CS489 Report: Flute Synthesis

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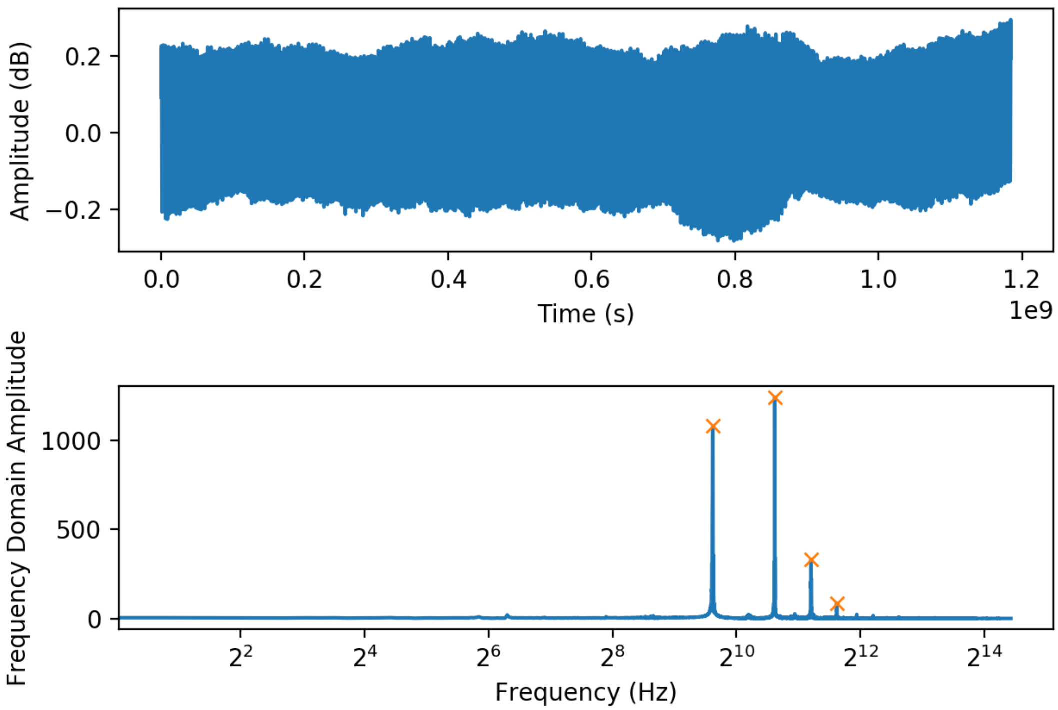
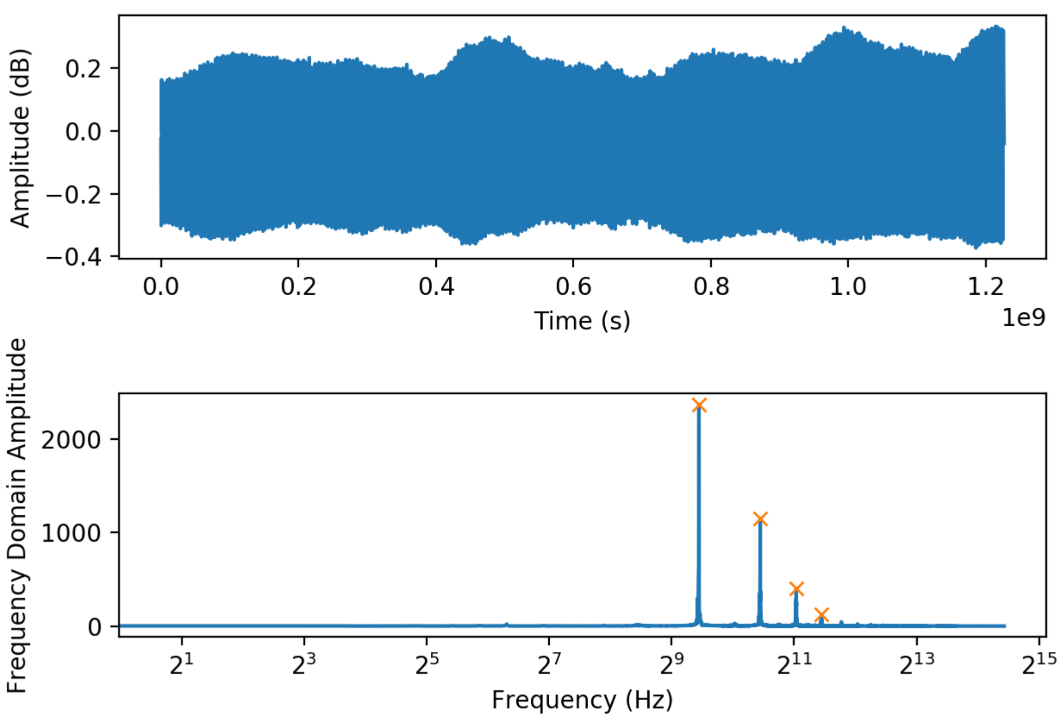
In 2018, I recorded a song [1] that featured recordings of myself playing the flute. I extracted four samples from one flute track in the song where a single note is held for around a second. There are samples for the notes G, F, E♭, and D. I processed each sample to get a sense of the harmonics present in the sound of the flute.

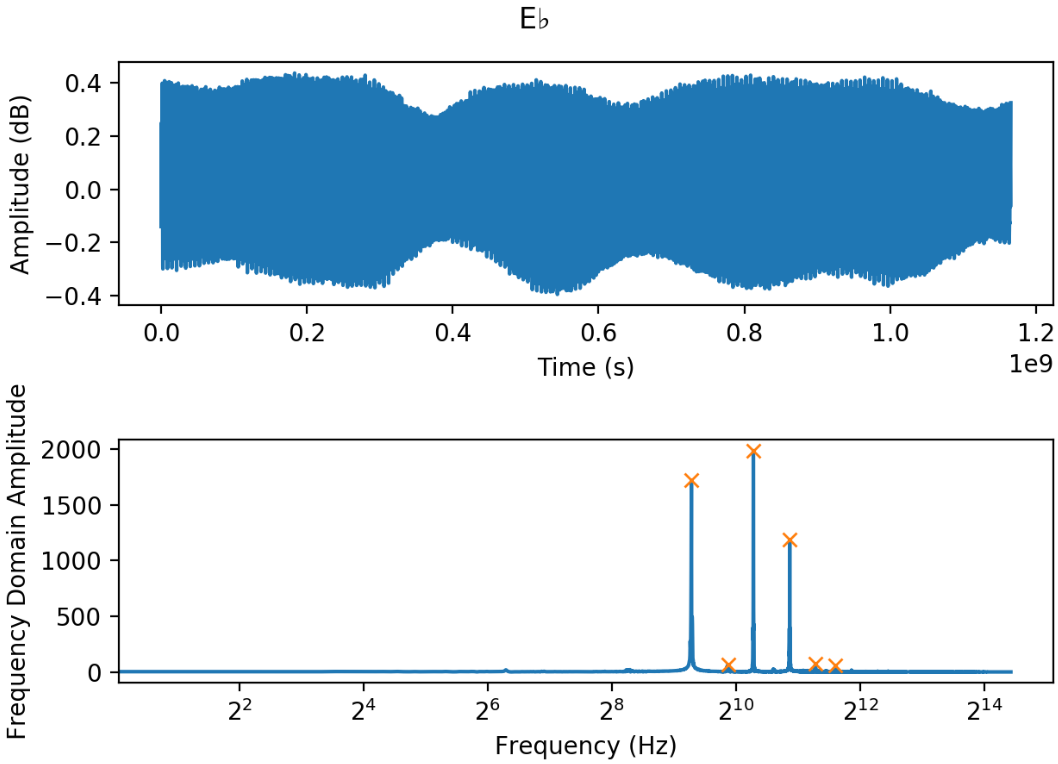
To do this, I ran each sample through a Fast Fourier Transform (FFT). (TODO describe input format.) This converts each sample into the frequency domain, showing, for each frequency contributing to the overall sound, the amplitude of its contribution. The frequency domain is slightly noisy due to the background noise in the recording and the shaky tuning of my own playing, so a peak at a given frequency typically also shows some contribution to the surrounding frequencies. I ran the FFT results through a peak finding algorithm to come up with the frequency and amplitude of each peak, each marked with an X in the figures below.

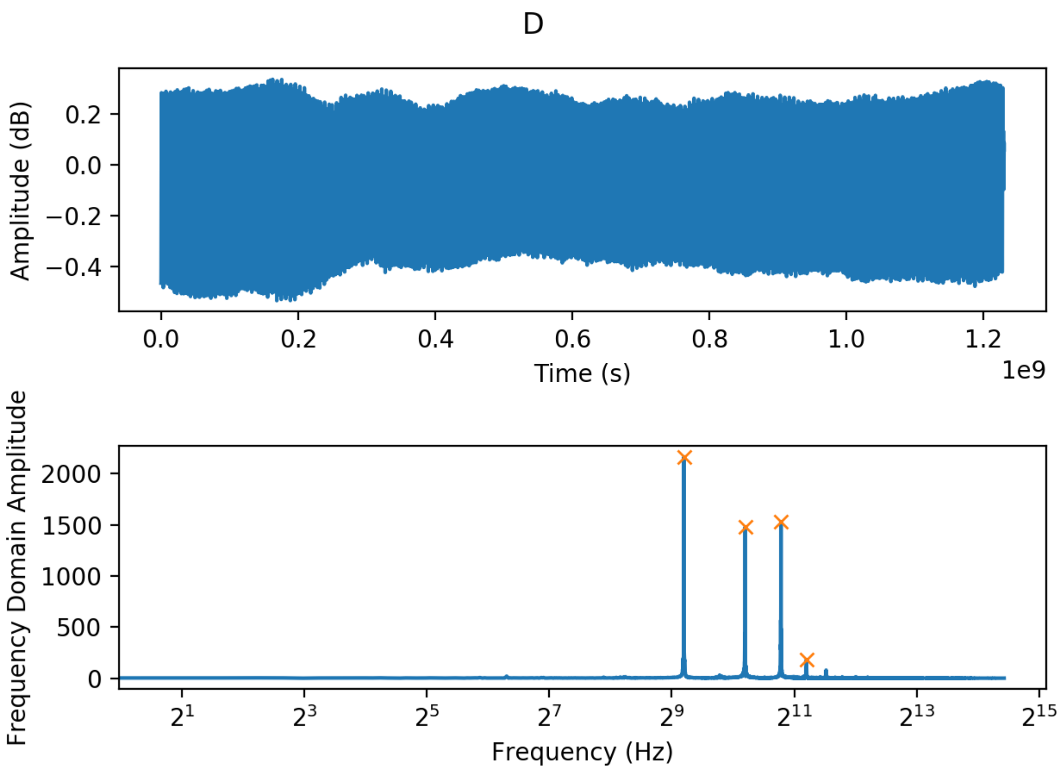
G


Amplitude (dB)
0.2
0.0
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Time (s)
1e9
Frequency Domain Amplitude
1000
500
0
2^2
2^4
2^6
2^8
2^10
2^12
2^14
Frequency (Hz)

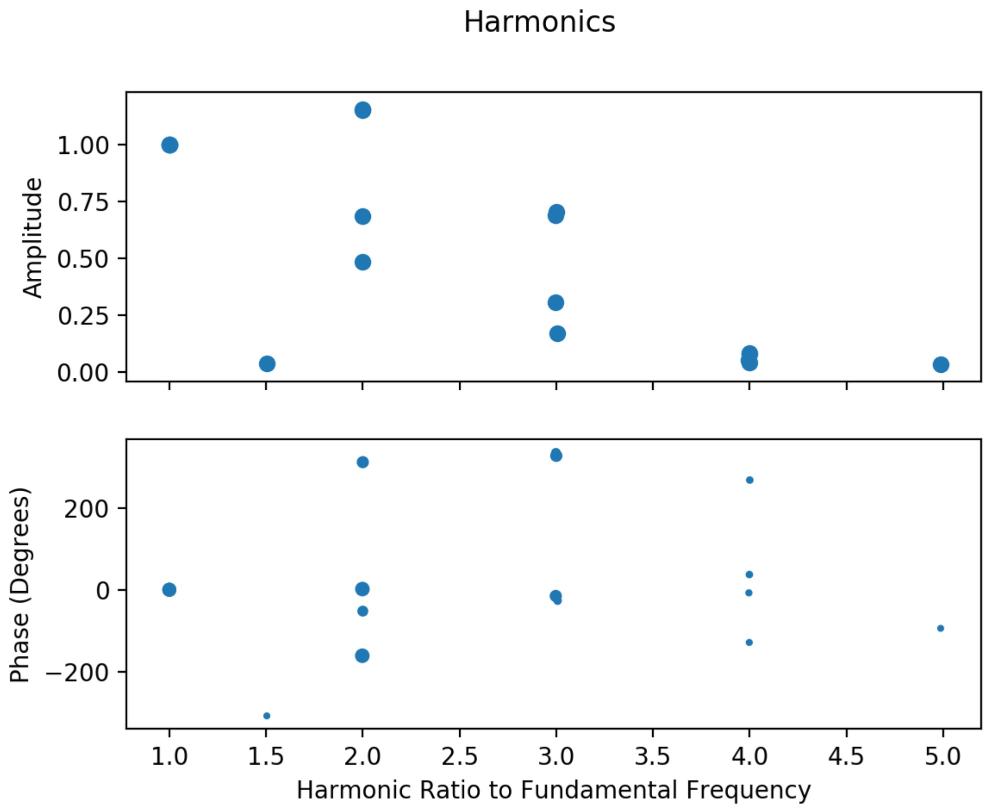
F


Amplitude (dB)
0.2
0.0
−0.2
−0.4
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Time (s)
1e9
Frequency Domain Amplitude
2000
1000
0
2^1
2^3
2^5
2^7
2^9
2^11
2^13
2^15
Frequency (Hz)





For each peak found, I graphed its amplitude and phase relative to its distance from the largest peak, which I consider to be the fundamental frequency.



Here we can see that the peaks come in multiples of 0.5. Why does this occur?

The flute is an open air column. Resonant frequencies arise in open air columns due to the standing pressure wave patterns that are able to form. The geometry of the column allows standing waves with integer number of nodes:

TODO insert node diagram

Each standing wave has a wavelength relative to the length of the instrument. The frequency of the note produced by each wave is found by the equation $f = \frac{v}{\lambda}$, where v is the speed of sound, and λ is the wavelength. This tells us that the lowest resonant frequency is proportional to twice the length of the instrument. Each successively higher frequency increases by a factor of half the length of the instrument. Given that the length of a flute is around 66 cm, we can estimate the lowest note, corresponding to one node:

$$\begin{aligned} f &= \frac{v}{\lambda} \\ &= \frac{340.27 \text{ m/s}}{2 \cdot 66 \text{ cm}} \quad = 257.78 \text{ Hz} \end{aligned}$$

This frequency corresponds to the note B3 and 74 cents. This agrees with the experience of playing the flute, where the lowest note that can be fingered is C3 by covering every hole. If we continue adding harmonics by doubling the frequency, corresponding to the addition of another node, we should see the following notes[1]:

[1] It is interesting to note here that the harmonic frequencies are not in tune. We perceive a doubling of a frequency to be a jump up an octave, so octaves are found to be proportional to 2^n for increasing n. Equal-tempered tuning divides the space between octaves into 12 equally spaced semitones, so semitones are proportional to $2^{\frac{n}{12}}$. Integer multiples of a base note do not always align with these twelfths. Some instruments purposefully detune notes that are intended to be played in a chord with other notes so that the harmonics do not interfere dissonantly with the rest of the chord. (TODO find citation for this)

Harmonic	Frequency	Note
1	261.6256	C3
2	523.2511	C4
3	784.8767	G4 and 2 cents
4	1046.5023	C5
5	1308.1278	D#5 and 86 cents
6	1569.7534	G5 and 2 cents
7	1831.3790	A6 and 69 cents
8	2093.0045	C6
9	2354.6301	D6 and 4 cents
10	2616.2557	D#6 and 86 cents

All of these frequencies are integer multiples of the fundamental. Since the second harmonic is double the frequency of the first, if we were to look at the higher harmonics relative to the second one, it would look like they are all multiples of 0.5, like what we saw in the harmonics from the recordings. This implies that the first peak frequency found in the recordings is actually the second harmonic of the flute for that fingering. Notes like this are played by overblowing to change the air pressure in the instrument, preventing the first harmonic from sounding.

TODO figure out which notes this happens for

TODO describe the oscillator setup

Noise component

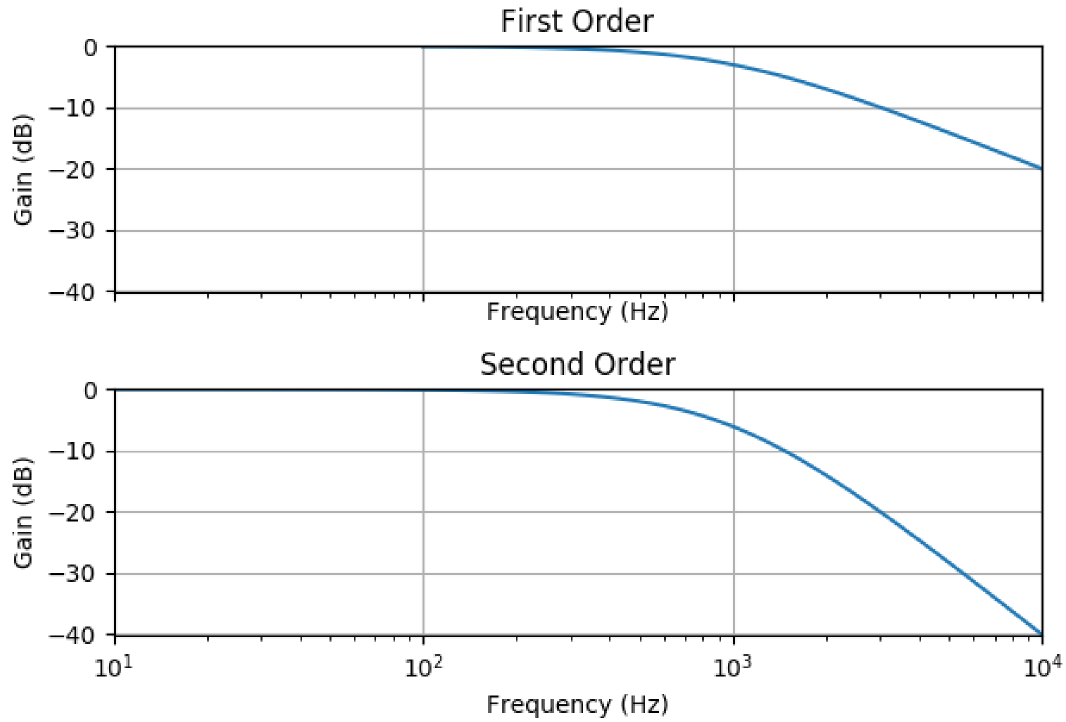
A spectrum analysis of recorded notes shows that there is a noise component to the sound. This low-energy noise is the wind sound from blowing into the instrument, picked up in the recording due to the close proximity of the microphone to the instrument. It makes the flute sound more intimate, so it is a sound I want to replicate in the synthesizer.

Looking at the spectrum produced by the recorded noise component, it is clear that frequencies do not all equally contribute to the noise, as the lower frequencies have higher magnitudes. This tells us that it is not *white noise*, which is defined to be noise where frequency contributions are the same within the region of interest. One way to generate the sort of noise seen in the recording is to use subtractive synthesis, where one starts with white noise, but then quiets down the higher frequencies by applying a filter.

White noise is defined by having equal contributions from each frequency, but there are multiple functions that generate discrete samples that create white noise. One could implement *Gaussian white noise*, where samples have a normal distribution with zero mean. However, Gaussian distributions have an unbounded range, but we need amplitudes to stay between -1 and 1. Instead, we can use a uniform distribution:

```
def noise():
    return np.random.uniform(-1, 1)
```

We then need to filter down this noise using some kind of lowpass filter, that leaves low frequencies the same, but reduces the importance of higher frequencies. A simple first-order system could work, but for frequencies above the bandwidth frequency ω_{BW}, high frequencies roll off at -20dB per decade. In order to have high frequencies roll off faster, higher order systems are required. A second-order system rolls off at -40dB per decade, shown below.



The steeper rolloff rate more closely matches the recorded data, so I decided to implement a second order low pass filter. Transfer functions for second order systems typically have the form $H(s) = \frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$. $\omega_n \approx \omega_{BW}$, which is the cutoff frequency after which we see rolloff. We do not need any extra gain, so set the gain $K = 1$. ζ dictates the oscillatory behaviour of the system: if it is greater than 1, the system is overdamped, and the filter can overshoot and oscillate when reaching to "catch up" with an input signal. We do not want this to happen, and when $\zeta \leq 1$, the system will not oscillate. We will pick $\zeta = 1$ so that the system is critically damped and will reach its equilibrium point as fast as it can without oscillating.

To actually implement this as a digital filter, we need to turn the transfer function into a discrete time equation relative to the past discrete samples.

$$
\begin{aligned}
H(s) &= \frac{\omega_{BW}}{s^2 + 2\omega_{BW}s + \omega_{BW}^2} \\
\frac{Y(s)}{X(s)} &= \frac{1}{\frac{s^2}{\omega_{BW}^2} + \frac{2s}{\omega_{BW}} + 1} \\
X(s) &= Y(s)\left(\frac{s^2}{\omega_{BW}^2} + \frac{2s}{\omega_{BW}} + 1\right) \\
X(s) &= \frac{s^2}{\omega_{BW}^2}Y(s) + \frac{2s}{\omega_{BW}}Y(s) + Y(s)
\end{aligned}
$$

Having separated input from output, we can bring the equation into the time domain using the inverse Laplace transform, assuming $y(0) = \left.\frac{dy(t)}{dt}\right|_{t=0} = \left.\frac{d^2y(t)}{dt^2}\right|_{t=0} = 0$:

$$
\begin{aligned}
\mathcal{L}^{-1}\left\{U(s)\right\} &= \mathcal{L}^{-1}\left\{\frac{s^2}{\omega_{BW}^2}Y(s) + \frac{2s}{\omega_{BW}}Y(s) + Y(s)\right\} \\
x(t) &= \frac{1}{\omega_{BW}^2}\left(\frac{d^2y(t)}{dt^2}\right) + \frac{2}{\omega_{BW}}\left(\frac{dy(t)}{dt}\right) + y(t)
\end{aligned}
$$

We can then bring everything into discrete time by approximating first and second derivatives and rearranging for $y(t)$:

$$x(t) = \frac{1}{\omega_{BW}^2}\left(\frac{y(t) - 2y(t-\Delta t) + y(t-2\Delta t)}{\Delta t^2}\right) + \frac{2}{\omega_{BW}}\left(\frac{y(t) - y(t-\Delta t)}{\Delta t}\right) + y(t)$$

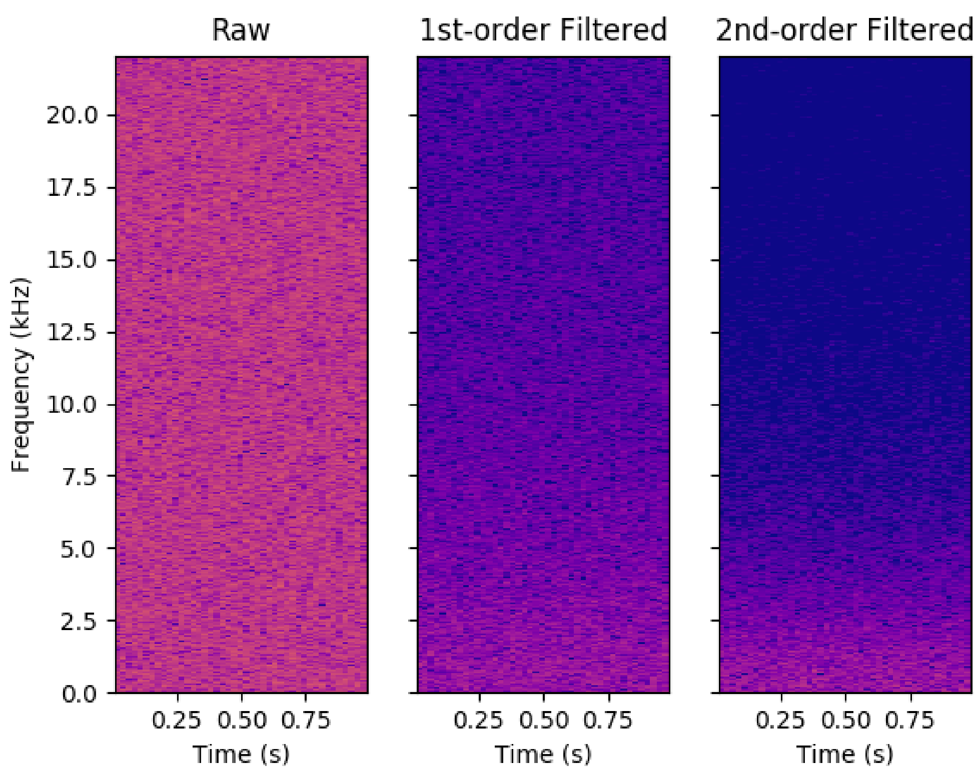
$$y(t) = \frac{x(t) - \left(\frac{-2}{\omega_{BW}\Delta t} - \frac{2}{\omega_{BW}^2\Delta t^2}\right)y(t-\Delta t) - \left(\frac{1}{\omega_{BW}^2\Delta t^2}\right)y(t-2\Delta t)}{1 + \frac{2}{\omega_{BW}\Delta t} + \frac{1}{\omega_{BW}^2\Delta t^2}}$$

$$\begin{aligned} y(t) = &\left(\frac{\omega_{BW}^2\Delta t^2}{\omega_{BW}^2\Delta t^2 + 2\omega_{BW}\Delta t + 1}\right)x(t) \\ &- \left(\frac{-2\omega_{BW}\Delta t - 2}{\omega_{BW}^2\Delta t^2 + 2\omega_{BW}\Delta t + 1}\right)y(t-\Delta t) \\ &- \left(\frac{1}{\omega_{BW}^2\Delta t^2 + 2\omega_{BW}\Delta t + 1}\right)y(t-2\Delta t) \end{aligned}$$

If we let $\alpha = \frac{\omega_{BW}\Delta t}{\omega_{BW}\Delta t + 1}$, then we can see that each new value is effectively a weighted average of the new input and the past two output values:

$$\begin{aligned} y(t) = &\ \alpha^2 x(t) \\ &- 2(\alpha - 1)y(t-\Delta t) \\ &- (1-\alpha)^2 y(t-2\Delta t) \end{aligned}$$

We can then find the value of α using $\Delta t = 44100\text{Hz}$, the sampling frequency, and $\omega_{BW} = 15000\text{Hz}$, the approximate frequency around which we want to start seeing rolloff. Here is what white noise looks like, subjected to a first-order lowpass filter and this second-order filter:



Time variance

Results

References

[1] D. Pagurek, “Throw the dice.” 2018 [Online]. Available: https://soundcloud.com/davidpvm/throw-the-dice