Revenue Equilibrium

- do not the number of bidder ?
- 2 bidders SPA <=> FPA.

$F(v) = v$ (v uniformly distributed through [0, 1])

$\underline{v} = 0 \quad \bar{v} = 1 \quad f(v) = 1$

FPA $b^*(v) = \frac{v}{2}$

conditional on a bidder of valuation v winning the auction payment equals $\frac{v}{2}$

SPA
- Conditional on winning the auction, the expected payment of a bidder of valuation v.
- pay second highest bid valuation, conditional on this valuation $\le v$.
- distribution of a bidder's valuation, Conditional on it $\le v$. uniform on $[0, v]$.
- expected valuation, conditional on valuation $\le v$, $\frac{v}{2}$?

$\Rightarrow$ expected payment is the same.

Rent Dissipation

- contests —— winner is randomly determined
- tournament — multiple prizes (if lose, may still have prize)
- all pay auction —

In many cases the efforts expended are not socially productive.

social waste.

rent seeking

Ex: theft.

- foregone output (input into theft $\Rightarrow$ social waste).
- outputs used in thievery
- protection (bank prevent theft).

Posner: rent seeking expenditure $\doteq$ valuation

all pay auction.

a bidder's bid = bidder's expense

expected ~~total~~ expenditure = seller's expected revenue =

$$R = n \int_{\underline{v}}^{\bar{v}} [v f(v) - (1-F(v))] G(v)\, dv$$

social value (wealth)

$$S = E[V_{(1)}]$$

$$= \int_{\underline{v}}^{\bar{v}} v f_{(1)}(v)\, dv = n \int_{\underline{v}}^{\bar{v}} v F^{n-1}(v) f(v)\, dv.$$

density of distribution of first order statistics.

$$= n \int_{\underline{v}}^{\bar{v}} v f(v) G(v)\, dv$$

waste.

$$S - R = n \int_{\underline{v}}^{\bar{v}} v f(v) G(v)\, dv - n \int_{\underline{v}}^{\bar{v}} [v f(v) - (1-F(v))] G(v)\, dv.$$

cancel

$$= n \int_{\underline{v}}^{\bar{v}} (1-F(v))\, G(v)\, dv > 0.$$

has social waste, but < social gain => social value created

what happens when $n \uparrow$ (infinite) $\to S \to \bar{v}$, $R \to \bar{v}$
↳ more competitions morderate more waste.

Information rent.
expected bidder payoff (surplus) ≥ 0

$xG(x)\Big|_{\underline{v}}^{v} - \int_{\underline{v}}^{v} G(x)\,dx$
$= vG(x) - 0 - \int_{\underline{v}}^{v} G(x)\,dx$

expected bidder surplus

$$U(v) = vG(v) - t^*(v) = vG(v) - \int_{\underline{v}}^{v} x g(x)\,dx$$
$$= vG(v) - vG(v) + \int_{\underline{v}}^{v} G(x)\,dx$$
$$= \int_{\underline{v}}^{v} G(x)\,dx$$

$U'(v) \geq 0$