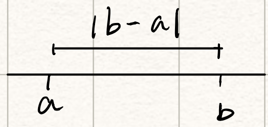
L4 01.29

§4 Graphs

non-examinable.

Real line

Distance



So, given $a, \varepsilon > 0$ $\{x \mid |x-a| < \varepsilon\}$.

Intervals

Let $a, b \in \mathbb{R}$

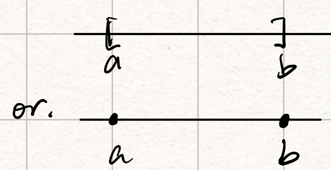
Open interval $(a,b) = \{x \mid a < x < b\}$.

If $b \leq a$, $(a,b) = \emptyset$.

Closed interval $[a,b] = \{x \mid a \leq x \leq b\}$.

If $b < a$, $[a,b] = \emptyset$

$b = a$, $[a,b] = \{a\}$.

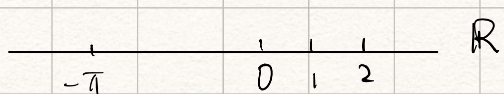


$(-\infty, a) = \{x \mid x < a\}$.

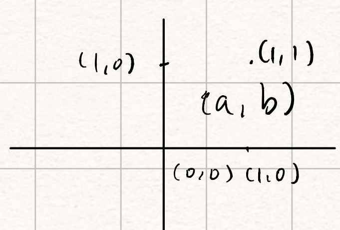
$[b, \infty) = \{x \mid x \geq b\}$.

L5 01.31

Graphs

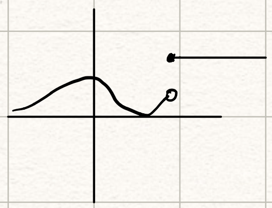


Plane $\mathbb{R}^2$



$\mathbb{R}^2 = \{(x,y) \mid x, y \in \mathbb{R}\}$.

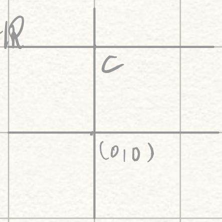
Graph in plane, for $f: \mathbb{R} \to \mathbb{R}$. $G_f = \{(x, f(x)) \mid x \in \mathbb{R}\}$.



As $f(x)$ is uniquely determined by x.

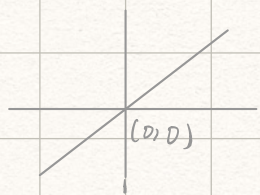
$(x, y_1), (x, y_2) \in G_f$ implies $y_1 = y_2 = f(x)$.

E.x. $f(x) = c$ $c \in \mathbb{R}$



$G_f = \{(x, c) \mid x \in \mathbb{R}\}$.

$f(x) = mx$, $m > 0$



$G_f = \{(x, mx) \mid x \in \mathbb{R}\}$.

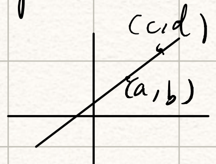
Not all curves are graph.

e.g. $x^2 + y^2 = 1$

$x = y^2$, not a graph for x, but a graph for y.

$\{(x, y) \mid x = y^2, y \geq 0\}$ is a graph over $\{x \geq 0\}$.

Straight Lines have a slope.



$$\text{slop} = \frac{(d-b)}{(c-a)} \quad \text{provided } c \neq a.$$

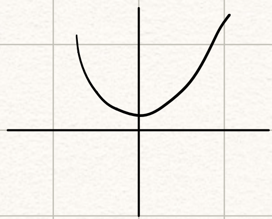
Distances in $\mathbb{R}^2$. Let $(a,b), (c,d) \in \mathbb{R}^2$

$$|(a,b) - (c,d)| = \sqrt{(c-a)^2 + (d-b)^2}$$

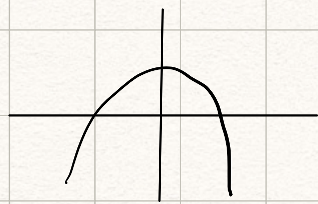
Polynomials

$f(x) = ax^2 + bx + c.$

$a > 0$

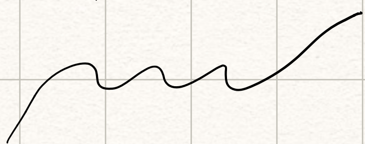


$a < 0$

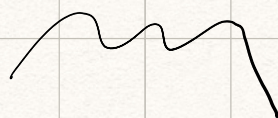


$p(x) = a_n x^n + \cdots \cdots + a_1 x + a_0.$

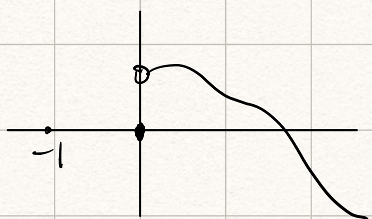
n add, $a_n > 0$



n even, $a_n < 0$



If $\mathrm{dom}(f) = \mathbb{R}$ $\quad G_f = \{(x, f(x)) \mid x \in \mathrm{dom}(f)\}$ $\quad \mathrm{dom}(f) = \{-1\} \cup \{x \geq 0\}.$



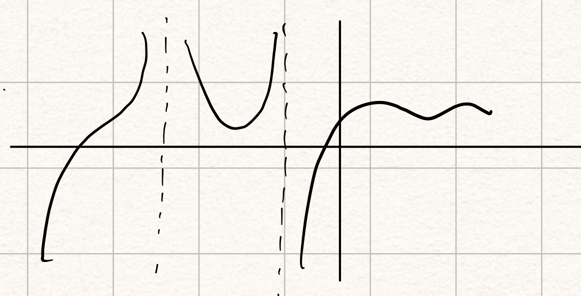
Rational Functions.

$\frac{p}{q}$, p, q poly.

domain $(\frac{p}{q}) = \{x \mid q(x) \neq 0\}.$

E.x. $f(x) = \frac{1}{x}$, $x \neq 0.$

Typically, Rational Functions looks like



Define f piecewise by $f(x) = \begin{cases} 1 & |x| \geq 1 \\ f(\frac{1}{n}) = (-1)^{n+1} & \text{for } n \in \mathbb{N}. \\ f(-\frac{1}{n}) = (-1)^{n+1} & \text{for } n \in \mathbb{N}. \end{cases}$

f is linear on each interval $[\frac{1}{n+1}, \frac{1}{n}]$, $[-\frac{1}{n}, -\frac{1}{n+1}]$.

E.x. i) Let $f(x) = \begin{cases} \sin(\frac{1}{x}), & x \neq 0 \\ 0 & , x = 0 \end{cases}$

$\sin(y) = 1 \quad y = \frac{\pi}{2} + 2k\pi.$

$\sin(\frac{1}{x}) = 1$ if $x = \frac{\pi}{2} k\pi$.

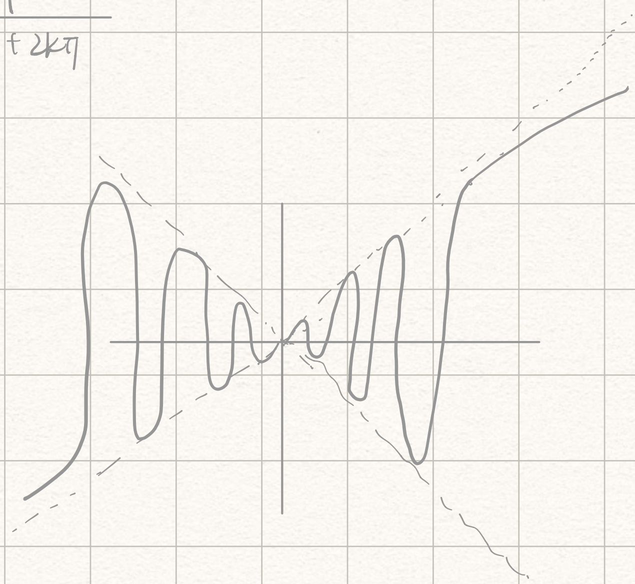
$\sin(\frac{1}{x}) = -1$ if $x = \frac{1}{\frac{3}{2}\pi + 2k\pi}$

ii) $g(x) = \begin{cases} x\sin(\frac{1}{x}), & x \neq 0 \\ 0 & x = 0 \end{cases}$

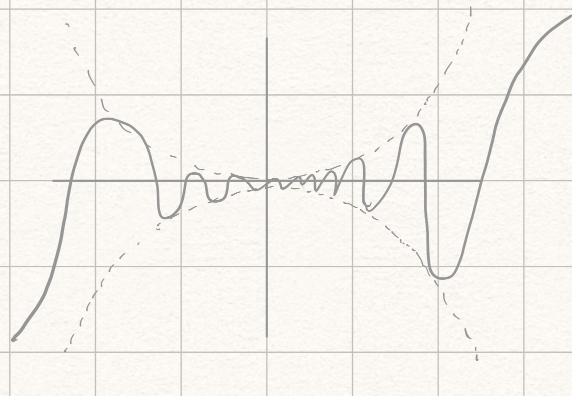
Note as $|\sin(y)| \leq 1$

$g(x) \leq |x| \, |\sin(\frac{1}{x})|$

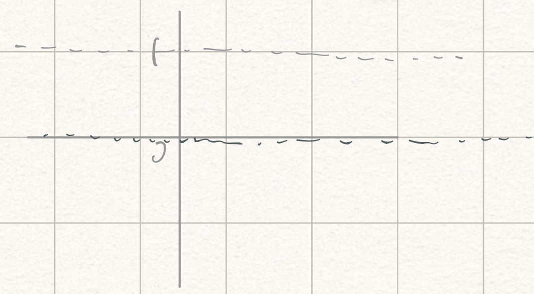
$\leq |x| \cdot 1 = x$



iii] $h(x) = \begin{cases} x^2 \sin(\frac{1}{x}) & x \neq 0 \\ 0 & x = 0 \end{cases}$



iv) $q(x) = \begin{cases} 1, & x \in Q. \\ 0, & x \notin Q. \end{cases}$



$$\begin{aligned} G_q &= \{(x, q(x))\} \\ &= \{(x, 1) \mid x \in Q\} \\ &\quad \cup \{(x, 0) \mid x \notin Q\}. \end{aligned}$$