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Fundamental Algorithms Problem Set 4

Q1. $T(n) = 9T(n/3) + n^2$ $T(1) = 1$

$T(3) = 9T(1) + 9 = 9 \times (1+1) = 9 \times 2 = 18$

$T(9) = 9T(3) + 9^2 = 9^2 \times (2+1) = 9^2 \times 3 = 243$

$T(27) = 9T(9) + 9^3 = 9^3 \times (3+1) = 9^3 \times 4 = 2916$

$T(81) = 9T(27) + 9^4 = 9^4 \times (4+1) = 9^4 \times 5 = 32805$

$T(243) = 9T(81) + 9^5 = 9^5 \times (5+1) = 9^5 \times 6 = 354294$

Guess the general formula for $T(3^i)$ and write as $T(n)$

Observe from the answer above, we have,

$T(3^i) = 9^i \times (i+1)$

$n = 3^i \Rightarrow i = \log_3 n$

$T(n) = (9^{\log_3 n}) \times (\log_3 n + 1)$

$= (3^{2\log_3 n}) \times (\log_3 n + 1)$

$= (3^{\log_3 n^2}) \times (\log_3 n + 1)$

$T(n) = n^2(\log_3 n + 1)$

Use the Master Theorem

$T(n) = 9T(n/3) + n^2$

$\because \log_3 9 = 2$

$\therefore$ Just right Overhead

$T(n) = \Theta(n^2 \lg n)$

As stated in the class, $\lg n$ and $\log_3 n$ are consistant

Answer to guessing the general formula and the Master Theorem are consistant

Q2. a) $T(n) = 6T(n/2) + n\sqrt{n}$

$\log_2 6 \approx 2.584963 > 1.5 \Rightarrow$ lower Overhead $T(n) = \Theta(n^{\log_2 6})$

b) $T(n) = 4T(n/2) + n^5$

$\log_2 4 = 2 < 5 \Rightarrow$ Higher Overhead $T(n) = \Theta(n^5)$

c) $T(n) = 4T(n/2) + 7n^2 + 2n + 1$

$\log_2 4 = 2 \Rightarrow$ Just right Overhead $T(n) = \Theta(n^2 \lg n)$

Q3. The recusion for the time $T(n)$ for Toom-3

$T(n) = 5T(n/3) + 30(O(n))$

Master Theorem

$\because \log_3 5 = 1.465 > 1$

$\therefore$ Lower Overhead $T(n) = \Theta(n^{\log_3 5})$

Compare with $\Theta(n^{\log_2 3})$ of karatsuba,

$\because \log_3 5 = 1.465 < \log_2 3 = 1.585$

$\therefore$ Toom-3 is faster when n is large.

Q4. Give $\Theta(g(n))$, c_1, c_2.

a) $n^2 + (n+1)^2 + \cdots (2n)^2$

$\leq n(2n)^2 = 4n^3 \rightarrow c_2 = 4$

$\geq \frac{n}{2}(\frac{3}{2}n)^2 = \frac{9}{8}n^3 \rightarrow c_1 = \frac{9}{8}$

$\Rightarrow \Theta(n^3)$ $c_1 = \frac{9}{8}$ $c_2 = 4$

b) $\lg^2(1) + \lg^2(2) + \cdots \lg^2(n)$

$\leq n\lg^2(n) \rightarrow c_2 = 1$

$\geq \frac{n}{2}(\lg^2(\frac{n}{2})) = \frac{n}{2}((\lg n - \lg 2)^2)$

$= \frac{n}{2}(\lg^2 n - 2\lg 2 \lg n + \lg^2 2) \rightarrow c_1 = \frac{1}{2}$

$\Rightarrow \Theta(n\lg^2(n))$ $c_1 = \frac{1}{2}$ $c_2 = 1$

c) $1^3 + \cdots + n^3$

$\leq n \cdot n^3 = n^4 \rightarrow c_2 = 1$

$\geq \frac{n}{2}(\frac{n}{2})^3 = \frac{n}{2} \cdot \frac{n^3}{8} = \frac{n^4}{16} \rightarrow c_1 = \frac{1}{16}$

$\Rightarrow \Theta(n^4)$ $c_1 = \frac{1}{16}$ $c_2 = 1$

Q5. Give an algorithm for subtracting two n-digit decimal #s.

Input: A [0 ... N]

B [0 ... N]

Output: C [0 ... N] Assume nonnegative result.

Algorithm:

$\theta(n)$

```
Sub ← 0
For i = N to 0
    C[i] = A[i] - B[i] - Sub;
    if ( C[i] < 0 )
        do Sub ← 1
        C[i] = A[i] + 10 - B[i];
    else
        Sub ← 0
End For
```

Input & Output assumption based on my Algorithm

① For Input: If the number A is 765, it will be stored in the Array as:

A = [7, 6, 5] in order.

② For Output: The result will be stored in order as well

eg. A = [7, 6, 5]

B = [3, 7, 1]

result is 394

Thus it will be stored in the Array C as:

C = [3, 9, 4]