ascending price auction (English auction) - Second price auction (SPA)
(dynamic, learning sth. about competition -> consider static)

descending price auction (Deutsch ~) - First - - - - - (FPA)
raise when have the highest price.

Proof: Ascending ... = Second.
↳ decide when to raise (flashhold)
optimal at beginning ⇒ optimal when learning.
assume : drop out price
↳ pay the highest than others drop out
second highest drop out price.

decide simontaneously (submit, don't know what opponent doing).
static auctions

strategy for player (bidder) i is to submit a bid $b_i \in \mathbb{R}_+$ (包括0)
n bidders $i \in \{1, 2, \cdots, n\}$
$(b_1, b_2, \cdots\cdots, b_n)$ = bid profile

rule of auction determines as a function of bid profile
who gives the goods and what everybody pays.

First Price Auction

utility fn

$$M_i(b_1, b_2, \cdots, b_n \mid v_i) = \begin{cases} \overset{\text{(surplus)}}{v_i - b_i} & b_i \text{ (highest bid) } > \max\limits_{j \neq i} b_j \\ \dfrac{1}{\underbrace{\#\{j : b_j = b_i\}}_{\text{(the number of)}}} (v_i - b_i) & b_i = \max\limits_{j \neq i} b_j \\ 0 & b_i < \max\limits_{j \neq i} b_j \end{cases}$$

Second Price Auction

$$\begin{cases} v_i - \max_{j \neq i} b_j & b_i > \max_{j\neq i} j \\ \frac{1}{\#\{j: b_j = b_i\}} (v_i - b_i) & b_i = \max_{j \neq i} j \\ 0 & \cdots < \cdots \end{cases}$$

equilibrium, incomplete information

Dominant Strategy Equilibrium.

Bid b_i is a DS for bidder i of valuation v_i

if bidding b_i is optimal for this bidder, regardless of what other bidders submit

b_{-i}

$(b_1, b_2 \cdots b_{i-1}, b_{i+1}, \cdots b_n)$

$\triangle\ u_i(b_i, b_{-i} \mid v_i) \geq u_i(b_i', b_{-i} \mid v_i), \forall b_i', \forall b_{-i}$

most games do not have a DSE

"battle of the sexes"

		Jill Bu	Jill Ba
Jack	Bu	4, 3	0, 0
Jack	Ba	0, 0	3, 4

(go together)
2 optimal.

decide independently, simultaneously.

base on guess of what the other do.

no DSE.

most auction do not have DSE.

However, SPA has DSE ⇒ goal: maximize the surplus X win!

It is a dominant strategy for bidder i of valuation v_i to bid v_i

the truthful bidding is a DSE.

Proof:

$\triangle\ v_i > \max_{j \neq i} b_j$

if bid v_i: $u_i(v_i, b_{-i} \mid v_i) = v_i - \max_{j\neq i} b_j$.

suppose $b_i' > \max_{j\neq i} b_j \Rightarrow u(b_i', b_{-i} \mid v_i) = v_i - \max_{j\neq i} b_j$ (not improve).

$b_i' < \max_{j\neq i} b_j \Rightarrow 0$ } worse

$b_i = \max_{j \neq i} b_j \Rightarrow \frac{1}{\#\{j: b_j = i\}}(v_i - \max_{j \neq i} b_j)$ worse

Δ $v_i < \max_{j \neq i} b_j$

if bidden i bid $v_i \Rightarrow 0$

$b_i' < \max_{j \neq i} b_j \Rightarrow 0$ not improve

$b_i' > \max_{j \neq i} b_j \Rightarrow < 0$ $(v_i - b_i') < 0$ negative $\Rightarrow$ lose

Intuition