 =$

$m[2,4] = \min(m[2,2] + m[3,4] + p_1 p_2 p_4) = 0 + 180 + 10 \times 3 \times 5 = 330$

$m[2,4] = \min(m[2,3] + m[4,4] + p_1 p_3 p_4) = 360 + 0 + 10 \times 12 \times 5 = 960$

$k=2$: $\min(m[1,2] + m[3,4] + p_0 p_2 p_4) = 150 + 180 + 5 \times 3 \times 5 = 405$

$k=3$: $\min(m[1,3] + m[4,4] + p_0 p_3 p_4) = 330 + 0 + 5 \times 12 \times 5 = 630$

Min of $m[3,5] = 930$

$k=3$: $\min(m[3,3] + m[4,5] + P_2P_3P_5) = 0 + 3000 + 3\times 12\times 50 = 4800$

$k=4$: $\min(m[3,4] + m[5,5] + P_2P_4P_5) = 180 + 0 + 3\times 5\times 50 = 930$

Min of $m[2,5] = 2430$

$k=2$: $\min(m[2,2] + m[3,5] + P_1P_2P_5) = 0 + 930 + 10\times 3\times 50 = 2430$

$k=3$: $\min(m[2,3] + m[4,5] + P_1P_3P_5) = 360 + 3000 + 10\times 12\times 50 = 9360$

$k=4$: $\min(m[2,4] + m[5,5] + P_1P_4P_5) = 330 + 0 + 10\times 5\times 50 = 2830$

Min of $m[1,5] = 1665$

$k=1$: $\min(m[1,1] + m[2,5] + P_0P_1P_5) = 0 + 2430 + 5\times 10\times 50 = 4930$

$k=2$: $\min(m[1,2] + m[3,5] + P_0P_2P_5) = 150 + 930 + 5\times 3\times 50 = 1830$

$k=3$: $\min(m[1,3] + m[4,5] + P_0P_3P_5) = 330 + 3000 + 5\times 12\times 50 = 6330$

$k=4$: $\min(m[1,4] + m[5,5] + P_0P_4P_5) = 405 + 0 + 5\times 5\times 50 = 1665$

Min of $m[4,6] = 1860$

$k=4$: $\min(m[4,4] + m[5,6] + P_3P_4P_6) = 0 + 1500 + 12\times 5\times 6 = 1860$

$k=5$: $\min(m[4,5] + m[6,6] + P_3P_5P_6) = 3000 + 0 + 12\times 50\times 6 = 6600$

Min of $m[3,6] = 1770$

$k=3$: $\min(m[3,3] + m[4,6] + P_2P_3P_6) = 0 + 1860 + 3\times 12\times 6 = 2076$

$k=4$: $\min(m[3,4] + m[5,6] + P_2P_4P_6) = 180 + 1500 + 3\times 5\times 6 = 1770$

$k=5$: $\min(m[3,5] + m[6,6] + P_2P_5P_6) = 930 + 0 + 3\times 50\times 6 = 1830$

Min of $m[2,6] = 1950$

$k=2$: $\min(m[2,2] + m[3,6] + P_1P_2P_6) = 0 + 1770 + 10\times 3\times 6 = 1950$

$k=3$: $\min(m[2,3] + m[4,6] + P_1P_3P_6) = 360 + 1860 + 10\times 12\times 6 = 2940$

$k=4$: $\min(m[2,4] + m[5,6] + P_1P_4P_6) = 330 + 1500 + 10\times 5\times 6 = 2130$

$k=5$: $\min(m[2,5] + m[6,6] + P_1P_5P_6) = 2430 + 0 + 10\times 50\times 6 = 5430$

Min of $m[1,6] = 2010$

$k=1$: $\min(m[1,1] + m[2,6] + P_0P_1P_6) = 0 + 1950 + 5\times 10\times 6 = 2250$

$k=2$: $\min(m[1,2] + m[3,6] + P_0P_2P_6) = 150 + 1770 + 5\times 3\times 6 = 2010$

$k=3$: $\min(m[1,3] + m[4,6] + P_0P_3P_6) = 330 + 1860 + 5\times 12\times 6 = 2550$

$k=4$: $\min(m[1,4] + m[5,6] + P_0P_4P_6) = 405 + 1500 + 5\times 5\times 6 = 2055$

$k=5$: $\min(m[1,5] + m[6,6] + P_0P_5P_6) = 1665 + 0 + 5\times 50\times 6 = 3165$

$s[1,6] = 2 \Rightarrow S[1,2] = 1 \Rightarrow (A_1A_2)$

$S[3,6] = 4 \Rightarrow (A_3A_4)(A_5A_6)$

$\Rightarrow$ Thus, the final sequence is $(A_1A_2)(A_3A_4)(A_5A_6)$

Q5.

a) $\lg(4^n/\sqrt{n})$ in simplest form

$= \lg(4^n) - \lg(\sqrt{n})$

$= n\lg 4 - \lg(\sqrt{n})$

$= n\lg 4 - \frac{1}{2}\lg n$

$= 2n - \frac{1}{2}\lg n \sim 2n$

b). $5^{313340} > 7^{271251}$

$\lg 5^{313340}$ $\quad\quad$ $\lg 7^{271251}$

$313340 \cdot \lg 5$ $\quad\quad$ $271251 \cdot \lg 7$

$\lg 5 = 2.321928$ $\quad\quad$ $\lg 7 = 2.807355$

$(313340 - 271251) = 42089 \times 0.5 = 21044.5$

$\because 21044.5 < 42089 \times 2$

$\therefore 313340 \cdot \lg 5 > 271251 \cdot \lg 7$

c) Simplify $n^2\lg(n^2)$ and $\lg^2(n^3)$

$n^2\lg(n^2) = 2n^2\lg n$

$\lg^2(n^3) = \lg(n^3) \cdot \lg(n^3) = (3\lg n)(3\lg n) = 9\lg^2 n$

d) $e^{-x^2/2} = \frac{1}{n}$ $\quad$ take $\log_e$ ($\ln$) on both sides

$-x^2/2 = \ln(\frac{1}{n})$

$-x^2 = 2\ln(\frac{1}{n})$

$-x^2 = 2((\ln 1) - (\ln n))$

$-x^2 = 2(0 - \ln n)$

$-x^2 = -2\ln n$

$x^2 = 2\ln n$

$x = \pm\sqrt{2}\ln^{\frac{1}{2}} n$

e) $\log_n 2^n$ and $\log_n n^2$ simplify

$\log_n 2^n = n\log_n 2$

$\log_n n^2 = 2\log_n n = 2$

f) Relationship between $\lg n$ and $\log_3 n$?

$\log_2 n = \log_3 n / \log_3 2$

Because $\log_3 2$ is constant. $O(\lg n) = O(\log_3 n)$

(g) Assume $i < n$ How many times need i be doubled before it reaches or (exceeds) n?

Suppose x times

$2^x i \geq n$

$2^x \geq \frac{n}{i}$

$x \geq \log_2(\frac{n}{i})$ assume $i \neq 0$

(h). $\lg[n^n e^{-n} \sqrt{2\pi n}]$ in sum simplest form.

$= \lg n^n + \lg e^{-n} + \lg\sqrt{2\pi n}$

$= n\lg n + (-n)\lg e + \frac{1}{2}\lg(2\pi n)$

$= n\lg n + (-n)\lg e + \frac{1}{2}\lg 2\pi + \frac{1}{2}\lg n$

$= (n+\frac{1}{2})\lg n - n\lg e + \frac{1}{2}\lg 2\pi \sim n\lg n$