Expected seller revenue in a SPA with revenue price r.

$$\pi(r) = \underbrace{n(r-c)(1-F(r))\,F(r)^{n-1}}_{\substack{\text{expected revenue from}\\ \text{binding reserve price}\\ \text{(only one bidder participate)}}} + \underbrace{\int_R^{\bar{v}} (v-c)\, f_{(2)}(v)\, dv}_{\substack{\text{expected revenue when reserve price}\\ \text{doesn't binding}}}$$

r↑ 前↑, 后↓

density function for SPA (evaluated for r).

$$f_{(2)}(r) = n(n-1)F^{n-2}(r)(1-F(r))\cdot f(r).$$

to find the max Revenue.

marginal revenue from single bidder

$$\frac{d\pi(r)}{dr} = nF(r)^{n-1}\frac{d}{dr}[(r-c)(1-F(r))] + n(n-1)F(r)^{n-2}\cdot f(r)(r-c)(1-F(r))$$
$$-(r-c)f_{(2)}(r)$$

抵消

r↑ π(r) 前后两项 ↑, ↓ 抵消, Density

$$= nF(R)^{n-1}\frac{d}{dr}[(r-c)(1-F(r))]$$

max : $\Downarrow$ $=0.$

if $r > \underline{v}$ (i.e reserve price is binding).

then, $F(r) > 0$

So, $\frac{d\pi}{dr} = 0$ iff $\frac{d}{dr}[(r-c)(1-F(r))] = 0.$

△ The optimal reserve price does not depend on n.

☆ (true for every distribution function)

WHY? Intuition?

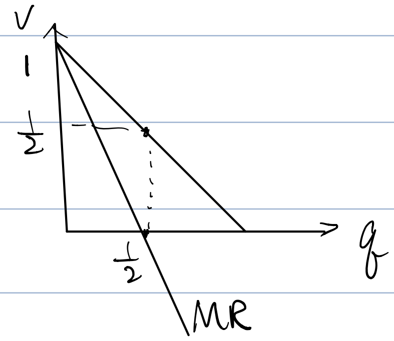
Expected marginal revenue from a reserve price r

is the expect MR of winning bidder

△ The optimal reserve price with n bidders can be calculated as the optimal reserve price with one bidder

↳ monoply pricing problem $(1-F(r))$ like demand curve

Ex. $\underline{v}=0$, $\bar{v}=1$, $F(v)=v$, $1-F(v)=v$



assume $c=0$.

reserve price $=\frac{1}{2}$

↳ $\frac{d}{dr}(r-c)(1-F(r)) = 0$

$\Rightarrow \frac{d}{dr}\left(r(1-r)\right)=0$

$\Rightarrow 1-2r \doteq 0 \quad \Rightarrow r=\frac{1}{2}$

First Price Auction

∟ bidders should bid below their valuation
must.

Seller expected revenue

SPA: bidders bid at valuations (higher)
Δ pay second highest bid
FPA: bidders bid below valuations
Δ pay highest bid.

Suggest other auction?

All pay auction

highest bidder win, everybody pay their bid (for chance to win)

auction: allocation

↳ One has the price and others spend things to win price
money, time, resourses ...

standard auction (collection of auction formats) w/o reserve price

① the highest valuation bidder gets the good

(efficient: if any buyer who buy the good, that should get the good)

SPA ; FPA (if eqm bid function is strictly increasing in valuation)

APA ⇒ then the above condition is satisfied

All auction give the good to the highest bidder

② expected payment of lowest bidder type ($\underline{v}$) = 0.

SPA