The Solow Model with Human Capital

Human Capital

- ▶ Human capital refers to anything embodied in people that makes them productive at work.
- Examples:

education experience

Adding Human Capital to the Growth Model

- ▶ Need to include human capital in the production function.
- ▶ Useful to stay close to the Solow model:
 - ► Solow model does a good job at accounting for growth in rich countries.
 - Don't want to lose that feature!
- ▶ Thus, continue to use Cobb-Douglas production function.
- ▶ But use a more general concept of labor supply.

- Adding Human Capital to the Growth Model

 Still have physical capital K_t and labor force (number on workers) N_t .

 Introduce new concept of human capital (per worker):

 Human capital measured by:

 If each worker has human capital h_t , human capital for the whole economy is:

 When the still have physical capital K_t and labor force (number on workers):

 Human capital measured by:

 Use this in Cobb-Douglas production function:

- Fraction of time spent working (as opposed to education):
- ► For now, let's abstract from productivity growth:

The Human Capital Model

- ► To complete the model, also need to specify laws of motion for physical and human capital.
- ▶ Use standard law of motion for physical capital:

$$K_{t+1} = (1 - \delta)K_t + sY_t.$$

- Assumption on production of human capital: Human capital is produced using existing human capital (abstract from physical input).
- Law of motion: gnotity of education

 Htt: 2 B(1- Ux) Ht

The Predictions of the Human Capital Model

► Key feature: Human capital grows on its own and does not interact with physical capital:

Human capital grows at rate:

Accumulation of physical capital works just as in the Solow model: permanent rureare in human capital equivalent to permanent rureare in productivity.

The Steady State for Physical Capital

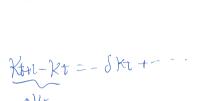
Httl: B(1-u)Ht. Physical capital will approach a steady state where the ratio of physical to human capital is constant. Define:

Law of motion in effective units:

$$K_{t+1} = (1-8) K_t + sK_t + (uH_t)^{1-\alpha},$$

$$k_{t+1} = (1-8) k_t + sK_t + sK_$$

This is the same as in the Solow model with positive productivity growth.



The Steady State for Physical Capital

The steady-state
$$\bar{k}$$
 satisfies $\bar{k} = \bar{k}$

The steady-state
$$\bar{k}$$
 satisfies: $\bar{k} = 80$

 $\bar{k} = B(1-h)$ $[(1-\delta)]$ h + s k q q(Bl1-U)-1+8) / = s 2× $\sqrt{k} = \left(\frac{S}{B(1-n)-1+\delta}\right)^{\frac{1}{2}} = \frac{k+1}{n+1} \left(\frac{physical capital}{human capital}\right)$ kt - WHt

| productivity > V kt.

The Steady State for Output

Output in the steady state is:

$$Y_{t} = R_{t}^{\alpha} (uH_{t})^{1-\alpha}$$

$$= \frac{R_{t}^{\alpha} (uH_{t})^{1-\alpha}}{uH_{t}} wH_{t}.$$

$$= \frac{R_{t}^{\alpha} uH_{t}}{uH_{t}^{\alpha}} wH_{t}$$

$$= \frac{R_{t}^{\alpha} uH_{t}}{uH_{t}^{\alpha}} wH_{t}.$$

- In the simple case where $N_t=1$ (constant population), this is also income per worker.
- ► Thus, outcome per worker in the long run is proportional to the level of human capital.

The Long-Run Growth Rate of Output

- ► The growth rate of output is the same as the growth rate of human capital:
- Important difference to Solow model: on people's actions! (investment rate in human capital 1-u).
- ► That's why we call this model an "endogenous growth" model.
 - 2 revisions of solor with in human capital.

 1 human capital accumulation affects the growth rate of GDP/GDP per person.

 2 human capital affect a gapita

Key Predictions of the H ${}^{'}$ uman C ${}^{'}$ pital Model - 1

Key Predictions of the Human Capital Model – 2