Transition from Stagnation to Growth

Properties of the Malthus Model

- Model predicts constant income per capita in steady state.
- Differences in productivity show up as differences in population density, not income per capita.
- Model provides an accurate description of the economic history of the world up to the Industrial Revolution.
- However, ultimately many countries managed to escape from the Malthusian trap.
- How can this escape be explained?

The Sources of Stagnation: A Growth Accounting Perspective

- Recall the agricultural production function from the previous lecture:

$$Y_t = (A_t X)^{\alpha} N_t^{1-\alpha}.$$

- Income per capita is given by:

$$y_t = \left(\frac{A_t X}{N_t}\right)^{\alpha}.$$

- The growth rate of income per capita can therefore be decomposed as follows:

growthr rate of productivity

$$\underbrace{\log(y_{t+1}) - \log(y_t)}_{\text{growth rate of } y.} = \alpha \left[\log(A_{t+1}) - \log(A_t)\right] - \alpha \left[\log(N_{t+1}) - \log(N_t)\right].$$

− " " of population

- Productivity growth enters with a positive sign, and population growth enters with a negative sign.

The Sources of Stagnation: A Growth Accounting Perspective

- In the Malthus model, we obtain the stagnation result because population growth always adjusts to just offset productivity growth.
- Based on the growth decomposition, there may be two ways to escape from Malthusian stagnation:
  - acceleration in productivity growth
  - slow down in population growth.

Escaping from Stagnation through Faster Productivity Growth

- In our version of the Malthus model, we assumed that fertility is proportional to income per capita.
- In that case, we get the stagnation result for any level of productivity growth.
- However, our model did not take account of the fact that human fertility is bounded (at perhaps 12-13 children per woman).
- If we take the upper bound on fertility into account, it is possible that productivity growth reaches a sufficiently high level to "outrun" population growth.

Escaping from Stagnation through Faster Productivity Growth

- Assume that the maximum number of children is given by $\bar{n}$. Let's say $\bar{n} = 5$ children per adult (10 per family).
- Then, population increases by a factor of 5 every generation, i.e., ca. every 25 years.
- If the amount of effective land increases faster than that, income per capita has to rise.
- On an annual basis, growth rate of effective land (i.e., productivity growth) would have to satisfy:

$$1+g > 5^{1/25} \approx 1.07$$

↑ × sustain for long.

Escaping from Stagnation through Faster Productivity Growth

- Thus, if productivity growth exceeds about ,
  population growth can no longer catch up with productivity growth, and income per capita will start to rise.
- Thus, an escape from the Malthusian trap is possible through a rise in productivity growth.
- As an empirical matter, no country has ever observed productivity growth at such a fast rate over multiple generations.
- Thus, while working fine in theory, this is probably not the right explanation in practice.

Escaping from Stagnation through Faster Productivity Growth

Nevertheless, faster productivity growth may be one of multiple factors that facilitated the escape from the trap:

- Productivity growth did indeed gradually accelerate throughout the Industrial Revolution (from about 0.1 to 2 percent per year).
- The practical limit for population growth may be a lot lower than our assumption of 10 children per woman).

Escaping from Stagnation through Lower Population Growth

- Perhaps the easiest way to escape from the trap would be through a reduction in population growth.
- Recall that the growth decomposition is:

$$\log(y_{t+1}) - \log(y_t) = \alpha\left[\log(A_{t+1}) - \log(A_t)\right] - \alpha\left[\log(N_{t+1}) - \log(N_t)\right].$$

- If (for whatever reason) fertility permanently falls to $n = 1$ in the Malthusian model (2 children per family), population will be constant, and income per capita will start to grow at the rate of productivity growth.
- The view that lower population growth would facilitate the transition to growth is widespread; see for example China's one-child policy.

Escaping from Stagnation through Lower Population Growth

- As an empirical matter, fertility did indeed decline in each and every country that underwent the transition from stagnation to growth: the demographic transition.
- Thus, the population-growth channel is potentially important.
- However, the timing is at odds with our hypothesis.

higher growth before ↓ fertility

20th century, early not.

I.R England ↑ growth 1770 ~1820

↓ fertility in early 19xxs

Escaping from Stagnation through Structural Change

$Y_t = (A_t X)^{\alpha} N_t^{1-\alpha}$

might not be constant.

- Mechanically, the Malthusian model implies that the income share of land stays constant over time (since α is a constant parameter).
- Is this a plausible assumption? Maybe we need to think about models where the land share changes over time. Why?
- The empirical evidence suggests that the share of land in output did indeed fall throughout the industrialization period.
- The reason is structural change: Agriculture was replaced by industry (and later by services) as the dominant sector of production.
- Of course, this raises the question of where the structural change originated.

A Model with Two Production Technologies

- We will consider a simple extension of the Malthus model where, in addition to the agricultural technology, a second technology is also available: industry.
- Both technologies are always available; productivity increases at constant rate in both sectors.
- Labor is allocated optimally between the two sectors.

The Model with Two Production Technologies

- Agricultural production function:

$$Y_t^a = (A_t X)^\alpha (N_t^a)^{1-\alpha}.$$

- Industrial production function: assume not require land

$$Y_t^i = A_t N_t^i$$

- Allocation of labor:

$$N_t = N_t^a + N_t^i$$

- Productivity growth:

$$A_{t+1} = (1+g) A_t$$

- Optimal fertility choice: $n_t = \frac{y_t}{2p}.$

The Model with Two Production Technologies

- Key question: Which technology will be used?
- At first sight, the industrial technology may seem more attractive: industry doesn't rely on the fixed factor land.
- However, the choice is not obvious:

  if land (effective unit of land) is abundent (relative to #people), then agriculture might be more productive than industry.

The Model with Two Production Technologies

- Marginal product of labor in agriculture:

$$\frac{\partial Y_t^a}{\partial N_t^a} = (1-\alpha)\left(\frac{A_t X}{N_t^a}\right)^{\alpha}$$

$N_t \uparrow$, $\downarrow$ marginal distribution to output.

- Marginal product of labor in industry:

$$\frac{\partial Y_t^i}{\partial N_t^i} = A_t. = \frac{Y_t^i}{N_t^i}$$

average output

- Marginal product of labor

is decreasing in the labor supply in agriculture,

but not industry.

The Optimal Allocation of Labor Across Sectors

- If agriculture yields higher marginal product of labor even when all workers are used in agriculture, only agriculture should be used.
- The condition for this to occur is: $(1-\alpha)\left(\frac{A_t X}{N_t}\right)^{\alpha} > A_t.$

or: $A_t^{1-\alpha} N_t^{\alpha} < \underbrace{(1-\alpha) X^{\alpha}}$

limited, finite.

- Thus, exclusive use of agriculture is optimal if population and/or productivity are low.

The Optimal Allocation of Labor Across Sectors

- Consider the reverse case:

$$(1-\alpha)\left(\frac{A_t X}{N_t}\right)^{\alpha} < A_t$$

- Then, only using agriculture is not optimal, because industry would yield higher output per worker.
- At least some workers should work in industry.
- Even so, it will always be optimal to employ at least some workers in agriculture, because marginal product of labor in that sector approaches infinity as number of workers approaches zero.
- Intuitively, it is always optimal to make at least some use of the fixed factor land.

The Optimal Allocation of Labor Across Sectors

- The optimal allocation if both sectors are used:

$$(1-\alpha)\left(\frac{A_t X}{N_t^a}\right)^{\alpha} = A_t,$$

$$(N_t^a)^{\alpha} = \frac{(1-\alpha)\, X^{\alpha}}{A_t^{1-\alpha}}.$$

$$N_t^a = \frac{(1-\alpha)^{\frac{1}{\alpha}}\, X}{A_t^{\frac{1-\alpha}{\alpha}}}$$

- The remaining workers will work in industry:

$$N_t^i = N_t - N_t^a = N_t - \frac{(1-\alpha)^{\frac{1}{\alpha}}\, X}{A_t^{\frac{1-\alpha}{\alpha}}}$$

- Since marginal products are equalized, marginal product of labor is equal to A_t in both sectors.

The Long-Run Evolution of the Economy

- Consider a version of this economy starting out 50,000 years ago.
- Back then, both productivity and population would be very low.
- As a result, the condition:

$$A_t^{1-\alpha}(N_t^a)^\alpha < (1-\alpha)X^\alpha$$

  would hold, and the economy would start out using only agriculture.
- If initial population size and productivity are very low, agriculture would be used exclusively for many generations.
- In other words, the model would be initially identical to our Malthus model, where industry does not exist at all!

The Malthusian Phase

- From analyzing the Malthus model earlier, we know that as long as agriculture is used exclusively, the economy will converge to a steady state with constant income per capita, given by:

$$\frac{Y_t^a}{N_t} = (1+g)2p.$$

- However, unlike in the Malthus model, in the two-sector model the steady state is not permanent.
- The industrial technology is also available, and it is getting better over time!
- At some point, productivity will reach a threshold where the industrial technology is introduced.

The Introduction of Industry

- Industry will be introduced once the following condition is satisfied:

$$A_t > (1-\alpha)\left(\frac{A_t X}{N_t}\right)^{\alpha}$$

- Recall that income per capita in the Malthusian steady state is:

$$\frac{Y_t^a}{N_t} = \left(\frac{A_t X}{N_t}\right)^{\alpha} = (1+g)2p.$$

- Thus, the condition for the introduction of industry can be written as:

$$A_t > (1-\alpha)(1+g)2p$$

- Since A_t is growing and the right-hand side is constant, this condition has to be satisfied at some point in time.

The Economy after Industry has been Introduced

- Once the industrial technology is in use, population growth no longer depresses productivity growth.
- Income per capita will start to grow at the rate of productivity growth.
- The economy escapes from the Malthusian trap.

Fertility : $N_T = \dfrac{y_T}{2p}$. ← no bound

↑

✱ no bound. out of control.

spread sheet

marginal product of N_t^α ↙ work in agriculture / industry

$$\frac{\partial Y_t^\alpha}{\partial N_t^\alpha} = \frac{(A_t X)^\alpha (N_t^\alpha)^{1-\alpha}}{\partial N_t^\alpha} = (1-\alpha) N_t^{\alpha^{-\alpha}} (A_t X)^\alpha = A_t .$$

$$\tilde{N}_t^\alpha = A_t^{\frac{-1-\alpha}{\alpha}} (1-\alpha)^{\frac{1}{\alpha}} X \qquad (N(\text{tilde\_t})).$$

N-t =

N_t(a) = min (N_t , N(tilde)_t).

N_{t_i} = $N\_t - N_{t(a)}$.

$y\_t = (N_{t\_i} * A_t + (A_t X)^\alpha (N_t^a$

$$Y_t = \frac{(A_t X)^\alpha (N_t^a)^{1-\alpha}}{N_t}$$

$$Y_0 = (A_t X)^\alpha N_t^{-\alpha}. \qquad (Y_0,\ N_t^a = N_t)$$

N(max)_t ← 假设所有人都到.