

# Bilateral Trade under asymmetric information (incomplete / private)

real world: only know your value  $\Rightarrow$  ? outcome.  
 $\hookrightarrow$  barrier to efficiency.  
decision on uncertainty.

model: who knows why / how

$\rightarrow$  single buyer  
single seller

no correlation

$\checkmark$  (independently)

$\rightarrow$  nature draw the valuation of the buyers, seller  
according to distribution given by distribution function

$$F(v) \text{ and } G(c) \Rightarrow P[\bar{c} \leq c]$$

(random)

$\hookrightarrow = P[\bar{v} \leq v]$  (possibility of the buyer to give)

$\rightarrow$  nature privately informed both traders about the evaluation, respective.

Bench Mark full info case

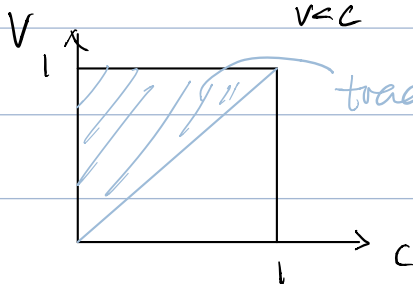
outcome -

$(c, v) \leftarrow$  know this, then know what happens.

$v > c$  trade occurs,  $p$  between  $c, v$ .

$v = c$  random

$v < c$  trade never occurs.



$\Delta$  Split-the-difference game.

A)  $v < c$  go home

$v > c$  ① machine designers set  $p = \frac{v+c}{2} \Rightarrow$  split equally.

(first best). ② buyer / seller each get to say yes or no

Outcome: if  $(y, y)$ , trade occurs at  $p$ .  
otherwise, go home

dominant strategy: say yes.  $\Rightarrow$  PD occurs

B) mechanism designer does not know  $c, v$ .

$\hookrightarrow$  split-the-difference mechanism.

buyer/seller independently report valuation:  $v', c'$

if  $v' > c' \Rightarrow p = \frac{v' + c'}{2}$

take  $p$  from  $p$  from buyer, give to the seller

if  $v' = c' \Rightarrow$  go home

(Prop) If the seller/buyers always tell the truth.

then, the split-the-difference mechanism is rec'd the first best outcomes.  $\leftarrow$  Happen?  
No!!!

Reality: E.g. buyer 20 100  $\Rightarrow (2, 2)$ .

seller 0 80  $\Rightarrow (2, 2)$ .

(if truth)  $\Rightarrow$  buyer reports  $\Rightarrow$  20 100

... reports

80	0, -	1, 90
0	1, 10	1, 50

$\leftarrow$  probably / price  
no trade.

For buyer seller  
For (100, 0)

A. If the buyer of evaluation 100 expect that the seller to always tell the truth.

and report  $v = 100$  surplus:  $\frac{1}{2}(100 - 50) + \frac{1}{2}(100 - 90) = 30$ .

B. If reports  $v = 20$ .

(02) seller surplus  $\frac{1}{2}(100 - (12)) = 90$   $\leftarrow$  trade  $\nRightarrow$  not trade with  $s_2$ .  
 the price to trade  $\downarrow \Rightarrow \uparrow$  surplus.

## Continuous Distribution of Evaluation

$F$  and  $G \sim [0, 1]$

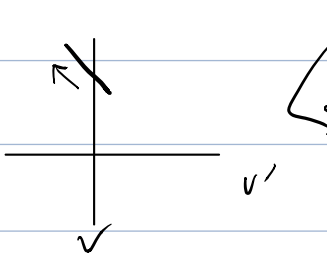
$$F(v) = v \quad G(c) = c$$

Expected Surplus of a buyer of valuation  $v$  when the report  $v'$ , and the seller always tell the truth.

suppose  $c$  random.

$$\hookrightarrow v < c \quad \text{surplus} = 0$$

$$\begin{aligned}
 v > c &\rightarrow \int_0^{v'} (v - \frac{v' + c}{2}) - g(c) dc \\
 \text{optimal} &: \frac{d}{dv} \left( \int_0^{v'} (v - \frac{v' + c}{2}) - g(c) dc \right) \\
 &= (v - \frac{v' + v'}{2}) g(v') - \frac{1}{2} \underbrace{\int_0^{v'} g(c) dc}_{G(v')}
 \end{aligned}$$


 if  $v' = v \quad = 0 - \frac{1}{2}G(v) < 0$  for all  $v > 0$ .  
 $\hookrightarrow$  Truth-telling is not optimal to any  $v > 0$ .  
 better to lower  $\Rightarrow$  optimal to understate the valuation