Permutations and Combinations

Permutations

Definitions

permutation of a set of distinct objects = an *ordered* arrangement of the objects

of ABC: ABC ACB BAC BCA CAB CBA

k-permutation = ordered arrangement of k elements of a set

CA: 2-permutation of ABC

P(n, k) = # of k-permutations of a set with n elements ($0 \le k \le n$)

$n(n-1)\cdots(n-(k-1))$ by product rule

$= \frac{n!}{(n-k)!}$

Note: when $k=n$, $P(n,n) = n!$

when $k=0$, $P(n,k) = 1$

Examples

1. How many ways are there to select a 1st prize winner, a 2nd prize winner, and a 3rd prize winner from 100 entries (if entries are only allowed to be picked at most once)?

3-permutations

$P(100,3) = 100 \times 99 \times 98$

2. How many permutations of the letters ABCDEFGH contain the string ABC?

$6! = 720$

not contain ABC $8! - 6!$

3. How many permutations of the letters ABCDEFGH contain the strings FH and CGA?

$5!$

Combinations

Definitions

combination – order does not matter.

k-combination of elements of a set = an unordered collection of k elements of the set
= subset containing k elements.

C(n, k) = # ways to pick k out of n elements (where order doesn't matter)
$= \binom{n}{k}$ n chooses k binomial coefficients.

$= \frac{n!}{k!(n-k)!}$

$= \binom{n}{n-k}$

$\binom{n}{k} = \frac{P(n,k)}{P(k,k)} = \frac{\#\text{ ordered ways}}{\#\text{ ways counted as same.}}$
$k! \to 1$ function.

Examples

1. How many ways are there to pick 3 colors from { red, blue, green, yellow }?
$\binom{4}{3} = 4$

2. How many poker hands of five cards can be dealt from a standard deck of 52 cards?
$C(52,5) = \binom{52}{5} = \frac{52 \times 51 \times 50 \times 49 \times 48}{5 \times 4 \times 3 \times 2 \times 1} = 2598960.$

3. Suppose a student group contains 9 graduate students and 11 undergraduate students. The group wants to form a committee consisting of 3 graduate students and 4 undergraduate students. How many ways are there to select this committee?
$\binom{9}{3} \times \binom{11}{4}$