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Eliminating the Effect of Magnetic Disturbances on the Inclination Estimates of Inertial Sensors ⋆

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Abstract: Inertial measurement units (IMUs) are a promising tool for realtime analysis of human motion in many application domains. We consider the standard sensor fusion problem in IMU orientation estimation. We highlight the malicious effects that inhomogeneous magnetic fields, which are often found in indoor environments, can have on the inclination portions (roll and pitch) of the orientation estimate. We then propose a novel method that uses an analytical solution of the sensor fusion problem and purely horizontal magnetometer-based corrections. The method assures that magnetic field measurements affect only the heading (yaw) component of the orientation estimate. Furthermore, we parametrize the algorithm such that the user can choose the time constant and aggressiveness with which the algorithm balances between gyroscope drift compensation and rejection of disturbances caused by inhomogeneous magnetic fields or by velocity changes.

Keywords: inertial measurement units, sensor fusion, orientation estimation, quaternion, alternating magnetic fields, magnetic disturbance, motion estimation, human motion analysis, biomedical engineering

1. INTRODUCTION AND MOTIVATION

Inertial measurement units are primarily known from aerial and submarine navigation. With the tremendous development of micro-electro-mechanical systems and chips, these sensors have become so small and affordable that they are nowadays not only found in every smartphone, they also enable us to develop completely new means for human motion analysis (Müller et al., 2016; Seel et al., 2014) and thus for realtime control of active motion support systems (Moreno et al., 2006; Seel et al., 2016). We simply attach an IMU to each segment we are interested in and measure its acceleration, angular rate and the magnetic field vector in the local coordinate system of the sensor, which rotates as the IMU rotates. In order to derive useful information (for example joint angles or gait phases) from these local measurements, it is required to transform them into a fixed reference coordinate system.

1.1 Representing Orientations by Unit Quaternions

According to Euler's rotation theorem, any rotation between two right-handed Cartesian coordinate systems can be expressed by a single rotation axis $\mathbf{x}_{\mathrm{rot}}, ||\mathbf{x}_{\mathrm{rot}}||_2 = 1$, and a corresponding rotation angle α_{rot}. Let the (vector representation of the) unit quaternion that represents this rotation be defined as

$$\mathbf{q} = \begin{bmatrix} \cos(\frac{\alpha_{\mathrm{rot}}}{2}) \\ \sin(\frac{\alpha_{\mathrm{rot}}}{2})\mathbf{x}_{\mathrm{rot}} \end{bmatrix} \in \mathbb{R}^4, \; ||\mathbf{q}||_2 = 1. \tag{1}$$

⋆ This work was conducted within the research project BeMobil, which is supported by the German Federal Ministry of Research and Education (FKZ 16SV7069K).

Consider a three-dimensional vector ${}^{\mathcal{A}}\mathbf{m} \in \mathbb{R}^3$ given in a coordinate system $\mathcal{A}$ and a quaternion ${}^{\mathcal{B}}_{\mathcal{A}}\mathbf{q}$ that describes the rotation between the coordinate system $\mathcal{A}$ and another coordinate system $\mathcal{B}$. Then interpret the vector ${}^{\mathcal{A}}\mathbf{m}$ as the last three entries of a quaternion with first entry zero. This allows us to transform the vector ${}^{\mathcal{A}}\mathbf{m}$ into the coordinate system $\mathcal{B}$ by calculating the quaternion product

$$\begin{bmatrix} 0 \\ {}^{\mathcal{B}}\mathbf{m} \end{bmatrix} = {}^{\mathcal{A}}_{\mathcal{B}}\mathbf{q} \otimes \begin{bmatrix} 0 \\ {}^{\mathcal{A}}\mathbf{m} \end{bmatrix} \otimes {}^{\mathcal{B}}_{\mathcal{A}}\mathbf{q}. \tag{2}$$

Therein, $\otimes$ denotes the quaternion multiplication and ${}^{\mathcal{A}}_{\mathcal{B}}\mathbf{q} = {}^{\mathcal{B}}_{\mathcal{A}}\mathbf{q}^*$ is the inverse quaternion of ${}^{\mathcal{B}}_{\mathcal{A}}\mathbf{q}$. Figure 1 illustrates the transformation of a vector measured in local sensor coordinates into a fixed global coordinate systems.

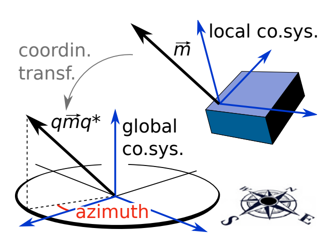

Fig. 1. Coordinate transformation of a locally measured quantity into the global reference coordinate system. Here, the fixed reference coordinate system is defined to have a vertical z-axis and an x-axis that points horizontally toward magnetic south.

The quaternion multiplication can furthermore be used to express concatenation of rotations. In addition to ${}^{\mathcal{B}}_{\mathcal{A}}\mathbf{q}$, consider a quaternion ${}^{\mathcal{C}}_{\mathcal{B}}\mathbf{q}$ that describes the rotation

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between the coordinate system $\mathcal{B}$ and another coordinate system $\mathcal{C}$. Then

$$ {}^{\mathcal{C}}_{\mathcal{A}}\mathbf{q} = {}^{\mathcal{B}}_{\mathcal{A}}\mathbf{q} \otimes {}^{\mathcal{C}}_{\mathcal{B}}\mathbf{q} \tag{3} $$

describes the rotation between the coordinate system $\mathcal{A}$ and the coordinate system $\mathcal{C}$. Furthermore,

$$ \begin{bmatrix} 0 \\ {}^{\mathcal{C}}\mathbf{m} \end{bmatrix} = {}^{\mathcal{B}}_{\mathcal{C}}\mathbf{q} \otimes {}^{\mathcal{A}}_{\mathcal{B}}\mathbf{q} \otimes \begin{bmatrix} 0 \\ {}^{\mathcal{A}}\mathbf{m} \end{bmatrix} \otimes {}^{\mathcal{B}}_{\mathcal{A}}\mathbf{q} \otimes {}^{\mathcal{C}}_{\mathcal{B}}\mathbf{q} \tag{4} $$

yields the vector ${}^{\mathcal{C}}\mathbf{m}$ in coordinates of the coordinate system $\mathcal{C}$.

1.2 Orientation Estimation by Sensor Fusion of Inertial Measurement Signals

Consider an IMU that is completely at rest within an ideally homogeneous magnetic field that is not perfectly vertical. Assume furthermore that a fixed reference coordinate system has been defined to have one vertical coordinate axis and one horizontal coordinate axis that is parallel to the projections of the magnetic field lines into the horizontal plane (cf. Figure 1). The orientation of the IMU with respect to this global coordinate system can be calculated from the two aforementioned measurement vectors. For example, the measured acceleration can be used to determine the inclination part of the orientation, i.e. the roll and pitch angle. The additional information of the magnetometer can then be used to determine the heading (or azimuth) part of the orientation.

When the IMU is accelerated or when the magnetic field is non-homogeneous, the IMU's orientation cannot be determined directly from the acceleration and the magnetic field vector. Strapdown integration of the measured angular rate vector yields an orientation estimate that is affected by drift (due to gyroscope bias) but is precise on short time scales. Therefore, IMU orientation estimation is typically performed by employing the sensor fusion scheme depicted in Figure 2 in one variation or another. The short-term accuracy of the gyroscope-based strapdown integration is combined with the long-term accuracy of the orientation information gained from accelerometer and magnetometer readings.

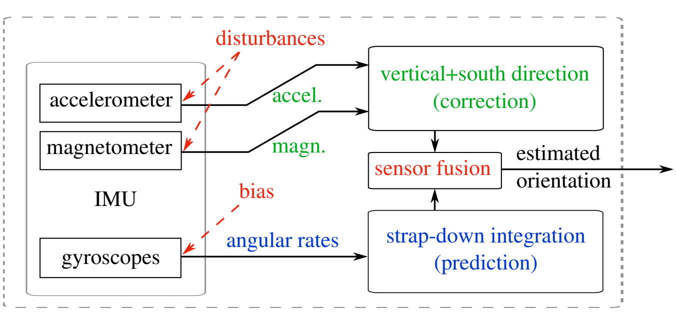

Fig. 2. Estimation of the orientation of an IMU with respect to a coordinate system with a vertical axis and a horizontally south-faced axis.

The large number of sensor fusion algorithms that have been proposed for IMU orientation estimation differ mainly by the mathematical framework they use and by the filter they employ for combining the measurement signals. With respect to the first aspect, rotation matrices and axis-angle representations are almost as commonly used as unit quaternions. With respect to the second aspect, different types of Kalman filters (Ang et al., 2004; Zhang et al., 2012), but also complementary filters (Mi et al., 2010; Fourati et al., 2014) and other filter types, such as gradient-descent methods (Madgwick et al., 2011) or sliding mode observers (El Hadri and Benallegue, 2009) have been employed.

Most fusion filters allow the user to specify weights or covariance matrices that influence the balance between prediction and correction. But comparatively little attention has been given to the question in which way the sensor fusion should rely on each of the raw measurement signals. In particular, recent research has demonstrated that magnetometers are hardly reliable in many indoor environments (De Vries et al., 2009). Therefore, magnetometer readings should be used only for the purpose for which they are essential, i.e. to remove drift in the azimuth part of the orientation, and should not influence the inclination part of the orientation. Thereby, in the worst case of a completely distorted magnetic field, one obtains orientation estimates that have at least precise inclination components. Only few authors address this problem. Won et al. (2010) avoid it by employing additional sensors; others solve the sensor-fusion problem only for roll and pitch. In constrast, Madgwick et al. (2011) propose a direct solution by means of a gradient-descent algorithm.

In the present contribution, we propose a sensor fusion algorithm that restricts magnetometer influence to the horizontal (heading/azimuth) part of the orientation. In simulations we compare the proposed algorithm to the gradient descent algorithm by Madgwick et al..

2. METHODS FOR ORIENTATION ESTIMATION

In accordance with Madgwick et al. (2011), consider the following sensor fusion problem. Denote the local sensor coordinate system by $\mathcal{S}$ and the fixed reference (earth) coordinate system by $\mathcal{E}$. Assume, without loss of generality, that the reference frame has been defined to have a vertical z-coordinate axis as well as a horizontal x-coordinate axis that is parallel to the projections of the magnetic field lines into the horizontal plane. Denote the unit vectors of the x-axis and z-axis of that reference frame, respectively, by ${}^{\mathcal{E}}\mathbf{r}_{\mathrm{mag}} = [1\ 0\ 0]^T$, ${}^{\mathcal{E}}\mathbf{r}_{\mathrm{acc}} = [0\ 0\ 1]^T$. Finally, assume that the coordinate systems $\mathcal{S}$ and $\mathcal{E}$ coincide initially, i.e. the orientation quaternion is ${}^{\mathcal{S}}_{\mathcal{E}}\mathbf{q}(t_0) = [1, 0, 0, 0]^T$.

Consider a sampling interval $[t - t_{\mathrm{s}}, t]$, where $t = t_0 + kt_{\mathrm{s}}, k \in \mathbb{N}_+$. For each such time interval, the norm and direction of the measured angular rates yield a quaternion that describes the change in orientation as detected by the gyroscopes. This quaternion is multiplied with the previous orientation estimate ${}^{\mathcal{S}}_{\mathcal{E}}\mathbf{q}(t - t_{\mathrm{s}})$ to obtain the prediction ${}^{\mathcal{S}}_{\mathcal{E}}\mathbf{q}_{\mathrm{gyr}}(t)$. Due to drift effects, this quaternion is only an approximation of the true orientation of the IMU. When the global z-axis ${}^{\mathcal{E}}\mathbf{r}_{\mathrm{acc}}$ is transformed into the sensor coordinate system using ${}^{\mathcal{S}}_{\mathcal{E}}\mathbf{q}_{\mathrm{gyr}}(t)$, then the resulting vector is not perfectly aligned with the current accelerometer readings ${}^{\mathcal{S}}\mathbf{a}(t)$. Likewise, when the global coordinate axis vector ${}^{\mathcal{E}}\mathbf{r}_{\mathrm{mag}}$ is transformed into the sensor coordinate system using ${}^{\mathcal{S}}_{\mathcal{E}}\mathbf{q}_{\mathrm{gyr}}(t)$, then the resulting vector is not perfectly aligned with the current magnetometer reading

${}^{\mathcal{S}}\mathbf{m}(t)$. Both disagreements can either be due to velocity changes and magnetic field disturbances, respectively, or due to integration drift effects. The sensor fusion task is to compensate the drift without being too sensitive to the disturbances.

Madgwick et al. (2011) capture these disagreements in the cost functions

$$f_{\mathrm{mag}}({}^{\mathcal{S}}_{\mathcal{E}}\mathbf{q}) := \left|\left| {}^{\mathcal{E}}_{\mathcal{S}}\mathbf{q} \begin{bmatrix} 0 \\ {}^{\mathcal{E}}\mathbf{r}_{\mathrm{mag}} \end{bmatrix} {}^{\mathcal{S}}_{\mathcal{E}}\mathbf{q} - \begin{bmatrix} 0 \\ {}^{\mathcal{S}}\mathbf{m}(t) \end{bmatrix} \right|\right|_2, \tag{5}$$

$$f_{\mathrm{acc}}({}^{\mathcal{S}}_{\mathcal{E}}\mathbf{q}) := \left|\left| {}^{\mathcal{E}}_{\mathcal{S}}\mathbf{q} \begin{bmatrix} 0 \\ {}^{\mathcal{E}}\mathbf{r}_{\mathrm{acc}} \end{bmatrix} {}^{\mathcal{S}}_{\mathcal{E}}\mathbf{q} - \begin{bmatrix} 0 \\ {}^{\mathcal{S}}\mathbf{a}(t) \end{bmatrix} \right|\right|_2. \tag{6}$$

To minimize these, they calculate the gradients of both cost functions in the full four-dimensional quaternion space and evaluate them at ${}^{\mathcal{S}}_{\mathcal{E}}\mathbf{q} = {}^{\mathcal{S}}_{\mathcal{E}}\mathbf{q}_{\mathrm{gyr}}(t)$. Finally, they correct ${}^{\mathcal{S}}_{\mathcal{E}}\mathbf{q}_{\mathrm{gyr}}(t)$ by a sufficiently small step into the direction of steepest descent of the cost functions, and project the result back into the subspace of unit quaternions. [1]

Unlike many other authors, Madgwick et al. are well aware that the magnetometer readings should not influence the inclination part of the orientation estimate. Therefore, at each sample instant, they define a modified magnetic reference vector ${}^{\mathcal{E}}\hat{\mathbf{r}}_{\mathrm{mag}}$ such that it

– has the same norm and the same z-component as the current magnetometer readings ${}^{\mathcal{E}}\mathbf{m}(t)$ in global coordinates,
– but its projection into the horizontal plane is aligned with the southbound global coordinate axis.

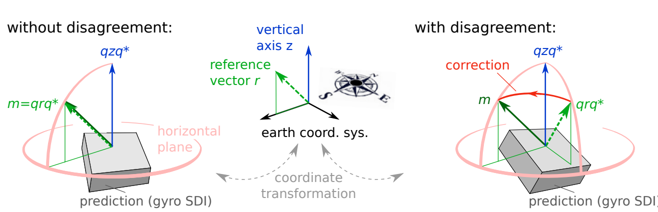

Fig. 3. Strapdown integration of the angular rates yields a prediction of the local sensor coordinate system (indicated by gray IMU box). If integration drift or magnetic disturbances occur, the measured magnetic field vector ${}^{\mathcal{S}}\mathbf{m}(t)$ and the transformed reference vector ${}^{\mathcal{S}}\mathbf{r}_{\mathrm{mag}}$ do not coincide. The prediction is then corrected by a small rotation into the direction of the measurement.

Figure 3 illustrates this aspect. It also shows that the gradient descent step corrects the predicted orientation by rotating the predicted coordinate system (indicated by the gray box) such that ${}^{\mathcal{E}}\mathbf{r}_{\mathrm{mag}}$ is rotated directly toward ${}^{\mathcal{E}}\mathbf{m}(t)$. Since both of these vectors have the same z-component, the correction affects mainly the heading if the disagreement is small. However, since the z-components are in general nonzero, the correcting rotation severely affects the inclination part of the orientation estimate if the disagreement is large.

[1] To some extent, this brief description is of course a simplification of the algorithm. For details, please refer to the original contribution (Madgwick et al., 2011).

2.1 *Analytical solution of the Sensor Fusion Problem*

Therefore, we propose an alternative method that uses an analytical solution for the minimization of (5) and (6). This method shall perform correction steps in such a way that the inclination is only affected by the measured accelerations, while the heading is only affected by the magnetometer readings. For convenience, denote the local sensor coordinates of the vertical unit vector by ${}^{\mathcal{S}}\mathbf{r}_{\mathrm{acc}}$, with

$$\begin{bmatrix} 0 \\ {}^{\mathcal{S}}\mathbf{r}_{\mathrm{acc}}(t) \end{bmatrix} = {}^{\mathcal{E}}_{\mathcal{S}}\mathbf{q}_{\mathrm{gyr}}(t) \otimes \begin{bmatrix} 0 \\ {}^{\mathcal{E}}\mathbf{r}_{\mathrm{acc}} \end{bmatrix} \otimes {}^{\mathcal{S}}_{\mathcal{E}}\mathbf{q}_{\mathrm{gyr}}(t). \tag{7}$$

The minimum of the cost function (5) is found analytically by determining the analytical-solution quaternion $\mathbf{q}_{\mathrm{as,acc}}(t)$ that rotates ${}^{\mathcal{S}}\mathbf{r}_{\mathrm{acc}}(t)$ onto ${}^{\mathcal{S}}\mathbf{a}(t)$. By calculating the angle [2] $\alpha_{\mathrm{err,acc}}(t) = \sphericalangle\left({}^{\mathcal{S}}\mathbf{a}(t), {}^{\mathcal{S}}\mathbf{r}_{\mathrm{acc}}(t)\right)$ between both vectors, as well as their vector product, we obtain

$$\mathbf{q}_{\mathrm{as,acc}}(t) = \begin{bmatrix} \cos\left(\frac{1}{2}\alpha_{\mathrm{err,acc}}(t)\right) \\ \sin\left(\frac{1}{2}\alpha_{\mathrm{err,acc}}(t)\right) \mathbf{x}_{\mathrm{corr,acc}}(t) \end{bmatrix}, \tag{8}$$

$$\mathbf{x}_{\mathrm{corr,acc}}(t) = \frac{{}^{\mathcal{S}}\mathbf{a}(t) \times {}^{\mathcal{S}}\mathbf{r}_{\mathrm{acc}}(t)}{||\,{}^{\mathcal{S}}\mathbf{a}(t) \times {}^{\mathcal{S}}\mathbf{r}_{\mathrm{acc}}(t)||_2}.$$

This correction eliminates the disagreement, since

$$\begin{bmatrix} 0 \\ {}^{\mathcal{S}}\mathbf{a}(t) \end{bmatrix} = \mathbf{q}_{\mathrm{as,acc}}(t)^* \otimes \begin{bmatrix} 0 \\ {}^{\mathcal{S}}\mathbf{r}_{\mathrm{acc}}(t) \end{bmatrix} \otimes \mathbf{q}_{\mathrm{as,acc}}(t). \tag{9}$$

Using (7), we furthermore conclude that the concatenation of $\mathbf{q}_{\mathrm{as,acc}}(t)$ and ${}^{\mathcal{S}}_{\mathcal{E}}\mathbf{q}_{\mathrm{gyr}}(t)$ minimizes the cost function (6).

As discussed above, it is desirable to balance between drift compensation and disturbance robustness. To this end, use only a small portion of the angle $\alpha_{\mathrm{err,acc}}(t)$ to correct the prediction in every sampling interval. Consequently, the accelerometer-corrected estimate of the IMU orientation is calculated as follows:

$$\begin{aligned} {}^{\mathcal{S}}_{\mathcal{E}}\mathbf{q}_{\mathrm{gyracc}}(t) &= {}^{\mathcal{S}}_{\mathcal{E}}\mathbf{q}_{\mathrm{gyr}}(t) \otimes \mathbf{q}_{\mathrm{corr,acc}}(t), \\ \mathbf{q}_{\mathrm{corr,acc}}(t) &= \begin{bmatrix} \cos\left(\frac{1}{2}k_{\mathrm{acc}}\,\alpha_{\mathrm{err,acc}}(t)\right) \\ \sin\left(\frac{1}{2}k_{\mathrm{acc}}\,\alpha_{\mathrm{err,acc}}(t)\right) \mathbf{x}_{\mathrm{corr,acc}}(t) \end{bmatrix}, \end{aligned} \tag{10}$$

where $k_{\mathrm{acc}} \in [0, 1]$ is an adjustable sensor fusion weight.

In order to correct the heading of ${}^{\mathcal{S}}_{\mathcal{E}}\mathbf{q}_{\mathrm{gyracc}}(t)$, we propose the following magnetometer-based correction. Use ${}^{\mathcal{S}}\mathbf{r}_{\mathrm{acc}}$ to project the measured magnetic field vector ${}^{\mathcal{S}}\mathbf{m}(t)$ into the horizontal plane, i.e.

$${}^{\mathcal{S}}\bar{\mathbf{m}}(t) = {}^{\mathcal{S}}\mathbf{m}(t) - ({}^{\mathcal{S}}\mathbf{m}(t) \cdot {}^{\mathcal{S}}\mathbf{r}_{\mathrm{acc}})\,{}^{\mathcal{S}}\mathbf{r}_{\mathrm{acc}}\,, \tag{11}$$

and (by analogy with (10)) use the correction step

$$\begin{aligned} {}^{\mathcal{S}}_{\mathcal{E}}\mathbf{q}_{\mathrm{gyraccmag}}(t) &= {}^{\mathcal{S}}_{\mathcal{E}}\mathbf{q}_{\mathrm{gyracc}}(t) \otimes \mathbf{q}_{\mathrm{corr,mag}}(t), \\ \mathbf{q}_{\mathrm{corr,mag}}(t) &= \begin{bmatrix} \cos\left(\frac{1}{2}k_{\mathrm{mag}}\,\alpha_{\mathrm{err,mag}}(t)\right) \\ \sin\left(\frac{1}{2}k_{\mathrm{mag}}\,\alpha_{\mathrm{err,mag}}(t)\right) \mathbf{x}_{\mathrm{corr,mag}}(t) \end{bmatrix}, \\ \alpha_{\mathrm{err,mag}}(t) &= \sphericalangle\left(\bar{\mathbf{m}}(t), {}^{\mathcal{S}}\mathbf{r}_{\mathrm{mag}}\right), \\ \mathbf{x}_{\mathrm{corr,mag}}(t) &= \frac{\bar{\mathbf{m}}(t) \times {}^{\mathcal{S}}\mathbf{r}_{\mathrm{mag}}(t)}{||\bar{\mathbf{m}}(t) \times {}^{\mathcal{S}}\mathbf{r}_{\mathrm{mag}}(t)||_2}, \end{aligned} \tag{12}$$

where $k_{\mathrm{mag}} \in [0, 1]$ is an adjustable sensor fusion weight. Note that the normalized vector product in the axis part of the correction quaternion is collinear with ${}^{\mathcal{S}}\mathbf{r}_{\mathrm{acc}}$. Therefore, the coordinate system is rotated around a vertical axis, which leaves the inclination portion of the orientation estimate unchanged. In Section 3 we will

[2] using for example the scalar product $\cdot$ and the standard formula $\sphericalangle(\mathbf{a}, \mathbf{b}) = \mathrm{acos}\left(\frac{\mathbf{a}\cdot\mathbf{b}}{||\mathbf{a}||_2\,||\mathbf{b}||_2}\right)$

demonstrate the efficacy of this approach by simulations of magnetic disturbances. Prior to this, we further enhance a few aspects of the proposed algorithm.

2.2 Bias Compensation by Integral Action

It has been pointed out in several previous contributions that integration drift cannot be compensated completely by a correction that is proportional to the observed disagreement. From a control engineer's point of view, this becomes evident if bias is interpreted as a step-type input disturbance on a plant with integral dynamics, for which we have so far only designed a proportional controller. Therefore, we propose the following bias estimation that exhibits integral action and is based on both observed disagreements:

$$\begin{aligned}\mathbf{b}(t) = \mathbf{b}(t-t_\mathrm{s}) &+ k_{\mathrm{bias,acc}}\, \alpha_{\mathrm{err,acc}}(t)\, \mathbf{x}_{\mathrm{corr,acc}}(t) \\ &+ k_{\mathrm{bias,mag}}\, \alpha_{\mathrm{err,mag}}(t)\, \mathbf{x}_{\mathrm{corr,mag}}(t),\end{aligned} \tag{13}$$

where $k_{\mathrm{bias,acc}}, k_{\mathrm{bias,mag}} \in [0,1]$ are adjustable gains. For each sampling interval, the estimated bias is used to calculate corrected angular rates

$${}^{\mathcal{S}}\breve{\mathbf{g}}(t) = {}^{\mathcal{S}}\mathbf{g}(t) + \mathbf{b}(t-t_\mathrm{s}). \tag{14}$$

By strapdown integration, we obtain an improved prediction ${}^{\mathcal{S}}_{\mathcal{E}}\breve{\mathbf{q}}_{\mathrm{gyr}}(t)$, which is then corrected as described by (10) and (12) above.

2.3 Interpretable Algorithm Parametrization

The sensor fusion algorithm that consists of (10), (12), and (14) balances between compensating integration drift and rejecting disturbances in the accelerometer and magnetometer readings. Obviously, this balance depends on the choice of the parameters $k_{\mathrm{acc}}, k_{\mathrm{mag}}, k_{\mathrm{bias,acc}}, k_{\mathrm{bias,mag}}$. The vast majority of methods proposed in the literature contain similar parameters, ranging from complementary filter weight to noise covariance in Kalman filter implementations. Interpretation of these parameters is often as difficult as assigning reasonable values to them. Moreover, in many cases, a change in the sampling frequency results in a need to adjust the filter weights.

For the present algorithm, we propose a parametrization that allows the user to choose a time scale and a level of aggressiveness for the correction. This is best explained using an example. Assume that the sensor is at rest, i.e. $\breve{\mathbf{g}}(t) = [0,0,0]^T$, but that a disagreement is observed between the prediction and the accelerometer readings, i.e. $\alpha_{\mathrm{err,acc}} > 0$. Then we would like to set the time constant τ_{acc} at which the orientation estimate converges to the orientation defined by the accelerometer readings. See Figure 4 for illustration. The larger τ_{acc} is, the less we trust the measured accelerations; or more precisely: we only trust their average over a larger period of time.

Another characteristic convergence parameter is associated with the bias estimation (13). While it helps to compensate drift errors, it may also lead to temporary overcompensation of the disagreements, which results in overshoot effects in the aforementioned convergence, as illustrated in Figure 4. Obviously, it is desirable to choose the characteristics ζ of this overshoot independently of the time scale τ_{acc}.

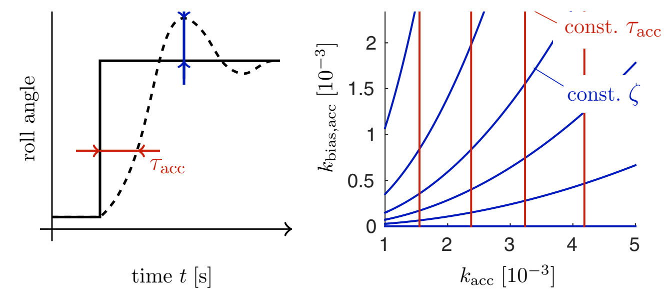

Fig. 4. When disagreements between the prediction and the measured acceleration occur, the convergence of the roll angle estimate can be characterized by a time constant τ_{acc} and an overshoot measure ζ, both of which depend on the correction gains $k_{\mathrm{acc}}, k_{\mathrm{bias,acc}}$.

We determine the convergence parameters τ_{acc} and ζ for a large set of combinations of $k_{\mathrm{acc}}, k_{\mathrm{bias,acc}}$. Based on the results of this process, which are depicted in Figure 4, we propose the following parameter mapping:

$$k_i = \frac{t_\mathrm{s}}{1.4\,\tau_i + t_\mathrm{s}}, \quad k_{\mathrm{bias},i} = \frac{\zeta^2}{160\,\tau_i} k_i, \quad \forall\, i \in \{\mathrm{acc}, \mathrm{mag}\},$$

wherein $\tau_{\mathrm{acc}}, \tau_{\mathrm{mag}}, \zeta$ are the new algorithm parameters and t_s is the sampling period. Note that the same mapping is proposed for the (mathematically equivalent) magnetometer-based correction. The same ζ value is chosen for both corrections, while different trust levels can be assigned to accelerometers and magnetometers by choosing different values for τ_{acc} and τ_{mag}.

In Section 3.2, we will verify that τ_{acc} and τ_{mag} are the approximate half-value periods of the corrections, while ζ characterizes the amount of overshoot independently of the chosen time constants and sampling periods.

3. EVALUATION OF THE METHOD

3.1 Analytical Solution vs. Gradient Descent

Above we briefly reviewed the approach of Madgwick et al. (2011) for assuring that magnetometer readings do not influence the inclination part of the orientation estimate. We then proposed an analytical solution for the sensor fusion problem and a modified projection of the magnetometer readings. We claimed that this approach leaves the inclination portion of the orientation estimate unchanged. In order to verify this claim, we consider the following simple scenario:

- The local sensor coordinate system and the fixed reference frame coincide as the sensor rests for a very long period of time $t \in [t_0, t_{\mathrm{dist}}]$ with $t_0 << t_{\mathrm{dist}}$.
- During that period of time, the measured acceleration is perfectly vertical and the projection of the measured magnetic field vector into the horizontal plane points perfectly into positive x-direction of both coordinate systems.
- For $t > t_{\mathrm{dist}}$, the magnetic field vector is disturbed such that its magnitude and its z-component remain unchanged but the x- and y-component (precisely the magnetic field heading) take new constant values.
- Consider a weak and a strong disturbance, during which the heading of the magnetic field changes by 50° and by approximately 180°, respectively.

Figure 5 illustrates the sensor fusion strategies of the conventional gradient descent algorithm and the proposed analytical solution for this magnetic disturbance scenario. As discussed previously, both algorithms should trust the magnetometer to some extent. Therefore, in this scenario with a constant magnetic disturbance, every reasonable sensor fusion algorithm must eventually correct the predicted coordinate system such that its x-axis points into the direction of the new magnetic field vector. There is, however, a remarkable difference in the way both algorithms achieve this result. The conventional gradient descent algorithm corrects the predicted coordinate system in such a way that the reference vector and the measured vector approach each other on the shortest path, i.e. along the geodesic line between them. In contrast, the proposed novel algorithm applies a pure heading correction to the predicted coordinate system, i.e. it rotates it only around the vertical axis.

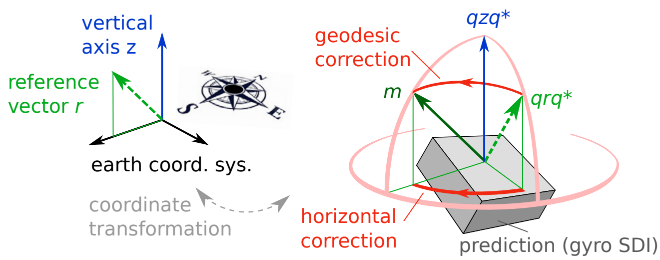

Fig. 5. When the magnetic field is disturbed, the predicted coordinate system disagrees with the magnetometer readings in the sense that the measured magnetic field vector ${}^{\mathcal{S}}\mathbf{m}(t)$ does not coincide with the transformed reference vector ${}^{\mathcal{S}}\mathbf{r}_{\mathrm{mag}}$. While the conventional gradient-descent correction corresponds to a correction along the shortest path (geodesic line), the proposed novel algorithm uses a pure heading (yaw) correction around the vertical axis.

Figure 6 shows exemplary results of both algorithms for the described magnetic disturbance scenario. The parameters of both algorithms were chosen to yield reasonably fast convergence (see dashed lines in figure) for the weak disturbance. The roll angle estimate of the conventional algorithm is affected by 4°. Moreover, for the strong disturbance described in the scenario above, the pitch angle is heavily disturbed from the very first seconds. Even if the considered magnetic disturbance would last only a few seconds, the accuracy of the pitch angle estimate would still be severely deteriorated.

As postulated in Section 2, the proposed novel algorithm exhibits none of this undesirable behavior. Regardless of the disturbance magnitude, the yaw angle converges smoothly with the same time constant, and the pitch and roll angle estimates remain unaffected.

3.2 Validating the Usefulness of the Parametrization

The scenario described in Section 3.1 is considered again, but with the slight modification that a gyroscope bias of $2\,°/\mathrm{s}$ is added for $t > t_{\mathrm{dist}}$. The proposed algorithm is employed with a large number of combinations of $\tau_{\mathrm{acc}}, \tau_{\mathrm{mag}}, \zeta$, and the convergence of the yaw angle estimate is analyzed for all of these combinations. Furthermore, all

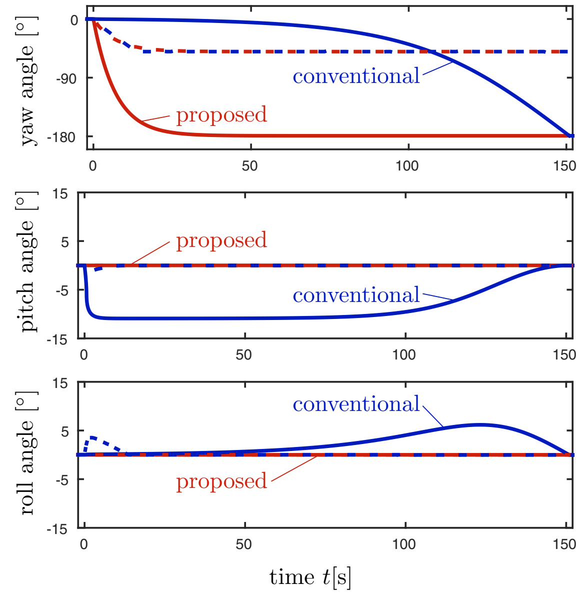

Fig. 6. Reaction of the conventional and the proposed algorithm to a weak (dashed) and a strong (solid) magnetic disturbance. Both algorithms eventually align with the new magnetic field direction, i.e. the yaw reaches approx. $-180°$. But the roll and pitch angle are severely affected when using the conventional gradient-descent algorithm. In contrast, the proposed novel method modifies only the yaw angle,

combinations are analyzed for different sampling periods. Figure 7 shows exemplary results of this parameter study.

Independent of ζ and t_{s}, the observed convergence time constant is in well accordance with the user-defined parameter τ_{mag}. Likewise, ζ well describes the amount of overshoot when set to values larger than zero. As postulated in Section 2.2, a disagreement remains for all time if ζ is set to zero. As the previously mentioned analogy (proportional control of an integral plant with input disturbance) suggests, this remaining disagreement is larger for smaller correction gains, i.e. for larger values of τ_{mag}.

For the accelerometer-based correction, an analogous study yields equivalent results, which are therefore not presented here.

4. SUMMARY AND CONCLUSIONS

We have considered the standard sensor fusion problem in orientation estimation of inertial measurement units. We highlighted the malicious effects that inhomogeneous magnetic fields, which are often found in indoor environments, can have on the inclination (roll and pitch) portions of the orientation estimate. We proposed a novel method that uses an analytical solution of the sensor fusion problem in combination with purely horizontal magnetometer-based corrections. The method assures that magnetic field measurements affect only the heading (yaw) component of the orientation estimate. Furthermore, the proposed parametrization of the proportional-integral correction scheme allows the user to choose the time constant and aggressiveness (amount of overshoot) with which the algorithm balances between gyroscope drift compensation and rejection of magnetic disturbances and velocity changes.

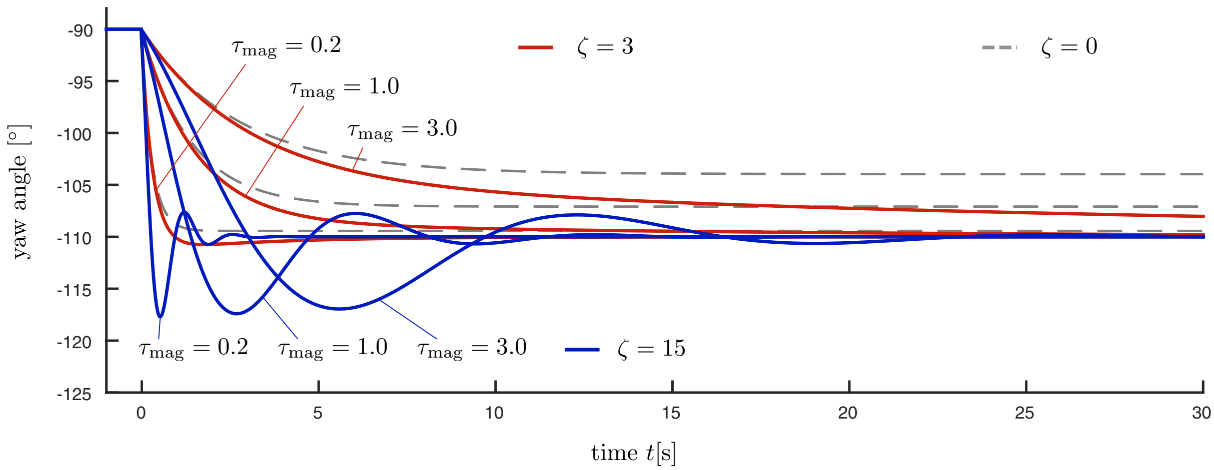

Fig. 7. Convergence of the yaw angle after disagreement between prediction and magnetometer readings. The user-defined algorithm parameters τ_{mag} and ζ well describe the convergence characteristics (i.e. the time constant and the amount of overshoot) of the sensor fusion.

We conclude that the proposed algorithm can be used in outdoor *and* indoor environments. If (at least the heading of) the magnetic field is homogeneous, then the entire orientation estimate will be highly accurate. If the magnetic field is heavily disturbed, only the yaw portion of the orientation estimate will be affected. Since the algorithm parameters are designed to correlate directly with important features of the sensor fusion, they are easily adjusted to the individual circumstances.

As a consequence of the discussed arguments, we consider the new method highly suitable for many applications of inertial sensors including human motion analysis in biomedical or sports context. Our current and future work is concerned with dynamic adaptation of the fusion weights, optimization of the computational load, and with validation against optical motion capture systems.

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