△ Continuous Distribution

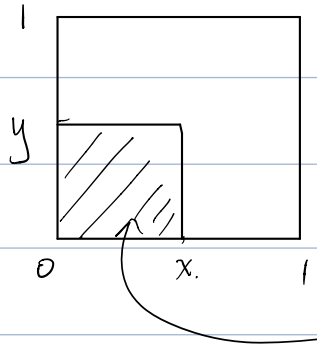
11/1 midterm sample exam this weekend.

2 answers - out of 3 x curve

result, proposition

Independent random variables. ← statistically independently.

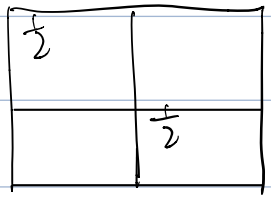
$F(x,y) = F(x) \cdot F(y)$.



$F(x) = x$ (possibility of falling between 0 and x).

$F(y) = y$

$F(x,y) = F(x) \cdot F(y) = xy$



⇐ not independent. (Discrete probability).

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n bidders $v_1, v_2 \cdots\cdots v_n$ (do not know others' v)

↓ (v_i are independent random variables ↙

| each valuations is drawn from the same distribution function $F(x)$ ↑

determining the optimal reserved price in a second price auction.

$n=1$ n^* maximizes $\pi(R) = (R-c)(1-F(R))$ (上一节课)

general n △ sometimes the seller get the R

↳ all but one bidders' valuation is above R.

(V_i highest ... V_m highest) $n \cdot F(R)^{(n-1)} \cdot (1 - F(R))$ — the bidder's valuation $> R$. $\Rightarrow$ Likelyhood.

(other bidders valuation lower than R)

$$\pi(R) = (R-c)\, n F(R)^{(n-1)} (1 - F(R))$$

△ second highest valuation above R

$V_{(2)}$

$$\int_R^{\bar{v}} (V_{(2)} - c)\, f_{(2)}(V_{(2)})\, dV_{(2)}$$

Density

What is the distribution of second highest valuation?

order statistic ↳ second order statistic.

$$F_{(2)}(x) = \mathbb{P}[V_{(2)} \le x]$$

(1) all of the r are below x $\quad \underbrace{F(x) \cdots F(x)}_{n} = F(x)^n$

(2) one valuation is above x and all others are below x.

$$(1 - F(x)) F(x)^{n-1}$$

$$F_{(2)}(x) = F(x)^n + n(1 - F(x)) F(x)^{n-1} = n F(x)^{n-1} - (n-1) F(x)^n$$

$$f_{(2)}(x) = \frac{d}{dx} F_{(2)}(x) = n(n-1) F(x)^{n-2} f(x) - (n-1) n F(x)^{n-1} f(x)$$

$$= (n-1) n F(x)^{n-2} (1 - F(x)) f(x)$$

Expected revenue to seller with a reserve price of r when there are n bidders.

$$\pi(r) = n(r-c) F(r)^{n-1} (1 - F(r)) + \int_r^{\bar{v}} (V_{(2)} - c) f_{(2)}(V_{(2)})\, dx$$

optimal : $\pi'(r) = 0$.