Externalities

Pigoawnian taxation (subsidination)

set the marginal tax rate (subsidy) equal to the marginal externality imposed on the offers

change the consumers' utility function

ix. the Restraurant Game

$V_i(x_1, x_2 \cdots x_n) = \sqrt{x_i} - \frac{1}{n}\sum_{j=1}^{n} x_j$

$\hat{V}_i(x_1, x_2 \cdots x_n) = V_i(x_1, x_2 \cdots x_n) - z x_i$ choose $z \Rightarrow \frac{\partial V_i}{\partial x_i} = \frac{\partial W}{\partial x_i}$

$z = \frac{n-1}{n}$

$\downarrow$

$\tilde{V}_i(x_1 \cdots x_n) = \sqrt{x_i} - \frac{1}{n}\sum_{j=1}^{n} x_j - \frac{n-1}{n} x_i$

add the term $\sum_{j\neq i} \sqrt{x_j} - \frac{n-1}{n}\sum_{j\neq i} x_j$

$\tilde{\tilde{V}}_i = \sum_{j=1}^{n} \sqrt{x_j} - \sum_{j=1}^{n} x_j = W(x_1, x_2 \cdots x_n)$

↳ acts so as to max W wrt x_i

Tradable pollution rights ⇒ other two ways can be on exam

$i = 1, 2 \cdots n$ individuals

(more produce, more pollute)

$j = 1, 2 \cdots m$ firm → pollute x_j: pollution level

$U_i(\sum_{j=1}^{m} x_j)$ $\quad u_i' < 0$

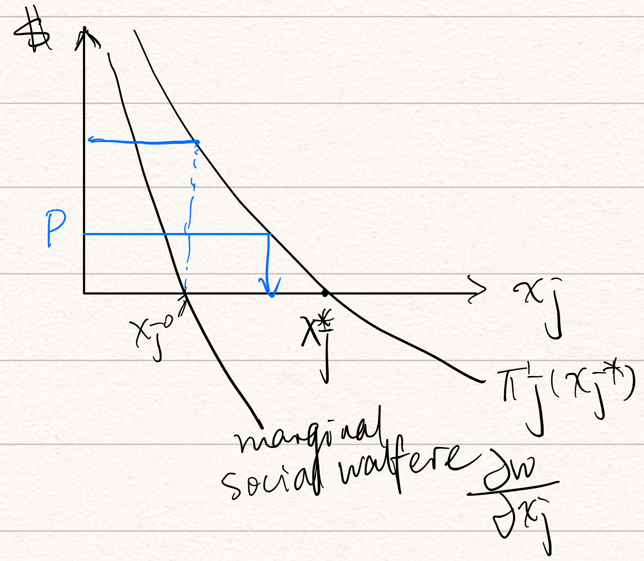
$\pi_j(x_j)$ $\quad \pi_j' > 0$ $\quad \pi_j'' < 0$ (marginal profit of pollution is ↓).

Social Optimum.

$W(x_1, x_2 \cdots x_n) = \sum_{i=1}^{n} \overset{\text{negotive}}{u_i}(\sum_{j=1}^{m} x_j) + \sum_{j=1}^{m} \pi_j(x_j)$

$\frac{\partial W}{\partial x_j} = \sum_{i=1}^{m} u_i'(\sum_{j=1}^{m} x_j^{\circ}) + \pi_j'(x_j^{\circ}) = 0$

equ $\Rightarrow \pi_j'(x_j^{*}) = 0$



government issue the pollution rights.

total number: $\sum_{j=1}^{m} x_j^{-*}$

auction Let p be equilibrium auction price

$$\max\ [\pi_j(x_j) - P x_j] \Rightarrow \pi_j'(x_j) = P$$

the demand of pollution right

What is the equ price

$$p^o = -\sum_{j=1}^{n} u_i'\left(\sum_{j=1}^{m} x_j^{-o}\right)$$

代入 得 $\frac{\partial w}{\partial x_j} = 0$ (相同式子)

prove ↑.

suppose $p > p^o$ $\pi_j'(x_j) = P > p^o$

$x_j < x_j^o$ $\sum_{n=1}^{m} x_j < x_j^{-o}$

demand supply of pollution right

Public Good. (eg of externality)

pure, conjestable, clubs

conjestable within (swimming pool, bridge, parks) pay to go to

provide Public good should not be sold in private market.

Eg. pure public good

large number of consumers i, n

provided by a competitive market (large number of firms) j, m

① Max: $u_i(x_i + \sum_{j \neq i} x_j) - Px_i$

$$u_i'(x_i + \sum_{j \neq i} x_j) - P \leq 0 \quad (= 0 \text{ if } x_i > 0)$$

② Firms, let $c(q)$ be the cost of q units of public good

In a CE, q_e must max

$$P(q) - c(q)$$

$$\hookrightarrow c(q^*) = P$$

Combine ①, ②

$$c(q^*) = P \geq u_i'(q^*) \quad (= \text{ if } x_i^* > 0)$$

$u_1' > u_2' > \cdots > u_n'$ (rank)

Suppose there exist at least one person which purchases a strictly > 0 amount, call it k

$$u_k'(q^*) = c'(q^*) \Rightarrow \boxed{k = 1} \Rightarrow X \text{ in competitive market}$$

$$\sum_{i=1}^{n} u_k'(q^*) > c'(q^*)$$

Walfare: $\sum_{i=1}^{n} u_i(q) - c(q)$

optimal: $\sum_{i=1}^{n} u_i'(q^0) = c'(q^0)$ sum of marginal utility