The Solow Model with Human Capital

Human Capital

- Human capital refers to anything embodied in people that makes them productive at work.
- Examples:

education

experience

Adding Human Capital to the Growth Model

- Need to include human capital in the production function.
- Useful to stay close to the Solow model:
  - Solow model does a good job at accounting for growth in rich countries.
  - Don't want to lose that feature!
- Thus, continue to use Cobb-Douglas production function.
- But use a more general concept of labor supply.

Adding Human Capital to the Growth Model

Human capital per worker h

$H_t = N_t h_t.$

$Y_t = A K_t^{\alpha} (H_t)^{1-\alpha}$

$= A K_t^{\alpha} (u_t H_t)^{1-\alpha}$

$\frac{\text{time school}}{\text{time work}}$

- Still have physical capital K_t and labor force (number of workers) N_t.
- Introduce new concept of human capital (per worker):
- Human capital measured by:
- If each worker has human capital h_t, human capital for the whole economy is:
- Use this in Cobb-Douglas production function:
- Fraction of time spent working (as opposed to education):
- For now, let's abstract from productivity growth:

The Human Capital Model

- To complete the model, also need to specify laws of motion for physical and human capital.
- Use standard law of motion for physical capital:

$$K_{t+1} = (1 - \delta)K_t + sY_t.$$

- Assumption on production of human capital: Human capital is produced using existing human capital (abstract from physical input).
- Law of motion:

quality of education

$$H_{t+1} = B(1 - u_{\cancel{t}})H_t$$

- $$= (B(1-u))^t H_0,$$

- $$\frac{H_{t+1} - H_t}{H_t} \qquad B(1-u) - 1$$

time education

The Predictions of the Human Capital Model

- Key feature: Human capital grows on its own and does not interact with physical capital:

$$H_{t+1} = B(1-u)H_t$$

$$H_{t+1} = (B(1-u))^t H_0$$

- Human capital grows at rate:

$$B(1-u) - 1$$

- Accumulation of physical capital works just as in the Solow model: permanent increase in human capital equivalent to permanent increase in productivity.

The Steady State for Physical Capital

$K_t = k_t \, uH_t$

$H_{t+1} = B(1-u)H_t$.

- Physical capital will approach a steady state where the ratio of physical to human capital is constant. Define:

$$k_t = \frac{K_t}{uH_t}$$

human capital used in productivity.

$\underbrace{K_{t+1} - K_t}_{\Delta K_t} = -\delta K_t + \cdots$

- Law of motion in effective units:

$$K_{t+1} = (1-\delta)K_t + sK_t^{\alpha}(uH_t)^{1-\alpha}.$$

$$k_{t+1}\, uH_{t+1} = (1-\delta)k_t uH_t + s(k_t uH_t)^{\alpha}(uH_t)^{1-\alpha},$$

$$\cdots B(1-u)H_t u = \cdots$$

$$k_{t+1} = \frac{1}{B(1-u)}\left[(1-\delta)k_t + sk_t^{\alpha}\right]$$

additional to?

- This is the same as in the Solow model with positive productivity growth.

The Steady State for Physical Capital

$k_{t+1} - k_t = \Delta k_t = 0$

$k_{t+1} = k_t = \bar{k}$

- The steady-state $\bar{k}$ satisfies:

$$\bar{k} = \frac{1}{B(1-u)} \left[(1-\delta)\bar{k} + s\bar{k}^{\alpha} \right].$$

$$(B(1-u) - 1 + \delta)\bar{k} = s\bar{k}^{\alpha}$$

$$\bar{k} = \left(\frac{s}{B(1-u)-1+\delta} \right)^{\frac{1}{1-\alpha}} \Rightarrow \frac{K_t}{uH_t} \left(\frac{\text{physical capital}}{\text{human capital}} \right)$$

same growth rate.

$$k_t = \frac{K_t}{uH_t}$$

$\uparrow$ productivity $\Rightarrow \downarrow k_t$.

The Steady State for Output

- Output in the steady state is:

$$Y_t = K_t^{\alpha} (uH_t)^{1-\alpha}.$$
$$= \frac{K_t^{\alpha} (uH_t)^{1-\alpha}}{uH_t} uH_t.$$
$$= \frac{K_t^{\alpha}}{uH_t^{\alpha}} uH_t$$
$$= \bar{k}^{\alpha} uH_t$$

- In the simple case where $N_t = 1$ (constant population), this is also income per worker.
- Thus, outcome per ~~worker~~ gdp in the long run is proportional to the level of human capital.

The Long-Run Growth Rate of Output

- The growth rate of output is the same as the growth rate of human capital: $\frac{\gamma_{t+1}}{\gamma_t} - 1 = B(1-u) - 1$
- Important difference to Solow model: on people's actions! (investment rate in human capital $1-u$).
- That's why we call this model an "endogenous growth" model.

2 versions of solow with in human capital.

① human capital accumulation affects the growth rate of GDP / GDP per person.

② human capital affect a $\frac{GDP}{capita}$

Key Predictions of the Human Capital Model – 1

Key Predictions of the Human Capital Model – 2