Propositions and Propositional Logic

Propositions

Goal: reason about discrete structures in a rigorous manner

Proof: chain of logic deductions. starting from some axioms and ending with proposition.

Axiom: basic facts that we take for grants; underlying assumptions.

e.g. If k is an even integer, then k is equal to 2i for some integer i

Proposition: A statement that is either true or false (not both)

Which of the following statements are propositions?

Truth Value:

1. Mark Hill is the chair of the CS Department. yes → false
2. Today is not Thursday. yes → True.
3. opinion ⇒ Serena Williams is the greatest women's tennis player ever. X deal with in 240.
4. It is snowing. Yes → False.
5. The matching algorithm described earlier produces a stable matching.
6. This sentence is false. no, paradox → can't assign T/F.
7. This sentence is true. no, paradox → can't assign single T/F.

self-referencial

Operations on propositions

High precedure

negation: ¬ not $\neg p$

conjunction: ∧ and $p \wedge q$

disjunction: ∨ or $p \vee q$

implication: ⇒ imply $p \rightarrow q$ if p, then q

premise, consequence

e.g. if you get ≥95%, you will get an A.
q, p

P	Q	P ⇒ Q
T	T	T
T	F	F
F	T	T
F	F	T

if premise is false, $p \Rightarrow q$ is true.

equivalence (or biconditional): ⇔ $p \Leftrightarrow q$ if and only if. (iff). $P \Rightarrow Q$ and $Q \Rightarrow P$.

P	Q	P ⇔ Q
T	T	T
T	F	F
F	T	F
F	F	T

Low precedure

Precedence

Propositional formulas

zybook 2.5

Properties of operators

Some of the most useful:

- associative $(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(\wedge)$
- commutative $p \vee q \equiv q \vee p$ $(\wedge)$
- distributive $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$, $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
- DeMorgan's Laws
- conditional identities $p \Rightarrow q \equiv \neg p \vee q$; $p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$

Examples

1. (¬A) ∨ B $\equiv A \Rightarrow B$
2. (A ⇒ B) ∧ (B ⇒ A) $\equiv A \Leftrightarrow B$
3. (A ∧ (A ⇒ B)) ⇒ B $\equiv (A \wedge (\neg A \vee B)) \Rightarrow B \equiv (A \wedge \neg A) \vee (A \wedge B) \Rightarrow B \equiv (A \wedge B) \Rightarrow B \equiv (\neg A \vee \neg B) \vee B \equiv T$
4. (A ∨ B) ∧ (A ∨ ¬B) ∧ ¬A contradiction

A	B	¬A	①	≡ A⇒B	B⇒A	② ≡ P⟺q	(A∧(A⇒B))	③
T	T	F	T	T	T	T	T	T
T	F	F	F	F	T	F	F	T
F	T	T	T	T	F	F	F	T
F	F	T	T	T	T	T	F	T

↑ tautology
always false: contradiction

Logical equivalence

A ≡ B iff they have exact same truth table

symbol: ≡

Show A ⇒ B is logically equivalent to ¬B ⇒ ¬A contrapositive. of A ⇒ B.

Show A ⇒ B is ***not*** logically equivalent to B ⇒ A converse

A	B	A⇒B	≡ ¬B⇒¬A	B⇒A
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Show (A ∨ B) ⇒ C is logically equivalent to (A ⇒ C) ∧ (B ⇒ C)

$(A \vee B) \Rightarrow C \equiv \neg(A \vee B) \vee C$ conditional identity

$\equiv (\neg A \wedge \neg B) \vee C$ D. morgen law

$\equiv C \vee (\neg A \wedge \neg B)$ commutive law.

$\equiv (C \vee \neg A) \wedge (C \vee \neg B)$ distributive law

$\equiv (\neg A \vee C) \wedge (\neg B \vee C)$ commutive law.

$\equiv (A \Rightarrow C) \wedge (B \Rightarrow C)$ conditional identity.

A	B	C	$A \vee B$	$(A \vee B) \to C$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	F	T

总

operation and propositions. $\neg \quad \wedge \quad \vee \quad \to \quad \leftrightarrow$

propositional formulas.

tautology: always true

contradition: always false

contrapositive: $P \Rightarrow Q$: $\neg Q \Rightarrow \neg P$.

converse $Q \Rightarrow P$.

logical equivalence : $\equiv$