# Externality

## "Restaurant" game

$n$. students $i = 1, 2 \cdots, n$.

$x$ = price (quality) of a meal ↙ measure by.

$m$ = money

$U_i(x, m) = \sqrt{x} + m$ (assume)

$y_i$ = student's budget

o share the bill

$$m_i = y_i - \frac{1}{n}\sum_{j=1}^{n} x_j$$

$$V_i(x_1, x_2 \cdots x_n) = \sqrt{x_i} + y_i - \frac{1}{n}\sum_{j=1}^{n} x_j$$

negative externality

Nash Equilibrium $\Rightarrow \frac{\partial V_i}{\partial x_i} = 0$

$$\frac{1}{2} x_i^{-\frac{1}{2}} - \frac{1}{n} = 0$$

$$\frac{1}{2\sqrt{x_i}} = \frac{1}{n}$$

$$x_i = \frac{n^2}{4}$$

overspending

$$W(x_1, x_2 \cdots x_n) = \sum_{i=1}^{n} \sqrt{x_i} - \sum_{i=1}^{n} x_i$$

↑ welfare (total surplus)

socially optimal $\Rightarrow \frac{\partial W}{\partial x_i} = 0$

$$\frac{1}{2\sqrt{x_i}} - 1 = 0 \Rightarrow \text{go by oneself.}$$

pay $\frac{1}{n}$ of actural cost → when $n\uparrow$ pay less for each dish

do take into account that (n-1) pay for it

⇒ over consumption

## Congestion

↙ model choice

continuuneous individules $i \in [0,1]$

going to work

bus

car

$S_i = \{0, 1\}$

0: car, 1: bus

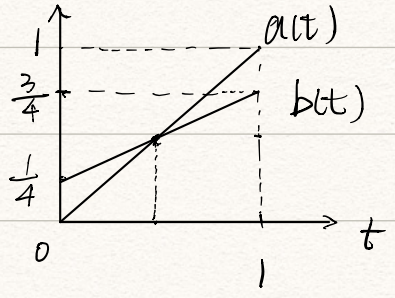
$t = \int_0^1 s(i)\, di$

↑ fraction of people on public transpotation (bus)

$$u_i = \begin{cases} a(t) & s(i)=0 \quad \text{car} \\ b(t) & s(i)=1 \quad \text{bus.} \end{cases}$$

$a(t) = t$

$b(t) = \frac{1}{4} + \frac{1}{2}t$ (assume)




Nash Equilibrium $t^*$ (equilibrium number people taking the bus)

$a(t^*) = b(t^*)$ $t^* = \frac{1}{2}$ ⇒ at this time: (indifferent between car/bus)

putting more people to the car, ↓ time to work.

↓

choose car ⇒ negative ext

Social Optimum $W(t) = (1-t)\,a(t) + t\,b(t)$

$= t - t^2 + \frac{1}{4}t + \frac{1}{2}t^2$

$$= \frac{5}{4}t - \frac{t^2}{2}$$

$$\frac{\partial W(t)}{\partial t} = \frac{5}{4} - t > 0 \quad \text{for } t \in [0,1]$$

$t^0 = 1$ (optimum)

Possible solutions to externality

1. Quantity based solution (quota $\begin{cases} \text{pos} \\ \text{neg} \end{cases}$ $\begin{matrix} \text{lower} \\ \text{upper} \end{matrix}$ bound)
2. Price based solution (taxes)
3. Market for property right

Ex. Restaurant game

$$x_i^* = \frac{n^2}{4} > x_i^0 = \frac{1}{4}$$

① set upper bound of $x_i$ to a quarter.

② $\sqrt{x_i} - \frac{1}{n}\sum_{j=1}^{n} x_j - \underbrace{t x_i}_{\text{marginal tax.}}$

negative ext = $\frac{n-1}{n} x_i$ $\Leftarrow$ set $t x_i$ $\Rightarrow$ take into account the ext on others

Ex. Congestion $t^* = \frac{1}{2} < t^0 = 1$

① don't allow anybody to use the car $\Rightarrow$ may not achieve good ending

② tax / subsidy (marginal)

individual set marginal property $> 0$.

Ex. the property right: the right to pollute