Empirical Application of the Solow Model: Development Accounting

Steady State of the Solow Model

- Recall the steady-state expressions in *per capita* terms:

$$k^*(t) = \left(\frac{s}{n+\delta+g}\right)^{\frac{1}{1-\alpha}} A(t)$$

increasing in A. (indirect)

$$y^*(t) = k^*(t)^{\alpha} * A(t)^{1-\alpha}$$

$$= \left(\frac{s}{n+\delta+g}\right)^{\frac{\alpha}{1-\alpha}} A(t)$$

increasing in A direct.

- Steady state output per capita takes into account direct effect of $A(t)$ on $y^*(t)$ for given $k^*(t)$ **plus** indirect effect of $A(t)$ through $k^*(t)$!

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"Decomposition" of Cross-Country Income Gap

- Interested in *relative* income gaps:

$$\frac{y_i^*(t)}{y_j^*(t)} = \frac{\left(\frac{s_i}{n_i+\delta_i+g_i}\right)^{\frac{\alpha}{1-\alpha}} A_i(t)}{\left(\frac{s_j}{n_j+\delta_j+g_j}\right)^{\frac{\alpha}{1-\alpha}} A_j(t)}$$

- Use *log differences* to measure size of gap:

$$\ln(y_i^*(t)) - \ln(y_j^*(t)) = \frac{\alpha}{1-\alpha}\left[\ln\left(\frac{s_i}{n_i+\delta_i+g_i}\right) - \ln\left(\frac{s_j}{n_j+\delta_j+g_j}\right)\right] + \ln(A_i(t)) - \ln(A_j(t)).$$

$$1 = \frac{\frac{\alpha}{1-\alpha}\left[\ln\left(\frac{s_i}{n_i+\delta_i+g_i}\right) - \ln\left(\frac{s_j}{n_j+\delta_j+g_j}\right)\right]}{\ln(y_i^*(t)) - \ln(y_j^*(t))} + \frac{\ln(A_i(t)) - \ln(A_j(t))}{\ln(y_i^*(t)) - \ln(y_j^*(t))}$$

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Stylized Example

Consider two countries with the following values for income, saving-to-depreciation rate, and productivity:

$$\frac{1.25}{9} = \frac{(1)^{\frac{\alpha}{1-\alpha}}\, 1.25}{(2.25)^{\frac{\alpha}{1-\alpha}}\, 6}$$

Now, take logs:

$$\underbrace{\ln(1.25) - \ln(9)}_{=-1.97} = \underbrace{\frac{\alpha}{1-\alpha}\left[\ln(1) - \ln(2.25)\right]}_{=-0.405} + \underbrace{\left[\ln(1.25) - \ln(6)\right]}_{=-1.569}$$

Then divide by the left hand side of the equation:

share of contribution of capital and productivity $\Rightarrow$

$$1 = \frac{\frac{\alpha}{1-\alpha}\left[\ln(1) - \ln(2.25)\right]}{\ln(1.25) - \ln(9)} + \frac{\left[\ln(1.25) - \ln(6)\right]}{\ln(1.25) - \ln(9)}$$

- In practice, we don't observe $A(t)$ and, for this reason, we don't know the value of g.
- Instead, use the equation

$$y^*(t) = k^*(t)^\alpha * A(t)^{1-\alpha}$$

  to compute $A(t)$.
- Take data from the **Penn World Tables** (data adjusted for differences in purchasing power parity) available at `https://www.rug.nl/ggdc/productivity/pwt/`
- Download the PWT 9.1 (most recent version) in Excel format.

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Real World Data Exercise: Some Additional Notes

$$y(t) = k(t)^{\alpha} A(t)^{1-\alpha}$$

$$A(t) = \left(\frac{y(t)}{k(t)^{\alpha}} \right)^{\frac{1}{1-\alpha}}$$

Empirical Application of the Solow Model: Development Accounting (Filling Some Gaps)

Development Accounting

Conclusions from Real World Data Exercise

Comment #1: arithmetic mean v.s. geometric mean.

Comment #2: confident: investment/capital are measured accurately.

Decomposition: Dealing with Logs and Means

Arithmatic mean $\Longleftrightarrow$ Geometric Mean.

$$y = k^{\alpha} A^{1-\alpha}.$$

e.g. 2 countries

$$y_1 = k_1^{\alpha} A_1^{1-\alpha}$$

$$y_2 = k_2^{\alpha} A_2^{1-\alpha}.$$

Ari: $y_1 + y_2 = \frac{1}{2}\left(k_1^{\alpha} A_1^{1-\alpha} + k_2^{\alpha} A_2^{1-\alpha}\right). \neq \frac{1}{2}\left(k_1^{\alpha} + k_2^{\alpha}\right)\left(A_1^{1-\alpha} + A_2^{1-\alpha}\right).$

Geo: $y_1^{\frac{1}{2}} y_2^{\frac{1}{2}} = \left(k_1^{\alpha} A_1^{1-\alpha}\right)^{\frac{1}{2}} \left(k_2^{\alpha} A_2^{1-\alpha}\right)^{\frac{1}{2}}.$

$$= \left(k_1^{\alpha} k_2^{\alpha}\right)^{\frac{1}{2}} \left(A_1^{1-\alpha} A_2^{1-\alpha}\right)^{\frac{1}{2}}$$

2 ways to deal with logs and means

1. start with geo mean, then take log.
2. start by taking log, then take arithmetric meen. of the log.

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Conclusions from Real World Data Exercise

Observation #1: productivity differences amount for 65%+ of the income different.

Observation #2: measurement of labor input doesnt contaminate the productivity residual.

- variation in labor force participation,
- hours of work. may vary across countries.
- human capital = efficiency unit of labor.

$Y = K^\alpha (AL)^{1-\alpha}$

$\frac{Y}{L} = y = k^\alpha L^{1-\alpha}$. 2) not everyone in the labor.

Economic Value of Education

Mincer Regression

- Human capital is measured in *efficiency units* of labor, denoted by h:
  1. someone with human captical h is economically equivalent to h worker with 1 unit of human capital
  2. Problem: don't observe h directly
  3. Sol: reverse-engineering the human capital per worker from info on earnings or wages.
- Human capital is a function of schooling: $\Rightarrow \ln(\exp(s)^{\rho}) = \rho s$

$$h = \exp(s \cdot \rho) = \exp(s)^{\rho}$$

ρ → return of schooling one more year

- Wage is a function of human capital and experience:

$$\ln w = \underbrace{\ln w_0}_{\substack{\text{log of wage with } no \text{ schooling} \\ \text{(raw labor)}}} \underbrace{+\rho_p s_p + \rho_s s_s + \rho_t s_t}_{\substack{\ln(\exp(s)^{\rho}) \\ \text{schooling}}} + \underbrace{\beta_1 x + \beta_2 x^2}_{\text{experiences}} + \varepsilon$$

Two-step process for "construction" of human capital stocks:

1. Estimate ρ using micro data (e.g. census data, surveys, ...):
   - $\rho_p = .134$ for $0 < s \leq 4$
   - $\rho_s = .101$ for $4 < s \leq 8$
   - $\rho_t = .068$ for $s > 8$
2. Use these Mincer returns to construct average human capital per worker for each country based on average educational attainment (from *Barro-Lee Educational Attainment Dataset*, for instance).

Neoclassical Production Function with Human Capital

$A(t)$ and $h(t)$ are *labor augmenting*:

$$Y(t) = K(t)^{\alpha}\left(A(t)h(t)L(t)\right)^{1-\alpha}$$

$$y(t) = k_t^{\alpha}\left(A(t)\,h(t)\right)^{1-\alpha}$$

The *Solow Residual* is given by:

$$A(t) = \left(\frac{y(t)}{k_t^{\alpha}\,h_t^{1-\alpha}}\right)^{\frac{1}{1-\alpha}} = \left(\frac{y(t)}{k_t^{\alpha}}\right)^{\frac{1}{1-\alpha}} \cdot \frac{1}{h(t)}$$

Productivity and Income Per Capita

Relative to the United States

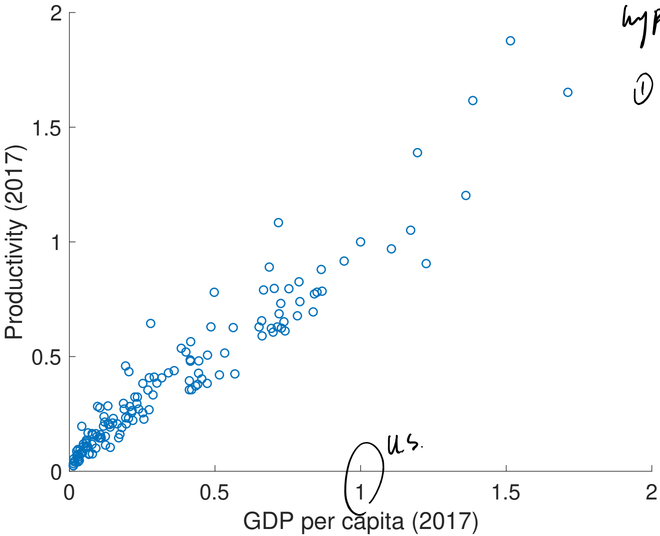
rich country: more productivity.

hypothesis:

① ↑productivity → ↑income.

② vice versa

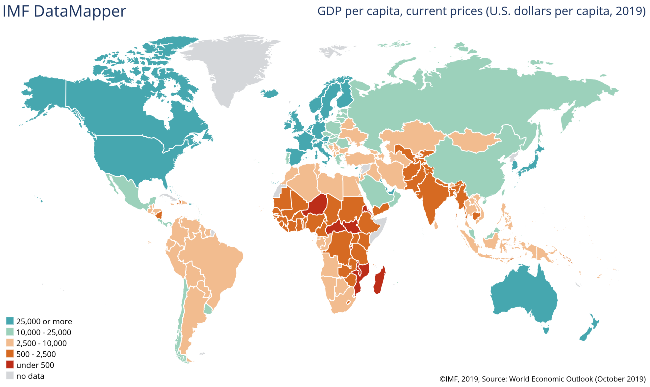
③ other → ↑A, ↑y



Productivity and Income Per Capita: Interpretation

1. high income causes high productivity
2. vice versa.
3. some other factor(s) effect both income/productivity.

Productivity and Income Per Capita: A Map



Productivity and Income Per Capita: Another Map

north / south Korea similar geography
but different lightening

Candidates for *Fundamental* Causes of Income Differences

geography. /climate

Insstitution