- Differentiate.
- Integrate.
- Maximize a func
- Calculate area under a curve.
- sum of an infinite series.

What is probability?

concrete example

40 balls in a urn, label 1 ~ 40.

chance?

① sum of 2 balls' num is 30

② the max of 2 ball is 10.

⇒ Experiment.

- random;
- repeatable;

e.g. Tossing a coin one time of several times

outcome space, Ω, is the set of all possible outcomes of an experiment.

⇒ Event: a subset of Ω

e.g. toss a coin two times and record the face up.

$\Omega = \{HH, TT, HT, TH\}$

A: the # of heads land up is ≥ 1

$A = \{HH, HT, TH\}$

e.g. An experiment in hospital of recording the sex of each newborn

infant until the birth of a male is observed. outcome space ?

$\{M, FM, FFM, \cdots\}$

infinite outcome space.

If Ω consists of finite number of outcomes, and each outcome is equally likely,

the $P(A) = \frac{\#(A)}{\#(\Omega)}$

where $\#(A)$ is the number of outcomes in A.

e.g. P(at least (A) one head when tossing coin 2 times) $= \frac{\#(A)}{\#(\Omega)} = \frac{3}{4}$.

Remark: ① $0 \le P(A) \le 1$.

② $P(A) = 0$, $P(\emptyset) = 0$ ($\emptyset$: empty set.) $A = \{\}$ impossibility.

③ $P(\Omega) = 1$ certainty

eg. 1: Suppose two dice are rolled. Find the probability

① the sum of the two number rolled is 6.

② the max of the two number rolled is less than / equal to 2.

(fair dice, six sided)

① $A = \{(1,5), (5,1), (3,3), (2,4), (4,2)\}$

$\#(\Omega) = 6 \times 6 = 36$

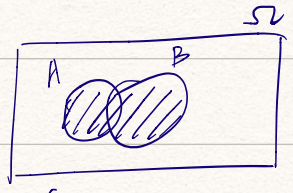
$P(A) = \frac{\#(A)}{\#(\Omega)} = \frac{5}{36}$

② $A = \{(1,2), (1,1), (2,2), (2,1)\}$

$P(A) = \frac{4}{36} = \frac{1}{9}$

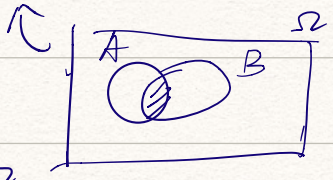
⇒ Operation on Events.

- A, B



① Union, $A \cup B$, the union of events A and B is the event consisting of all outcomes either in A or in B or in both events.

② Intersection, $A \cap B$, the event consisting of all outcomes in both A and B.



③ complement, A^c, is the set of all outcomes in Ω but not in A

E.g. We toss a coin 10 times and observe the number of heads on.

$\Omega = \{0, 1, \cdots 10\}$

Denote A - even number of heads up $= \{0, 2, 4, 6, 8, 10\}$

B - number of heads up ≤ 5. $= \{0, 1, 2, 3, 4, 5\}$

C - number of heads up ≥ 6. $= \{6, 7, 8, 9, 10\}$

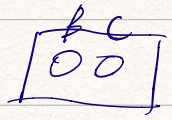
$A \cup B = \{0, 1, 2, 3, 4, 5, 6, 8, 10\}$

$A \cap B = \{0, 2, 4\}$

$B \cap C = \phi$

$A^c = \{1, 3, 5, 7, 9\}$

⇒ If $BC = \phi$, then B, C are mutually exclusive or disjoint



⇒ properties

① $A \cup B = B \cup A$

$A \cap B = B \cap A$

② $(A \cup B) \cup C = A \cup (B \cup C)$

$(A \cap B) \cap C = A \cap (B \cap C)$

③ $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$.

$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

④ $(A \cap B)^C = A^C \cup B^C$

$(A \cup B)^C = A^C \cap B^C \rightarrow$