

RET

In every standard auction, the expected payment of a bidder of valuation v equals $t^*(v) = \int_{\underline{v}}^v x g(x) dx$

$$g(x) = G'(x)$$

$$G(x) = F^{n-1}(x) \rightarrow \text{depend on number of bidder.}$$

x on format of auction

and so the seller's expected revenue is the same.

△ important use: derive equilibrium bid function in SI

↳ First Price Auction:

$b^*(v)$ common equilibrium bid function.

expected payment of a bidder of type v .

$$t^*(v) = G(v) b^*(v) + (1 - G(v)) \cdot 0$$

↓
prob of winning

$$G(v) b^*(v) = \int_{\underline{v}}^v x g(x) dx.$$

$$\Delta \quad b^*(v) = \frac{\int_{\underline{v}}^v x g(x) dx}{G(v)}$$

(1) $b^*(v)$ is strictly increasing

(2) $b^*(v) < v$ for all $v > \underline{v}$

bid shading

$$3) F(v) = v \quad \underline{v} = 0, \bar{v} = 1$$

$$G(v) = v^{n-1}$$

$$g(v) = (n-1) v^{n-2}$$

$$\begin{aligned} \int_{\underline{v}}^v x g(x) dx &= \int_{\underline{v}}^v (n-1) x^{n-1} dx = (n-1) \frac{x^n}{n} \Big|_{\underline{v}}^v \\ &= \frac{(n-1) v^n}{n} \end{aligned}$$

$b^*(v) = \frac{n-1}{n} v \Rightarrow$ how much shades the bid.

$\hookrightarrow n \uparrow \rightarrow b^*(v) \uparrow$

All pay auction

$$b^*(v) = t^*(v) = \int_v^v x q(x) dx$$

ex. $F(v) = v$. ($v \in [0, 1]$)

$$b^*(v) = \int_v^v x q(x) dx = \frac{(n-1)v^n}{n}$$

Review

Start: efficiency

\hookrightarrow exchange market

Pareto efficiency (define efficiency)

Δ allocation that can not be pareto improved.

another allocation that makes no person worse off

and makes at least one person strictly better off.

characterize: if changed, at least one is worse off.

\hookrightarrow gains from trade

Δ if an allocation is PO, then it maximizes gains from trade.

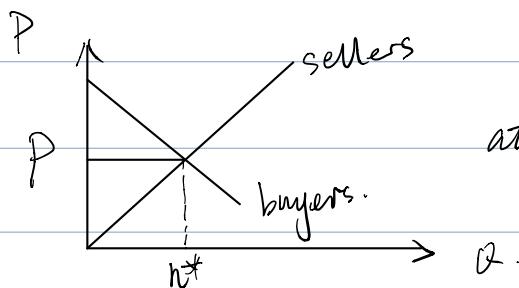
$$V(n) = \sum_{i=1}^n (v_i - c_i)$$

n^* maximizes $V(n)$ increasing n^* up to when $v_i - c_i < 0$.

\uparrow not uniquely determined if $v_i = c_i$ happens.

if volume is determined, the pattern is not uniquely determined.
(if $v_i \neq c_i$) (who hold the good)

Δ Correction between PO and CE (competitive equilibrium)



Bench Mark! under perfect info,

(many PO \rightarrow same \uparrow final holdings of goods).
highest valuations.

Will ever get there?

Brokers. monopoly (single broker necessarily match).

\hookrightarrow the objective: max brokerage commission (not for society).

{ fixed fee: δ
cost: k

2 effects

1) traditional monopoly: broker want to set $\delta > k$.

2) other effect: broker wants at high of a transaction volume as possible.

\swarrow achieve by opposite matching
(high $b \Leftrightarrow$ high s)

If $\delta \geq k$, the broker volume \geq efficient volume.

Asymmetric info \Rightarrow can be a barrier to trade.

intuition explicit the difference between the asking & offer price
{ buyer will shade the offer
seller will overstate the cost.

case where $v \in [0, 1]$, $c \in [0, 1]$

\Rightarrow revelation principle

$p(c, v)$ $t(c, v)$
 \uparrow \uparrow
prob of trade expected payment.

\rightarrow truth telling is DSE in deterministic direct mechanism.

iff it is a single price equilibrium (only one).

when is an asymmetric bad enough \Rightarrow inefficiency.

Auction

Δ SPA

① bid truthfully \Rightarrow DSE (why)

(bid not effect the price you pay).

② reserve price

③ optimal RP not depend on number of bidders.

bn !!!

\Rightarrow (marginal price of winning bidder is 0)

only when RP is binding.

$$\pi(r) = (r - c)(1 - F(r))$$

\uparrow RP is binding