Functions

Definitions

function A function f from A to B ($f: A \to B$) is a relation R between A and b where $\forall a \in A$ there is at most one $b \in B$. s.t. aRb $f(a) = b$.

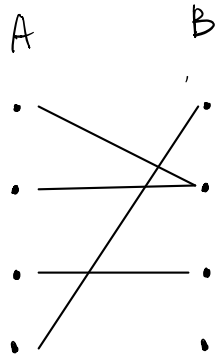
total function A function is total iff $\forall a \in A$ there is exactly one $b \in B$ s.t. aRb

inverse relation (R^{-1}) = inverse relation of R from A to B.
= a relation from B to A s.t $(\forall a \in A)(\forall b \in B)(aRb \equiv bR^{-1}a)$
all relations have an inverse
not all functions have an inverse that is a function.

injective / one-to-one $(\forall a_1, a_2 \in A)\ (f(a_1) = f(a_2) \Rightarrow a_1 = a_2)$.

surjective / onto $(\forall b \in I)(\exists a \in A)\ (f(a) = b)$.

bijective = one to one and onto.



⇒ function
nodes on A side have at most one edge.

⇒ one-to-one (1-1)
nodes on B have at most 1 edge.

⇒ onto:
nodes on B have at least 1 edge.

⇒ bijective.
nodes on B have exactly 1 edge

⇒ total function:
nodes on A side have exact one edge.

Examples

Define the following functions:

- $F_1 : \mathbf{Z} \to \mathbf{Z}$ where $F_1 = \{(a, b) \mid a + b = 0\}$ $b = -a$.
- $F_2 : \mathbf{Z} \to \mathbf{Q}$ where $F_2 = \{(a, b) \mid a \cdot b = 1\}$
- $F_3 : \mathbf{R} \to \mathbf{R}$ where $F_3 = \{(a, b) \mid |a| = b\}$

Function	Total?	1-1?	Onto?
F_1	✓	✓	✓
F_2	X ($a=0$)	✓	X ($b=0$)
F_3	✓	X	X ($b<0$)