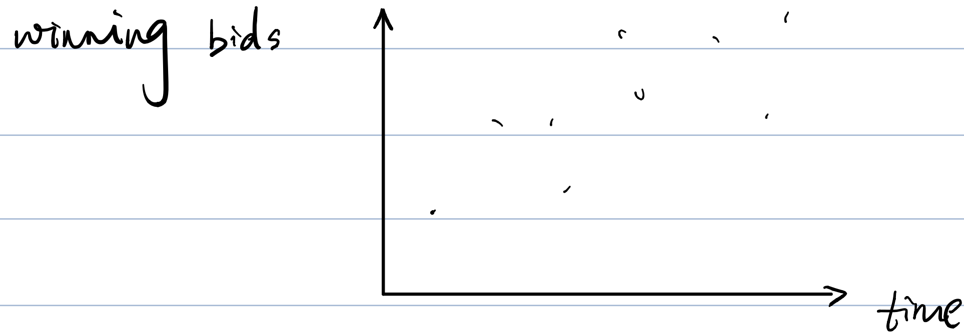
Standard Auction

(*) object goes to highest valuation biders
the lowest bidder type has zero expected payment.

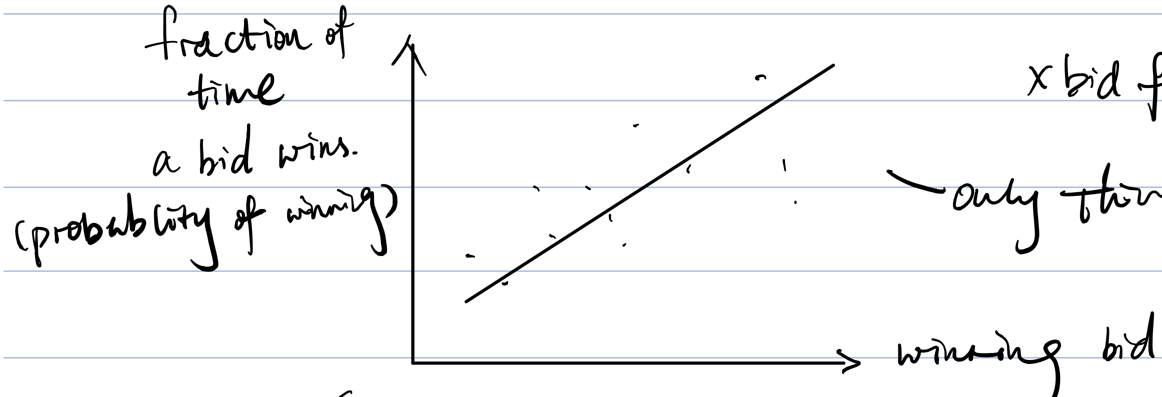
to prove the auction is standard

If the equilibrium bid function is strictly increasing in evaluation

bidding for contracts (How can bidders know others $F(v)$).



assume auction has same number of bidders



x bid function / distribution of valuation.
only thing bidders care about.

image of the distribution of <u>first order statistic</u>

$b^*(v)$

$F(v)$ conduces a distribution of bids

$H(b) = \mathbb{P}_F[b^*(v) \le b] = \Pr[v \le (b^*)^{-1}(b)] = F((b^*)^{-1}(b))$

Distribution of 1 bidder's bid.   strictly ↑ in v equilibrium.

Surplus $= (b(-c))\,\mathbb{P}r[b \text{ wins}]$ ⇒ have $F(v)$ potentially

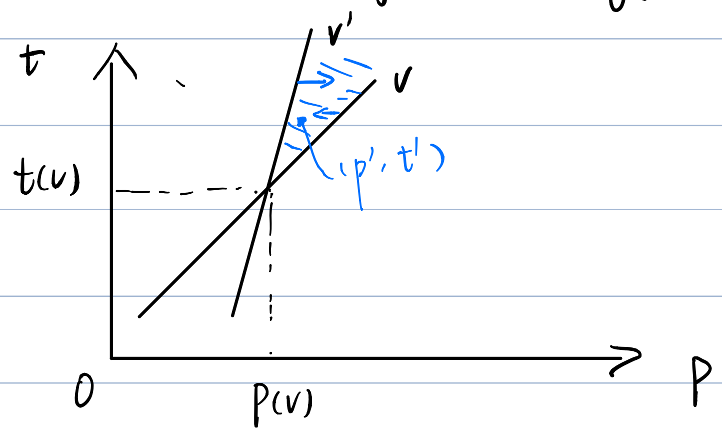
contrasts' cost (random) ⇐ don't know.

Do not care how they bid, just care the distribution of bid

higher cost → bid higher ⇔ higher ⇐ higher valuation.

$(v-b)\,\mathbb{P}[b \text{ wins}] = vp - t := U$ (utility).

when v↑ tradeoff: winning probablity↑ + lose more when lose.



single crossing.

$t(v')$ $p(v')$

t' p'

tradeoff: probablity (p) ⇔ payment (+).

like

$t = vp - U$

$\frac{dt}{dp} = v$

$t' \geq t$ $p' \geq p$

⇑

Δ Given $F()$ and given $b^*()$ ⇒ how should a bidder of type v bid

how this depends on v

bidder's best response

$n=2$ $U_i(b_1, b_2 | v_i)$ — b_2: not know (random) ⇐ F choose $\uparrow$ know $\uparrow$

$$\underset{v_2}{E}\left[U_1(b_1, b_2^*(v_2) \mid v_1) \right] = \int_{\underline{v}_2}^{\bar{v}_2} U_1(b_1, b_2^*(v_2) \mid v_1) f_2(v_2) dv_2$$

expected value

$\max_{b_1} \rightarrow b_1^*(v_1)$. $\leftarrow$ bidder 1's best response

optimal bid function of valuation

NE in bid function $\Rightarrow$ $b_1^*(\ )$ is a best response to $b_2^*(\ )$

$b_2^*(\ )$ .... $b_1^*(\ )$

Bayes - Nash equilibrium.

$F_1 = F_2$ ...... F_n.

$b_1^*(\ ) = b_2^*(\ ) = \cdots\cdots$

Revenue Equilibrium Theorem

☆ Prove + result

In any standard auction, a bidder of type (valuation) v has the same expected payment $t^*(v)$, and the expected Revenue of the seller is the same

Prove: let $U(v' \mid v)$ be the expected utility of a bidder of type v who follow the bid strategy of type v'

$$u(v' \mid v) = G(v')\, \overset{(\text{surplus})}{v} - t^*(v')$$

$G(v')$ = prob that v' is highest valuation among n bidders.

= $F(v')^{n-1}$

equilibrium requires that optimal v' equals v

$v' = v$ maximizes $U(v' \mid v)$

$$\left.\frac{dU}{dv'}\right| = 0 \quad \text{at } v' = v$$

$$\| $$

$$g(v')\,v - \frac{dt^*}{dv}(v') = 0 \quad \text{at } v' = v$$

$$\frac{dt^*}{dv} = v\,g(v)$$

$$t^*(v) - \underbrace{t^*(\underline{v})}_{=0} = \int_{\underline{v}}^{v} x\,g(x)\,dx$$