Practice Questions for Midterm #2

November 18, 2019

1. Write down the profit function of a representative final good producer.

2. Write down the profit function of the capital good producer j.

3. Take the two first order conditions (F.O.C.s) associated with the final good producer's problem.

4. Using the F.O.C. with respect to x_j, show that the price elasticity of demand is constant (i.e. show that $\frac{\partial p_j(x_j)}{\partial x_j}\frac{x_j}{p_j}$ is constant).

5. Again, using the F.O.C. with respect to x_j show that $\frac{\partial p_j(x_j)}{\partial x_j}\frac{x_j}{p_j} = \frac{\partial \ln\left(p_j(x_j)\right)}{\partial \ln(x_j)}$.

6. Use the constant price elasticity to solve for $p_j(x_j)$ as a function of r and model parameters. Show that the rental rates for all varieties $j \in [0, A]$ are equal.

7. Use your previous result to characterize $x = x_j$ for $j \in [0, A]$ as a function of K and A.

8. Use your answer above to characterize the rental rate r as a function of the capital-labor ratio in production $\frac{K}{L_Y}$, A, and parameters of the model. Rewrite the production function for final goods as a function of these aggregate variables.

9. Write a capital producer's profit as a function of aggregate variables (K, L_Y, A) and the parameters of the model.

10. Using $s'_R = 0$, write tomorrow's total output Y' as a function of today's aggregate variables (K, A, L), the population growth rate n, and s_R (today's employment share of research).

11. Take the F.O.C. with respect to s_R and simplify the expression as much as you can. The condition does not have a closed form solution, but try to collect all the s_R terms on one side of the equation and all the terms that don't involve s_R on the other side.

12. If you can, show that the non-constant side of the equation is monotonically increasing (or decreasing) in s_R. Show also that $s_R = 1$ and $s_R = 0$ do not satisfy the F.O.C.

13. Is s_R increasing or decreasing in today's stock of knowledge A? What's the economic intuition?