Lecture Note on

# Human Capital and Growth*

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The neoclassical growth model is at the same time a remarkable success and a remarkable failure. The success is, of course, that the model is able to account for each and every one of the stylized facts of growth in developed countries. In all appearance, the model captures some key characteristics of the actual economic system of advanced industrial countries. For this reason, the neoclassical growth model is the underlying structure used for most macroeconomic research concerning developed countries.

The neoclassical growth model is also a failure, however, because it does not provide any explanation for the lack of convergence between rich and poor countries during the last 50 years. Evidently, there is something that keeps most of the world from growing, and whatever it is, it is not captured in the neoclassical growth model.

The question arises whether we need to abandon the standard growth model completely in order to understand stagnation in developing countries. In this section, we will see that it is possible to extend the standard model in a way that keeps all its implications for rich countries intact, while also offering an explanation for stagnation in the poorer countries of the world. The key ingredient for this explanation is human capital.

In order to develop a growth theory that accounts for the role of human capital, a natural starting point is the aggregate production function. Assume that the production function takes the form:

$$Y_t = K_t^{\alpha}(uH_t)^{1-\alpha}.$$

Here $K_t$ is physical capital, $H_t$ is human capital, and $u$ is the fraction of time spend in production (the alternative is education). Capital evolves according to the usual law of motion:

$$K_{t+1} = (1-\delta)K_t + sY_t. \tag{1}$$

The law of motion for human capital takes a different form. Unlike physical capital, human capital is not just a part of output that gets converted into productive capital. Rather, producing new human capital requires the use of existing human capital. Human capital is produced through education, and existing knowledge is the key ingredient in the production of education. We therefore assume that future human capital is produced using only human, but not physical capital. The amount of future human capital depends on the fraction of time $1-u$ spent on education:

$$H_{t+1} = B(1-u)H_t. \tag{2}$$

Here $B$ is a measure of the overall productivity of the education sector.

The human capital economy has the feature that output grows continuously, as opposed to approaching a steady state. The evolution of human capital is easy to determine. According to (2) $H_t$ grows at the constant rate $\gamma$:

$$\frac{H_{t+1}}{H_t} - 1 = B(1-u) - 1 = \gamma.$$

To analyze the dynamics of physical capital, it is once again useful to rewrite the law of motion in terms of a variable that is constant in the long run. In this model, such a variable is given by the ratio of physical to human capital. Let:

$$k_t = \frac{K_t}{H_t}.$$

Dividing both sides of (1) by $H_t$ gives:

$$\frac{K_{t+1}}{H_t} = (1-\delta)\frac{K_t}{H_t} + s\left(\frac{K_t}{H_t}\right)^{\alpha} u^{1-\alpha}.$$

Since $H_{t+1} = (1+\gamma)H_t$, this equation can be written as:

$$\frac{K_{t+1}}{H_{t+1}/(1+\gamma)} = (1-\delta)\frac{K_t}{H_t} + s\left(\frac{K_t}{H_t}\right)^{\alpha} u^{1-\alpha}$$

or:

$$(1+\gamma)k_{t+1} = (1-\delta)k_t + sk_t^{\alpha}u^{1-\alpha}.$$

We therefore arrive at a law of motion that takes a familiar form. It implies that there exists a steady state $\bar{k}$ at which $k_t$ is constant. Regardless of the initial position, $k_t$ converges to $\bar{k}$, that is, $k_t$ increases over time if $k_t < \bar{k}$, and it decreases for $k_t > \bar{k}$.

To solve for the steady state, we plug $\bar{k}$ into both sides of the law of motion:

$$\begin{aligned}
(1+\gamma)\bar{k} &= (1-\delta)\bar{k} + s\bar{k}^{\alpha}u^{1-\alpha}, \\
(\gamma+\delta)\bar{k} &= s\bar{k}^{\alpha}u^{1-\alpha}, \\
\bar{k} &= \left(\frac{s}{\gamma+\delta}\right)^{\frac{1}{1-\alpha}} u.
\end{aligned}$$

To see what happens to output in the steady state, it is useful to rewrite the production function in terms of $k_t$:

$$\begin{aligned} Y_t &= K_t^\alpha (uH_t)^{1-\alpha} \\ &= K_t^\alpha (uH_t)^{1-\alpha} \frac{H_t^\alpha}{H_t^\alpha} \\ &= \left(\frac{K_t}{H_t}\right)^\alpha u^{1-\alpha} H_t \\ &= k_t^\alpha u^{1-\alpha} H_t \end{aligned}$$

Therefore, in the steady state when $k_t$ is constant, output $Y_t$ grows at the same rate $\gamma$ as human capital, namely, $\gamma = B(1-u) - 1$. Notice that even though the growth rate is constant, it does depend on an action of the people in the economy, namely, the education time $1 - u$. This is a crucial difference to the Solow model. In the Solow model, the long-run growth rate was determined by productivity growth, and there was no possibility of altering the growth rate. Since the growth rate is not determined inside the model, the Solow model is often classified as an *exogenous growth model*. In the human capital model, the growth rate depends on the education time, and therefore in effect can be chosen by the people in the model economy. Therefore, the human capital model is counted among the *endogenous growth models*.

What does the human capital model imply for growth? Consider first the case of two countries A and B with the same amount of human capital, $H_A = H_B$. Let us also assume that the two countries are identical in every respect apart from their initial level of physical capital. Since the growth rate of human capital is constant and identical in the two economies, we have the immediate conclusion that the level of human capital in the two economies will always be the same. We also know that the ratio of physical to human capital $k_t$ is constant in the long run as well. Therefore, in the steady state both economies have identical amounts of human and physical capital, and therefore the same level of output. If one country should happen to start with a lower level of physical capital, it will grow faster during the transition to the steady state and ultimately converge to the other country. In other words, for countries with the same level of human capital the model works just like the Solow model. This feature is one of the main advantages of the human capital model: it leads to additional implications without sacrificing any of the successes of the Solow model in terms of explaining growth in rich countries.

The situation is different if two countries start out with different initial levels of human capital. Assume, for example, that country B has only half as much human capital as country A. Since human capital grows at the same constant rate in both countries, the ratio of human capital between the countries will always be two to one. Since in each country, the ratio of physical to human capital approaches a constant, the long-run ratio of physical capital between the two countries will be two to one as well. In other words, the country that starts out with less human capital never catches up to the rich country: convergence does not occur.

We therefore see that the human capital is, in principle, consistent with most of the evidence on the world income distribution. The model does predict convergence among rich countries (under the assumption that they have similar levels of human capital), but also relative stagnation of poor countries that start with little human capital. The predictions for poor countries are not entirely pessimistic, however. Since the growth rate is endogenous, the model points to a potential path out of poverty: a country that invests a higher share of resources into education can increase its long-run growth rate, and thereby converge to rich countries even if initial human capital is lower.

In practice, growth accounting exercises show that the human capital model overstates the role of human capital, in the sense that there are sizable cross-country differences in income levels that are explained neither by physical nor by human capital, and that investing in education alone does not seem to be sufficient to enhance growth in poor countries. Nevertheless, accounting for human capital is an important first step towards answering the questions about international income differences left open by the Solow model.

## References

LUCAS, R. E. (1988): "On the mechanics of economic development," *Journal of Monetary Economics*, 22, 3–42.