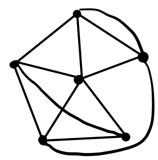
Graph Applications

Spanning trees

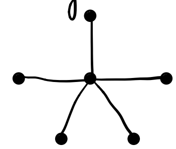
Definition : Let $G = (V, E)$ be a graph and $T = (V, F)$ be a *connected* subgraph of G. If T is a tree, we call it a ***spanning tree*** of G.

$F \subseteq E$.

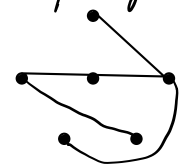
Example



one spanning tree



another spanning tree.

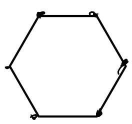


Theorem : Every connected graph G has a spanning tree.

Proof (outline) : start $T = G$

If T is a tree, we are done.

Otherwise, find a simple cycle in G, pick any edge in that cycle & remove it. to obtain T_1.



T_1 is still connected and T_1 has same vertices as G.

(So T_1 is a subgraph of G)

Set $T = T_1$ & repeat the process

Removing edges will st > when T has $|V|-1$ edges

T will be a spanning tree

How can we find a spanning tree?

In zybook / CS 400, depth-first-traversal & breath-first-traversal.

Graph coloring

Definitions

coloring : an assignment of a color to each vertex in a graph so that no two adjacent vertices have the same color

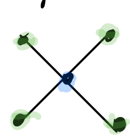
***k*-coloring**: a coloring that uses k colors

chromatic number of a graph G is the fewest number of colors needed for a coloring, i.e., the smallest k for which G has a k-coloring

denoted $\chi(G)$ (χ is the greek chi)

• $\chi(G) = 1$

•—• $\chi(G) = 2$

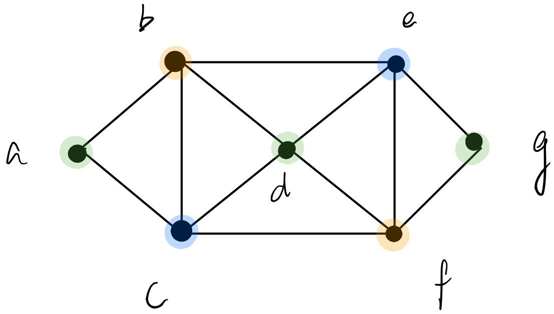


$\chi(G) = 2$

Theorem: A graph is 2 colorable if and only if it is bipartite.

Examples

Example 1 : What is the chromatic number of this graph?

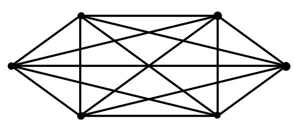


$\chi(G) \geq 3$ since a, b, c requires 3 colors

$\chi(G) = 3$

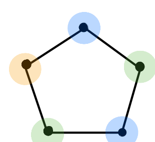
what if add an edge $\{a, g\}$ → g require a new color.

Example 2 : K_6, C_5, C_8



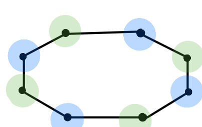
K_6 = complete graph in 6 vertices

$\chi(K_6) = 6$ since every vertex is connected to every other vertex.



C_5 = cycle with 5 vertices

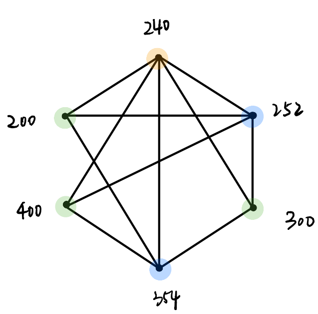
$\chi(C_5) = 3$



C_8 = cycle with 8 vertices

$\chi(C_8) = 2$

Example 3 : Scheduling problem



How many different exam times are required so there are not any conflicts?

each color = one exam time

So $\chi(G)$ is the fewest ~~#of~~ exam time required

know $\chi(G) \geq 3$,

$\chi(G) = 3$

In general, colorings are not necessarily unique