上节:

$n=2$ 2 buyers $v \in \{3, 6, 10\}$

$\{3, 6, 10\}$	probability (likehood)	no reserve $(=r=3)$			$r=6$			$r=10$	
$(3,3)$	$\frac{1}{9}$	3			0			0	
$(3,6)$	$\frac{2}{9}$	3	$\frac{5}{9}$		6			0	
$(3,10)$	$\frac{2}{9}$	3		$\frac{43}{9}$	6	$\frac{7}{9}$	$\frac{52}{9}$	10	$\frac{50}{9}$
$(6,6)$	$\frac{1}{9}$	6	$\frac{3}{9}$		6			0	
$(6,10)$	$\frac{2}{9}$	6			6			10	
$(10,10)$	$\frac{1}{9}$	10	$\frac{1}{9}$		10	$\frac{1}{9}$		10	

So, the optimal reserve is 6.

for 2 bidder is 6

1 bidder is 6.

try 3 bidder?

Resever price in SPA the same for every n? and if so, what is the optimal reserve?

$n=1$ $v \sim$ (distributed) distribution funtion $F(v)$, $F'(v) = f(v)$ (continnous function)

$f(v) > 0$ on $[\underline{v}, \bar{v}]$, $f(v) = 0 \mid v \notin [\underline{v}, \bar{v}]$ (support)

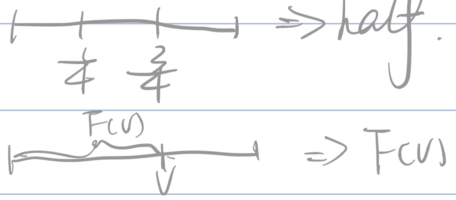
(Explain distribution function) $F(v) = \mathbb{P}[\hat{v} \leq v] \Rightarrow$ between 0, 1.

$v \in \{3, 6, 10\} \Rightarrow F(v) = \begin{cases} 0 & v < 3 \\ 1/3 & 3 \leq v < 6 \\ 2/3 & 6 \leq v < 10 \\ 1 & v \geq 10 \end{cases}$

Ex: valuation are uniformly distributed on $[0, 1]$.

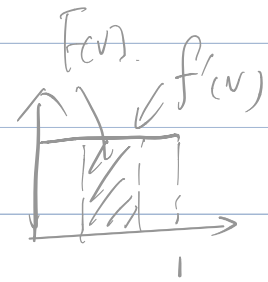
(likehood of picking a point is zero)

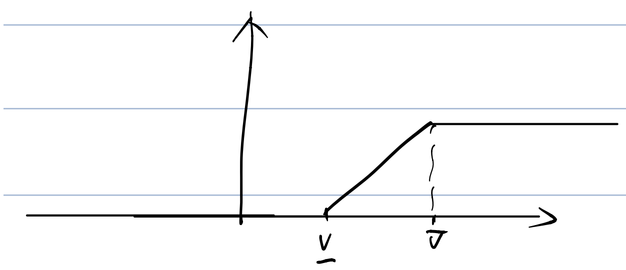
↳ likelihood of picking a point is zero.



$\Rightarrow F(v) = v$

$F'(v) = f(v) = 1$.





suppose seller set a reserve r.

$v < r$, seller get nothing $0 \times F(r)$

$v > r$ get $(r-c)(1-F(R))$

$\pi(R) = 0 F(r) + (r-c)(1-F(r))$.

Which is optimal r?

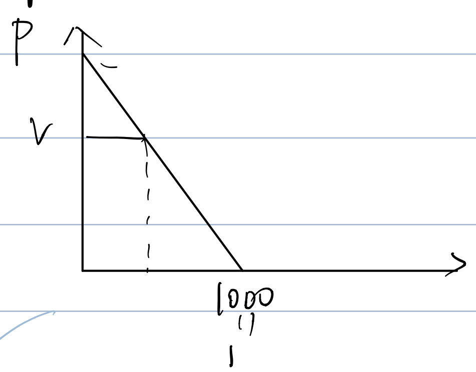
$\pi'(r) = 0$ marginal.

$= \underbrace{(1-F(r))}_{\text{Marginal Benefit}} + \underbrace{(r-c)(-f(r))}_{\text{Marginal cost}} = 0$ $\quad \leq f(v)$.

one buyer with continuous range valuation

buyers with continuous valuation

Monopolic pricing. same problem with auction with one bidder.



$F(v)$ = fraction of customs valuations $\leq v$

$D(v) = (1-F(v))$ is demand at price v.

└ Demand Function.

$\pi(r) = (r-c)(1-F(r))$

lose buyers

gain more from existing buyers.

SPA with $n > 1$ bidder.

expected profits of a seller who sets a reserve price r.

2 possibility

① either reserve price r is bunding ← revenue get from is reserved p. for winning bidder.

probability ↑

winning bidder's valuation $> r$.

all losing - - - - - - - - - - $\le r$.

If bidder 1 is winning bidder.

$\mathbb{P}[v_1 \ge r, v_2 \le R \cdots v_m \le R]$

Satistical independence

Two random x, y.

$H(x, y) = \mathbb{P}[\check{x} \le x, \check{y} \le y]$

$F(x) = \mathbb{P}[\check{x} \le x]$ marginal distribution of x.

$G(x) = \mathbb{P}[\check{y} < y]$ of y.

The random variables $\check{x}$ and $\check{y}$ are independent

if $H(x, y) = F(x)\, G(x)$

new — ② reserve price is not bunding for winning bidder.