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```

1. #Template
    1.1. C++ Template
1. #define optimizar_io ios_base::sync_with_stdio(0);cin.tie(0);
2. #include <inttypes.h>
3. static void main2() {
4.     char *ppp;
5.     printf("hello world %p\n", &ppp);
6. }
7. static void run_with_stack_size(void (*func)(),size_t stsize){
8.     char *stack, *send;
9.     stack=(char *)malloc(stsize);
10.    send=stack+stsize-16;
11.    send=(char *)((uintptr_t)send/16*16);
12.    asm volatile(
13.        "mov %%rsp, (%0)\n"
14.        "mov %0, %%rsp\n"
15.        :
16.        : "r" (send));
17.    func();
18.    asm volatile(
19.        "mov (%0), %%rsp\n"
20.        :
21.        : "r" (send));
22.    free(stack);
23. }
24. int main() {
25.    run_with_stack_size(main2, 64*1024*1024);
26. }

```

```

1.2. Java Template
1. import java.io.IOException;
2. import java.math.*;
3. import java.util.*;
4.
5. public class main {
6.
7.     public static void main(String[] args)throws IOException{
8.         //FileReader rd = new FileReader("a.in");
9.         Scanner cin = new Scanner(System.in);
10.
11.         while( cin.hasNext() )
12.             int y = cin.nextInt();
13.         List<Integer> B = new ArrayList<Integer>();
14.
15.         int [] C = new int[10];
16.
17.         for( int i = 1; i <= 100; i += 5 ) B.add(i);
18.
19.         Collections.sort(B);

```

```

20.         int a = Collections.binarySearch(B, 7);
21.
22.         B.set(2, 7);
23.         BigInteger d = cin.nextBigInteger();
24.
25.         System.out.println(B.get(2));
26.         System.out.printf("%d", 5);
27.         cin.close();
28.     }
29. }

```

1.3. Python Template

```

1. import string
2. import math
3. import fractions
4.
5. @memoize
6. def funcion( s ):
7.     print s[0:2]
8.     s = sorted( s )
9.     return s
10.
11. arr = []
12. arr.append( 5 )
13. arr.append( 1 )
14. arr = funcion(arr)
15. print arr[0:5]
16. for x in range(0, 10):
17.     arr.append( x )
18. gg = fractions.gcd(10, 65)
19.
20. while True:
21.     try:
22.         n, c, d = raw_input().split()
23.         import math
24.         print pow( long(c), long(n), long(d) )
25.     except EOFError:
26.         break

```

2. Data Structures

2.1. 2D RMQ

```

1. void build( ){ // O(n*m*log(n)*log(m))
2.     for(int i = 0; i < n ; i ++){
3.         for(int j = 0; j < m ; j ++){
4.             table[0][i][j] = Matrix[i][j];
5.             for(int lj = 1; lj <= log2( m ); lj ++){
6.                 for(int j = 0; j + (1<<(lj-1)) < m; j ++){
7.                     table[0][lj][i][j] =
8.                         min(table[0][lj-1][i][j],
9.                             table[0][lj-1][i][j+(1<<(lj-1))]);
10.                }
11.            for(int li = 1; li <= log2(n); li ++ )
12.                for(int i = 0; i < n; i ++ )
13.                    for(int lj = 0; lj <= log2(m); lj++ )
14.                        for(int j = 0; j < m; j ++ )

```

```

15.                table[li][lj][i][j] =
16.                    min(table[li-1][lj][i][j],
17.                        table[li-1][lj][i+(1<<(li-1))][j]);
18.            }
19.        int Query(int x1,int y1,int x2,int y2){
20.            int lenx=x2-x1+1;
21.            int kx=log2(lenx);
22.            int leny=y2-y1+1;
23.            int ky=log2(leny);
24.            int min_R1 = min ( table[kx][ky][x1][y1] ,
25.                                table[kx][ky][x1][y2-(1<<ky) + 1] );
26.            int min_R2 = min ( table[kx][ky][x2-(1<<kx) + 1][y1],
27.                                table[kx][ky][x2-(1<<kx)+1][y2-(1<<ky)+1] );
28.            return min ( min_R1, min_R2 );
29.        }

```

2.2. Convex Hull Optimization

```

1. //para buscar maximo
2. typedef complex<ll> point;
3. typedef vector<point> hull;
4. ll cross(point a, point b){return imag(conj(a) * b);}
5. ll dot(point a, point b){ return real(conj(a) * b); }
6. void add(point a, hull &ch){
7.     for(int n = (int)ch.size(); n > 1 &&
8.         cross(ch[n-1]-ch[n-2], a-ch[n-2]) >= 0; n--)
9.         ch.pop_back();
10.    ch.push_back(a);
11. }
12. ll eval(point a, hull &ch){
13.     int lo = 0, hi = (int)ch.size()-1;
14.     while(lo < hi){
15.         int m = (lo + hi)/2;
16.         if( dot(ch[m], a) >= dot(ch[m+1], a) ) hi = m;
17.         else lo = m + 1;
18.     }
19.     return dot(ch[lo], a);
20. }
21. hull merge(const hull &a, const hull &b){
22.     int n =(int)a.size(), m =(int)b.size(), x=0, y=0;
23.     hull c;
24.     while(x < n && y < m){
25.         if(real(a[x]) <= real(b[y])) add(a[x++], c);
26.         else add(b[y++], c);
27.     }
28.     while (x < n) add(a[x++], c);
29.     while (y < m) add(b[y++], c);
30.     return c;
31. }
32. struct dyn{
33.     vector<hull> H;
34.     void add(point p){
35.         hull h; h.push_back(p);
36.         for (int i = 0; i < (int)H.size(); ++i){
37.             hull &ch = H[i];
38.             if (ch.empty()){ ch = h; return; }

```

```

39.         h = merge(h, ch);
40.         ch.clear();
41.     }
42.     if (!h.empty()) H.push_back(h);
43. }
44. ll query(point p){
45.     ll answer = -1ll<<60;
46.     for (int i = 0; i < (int)H.size(); ++i){
47.         hull &ch = H[i];
48.         if(ch.empty()) continue;
49.         answer = max( answer, eval(p, ch) );
50.     }
51.     return answer;
52. }
53. };

```

2.3. KD Tree

```

1. struct point {
2.     int x, y;
3. } P[maxn];
4. bool cmpx ( const point &a, const point &b ) {
5.     return a.x < b.x;
6. }
7. bool cmpy ( const point &a, const point &b ) {
8.     return a.y < b.y;
9. }
10. inline ll dist ( point a, point b ) {
11.     return 1ll*(a.x-b.x)*(a.x-b.x)+1ll*(a.y-b.y)*(a.y-b.y);
12. }
13. struct kd {
14.     kd *h1, *h2;
15.     point p;
16. }*KD;
17. void init ( int ini, int fin, kd *nod, int split ) {
18.     sort ( P+ini, P+1+fin, (!split)?cmpx : cmpy );
19.     int piv = ( ini+fin )>> 1;
20.     nod->p = P[piv];
21.     if ( ini < piv ) {
22.         nod->h1 = new kd();
23.         init ( ini, piv-1, nod->h1, split^1 );
24.     }
25.     if ( piv+1 <= fin ) {
26.         nod->h2 = new kd();
27.         init ( piv+1, fin, nod->h2, split^1 );
28.     }
29. }
30. ll best;
31. void query ( kd *nod, point p, int split ) {
32.     best = min ( best, dist ( p, nod->p ) );
33.     ll tmp = ( !split )? p.x - nod->p.x : p.y - nod->p.y;
34.     if ( tmp < 0 ) {
35.         if ( nod->h1 )
36.             query ( nod->h1, p, split^1 );
37.         if ( nod->h2 && tmp*tmp < best )
38.             query ( nod->h2, p, split^1 );

```

```

39.     } else {
40.         if ( nod->h2 )
41.             query ( nod->h2, p, split^1 );
42.         if ( nod->h1 && tmp*tmp < best )
43.             query ( nod->h1, p, split^1 );
44.     }
45. }

```

2.4. Persistent Treap

```

1. /* Careful with memory and recommended
2.    to use Garbage Collection */
3. typedef struct item* pitem;
4. struct item {
5.     int val, sz;
6.     pitem l, r;
7.     item ( ) {
8.         val = 0;
9.         sz = 1;
10.        l = r = 0;
11.    }
12. };
13. int sz ( pitem t ) { return (t)? t->sz : 0; }
14. void upd_sz ( pitem t ) {
15.     t->sz = sz(t->l) + sz(t->r) + 1;
16. }
17. typedef tuple<pitem,pitem> tupla;
18. tupla split ( pitem v, int k ) {
19.     if ( !v ) return make_tuple ( v, v );
20.     pitem l, r, ret;
21.     ret = new item(); //ret = v ( treap )
22.     ret->val = v->val;
23.     if ( k >= sz(v->l) + 1 ) {
24.         tie(l,r) = split ( v->r, k-sz(v->l)-1 );
25.         ret->l = v->l; //
26.         ret->r = l;
27.         upd_sz ( ret );
28.         return make_tuple ( ret, r );
29.     } else {
30.         tie(l,r) = split ( v->l, k );
31.         ret->r = v->r; //
32.         ret->l = r;
33.         upd_sz ( ret );
34.         return make_tuple( l, ret );
35.     }
36. }
37. pitem merge ( pitem l, pitem r ) {
38.     if ( !l ) return r;
39.     if ( !r ) return l;
40.     pitem clone = new item(); //no crear
41.     int tl = sz(l), tr = sz(r);
42.     if ( rand() % (tl+tr) < tl ) {
43.         clone->val = l->val; //clone = l
44.         clone->l = l->l;
45.         clone->r = merge ( l->r, r );
46.     } else {

```

```

47.         clone->val = r->val; //clone = r
48.         clone->r = r->r;
49.         clone->l = merge ( l, r->l );
50.     }
51.     upd_sz ( clone );
52.     return clone;
53. }

```

2.5. RB Tree

```

1. #include <ext/pb_ds/assoc_container.hpp>
2. #include <ext/pb_ds/tree_policy.hpp>
3. using namespace __gnu_pbds;
4. typedef tree<
5.     int,
6.     null_type,
7.     less<int>,
8.     rb_tree_tag,
9.     tree_order_statistics_node_update>
10. ordered_set;
11. ordered_set X; //declaracion
12. X.insert(1); // insertar
13. X.erase( X.find( 2 ) ); //eliminar
14. cout<<X.find_by_order(1)<<endl; // k-th menor elemento
15. cout<<X.order_of_key(-5)<<endl; //lower_bound(cant. de menores hay)

```

2.6. Rectangle Union $O(n \log n)$

```

1. struct rectangle {
2.     ll x1, y1, xh, yh;
3. };
4. ll rectangle_area(vector<rectangle> &rs) {
5.     vector<ll> ys; // coordinate compression
6.     for (auto r : rs) {
7.         ys.push_back(r.y1);
8.         ys.push_back(r.yh);
9.     }
10.    sort(ys.begin(), ys.end());
11.    ys.erase(unique(ys.begin(), ys.end()), ys.end());
12.    int n = ys.size(); // measure tree
13.    vector<ll> C(8 * n), A(8 * n);
14.    function<void(int, int, int, int, int, int)> aux =
15.        [&](int a, int b, int c, int l, int r, int k) {
16.            if ((a = max(a, l)) >= (b = min(b, r)))
17.                return;
18.            if (a == l && b == r) C[k] += c;
19.            else {
20.                aux(a, b, c, l, (l+r)/2, 2*k+1);
21.                aux(a, b, c, (l+r)/2, r, 2*k+2);
22.            }
23.            if (C[k]) A[k] = ys[r] - ys[l];
24.            else A[k] = A[2*k+1] + A[2*k+2];
25.        };
26.    struct event {
27.        ll x, l, h, c;
28.    };
29.    vector<event> es;

```

```

30.    for (auto r : rs) {
31.        int l = lower_bound(ys.begin(), ys.end(), r.y1)
32.            - ys.begin();
33.        int h = lower_bound(ys.begin(), ys.end(), r.yh)
34.            - ys.begin();
35.        es.push_back({ r.x1, l, h, +1 });
36.        es.push_back({ r.xh, l, h, -1 });
37.    }
38.    sort(es.begin(), es.end(), [](event a, event b)
39.        {return a.x != b.x ? a.x < b.x : a.c > b.c;});
40.    ll area = 0, prev = 0;
41.    for (auto &e : es) {
42.        area += (e.x - prev) * A[0];
43.        prev = e.x;
44.        aux(e.l, e.h, e.c, 0, n, 0);
45.    }
46.    return area;
47. }

```

2.7. Rectilinear MST $O(n \log n)$

```

1. typedef complex<ll> point;
2. ll rectilinear_mst(vector<point> ps){
3.     vector<int> id(ps.size());
4.     iota(id.begin(), id.end(), 0);
5.     struct edge{
6.         int src, dst;
7.         ll weight;
8.     };
9.     vector<edge> edges;
10.    for (int s = 0; s < 2; ++s){
11.        for (int t = 0; t < 2; ++t){
12.            sort(id.begin(), id.end(), [&](int i, int j){
13.                return real(ps[i] - ps[j]) <
14.                    imag(ps[j] - ps[i]);
15.            });
16.            map<ll, int> sweep;
17.            for (int i : id){
18.                for (auto it = sweep.lower_bound(-imag(ps[i]));
19.                    it != sweep.end(); sweep.erase(it++)){
20.                    int j = it->second;
21.                    if (imag(ps[j] - ps[i]) < real(ps[j] - ps[i]))
22.                        break;
23.                    ll d = abs(real(ps[i] - ps[j]))
24.                        + abs(imag(ps[i] - ps[j]));
25.                    edges.push_back({ i, j, d });
26.                }
27.                sweep[-imag(ps[i])] = i;
28.            }
29.            for (auto &p : ps)
30.                p = point(imag(p), real(p));
31.        }
32.        for (auto &p : ps)
33.            p = point(-real(p), imag(p));
34.    }
35.    ll cost = 0;

```

```

36. sort(edges.begin(), edges.end(), [](edge a, edge b){
37.     return a.weight < b.weight;
38. });
39. union_find uf(ps.size());
40. for (edge e : edges)
41.     if (uf.join(e.src, e.dst))
42.         cost += e.weight;
43. return cost;
44. }

```

3. Geometry

3.1. Delaunay Triangulation

```

1. /*Incremental Randomized Expected O(NlogN)*/
2. int n; point P[maxn];
3. struct edge {
4.     int t, side;
5.     edge ( ) { t = -1, side = 0; }
6.     edge ( int tt, int s ) { t = tt, side = s; }
7. };
8. struct triangle {
9.     point p[3]; edge e[3]; int child[3];
10.    triangle ( ) {}
11.    triangle(const point&p0,const point&p1,const point&p2){
12.        p[0] = p0, p[1] = p1, p[2] = p2;
13.        child[0] = child[1] = child[2] = 0;
14.    }
15.    bool inside(const point &pp) const {
16.        point a = p[0]-pp, b = p[1]-pp, c = p[2]-pp;
17.        return cross(a, b) >= 0 &&
18.            cross(b, c) >= 0 &&
19.            cross(c, a) >= 0;
20.    }
21. };
22. triangle T[maxn*3]; int ct;
23. bool is_leaf ( int t ) {
24.     return !T[t].child[0]&&!T[t].child[1]&&!T[t].child[2];
25. }
26. void add_edge ( edge a, edge b ) {
27.     if ( a.t != -1 ) T[a.t].e[a.side] = b;
28.     if ( b.t != -1 ) T[b.t].e[b.side] = a;
29. }
30. struct Triangulation {
31.     Triangulation ( ) {
32.         int M = 1e5 * 3; //multiplicar el maximo valor por 3
33.         T[0]=triangle(point(-M,-M),point(M,-M),point(0,M));
34.         ct = 1;
35.     }
36.     int find ( int t, const point &p ) {
37.         while ( !is_leaf(t) ) {
38.             for ( int i = 0; i < 3; i ++ )
39.                 if (T[t].child[i]&&T[T[t].child[i]].inside(p)){
40.                     t = T[t].child[i]; break;
41.                 }
42.             return t;
43.         }

```

```

44. void add_point ( const point &p ) {
45.     int t = find ( 0, p ), tab, tbc, tca;
46.     tab = ct;
47.     T[ct++] = triangle ( T[t].p[0], T[t].p[1], p );
48.     tbc = ct;
49.     T[ct++] = triangle ( T[t].p[1], T[t].p[2], p );
50.     tca = ct;
51.     T[ct++] = triangle ( T[t].p[2], T[t].p[0], p );
52.     add_edge ( {tab,0}, {tbc,1} );
53.     add_edge ( {tbc,0}, {tca,1} );
54.     add_edge ( {tca,0}, {tab,1} );
55.     add_edge ( {tab,2}, T[t].e[2] );
56.     add_edge ( {tbc,2}, T[t].e[0] );
57.     add_edge ( {tca,2}, T[t].e[1] );
58.     T[t].child[0] = tab; T[t].child[1] = tbc;
59.     T[t].child[2] = tca;
60.     flip ( tab, 2 ); flip ( tbc, 2 ); flip ( tca, 2 );
61. }
62. void flip ( int ti, int pi ) {
63.     int tj = T[ti].e[pi].t;
64.     int pj = T[ti].e[pi].side;
65.     if ( tj == -1 ) return;
66.     if (!incircle(T[ti].p[0],T[ti].p[1],
67.                 T[ti].p[2],T[tj].p[pj])) return;
68.     int tk = ct;
69.     T[ct++] = triangle(T[ti].p[(pi+1)%3],
70.                       T[tj].p[pj],T[ti].p[pi]);
71.     int tl = ct;
72.     T[ct++] = triangle ( T[tj].p[(pj+1)%3],
73.                         T[ti].p[pi], T[tj].p[pj]
74. );
75.     add_edge ( {tk,0}, {tl,0} );
76.     add_edge ( {tk,1}, T[ti].e[(pi+2)%3] );
77.     add_edge ( {tk,2}, T[tj].e[(pj+1)%3] );
78.     add_edge ( {tl,1}, T[tj].e[(pj+2)%3] );
79.     add_edge ( {tl,2}, T[ti].e[(pi+1)%3] );
80.     T[ti].child[0] = tk, T[ti].child[1] = tl,
81.     T[ti].child[2] = 0;
82.     T[tj].child[0] = tk, T[tj].child[1] = tl,
83.     T[tj].child[2] = 0;
84.     flip ( tk, 1 ); flip ( tk, 2 );
85.     flip ( tl, 1 ); flip ( tl, 2 );
86. }
87. void delaunay;
88. void triangulate ( ) {
89.     delaunay = Triangulation();
90.     random_shuffle ( P+1, P+1+n );
91.     for ( int i = 1; i <= n; i ++ )
92.         delaunay.add_point ( P[i] );
93. }

```

3.2. Minimum Enclosing Disk O(N) expected time

```

1. circle circumcircle ( const point &a,
2.     const point &b, const point &c ) {

```

```

3.  if ( abs( cross( a - c, b - c ) ) > eps ) {
4.      point o = three_point_circle ( a, b, c );
5.      return { o, abs ( o - a ) };
6.  }
7.  point p = min ( { a, b, c } );
8.  point q = max ( { a, b, c } );
9.  return circle { (p+q)*0.5, abs(p-q)*0.5 };
10. }
11. circle min_enclosing_disk_with_2_points ( vector<point> &p,
12.      int n, int a, int b ) {
13.     circle ret = circle { (p[a]+p[b])*0.5, abs(p[a]-p[b])*0.5 };
14.     for ( int i = 0; i <= n; i ++ ) {
15.         db d = abs ( ret.p - p[i] );
16.         if ( d <= ret.r + eps ) continue;
17.         ret = circumcircle ( p[a], p[b], p[i] );
18.     }
19.     return ret;
20. }
21. circle min_enclosing_disk_with_1_point ( vector<point> &p,
22.      int n, int a ) {
23.     circle ret = circle { p[a], 0 };
24.     for ( int i = 0; i <= n; i ++ ) {
25.         db d = abs ( ret.p - p[i] );
26.         if ( d <= ret.r + eps ) continue;
27.         ret = min_enclosing_disk_with_2_points( p, i, a, i );
28.     }
29.     return ret;
30. }
31. circle min_enclosing_disk ( vector<point> &p ) {
32.     srand(42);
33.     random_shuffle ( p.begin(), p.end() );
34.
35.     int n = p.size() - 1;
36.     circle ret = circle { p[0], 0 };
37.     for ( int i = 1; i <= n; i ++ ) {
38.         db d = abs ( ret.p - p[i] );
39.         if ( d <= ret.r + eps ) continue;
40.         ret = min_enclosing_disk_with_1_point ( p, i, i );
41.     }
42.     return ret;
43. }

```

3.3. Pick Theorem $O(n)$

```

1.  /*A = I + B/2 - 1:
2.   A = Area of the polygon
3.   I = Number of integer coordinates points inside
4.   B = Number of integer coordinates points on the boundary
5.   Polygon's vertex must have integer coordinates */
6.  ll points_on_segment(const line &s){
7.      point p = s[0] - s[1];
8.      return __gcd(abs(p.x), abs(p.y));
9.  }
10. pair<ll, ll> pick_theorem(polygon &P){
11.     ll A = area2(P), B = 0, I = 0;
12.     for (int i = 0, n = P.size(); i < n; ++i)

```

```

13.         B += points_on_segment({P[i], P[NEXT(i)]});
14.     A = abs(A);
15.     I = (A - B) / 2 + 1;
16.     return {I, B}; // < points inside, points in boundary>
17. }
18.

```

3.4. Primitives

```

1.  /**1- Base element
2.   2- The traveling direction of the point (ccw)
3.   3- Intersection
4.   4- Distance.
5.   5- Polygon inclusion decision point
6.   6- Area of a polygon
7.   7- Scale a polygon
8.   8- triangulation possible non convex poly  $O(n^2)$ 
9.   9-Convex hull (Andrew's Monotone Chain)
10.  10-Cutting of a convex polygon
11.  11-Convex polygon inclusion decision point
12.  12-Incircle
13.  13-Closest Pair Point
14.  14-Three Point Circle
15.  15-Circle_circle_intersect
16.  16-Tangents Point Circle
17.  17-Circle-Line-Intersection
18.  18-Centroid of a (possibly nonconvex) Polygon
19.  19-Point rotate */
20. ///----1-Base element----
21. struct point {
22.     db x, y;
23.     point ( db xx = 0, db yy = 0 ): x(xx), y(yy) { }
24.     point operator + ( const point &a ) const {
25.         return { x+a.x, y+a.y };
26.     }
27.     point operator - ( const point &a ) const {
28.         return { x-a.x, y-a.y };
29.     }
30.     point operator * ( const db &c ) const {
31.         return { x*c, y*c };
32.     }
33.     point operator * ( const point &p ) const {
34.         return { x*p.x - y*p.y, x*p.y + y*p.x };
35.     }
36.     point operator / ( const db &c ) const {
37.         return { x/c, y/c };
38.     }
39.     point operator / ( const point &a ) const {
40.         return point { x*a.x + y*a.y, y*a.x - x*a.y } /
41.             /*divide 2 complejos*/( a.x*a.x + a.y*a.y );
42.     }
43.     bool operator < ( const point &a ) const {
44.         if ( abs( x-a.x ) > eps )
45.             return x+eps < a.x;
46.         return y+eps < a.y;
47.     }

```

```

48. };
49. typedef vector<point> polygon;
50. struct line : public vector<point> {
51.     line(const point &a, const point &b) {
52.         push_back(a); push_back(b);
53.     }
54. };
55. struct circle { point p; db r; };
56. db cross ( const point &a, const point &b ) {
57.     return a.x*b.y - a.y*b.x;
58. }
59. db dot ( const point &a, const point &b ) {
60.     return a.x*b.x + a.y*b.y;
61. }
62. db norm ( const point &p ) {
63.     return dot ( p, p );
64. }
65. db abs ( const point &p ) {
66.     return sqrt ( norm(p) );
67. }
68. db arg ( const point &p ) {
69.     return atan2 ( p.y, p.x );
70. }
71. point conj ( const point &p ) {
72.     return point { p.x, -p.y };
73. }
74. point crosspoint(const line &l, const line &m) {
75.     db A = cross(l[1] - l[0], m[1] - m[0]);
76.     db B = cross(l[1] - l[0], l[1] - m[0]);
77.     if (abs(A)<eps&&abs(B)<eps) return m[0]; //same line
78.     if (abs(A)<eps) assert(false); //PRECONDITION NOT SATISFIED
79.     return m[0] + (m[1] - m[0])* B / A;
80. }
81. ///---2-The traveling direction of the point-----
82. int ccw(point a, point b, point c) {
83.     b = b-a; c = c-a;
84.     if (cross(b, c) > 0) return +1; // counter clockwise
85.     if (cross(b, c) < 0) return -1; // clockwise
86.     if (dot(b, c) < 0) return +2; // c--a--b on line
87.     if (norm(b) < norm(c)) return -2; // a--b--c on line
88.     return 0;
89. }
90. ///---3-Intersection-----
91. bool intersectLL(const line &l, const line &m) {
92.     return abs(cross(l[1]-l[0],m[1]-m[0]))>eps//non-parallel
93.         ||abs(cross(l[1]-l[0],m[0]-l[0]))<eps; //same line
94. }
95. bool intersectLS(const line &l, const line &s) {
96.     return cross(l[1]-l[0], s[0]-l[0])* // s[0] is left of l
97.         cross(l[1]-l[0],s[1]-l[0])<eps; //s[1] is right of l
98. }
99. bool intersectLP(const line &l, const point &p) {
100.     return abs(cross(l[1]-p, l[0]-p)) < eps;
101. }
102. bool intersectSS(const line &s, const line &t) {

```

```

103.     return ccw(s[0],s[1],t[0])*ccw(s[0],s[1],t[1]) <= 0 &&
104.         ccw(t[0],t[1],s[0])*ccw(t[0],t[1],s[1]) <= 0;
105. }
106. bool intersectSP(const line &s, const point &p) {
107.     return abs(s[0]-p)+abs(s[1]-p)-abs(s[1]-s[0])<eps;
108. }
109. ///---4-Distance-----
110. point projection(const line &l, const point &p) {
111.     db t = dot(p-l[0], l[0]-l[1]) / norm(l[0]-l[1]);
112.     return l[0] + (l[0]-l[1])*t;
113. }
114. point reflection(const line &l, const point &p) {
115.     return p + point(2,0)*(projection(l, p) - p);
116. }
117. double distanceLP(const line &l, const point &p) {
118.     return abs(p - projection(l, p));
119. }
120. double distanceLL(const line &l, const line &m) {
121.     return intersectLL(l, m) ? 0 : distanceLP(l, m[0]);
122. }
123. double distanceLS(const line &l, const line &s) {
124.     if (intersectLS(l, s)) return 0;
125.     return min(distanceLP(l, s[0]), distanceLP(l, s[1]));
126. }
127. double distanceSP(const line &s, const point &p) {
128.     const point r = projection(s, p);
129.     if (intersectSP(s, r)) return abs(r - p);
130.     return min(abs(s[0] - p), abs(s[1] - p));
131. }
132. double distanceSS(const line &s, const line &t) {
133.     if (intersectSS(s, t)) return 0;
134.     return min(min(distanceSP(s, t[0]), distanceSP(s, t[1])),
135.         min(distanceSP(t, s[0]), distanceSP(t, s[1])));
136. }
137. ///---5-Polygon inclusion decision point---
138. #define curr(G, i) G[i]
139. #define next(G, i) G[(i+1)%G.size()]
140. enum { OUT, ON, IN };
141. int contains(const polygon &G, const point& p) {
142.     bool in = false;
143.     for (int i = 0; i < (int)G.size(); ++i) {
144.         point a = curr(G,i) - p, b = next(G,i) - p;
145.         if (a.y > b.y) swap(a, b);
146.         if (a.y <= 0 && 0 < b.y)
147.             if (cross(a, b) < 0) in = !in;
148.         if (cross(a, b) == 0 && dot(a, b) <= 0) return ON;
149.     }
150.     return in ? IN : OUT;
151. }
152. ///---6-Area of a polygon-----
153. double area2(const polygon& G) {
154.     double A = 0;
155.     for (int i = 0; i < (int)G.size(); ++i)
156.         A += cross(curr(G, i), next(G, i));
157.     return A;

```



```

158. }
159. ///-----7-Scale a polygon---
160. #define prev(G,i) G[(i-1+G.size())%G.size()]
161. polygon shrink_polygon(const polygon &G, double len) {
162.     polygon res;
163.     for (int i = 0; i < (int)G.size(); ++i) {
164.         point a = prev(G,i), b = curr(G,i), c = next(G,i);
165.         point u = (b - a) / abs(b - a);
166.         double th = arg((c - b)/ u) * 0.5;
167.         res.push_back( b + u * point(-sin(th), cos(th))
168.                        * len / cos(th) );
169.     }
170.     return res;
171. }
172. ///-----8-triangulation possibly non convex poly O(n^2)--
173. polygon make_triangle(const point&a,const point&b,
174.                       const point&c){
175.     polygon ret(3);
176.     ret[0] = a; ret[1] = b; ret[2] = c;
177.     return ret;
178. }
179. bool triangle_contains(const polygon&tri,const point&p){
180.     return ccw(tri[0], tri[1], p) >= 0 &&
181.            ccw(tri[1], tri[2], p) >= 0 &&
182.            ccw(tri[2], tri[0], p) >= 0;
183. }
184. bool ear_Q(int i, int j, int k, const polygon& G) {
185.     polygon tri = make_triangle(G[i], G[j], G[k]);
186.     if (ccw(tri[0], tri[1], tri[2]) <= 0) return false;
187.     for (int m = 0; m < (int)G.size(); ++m)
188.         if (m != i && m != j && m != k)
189.             if (triangle_contains(tri, G[m]))
190.                 return false;
191.     return true;
192. }
193. void triangulate(const polygon& G, vector<polygon>& t) {
194.     const int n = G.size();
195.     vector<int> l, r;
196.     for (int i = 0; i < n; ++i) {
197.         l.push_back( (i-1+n) % n );
198.         r.push_back( (i+1+n) % n );
199.     }
200.     int i = n-1;
201.     while ((int)t.size() < n-2) {
202.         i = r[i];
203.         if (ear_Q(l[i], i, r[i], G)) {
204.             t.push_back(make_triangle(G[l[i]], G[i], G[r[i]]));
205.             l[ r[i] ] = l[i];
206.             r[ l[i] ] = r[i];
207.         }
208.     }
209. }
210. ///---9-Convex_hull-----
211. vector<point> convex_hull(vector<point> ps) {
212.     int n = ps.size(), k = 0;

```

```

213.     sort(ps.begin(), ps.end());
214.     vector<point> ch(2*n);
215.     for (int i = 0; i < n; ch[k++] = ps[i++]) // lower-hull
216.         while (k >= 2 && ccw(ch[k-2], ch[k-1], ps[i]) <= 0)--k;
217.     for (int i = n-2, t = k+1; i >= 0; ch[k++] = ps[i--]) //upper-hull
218.         while (k >= t && ccw(ch[k-2], ch[k-1], ps[i]) <= 0)--k;
219.     ch.resize(k-1);
220.     return ch;
221. }
222. ///---10-Cutting of a convex polygon-----
223. polygon convex_cut(const polygon& G, const line& l) {
224.     polygon Q;
225.     for (int i = 0; i < (int)G.size(); ++i) {
226.         point A = curr(G, i), B = next(G, i);
227.         if (ccw(l[0], l[1], A) != -1) Q.push_back(A);
228.         if (ccw(l[0], l[1], A)*ccw(l[0], l[1], B) < 0)
229.             Q.push_back(crosspoint(line(A, B), l));
230.     }
231.     return Q;
232. }
233. ///---11-Convex polygon inclusion decision point-----
234. int convex_contains(const polygon &G, const point &p) {
235.     //G[0] must be the lowest right vertex
236.     int b = 1, e = G.size() - 1;
237.     while(b < e){
238.         int mid = (b + e) / 2;
239.         if(cross( G[0]-p, G[mid]-p) <= eps){
240.             e = mid;
241.         }
242.         else b = mid + 1;
243.     }
244.     if(cross(G[b]-p,G[b-1]-p)<=eps&&
245.        cross(G[0]-p,G[b]-p)<=eps) if(b > 1
246.        or (G[0].y <= p.y + eps && p.y <= G[1].y + eps))
247.         return IN; // IN or ON
248.     return OUT;
249. }
250. ///-----12-Incircle-----
251. bool incircle(point a, point b, point c, point p) {
252.     a = a-p; b = b-p; c = c-p;
253.     return norm(a) * cross(b, c)
254.            + norm(b) * cross(c, a)
255.            + norm(c) * cross(a, b) >= 0;
256.     // < : inside, = cocircular, > outside
257. }
258. ///---13-closestPair-----
259. double closest_pair_points(vector<point> &P) {
260.     auto cmp = [](point a, point b) {
261.         return make_pair(a.y, a.x)
262.                < make_pair(b.y, b.x);
263.     };
264.     int n = P.size();
265.     sort(P.begin(), P.end());
266.     set<point, decltype(cmp)> S(cmp);
267.     const double oo = 1e9; // adjust

```



```

268. double ans = oo;
269. for (int i = 0, ptr = 0; i < n; ++i) {
270.     while (ptr < i && abs(P[i].x - P[ptr].x) >= ans)
271.         S.erase(P[ptr++]);
272.     auto lo = S.lower_bound(point(-oo, P[i].y - ans - eps));
273.     auto hi = S.upper_bound(point(-oo, P[i].y + ans + eps));
274.     for (decltype(lo) it = lo; it != hi; ++it)
275.         ans = min(ans, abs(P[i] - *it));
276.     S.insert(P[i]);
277. }
278. return ans;
279. }
280. ///----14-Three Point Circle-----
281. point three_point_circle(const point&a, const point&b,
282.                          const point&c) {
283.     point x = (b - a) / norm(b - a), y = (c - a) / norm(c - a);
284.     return (y - x) / (conj(x) * y - x * conj(y)) + a;
285. }
286. ///--15-Circle_circle_intersect-----
287. pair<point, point> circle_circle_intersect(const point&c1,
288.     const double& r1, const point& c2, const double& r2) {
289.     point A = conj(c2 - c1);
290.     point B = ((c2 - c1) * conj(c2 - c1)) * -1.0 + r2 * r2 - r1 * r1;
291.     point C = (c2 - c1) * r1 * r1;
292.     point D = B * B - A * C * 4.0;
293.     complex<db> q ( D.x, D.y );
294.     q = sqrt(q);
295.     D = { real(q), imag(q) };
296.     point z1 = (B * -1.0 + D) / (A * 2.0) + c1,
297.           z2 = (B * -1.0 - D) / (A * 2.0) + c1;
298.     return pair<point, point>(z1, z2);
299. }
300. ///--16-Tangents Point Circle-----
301. vector<point> tangent(point p, circle c) {
302.     double D = abs(p - c.p);
303.     if (D + eps < c.r) return {};
304.     point t = c.p - p;
305.     double theta = asin( c.r / D );
306.     double d = cos(theta) * D;
307.     t = t / abs(t) * d;
308.     if ( abs(D - c.r) < eps ) return {p + t};
309.     point rot( cos(theta), sin(theta) );
310.     return {p + t * rot, p + t * conj(rot)};
311. }
312. ///-17-Circle-Line-Intersection-----
313. vector<point> intersectLC( line l, circle c ) {
314.     point u = l[0] - l[1], v = l[0] - c.p;
315.     double a = dot(u, u), b = dot(u, v),
316.           cc = dot(v, v) - c.r * c.r;
317.     double det = b * b - a * cc;
318.     if ( det < eps ) return {};
319.     else return { l[0] + u * (-b + sqrt(det)) / a,
320.                 l[0] + u * (-b - sqrt(det)) / a };
321. }
322. ///-18--Centroid of a (possibly nonconvex) Polygon

```

```

323. point centroid(const polygon &poly) {
324.     point c(0, 0);
325.     double scale = 3.0 * area2(poly);
326.     for (int i = 0, n = poly.size(); i < n; ++i) {
327.         int j = (i + 1) % n;
328.         c = c + (poly[i] + poly[j]) * (cross(poly[i], poly[j]));
329.     }
330.     return c / scale;
331. }
332. ///-19-Point rotate-----
333. inline point rotate(point A, double ang) { //respect to origin
334.     double r = sqrt(dot(A, A));
335.     double oang = atan2(A.y, A.x);
336.     return (point){cos(ang + oang), sin(ang + oang)} * r;
337. }
338.

```

4. Graph

4.1. Dinic

```

1. int pos, Index[MAXN]; //index = -1, pos = 0
2. int lv[MAXN], Id[MAXN], in, fin, n;
3. struct edges { //N cant de nodos
4.     int nod, newn, cap, next;
5.     edges( int a = 0, int b = 0, int c = 0, int e = 0 ) {
6.         nod = a, newn = b, cap = c, next = e;
7.     }
8.     int nextn ( int a ) {
9.         return ( nod == a ) ? newn : nod;
10.    }
11. } G[MAXE];
12. //nod, newn, cap
13. void insertar( int a, int b, int c ) {
14.     G[pos] = edges( a, b, c, Index[a] );
15.     Index[a] = pos++;
16.     G[pos] = edges( b, a, 0, Index[b] );
17.     Index[b] = pos++;
18. }
19. queue<int> Q;
20. bool Bfs( int limt ) {
21.     while( !Q.empty() ) Q.pop();
22.     fill( lv, lv + n + 1, 0 );
23.     lv[in] = 1;
24.     Q.push( in );
25.     while( !Q.empty() ) {
26.         int nod = Q.front();
27.         Q.pop();
28.         for( int i = Index[nod]; i != -1; i = G[i].next ) {
29.             int newn = G[i].newn;
30.             if( lv[newn] != 0 || G[i].cap < limt ) continue;
31.             lv[newn] = lv[nod] + 1;
32.             Q.push( newn );
33.             if( newn == fin ) return true;
34.         }
35.     }
36.     return false;

```

```

37. }
38. bool Dfs( int nod, int limit ){
39.     if( nod == fin ) return true;
40.     for( ; Id[nod] != -1; Id[nod] = G[Id[nod]].next ){
41.         int newn = G[Id[nod]].newn;
42.         if( lv[nod] + 1 == lv[newn] &&
43.             G[Id[nod]].cap >= limit && Dfs( newn, limit ) ){
44.             G[Id[nod]].cap -= limit;
45.             G[Id[nod]^1].cap += limit;
46.             return true;
47.         }
48.     }
49.     return false;
50. }
51. int Dinic( ){
52.     int flow = 0;
53.     for( int limit = 1024; limit > 0; ){
54.         if( !Bfs( limit ) ){
55.             limit >>= 1;
56.             continue;
57.         }
58.         for( int i = 0; i <= n; i ++ )
59.             Id[i] = Index[i];
60.         while( limit > 0 && Dfs( in, limit ) )
61.             flow += limit;
62.     }
63.     return flow;
64. }

```

4.2. Dominator Tree $O((N+M)\log N)$

```

1. struct graph{
2.     int n;
3.     vector<vector<int>> > adj, radj, to;
4.     graph(int n) : n(n), adj(n), radj(n), to(n) {}
5.     void add_edge(int src, int dst){
6.         adj[src].push_back(dst);
7.         radj[dst].push_back(src);
8.     }
9.     vector<int> rank, semi, low, anc;
10.    int eval(int v){
11.        if (anc[v] < n && anc[anc[v]] < n){
12.            int x = eval(anc[v]);
13.            if (rank[semi[low[v]]] > rank[semi[x]])
14.                low[v] = x;
15.            anc[v] = anc[anc[v]];
16.        }
17.        return low[v];
18.    }
19.    vector<int> prev, ord;
20.    void dfs(int u){
21.        rank[u] = ord.size();
22.        ord.push_back(u);
23.        for (int i = 0; i < (int) adj[u].size(); ++i){
24.            int v = adj[u][i];
25.            if (rank[v] < n)

```

```

26.                continue;
27.                dfs(v);
28.                prev[v] = u;
29.            }
30.        }
31.        vector<int> idom; // idom[u] is an immediate dominator of u
32.        void dominator_tree(int r){
33.            idom.assign(n, n);
34.            prev = rank = anc = idom;
35.            semi.resize(n);
36.            for (int i = 0; i < n; ++i)
37.                semi[i] = i;
38.            low = semi;
39.            ord.clear();
40.            dfs(r);
41.            vector<vector<int>> > dom(n);
42.            for (int x = (int) ord.size() - 1; x >= 1; --x){
43.                int w = ord[x];
44.                for (int j = 0; j < (int) radj[w].size(); ++j){
45.                    int v = radj[w][j];
46.                    int u = eval(v);
47.                    if (rank[semi[w]] > rank[semi[u]])
48.                        semi[w] = semi[u];
49.                }
50.                dom[semi[w]].push_back(w);
51.                anc[w] = prev[w];
52.                for (int i = 0; i < (int) dom[prev[w]].size(); ++i){
53.                    int v = dom[prev[w]][i];
54.                    int u = eval(v);
55.                    idom[v] = (rank[prev[w]] > rank[semi[u]]?
56.                                u : prev[w]);
57.                }
58.                dom[prev[w]].clear();
59.            }
60.            for (int i = 1; i < (int) ord.size(); ++i){
61.                int w = ord[i];
62.                if (idom[w] != semi[w])
63.                    idom[w] = idom[idom[w]];
64.            }
65.        }
66.        vector<int> dominators(int u){
67.            vector<int> S;
68.            for ( ; u < n; u = idom[u])
69.                S.push_back(u);
70.            return S;
71.        }
72.        void tree( ){
73.            for (int i = 0; i < n; ++i){
74.                if (idom[i] < n)
75.                    to[ idom[i] ].push_back( i );
76.            }
77.        }
78.    };

```

4.3. Heavy Light Decomposition

```

1. vector<int> V[MAXN];
2. int n, sz[MAXN], lv[MAXN], P[MAXN], A[MAXN], B[MAXN], C[MAXN];
3. // P: padre A: ult hoja B: pos C:cant
4. // G[i] = vector<int>( 4*C[i], 0 );
5. // lv[1] = 1;
6. void Dfs( int nod = 1, int pad = 0 ){
7.     int mej = nod;
8.     A[nod] = nod;
9.     for( auto i : V[nod] ){
10.        if( i == pad ) continue;
11.        lv[i] = lv[nod]+1;
12.        Dfs( i, nod );
13.        if( sz[i] > sz[mej] ) mej = i;
14.        sz[nod] += sz[i];
15.    }
16.    mej = A[mej];
17.    sz[nod] ++;
18.    P[mej] = pad;
19.    A[nod] = mej, B[nod] = C[mej];
20.    C[mej] ++;
21. }
22. int sol;
23. void solve( int a, int b ){
24.     int a1 = a, b1 = b, dist = 0;
25.     while( A[a1] != A[b1] ){
26.         if( lv[ P[ A[a1] ] ] > lv[ P[ A[b1] ] ] )
27.             dist += lv[a1] - lv[ P[ A[a1] ] ], a1 = P[ A[a1] ];
28.         else
29.             dist += lv[b1] - lv[ P[ A[b1] ] ], b1 = P[ A[b1] ];
30.     }
31.     dist += abs( lv[ a1 ] - lv[ b1 ] );
32.     int lca = ( lv[a1] > lv[b1] ) ? b1 : a1;
33.
34.     sol = 0;
35.     while( A[a] != A[lca] ){
36.         sol = __gcd(sol, query(A[a], 0, C[A[a]]-1, 1, B[a], C[A[a]]-1));
37.         a = P[ A[a] ];
38.     }
39.
40.     sol = __gcd(sol, query(A[a], 0, C[A[a]]-1, 1, B[a], B[lca]-1 ));
41.
42.     while( A[b] != A[lca] ){
43.         sol = __gcd(sol, query(A[b], 0, C[A[b]]-1, 1, B[b], C[A[b]]-1));
44.         b = P[ A[b] ];
45.     }
46.
47.     sol = __gcd(sol, query(A[b], 0, C[A[b]]-1, 1, B[b], B[lca] ));
48. }

```

4.4. Hopcroft-Karp Bipartite Matching $O(M\sqrt{N})$

```

1. const int MAXV = 1001;
2. const int MAXV1 = 2*MAXV;
3. vector<int> ady[MAXV];
4. int D[MAXV1], Mx[MAXV], My[MAXV];
5. bool BFS(){

```

```

6.     int u, v, i, e;
7.     queue<int> cola;
8.     bool f = 0;
9.     for (i = 0; i < N+M; i++) D[i] = 0;
10.    for (i = 0; i < N; i++)
11.        if (Mx[i] == -1) cola.push(i);
12.    while (!cola.empty()){
13.        u = cola.front(); cola.pop();
14.        for (e = ady[u].size()-1; e >= 0; e--) {
15.            v = ady[u][e];
16.            if (D[v + N]) continue;
17.            D[v + N] = D[u] + 1;
18.            if (My[v] != -1){
19.                D[My[v]] = D[v + N] + 1;
20.                cola.push(My[v]);
21.            }else f = 1;
22.        }
23.    }
24.    return f;
25. }
26. int DFS(int u){
27.     for (int v, e = ady[u].size()-1; e >= 0; e--){
28.         v = ady[u][e];
29.         if (D[v+N] != D[u]+1) continue;
30.         D[v+N] = 0;
31.         if (My[v] == -1 || DFS(My[v])){
32.             Mx[u] = v; My[v] = u; return 1;
33.         }
34.     }
35.     return 0;
36. }
37. int Hopcroft_Karp(){
38.     int i, flow = 0;
39.     for (i = max(N,M); i >= 0; i--) Mx[i] = My[i] = -1;
40.     while (BFS())
41.         for (i = 0; i < N; i++)
42.             if (Mx[i] == -1 && DFS(i))
43.                 ++flow;
44.     return flow;
45. }

```

4.5. Hungarian $O(N^3)$

```

1. #define MAXN 300
2. int N, A[MAXN+1][MAXN+1], p, q, oo = 1 << 30;
3. int fx[MAXN+1], fy[MAXN+1], x[MAXN+1], y[MAXN+1];
4. int hungarian(){
5.     memset(fx, 0, sizeof(fx));
6.     memset(fy, 0, sizeof(fy));
7.     memset(x, -1, sizeof(x));
8.     memset(y, -1, sizeof(y));
9.     for(int i = 0; i < N; ++i)
10.        for(int j = 0; j < N; ++j) fx[i] = max(fx[i], A[i][j]);
11.     for(int i = 0; i < N; ){
12.         vector<int> t(N, -1), s(N+1, i);
13.         for(p = q = 0; p <= q && x[i]<0; ++p)

```

```

14.         for(int k = s[p], j = 0; j < N && x[i]<0; ++j)
15.             if (fx[k]+fy[j]==A[k][j] && t[j]<0)
16.             {
17.                 s[++q]=y[j];
18.                 t[j]=k;
19.                 if(s[q]<0)
20.                     for(p=j; p>=0; j=p)
21.                         y[j]=k=t[j], p=x[k], x[k]=j;
22.             }
23.         if (x[i]<0){
24.             int d = oo;
25.             for(int k = 0; k < q+1; ++k)
26.                 for(int j = 0; j < N; ++j)
27.                     if(t[j]<0) d=min(d,fx[s[k]]+fy[j]-A[s[k]][j]);
28.             for(int j = 0; j < N; ++j) fy[j]+=(t[j]<0?0:d);
29.             for(int k = 0; k < q+1; ++k) fx[s[k]]-=d;
30.         }
31.         else ++i;
32.     }
33.     int ret = 0;
34.     for(int i = 0; i < N; ++i) ret += A[i][x[i]];
35.     return ret;
36. }

```

4.6. Max Flow Min Cost

```

1. namespace MaxFlowMinCost{
2.     #define MAXE 1000005
3.     #define MAXN 100010
4.     #define oo 1e9
5.     int pos, Index[MAXN], In, Fin, NN;///index = -1
6.     typedef int type_cost;
7.     typedef pair<type_cost, int> par;
8.     type_cost Phi[MAXN];
9.     struct edges{
10.         int nod, newn, cap, next;
11.         type_cost cost;
12.         edges( int a=0,int b=0,int c=0,type_cost d=0,int e=0 ){
13.             nod = a, newn = b, cap = c, cost = d, next = e;
14.         }
15.     }G[MAXE];
16.     void initialize( int cnod, int source, int sink ){
17.         In = source, Fin = sink, NN = cnod;
18.         memset( Index, -1, sizeof(Index) );
19.         pos = 0;
20.     }
21.     ///nod, newn, cap, cost
22.     void insertar( int a, int b, int c, type_cost d ){
23.         G[pos] = edges( a, b, c, d, Index[a] );
24.         Index[a] = pos ++;
25.         G[pos] = edges( b, a, 0, -d, Index[b] );
26.         Index[b] = pos ++;
27.     }
28.     priority_queue<par, vector<par>, greater<par> >Qp;
29.     int F[MAXN], parent[MAXN];
30.     type_cost dist[MAXN];

```

```

31.     par Max_Flow_Min_Cost( ){
32.         int FlowF = 0;
33.         type_cost CostF = 0;
34.         int nod, newn, flow;
35.         type_cost newc, cost;
36.         memset( Phi, 0, sizeof(Phi) );
37.         for( ; ; ){
38.             fill( F, F + 1 + NN, 0 );
39.             fill( dist, dist + 1 + NN, oo );
40.             F[In] = oo, dist[In] = 0;
41.             Qp.push( par( 0, In ) );
42.             while( !Qp.empty() ){
43.                 nod = Qp.top().second, cost = Qp.top().first;
44.                 Qp.pop();
45.                 flow = F[nod];
46.                 for( int i = Index[nod]; i != -1; i = G[i].next ){
47.                     newn = G[i].newn;
48.                     newc = cost + G[i].cost + Phi[nod] - Phi[newn];
49.                     if( G[i].cap > 0 && dist[newn] > newc ){
50.                         dist[newn] = newc;
51.                         F[newn] = min( flow, G[i].cap );
52.                         parent[newn] = i;
53.                         Qp.push( par( newc, newn ) );
54.                     }
55.                 }
56.             }
57.             if( F[Fin] <= 0 ) break;
58.             CostF += (( dist[Fin] + Phi[Fin] ) * F[Fin] );
59.             FlowF += F[Fin];
60.             for( int i = In; i <= Fin; i ++ )
61.                 if( F[i] ) Phi[i] += dist[i];
62.             nod = Fin;
63.             while( nod != In ){
64.                 G[parent[nod]].cap -= F[Fin];
65.                 G[parent[nod]^1].cap += F[Fin];
66.                 nod = G[parent[nod]].nod;
67.             }
68.         }
69.         return par( CostF, FlowF );
70.     }
71. }

```

4.7. Minimum Arborescences $O(M \log N)$

```

1. template<typename T>
2. struct minimum_aborescense{
3.     struct edge{
4.         int src, dst;
5.         T weight;
6.     };
7.     vector<edge> edges;
8.     void add_edge(int u, int v, T w){
9.         edges.push_back({ u, v, w });
10.    }
11.    T solve(int r){
12.        int n = 0;

```

```

13.     for (auto e : edges)
14.         n = max(n, max(e.src, e.dst) + 1);
15.     int N = n;
16.     if( N == 0 ) return 0;
17.     for (T res = 0;;){
18.         vector<edge> in(N,{-1,-1,numeric_limits<T>::max()});
19.         vector<int> C(N, -1);
20.         for (auto e : edges)
21.             if (in[e.dst].weight > e.weight)
22.                 in[e.dst] = e;
23.         in[r] = {r, r, 0};
24.         for (int u = 0; u < N; ++u){
25.             if (in[u].src < 0)
26.                 return numeric_limits<T>::max();
27.             res += in[u].weight;
28.         }
29.         vector<int> mark(N, -1);
30.         int index = 0;
31.         for (int i = 0; i < N; ++i) {
32.             if (mark[i] != -1) continue;
33.             int u = i;
34.             while (mark[u] == -1){
35.                 mark[u] = i;
36.                 u = in[u].src;
37.             }
38.             if (mark[u] != i || u == r)
39.                 continue;
40.             for(int v=in[u].src;u!=v;v=in[v].src)
41.                 C[v] = index;
42.             C[u] = index++;
43.         }
44.         if (index == 0) return res;
45.         for (int i = 0; i < N; ++i)
46.             if (C[i] == -1) C[i] = index++;
47.         vector<edge> next;
48.         for (auto &e : edges)
49.             if(C[e.src]!=C[e.dst]&&C[e.dst]!=C[r])
50.                 next.push_back({C[e.src], C[e.dst],
51.                                 e.weight-in[e.dst].weight});
52.         edges.swap(next);
53.         N = index;
54.         r = C[r];
55.     }
56. }
57. };

```

4.8. Punto de Art. y Bridges O(N)

```

1. void bridges_PtoArt ( int nod ){
2.     Td[nod] = low[nod] = ++ k;
3.     for( auto num : V[nod] ){
4.         int newn = G[num].nextn( nod );
5.         if( G[num].band ) continue;
6.         G[num].band = true;
7.         if( Td[newn] ){
8.             low[nod] = min( low[nod], Td[newn] );

```

```

9.             continue;
10.         }
11.         bridges_PtoArt( newn );
12.         low[nod] = min( low[nod], low[newn] );
13.         if(Td[nod] < low[newn])
14.             puente.push(par( nod, newn ));
15.         if( (Td[nod] == 1 && Td[newn] > 2 ) ||
16.             ( Td[nod] != 1 && Td[nod] <= low[newn] ) )
17.             Punto_art[nod] = true;
18.     }
19. }

```

4.9. SQRT On Tree

```

1. void Dfs( int nod, int pad ){
2.     P[nod] = pad;
3.     if( lv[nod] % 2 ) G[nod] = ++k;
4.     for( auto i : V[nod] ){
5.         if( pad == i ) continue;
6.         lv[i] = lv[nod]+1;
7.         Dfs( i, nod );
8.     }
9.     if( lv[nod] % 2 == 0 ) G[nod] = ++k;
10. }
11. struct r{ int f, s, id; } Q[MAXA]; // f <= s
12. int R, kk;
13. bool comp ( const r s1, const r s2 ){
14.     if( G[s1.f] / R != G[s2.f] / R )
15.         return G[s1.f] / R < G[s2.f] / R;
16.     return G[s1.s] < G[s2.s];
17. }
18. void mov( int x, int y ){
19.     int p, cant = 0;
20.     while( x != y ){
21.         kk ++;
22.         if( lv[x] >= lv[y] ){
23.             p = P[x];
24.             if( mark[p] )
25.                 mark[x] = false, remove( A[x] );
26.             else
27.                 mark[p] = true, add( A[p] );
28.             x = p;
29.         }else{
30.             tmp[++cant] = y;
31.             y = P[y];
32.         }
33.     }
34.     for( int i = cant; i >= 1; i -- ){
35.         p = tmp[i];
36.         if( mark[p] )
37.             mark[x] = false, remove( A[x] );
38.         else
39.             mark[p] = true, add( A[p] );
40.         x = p;
41.     }
42. }

```

4.10. Stable Marriage

```

1. typedef vector<int> vi; typedef vector<vi> vvi;
2. #define rep(i,a,b) for ( __typeof(a) i=a; i<b; ++i)
3. vi stable_marriage(int n, int **m, int **w){
4.     queue<int> q;
5.     vi at(n, 0), eng(n, -1), res(n, -1); vvi inv(n, vi(n));
6.     rep(i,0,n) rep(j,0,n) inv[i][w[i][j]] = j;
7.     rep(i,0,n) q.push(i);
8.     while (!q.empty()) {
9.         int curm = q.front(); q.pop();
10.        for (int &i = at[curm]; i < n; i++) {
11.            int curw = m[curm][i];
12.            if (eng[curw] == -1) { }
13.            else if (inv[curw][curm] < inv[curw][eng[curw]])
14.                q.push(eng[curw]);
15.            else continue;
16.            res[eng[curw]] = curm = curw, ++i; break;
17.        }
18.    }
19.    return res;
20. }

```

4.11. StoerWagner $O(N^3)$

```

1. //maximo flujo seleccionando la mejor fuente y mejor sumidero
2. int G[MAXN][MAXN], w[MAXN], N;
3. bool A[MAXN], merged[MAXN];
4. int StoerWagner(int n){
5.     int best = 1e8;
6.     for(int i=1; i<n; ++i) merged[i] = 0;
7.     merged[0] = 1;
8.     for(int phase=1; phase<n; ++phase){
9.         A[0] = 1;
10.        for(int i=1; i<n; ++i){
11.            if(merged[i]) continue;
12.            A[i] = 0;
13.            w[i] = G[0][i];
14.        }
15.        int prev = 0, next;
16.        for(int i=n-1-phase; i>=0; --i){
17.            // hallar siguiente vertice que no esta en A
18.            next = -1;
19.            for(int j=1; j<n; ++j)
20.                if(!A[j] && (next==-1 || w[j]>w[next]))
21.                    next = j;
22.            A[next] = true;
23.            if(i>0){
24.                prev = next;
25.                // actualiza los pesos
26.                for(int j=1; j<n; ++j) if(!A[j])
27.                    w[j] += G[next][j];
28.            }
29.        }
30.        if(best>w[next]) best = w[next];
31.        for(int i=0; i<n; ++i){ // mezcla s y t

```

```

32.            G[i][prev] += G[next][i];
33.            G[prev][i] += G[next][i];
34.        }
35.        merged[next] = true;
36.    }
37.    return best;
38. }

```

4.12. Tree Isomorphism $O(N \log N)$

```

1. #define all(c) (c).begin(), (c).end()
2. struct tree{
3.     int n;
4.     vector<vector<int>> adj;
5.     tree(int n) : n(n), adj(n) {}
6.     void add_edge(int src, int dst){
7.         adj[src].push_back(dst);
8.         adj[dst].push_back(src);
9.     }
10.    vector<int> centers(){
11.        vector<int> prev;
12.        int u = 0;
13.        for (int k = 0; k < 2; ++k) {
14.            queue<int> q;
15.            prev.assign(n, -1);
16.            for (q.push(prev[u] = u); !q.empty(); q.pop()){
17.                u = q.front();
18.                for (auto v : adj[u]){
19.                    if (prev[v] >= 0) continue;
20.                    q.push(v);
21.                    prev[v] = u;
22.                }
23.            }
24.        }
25.        vector<int> path = { u };
26.        while (u != prev[u])
27.            path.push_back(u = prev[u]);
28.        int m = path.size();
29.        if (m % 2 == 0)
30.            return {path[m/2-1], path[m/2]};
31.        else
32.            return {path[m/2]};
33.    }
34.    vector<vector<int>> layer;
35.    vector<int> prev;
36.    int levelize(int r){
37.        prev.assign(n, -1);
38.        prev[r] = n;
39.        layer = {{r}};
40.        while (1){
41.            vector<int> next;
42.            for (int u : layer.back()){
43.                for (int v : adj[u]){
44.                    if (prev[v] >= 0)
45.                        continue;
46.                    prev[v] = u;

```

```

47.         next.push_back(v);
48.     }
49.     if (next.empty()) break;
50.     layer.push_back(next);
51. }
52. return layer.size();
53. }
54. };
55. bool isomorphic(tree S, int s, tree T, int t){
56.     if (S.n != T.n) return false;
57.     if (S.levelize(s) != T.levelize(t)) return false;
58.
59.     vector<vector<int>> longcodeS(S.n + 1), longcodeT(T.n + 1);
60.     vector<int> codeS(S.n), codeT(T.n);
61.     for (int h = (int) S.layer.size() - 1; h >= 0; --h) {
62.         map<vector<int>, int> bucket;
63.         for (int u : S.layer[h]){
64.             sort(all(longcodeS[u]));
65.             bucket[longcodeS[u]] = 0;
66.         }
67.         for (int u : T.layer[h]){
68.             sort(all(longcodeT[u]));
69.             bucket[longcodeT[u]] = 0;
70.         }
71.         int id = 0;
72.         for (auto &p : bucket) p.second = id++;
73.         for (int u : S.layer[h]){
74.             codeS[u] = bucket[longcodeS[u]];
75.             longcodeS[S.prev[u]].push_back(codeS[u]);
76.         }
77.         for (int u : T.layer[h]){
78.             codeT[u] = bucket[longcodeT[u]];
79.             longcodeT[T.prev[u]].push_back(codeT[u]);
80.         }
81.     }
82.     return codeS[s] == codeT[t];
83. }
84. bool isomorphic(tree S, tree T){
85.     auto x = S.centers(), y = T.centers();
86.     if (x.size() != y.size()) return false;
87.     if (isomorphic(S, x[0], T, y[0])) return true;
88.     return x.size() > 1 && isomorphic(S, x[1], T, y[0]);
89. }

```

5. Number Theory

5.1. Algoritmo Shanka-Tonelli ($x^2 = a \pmod{p}$)

```

1. //devuelve x (mod p) tal que x^2 = a (mod p)
2. long long solve_quadratic( long long a, int p ){
3.     if( a == 0 ) return 0;
4.     if( p == 2 ) return a;
5.     if( powMod(a,(p-1)/2, p) != 1 ) return -1;
6.     int phi = p-1, n = 0, k = 0, q = 0;
7.     while( phi%2 == 0 ) phi/=2, n++;
8.     k = phi;
9.     for( int j = 2; j < p; j ++ )

```

```

10.         if( powMod( j, (p-1)/2, p ) == p-1 ){
11.             q = j; break;
12.         }
13.     long long t = powMod( a, (k+1)/2, p );
14.     long long r = powMod( a, k, p );
15.     while( r != 1 ){
16.         int i = 0, v = 1;
17.         while( powMod( r, v, p ) != 1 ) v *= 2, i++;
18.         long long e = powMod( 2, n-i-1, p );
19.         long long u = powMod( q, k*e, p );
20.         t = (t*u)%p;
21.         r = (r*u*u)%p;
22.     }
23.     return t;
24. }

```

5.2. Extended GCD ($ax+by = \gcd(a,b)$)

```

1. //devuelve x,y tal que ax+by = gcd(a,b)
2. int64 extended_euclid( int64 a, int64 b, int64& x, int64& y ) {
3.     int64 g = a;
4.     x = 1, y = 0;
5.     if ( b != 0 ) {
6.         g = extended_euclid( b, a % b, y, x );
7.         y -= ( a / b ) * x;
8.     }
9.     return g;
10. }

```

5.3. FFT $O(N \log N)$

```

1. #define PI acos(-1)
2. typedef complex<double> base;
3. void fft (vector<base> &a, int invert){
4.     int n = (int) a.size();
5.     for (int i = 1, j = 0; i < n-1; ++i){
6.         for (int k = n >> 1; (j ^= k) < k; k >>= 1);
7.         if (i < j) swap(a[i], a[j]);
8.     }
9.     for (int len=2; len <= n; len<=1) {
10.         double ang = 2*PI/len * invert;
11.         base wlen(cos(ang), sin(ang)), w(1);
12.         for (int i=0; i < n; i += len, w = base(1) )
13.             for (int j=0; j<len/2; ++j, w *= wlen) {
14.                 base u = a[i+j], v = a[i+j+len/2] * w;
15.                 a[i+j] = u + v;
16.                 a[i+j+len/2] = u - v;
17.             }
18.     }
19.     if (invert == -1){ for (int i=0; i<n; ++i) a[i] /= n; }
20. } //a la hora de conv. de complex a int real + o - 0.5

```

5.4. Fast Modulo Transform $O(N \log N)$

```

1. const int mod = 167772161;
2. // so the algorithm works until n = 2 ^ 17 = 131072
3. const int G = 3; // primitive root
4. //const int MOD = 1073872897 = 2 ^ 30 + 2 ^ 17 + 1, g = 7
5. // another good choice is MOD = 167772161 = 2^27+2^25+1, g = 3

```



```

6. // a bigger choice would be MOD = 3221225473 = 2^31+2^30+1, g = 5
7. // but it requires unsigned long long for multiplications
8. // n must be a power of two
9. // sign = 1
10. // sign = -1
11. // fast modulo transform
12. // (1) n = 2^k < 2^23
13. // (2) only predetermined mod can be used
14. // (3) Inverso Modular */
15. void fmt(vector<ll> &x, int sign = +1){
16.     int n = x.size();
17.     for (int i = 0, j = 1; j < n - 1; ++j){
18.         for (int k = n >> 1; k > (i ^ k); k >>= 1);
19.         if (j < i) swap(x[i], x[j]);
20.     }
21.     ll h = pow(G, (mod - 1) / n, mod);
22.     if (sign < 0) h = inv(h, mod);
23.     for (int m = 1; m < n; m *= 2){
24.         ll w = 1, wk = pow(h, n / (2 * m), mod);
25.         for (int i = 0; i < m; ++i){
26.             for (int j = i; j < n; j += 2 * m){
27.                 ll u = x[j], d = (x[j + m] * w) % mod;
28.                 x[j] = (u + d) % mod;
29.                 x[j + m] = (u - d + mod) % mod;
30.             }
31.             w = w * wk % mod;
32.         }
33.     }
34.     if (sign < 0){
35.         ll n_inv = inv(n, mod);
36.         for (auto &a : x) a = (a * n_inv) % mod;
37.     }
38. }

```

5.5. Find a primitive root of a prime number

```

1. // Assuming the Riemman Hypothesis it runs in O(log^6(p)*sqrt(p))
2. int generator (int p){
3.     vector<int> fact;
4.     int phi = p-1, n = phi;
5.     for (int i=2; i*i<=n; ++i)
6.         if (n % i == 0){
7.             fact.push_back (i);
8.             while (n % i == 0)
9.                 n /= i;
10.        }
11.    if (n > 1) fact.push_back (n);
12.    for (int res=2; res<=p; ++res){
13.        bool ok = true;
14.        for (size_t i=0; i<fact.size() && ok; ++i)
15.            ok &= powmod (res, phi / fact[i], p) != 1;
16.        if (ok) return res;
17.    }
18.    return -1;
19. }

```

5.6. Floyds Cycle-Finding algorithm

```

1. par find_cycle() {
2.     int t = f(x0), h = f(t), mu = 0, lam = 1;
3.     while (t != h) t = f(t), h = f(f(h));
4.     h = x0;
5.     while (t != h) t = f(t), h = f(h), mu++;
6.     h = f(t);
7.     while (t != h) h = f(h), lam++;
8.     return par(mu, lam);
9. }

```

5.7. Gauss O(N^3)

```

1. const int oo = 0x3f3f3f3f;
2. const double eps = 1e-9;
3. int gauss(vector<vector<double>> a, vector<double> &ans){
4.     int n = (int) a.size();
5.     int m = (int) a[0].size() - 1;
6.     vector<int> where(m, -1);
7.     for (int col = 0, row = 0; col < m && row < n; ++col){
8.         int sel = row;
9.         for (int i = row; i < n; ++i)
10.            if (abs(a[i][col]) > abs(a[sel][col])) sel = i;
11.         if (abs(a[sel][col]) < eps) continue;
12.         for (int i = col; i <= m; ++i)
13.            swap(a[sel][i], a[row][i]);
14.         where[col] = row;
15.         for (int i = 0; i < n; ++i)
16.            if (i != row){
17.                double c = a[i][col] / a[row][col];
18.                for (int j = col; j <= m; ++j)
19.                    a[i][j] -= a[row][j] * c;
20.            }
21.         ++row;
22.     }
23.     ans.assign(m, 0);
24.     for (int i = 0; i < m; ++i)
25.         if (where[i] != -1)
26.             ans[i] = a[where[i]][m] / a[where[i]][i];
27.     for (int i = 0; i < n; ++i) {
28.         double sum = 0;
29.         for (int j = 0; j < m; ++j)
30.             sum += ans[j] * a[i][j];
31.         if (abs(sum - a[i][m]) > eps)
32.             return 0;
33.     }
34.     for (int i = 0; i < m; ++i)
35.         if (where[i] == -1) return oo;
36.     return 1;
37. }

```

5.8. Inverso Modular

```

1. ll inv(ll b, ll M){ //mcd(b,m)==1
2.     ll u = 1, x = 0, s = b, t = M;
3.     while( s ){
4.         ll q = t / s;

```

```

5.         swap(x -= u * q, u);
6.         swap(t -= s * q, s);
7.     }
8.     return (x % M) >= 0 ? x : x + M;
9. }

```

5.9. Josephus

```

1. // n-cantidad de personas, m es la longitud del salto.
2. // comienza en la k-esima persona.
3. ll josephus(ll n, ll m, ll k) {
4.     ll x = -1;
5.     for (ll i = n - k + 1; i <= n; ++i) x = (x + m) % i;
6.     return x;
7. }
8. ll josephus_inv(ll n, ll m, ll x){
9.     for (ll i = n; i--){
10.         if (x == i) return n - i;
11.         x = (x - m % i + i) % i;
12.     }
13.     return -1;
14. }

```

5.10. Linear Recurrence Solver $O(N^2 \log K)$

```

1. /* x[i+n] = a[0] x[i] + a[1] x[i+1] + ... + a[n-1] x[i+n-1]
2. with initial solution x[0], x[1], ..., x[n-1]
3. Complexity:  $O(n^2 \log k)$  time,  $O(n \log k)$  space */
4. ll linear_recurrence(vector<ll> a, vector<ll> x, ll k){
5.     int n = a.size();
6.     vector<ll> t(2 * n + 1);
7.     function<vector<ll>(ll)> rec = [&](ll k){
8.         vector<ll> c(n);
9.         if (k < n) c[k] = 1;
10.        else{
11.            vector<ll> b = rec(k / 2);
12.            fill(t.begin(), t.end(), 0);
13.            for (int i = 0; i < n; ++i)
14.                for (int j = 0; j < n; ++j){
15.                    t[i+j+(k&1)] += (b[i]*b[j])%mod;
16.                    t[i+j+(k&1)] %= mod;
17.                }
18.            for (int i = 2*n-1; i >= n; --i)
19.                for (int j = 0; j < n; ++j){
20.                    t[i-n+j] += (a[j]*t[i])%mod;
21.                    t[i-n+j] %= mod;
22.                }
23.            for (int i = 0; i < n; ++i)
24.                c[i] = t[i];
25.        }
26.        return c;
27.    };
28.    vector<ll> c = rec(k);
29.    ll ans = 0;
30.    for (int i = 0; i < x.size(); ++i){
31.        ans += (c[i] * x[i])%mod;
32.        ans %= mod;

```

```

33.     }
34.     return ans;
35. }

```

5.11. Matrix Exponentiation $O(N^3 \log N)$

```

1. typedef vector<ll> vect;
2. typedef vector< vect > matrix;
3. matrix identity (int n) {
4.     matrix A(n, vect(n));
5.     for (int i = 0; i < n; ++i) A[i][i] = 1;
6.     return A;
7. }
8. matrix mul(const matrix &A, const matrix &B) {
9.     matrix C(A.size(), vect(B[0].size()));
10.    for (int i = 0; i < C.size(); ++i)
11.        for (int j = 0; j < C[i].size(); ++j)
12.            for (int k = 0; k < A[i].size(); ++k){
13.                C[i][j] += (A[i][k] * B[k][j])%mod;
14.                C[i][j] %= mod;
15.            }
16.    return C;
17. }
18. matrix powm(const matrix &A, ll e) {
19.    return (e == 0) ? identity(A.size()) :
20.        (e % 2 == 0) ? powm(mul(A, A), e/2) :
21.        mul(A, powm(A, e-1));
22. }

```

5.12. Miller-Rabin is prime (probability test)

```

1. bool suspect(ll a, int s, ll d, ll n) {
2.     ll x = powMod(a, d, n);
3.     if (x == 1) return true;
4.     for (int r = 0; r < s; ++r) {
5.         if (x == n - 1) return true;
6.         x = mulmod(x, x, n);
7.     }
8.     return false;
9. }
10. // {2,7,61,0} is for n < 4759123141 (= 2^32)
11. // {2,3,5,7,11,13,17,19,23,0} is for n < 10^16 (at least)
12. unsigned test[] = { 2, 3, 5, 7, 11, 13, 17, 19, 23, 0 };
13. bool miller_rabin(ll n) {
14.     if (n <= 1 || (n > 2 && n % 2 == 0)) return false;
15.     ll d = n - 1; int s = 0;
16.     while (d % 2 == 0) ++s, d /= 2;
17.     for (int i = 0; test[i] < n && test[i] != 0; ++i)
18.         if (!suspect(test[i], s, d, n))
19.             return false;
20.     return true;
21. }

```

5.13. Modular Equations (ax = b(n))

```

1. /* Modular Linear Equation Solver  $O(\log n)$ 
2. * Given a, b and n, solves the equation ax = b(n)
3. * for x. Returns the vector of solutions, all smaller

```

```

4.  * than n and sorted in increasing order. */
5.  vector< int > msolve( int a, int b, int n ){
6.      if( n < 0 ) n = -n;
7.      int d, x, y;
8.      d = extended_euclid( a, n, x, y );
9.      vector< int > r;
10.     if( b % d ) return r;
11.     int x0 = ( b / d * x ) % n;
12.     if( x0 < 0 ) x0 += n;
13.     x0 = x0 % ( n / d );
14.     for( int i = 0; i < d; i++ )
15.         r.push_back( ( x0 + i * n / d ) % n );
16.     return r;
17. }

```

5.14. Newton Raphston

```

1.  double eval(double P[],int n, double x){
2.      double r = 0;
3.      for(int i = n - 1; i >= 0; i--){
4.          r*=x;
5.          r+=P[i];
6.      }
7.      return r;
8.  }
9.  int main() {
10.     int test = 1, n;
11.     while(scanf("%d", &n) && n) {
12.         double a[10] = {};
13.         for(int i = n; i >= 0; i--) scanf("%lf", &a[i]);
14.         double ret[10];
15.         int m = n;
16.         for(int i = 0; i < m; i++) {
17.             double b[10] = {}; // f'(x)
18.             for(int j = 0; j <= n; j++)
19.                 b[j] = a[j+1]*(j+1);
20.             double x = 25, tx; //max_value
21.             if(i) x = ret[i-1];
22.             while(true) {
23.                 double fx =eval(a,n+1,x),ffx =eval(b,n,x);
24.                 tx = x - fx/ffx;
25.                 if(fabs(fx) < 1e-8)
26.                     break;
27.                 x = tx;
28.             }
29.             ret[i] = x;
30.             for(int j = n; j >= 0; j--)
31.                 a[j] = a[j] + a[j+1]*x;
32.             for(int j = 0; j <= n; j++)
33.                 a[j] = a[j+1];
34.             n--;
35.         }
36.         printf("Equation %d:", test++);
37.         n = m;
38.         sort(ret, ret+n);
39.         for(int i = 0; i < n; i++) printf(" %.4lf", ret[i]);

```

```

40.         printf("\n");
41.     }
42. }
43.

```

5.15. Newton's Method

```

1.  template<class F, class G>
2.  double find_root(F f, G df, double x){
3.      for (int iter = 0; iter < 100; ++iter){
4.          double fx = f(x), dfx = df(x);
5.          x -= fx / dfx;
6.          if (fabs(fx) < 1e-12)
7.              break;
8.      }
9.      return x;
10. }

```

5.16. Parametric Self-Dual Simplex method O(n+m)

```

1.  /* - Solve a canonical LP:
2.      min. c x
3.      s.t. A x <= b
4.      x >= 0 */
5.  const double eps = 1e-9, oo = numeric_limits<double>::infinity();
6.  typedef vector<double> vec;
7.  typedef vector<vec> mat;
8.  double simplexMethodPD(mat &A, vec &b, vec &c){
9.      int n = c.size(), m = b.size();
10.     mat T(m + 1, vec(n + m + 1));
11.     vector<int> base(n + m), row(m);
12.     for(int j = 0; j < m; ++j){
13.         for (int i = 0; i < n; ++i)
14.             T[j][i] = A[j][i];
15.         T[j][n + j] = 1;
16.         base[row[j] = n + j] = 1;
17.         T[j][n + m] = b[j];
18.     }
19.     for (int i = 0; i < n; ++i) T[m][i] = c[i];
20.     while (1){
21.         int p = 0, q = 0;
22.         for (int i = 0; i < n + m; ++i)
23.             if (T[m][i] <= T[m][p]) p = i;
24.         for (int j = 0; j < m; ++j)
25.             if (T[j][n + m] <= T[q][n + m]) q = j;
26.         double t = min(T[m][p], T[q][n + m]);
27.         if (t >= -eps) {
28.             vec x(n);
29.             for (int i = 0; i < m; ++i)
30.                 if (row[i] < n) x[row[i]] = T[i][n + m];
31.             // x is the solution
32.             return -T[m][n + m]; // optimal
33.         }
34.         if (t < T[q][n + m]){
35.             // tight on c -> primal update
36.             for (int j = 0; j < m; ++j)
37.                 if (T[j][p] >= eps)

```

```

38.         if (T[j][p] * (T[q][n + m] - t) >=
39.             T[q][p] * (T[j][n + m] - t))
40.             q = j;
41.
42.         if (T[q][p] <= eps)
43.             return oo; // primal infeasible
44.     }else{
45.         // tight on b -> dual update
46.         for (int i = 0; i < n + m + 1; ++i)
47.             T[q][i] = -T[q][i];
48.         for (int i = 0; i < n + m; ++i)
49.             if (T[q][i] >= eps)
50.                 if (T[q][i] * (T[m][p] - t) >=
51.                     T[q][p] * (T[m][i] - t))
52.                     p = i;
53.         if (T[q][p] <= eps)
54.             return -oo; // dual infeasible
55.     }
56.     for (int i = 0; i < m + n + 1; ++i)
57.         if (i != p) T[q][i] /= T[q][p];
58.     T[q][p] = 1; // pivot(q, p)
59.     base[p] = 1;
60.     base[row[q]] = 0;
61.     row[q] = p;
62.     for (int j = 0; j < m + 1; ++j)
63.         if (j != q){
64.             double alpha = T[j][p];
65.             for (int i = 0; i < n + m + 1; ++i)
66.                 T[j][i] -= T[q][i] * alpha;
67.         }
68. }
69. return oo;
70. }

```

5.17. Pollard Rho $O(\sqrt{s(n)})$ expected

```

1. #define func(x)(mulmod(x, x+B, n)+ A )
2. ll pollard_rho(ll n) {
3.     if( n == 1 ) return 1;
4.     if( miller_rabin( n ) )
5.         return n;
6.     ll d = n;
7.     while( d == n ){
8.         ll A = 1 + rand()%(n-1), B = 1 + rand()%(n-1);
9.         ll x = 2, y = 2;
10.        d = -1;
11.        while( d == 1 || d == -1 ){
12.            x = func(x), y = func(func(y));
13.            d = __gcd( x-y, n );
14.        }
15.    }
16.    return abs(d);
17. }

```

5.18. Shanks' Algorithm $O(\sqrt{N})$ ($a^x = b(\text{mod } m)$)

```

1. //return x such that  $a^x = b(\text{mod } m)$ 

```

```

2. int solve ( int a, int b, int m ){
3.     int n = (int)sqrt( m + .0 )+1, an = 1;
4.     for ( int i = 0; i < n; i++ )
5.         an = (an * a)%m;
6.     map<int, int>vals;
7.     for ( int i = 1, cur = an; i <= n; ++ i ){
8.         if ( ! vals.count ( cur ) )
9.             vals [ cur ] = i ;
10.        cur = (cur * an)%m;
11.    }
12.    for ( int i = 0, cur = b; i <= n; ++ i ){
13.        if ( vals.count ( cur ) ){
14.            int ans = vals [ cur ] * n - i ;
15.            if ( ans < m )return ans;
16.        }
17.        cur = (cur * a)%m;
18.    }
19.    return -1;
20. }

```

5.19. Simpson Rule

```

1. // Error =  $O(\Delta x)^4$ 
2. const int ITR = 1e4; //must be an even number
3. double Simpson(double a, double b, double f(double)){
4.     double s = f(a) + f(b), h = (b - a) / ITR;
5.     for (int i = 1; i < ITR; ++i) {
6.         double x = a + h * i;
7.         s += f(x)*( i&1 ? 4 : 2);
8.     }
9.     return s * h/3;
10. }

```

5.20. Teorema Chino del Resto

```

1. int resto_chino (vector<int> x, vector<int> m, int k){
2.     int i, tmp, MOD = 1, RES = 0;
3.     for (i=0; i < k; i++) MOD *= m[i];
4.     for (i = 0; i < k; i++){
5.         tmp = MOD/m[i];
6.         tmp *= inverso_mod(tmp, m[i]);
7.         RES += (tmp*x[i]) % MOD;
8.     }
9.     return RES % MOD;
10. }

```

6. String

6.1. Aho Corasick

```

1. int tree[MAXN][26], fail[MAXN];
2. int termina[MAXN], size = 1;
3. void addWord( string pal ){
4.     int p = 0;
5.     for(char c : pal){
6.         if( !tree[p][c-'a'] )
7.             tree[p][c-'a'] = size++;
8.         p = tree[p][c-'a'];
9.     }

```

```

10. //termina[p].push_back( pal_id );
11. termina[p] = pal.size();
12. }
13. void buildersuffix(){
14.     queue<int> Q;
15.     for(int i = 0; i < 26; i++)
16.         if( tree[0][i] ) Q.push(tree[0][i]);
17.     while( !Q.empty() ){
18.         int u, v = Q.front(); Q.pop();
19.         //for( auto i : termina[fail[v]] )
20.         //    termina[v].push_back( i );
21.         termina[v] = max(termina[v], termina[fail[v]]);
22.         for( int i = 0; i < 26; i++ )
23.             if(u = tree[v][i]){
24.                 fail[u] = tree[fail[v]][i];
25.                 Q.push( u );
26.             }else
27.                 tree[v][i] = tree[fail[v]][i];
28.     }
29. }

```

6.2. Lyndon Decomposition $O(N)$

```

1. /*s = w1w2w3..wk, w1 >= w2 >= ... >= wk.
2. > Menor Rotaci3n Lexicogr4fica:Es el mayor valor
3. de i, tal que i < n, en la descomposicion de lyndon
4. de la cadena s+s, n = |s| */
5. void lyndon( string s ){
6.     int n = (int)s.length(), i = 0;
7.     while( i < n ){
8.         int j = i+1, k = i;
9.         while( j < n && s[k] <= s[j] ){
10.            if( s[k] < s[j] ) k = i;
11.            else k ++;
12.            j ++;
13.        }
14.        while( i <= k ){
15.            cout << s.substr( i, j-k )<<endl;
16.            i += j-k;
17.        }
18.    }
19. }

```

6.3. Manacher $O(N)$

```

1. int rad[ 2 * MAXLEN ], n;
2. char s[MAXLEN];
3. void manacher( ){ /// i%2!=0 par, i%2==0 impar
4.     int i, j, k; /// i -> 2*i o 2*i+1
5.     for ( i = 0, j = 0; i < 2 * n - 1; i += k ) {
6.         while ( i - j >= 0 && i + j + 1 < 2 * n &&
7.             s[(i - j)/2] == s[(i + j + 1)/2] )
8.             j++;
9.         rad[i] = j;
10.        for(k = 1; k <= rad[i] && rad[i-k] != rad[i]-k; k++)
11.            rad[ i + k ] = min( rad[ i - k ], rad[i] - k );
12.        j = max( j - k, 0 );
13.    }
14. }

```

6.4. Palindrome Tree $O(N)$

```

1. struct PalindromicTree{
2.     int tree[MAXN][30], link[MAXN], length[MAXN], sz, ult;
3.     int diff[MAXN], slink[MAXN], ans[MAXN], sans[MAXN];
4.     string s;
5.     void ini( ){
6.         memset( tree, 0, sizeof(tree) );
7.         memset( link, 0, sizeof(link) );
8.         memset( length, 0, sizeof(length) );
9.         length[1] = -1, link[1] = 1;
10.        length[2] = 0, link[2] = 1;
11.        sz = ult = 2, s.clear();
12.    }
13.    int find_x( int suff, int p ){
14.        int len = length[suff];
15.        while( p - len < 1 || s[p] != s[p-len-1] )
16.            suff = link[suff], len = length[suff];
17.        return suff;
18.    }
19.    void insertar( char c ){
20.        int p = s.size();
21.        s.push_back( c );
22.        int suff = find_x( ult, p );
23.        if( tree[suff][c-'a'] == 0 ){
24.            tree[suff][c-'a'] = ++sz;
25.            length[sz] = length[suff] + 2;
26.            link[sz] = ( length[sz] == 1 ) ? 2 :
27.                tree[find_x( link[suff], p )][c-'a'];
28.            diff[sz] = length[sz]-length[link[sz]];
29.            slink[sz] = ( diff[sz]!=diff[link[sz]] ) ?
30.                link[sz] : slink[link[sz]];
31.        }
32.        ult = tree[suff][c-'a'];
33.    }
34.    void descomponer( int i ){
35.        ans[i] = 1 << 30;
36.        for(int v = ult; length[v]>0; v = slink[v]){
37.            sans[v] = ans[i -(length[slink[v]] + diff[v])];
38.            if(diff[v] == diff[link[v]])
39.                sans[v] = min(sans[v], sans[link[v]]);
40.            ans[i] = min(ans[i], sans[v] + 1);
41.        }
42.    }
43. }palin;

```

6.5. Suffix Array $O(N \log N)$

```

1. int n, _sa[LEN], _b[LEN], top[LEN], _tmp[LEN];
2. int LCP[LEN], *SA = _sa, *B = _b, *tmp = _tmp;
3. char s[LEN];
4. void build_lcp ( ){
5.     for(int i = 0, k = 0; i < n; ++i){
6.         if(B[i] == n - 1)
7.             continue;
8.         for(int j = SA[B[i] + 1]; i + k < n &&
9.             j + k < n && s[i+k] == s[j + k]; k++);

```

```

10.     LCP[B[i]] = k;
11.     if( k ) k--;
12. }
13. }
14. void build_sa (){
15.     //memset 0 -> _sa, _b, _tmp, top, LCP
16.     s[n] = '\0', n++;
17.     int na = (n < 256 ? 256 : n);
18.     for (int i = 0; i < n; i++)
19.         top[B[i]] = s[i]++;
20.     for (int i = 1; i < na; i++)
21.         top[i] += top[i - 1];
22.     for (int i = 0; i < n; i++)
23.         SA[--top[B[i]]] = i;
24.     for (int ok = 1, j = 0; ok < n && j < n-1; ok <= 1){
25.         for (int i = 0; i < n; i++){
26.             j = SA[i] - ok;
27.             if (j < 0)
28.                 j += n;
29.             tmp[top[B[j]]++] = j;
30.         }
31.         SA[tmp[top[0] = 0]] = j = 0;
32.         for (int i = 1; i < n; i++){
33.             if (B[tmp[i]] != B[tmp[i - 1]] ||
34.                 B[tmp[i]+ok] != B[tmp[i-1] + ok])
35.                 top[++j] = i;
36.             SA[tmp[i]] = j;
37.         }
38.         swap(B, SA), swap(SA, tmp);
39.     }
40.     build_lcp();
41.     n--, s[n] = '\0';
42. }

```

6.6. Suffix Automata O(N)

```

1. // Construct:
2. // Automaton sa; for(char c : s) sa.extend(c);
3. // 1. Number of distinct substr O( N ):
4. // - Find number of different paths --> DFS on SA
5. // - f[u] = 1 + sum( f[v] for v in s[u].next
6. // 2. Number of occurrences of a substr O( N ):
7. // - Initially, in extend: s[cur].cnt = 1; s[clone].cnt = 0;
8. // - for( auto it = base.rbegin(); it != base.rend(); it ++ ){
9. //     int p = st[it->second].link;
10. //     cnt[p] += cnt[it->second]; }
11. // 3. Find total length of different substrings O( N ):
12. // - We have f[u] = number of strings starting from node u
13. // - ans[u] = sum(ans[v] + d[v] for v in next[u])
14. // 4. Lexicographically k-th substring O(N)
15. // - Based on number of different substring
16. // 5. Find first occurrence O(N)
17. // - firstpos[cur] = len[cur] - 1, firstpos[clone] = firstpos[q]
18. // 6. Longest common substring of two strings s, t O(N).
19. struct state {
20.     int len, link;

```

```

21.     int fpos;///
22.     map<char,int>next;
23.     state() {
24.         len = 0, link = -1, fpos = 0;
25.         next.clear();
26.     }
27. };
28. const int MAXLEN = 100002;
29. state st[MAXLEN*2];
30. int sz, last;
31. set<pair<int,int>> base ;///
32. int cnt[MAXLEN*2];///
33. void sa_init() {
34.     sz = last = 0;
35.     st[0] = state();
36.     cnt[0] = 0;
37.     sz++;
38.     base.clear();
39. }
40. void sa_extend (char c) {
41.     int cur = sz++;
42.     st[cur] = state();
43.     st[cur].len = st[last].len + 1;
44.     st[cur].fpos = st[cur].len - 1;///
45.     cnt[cur] = 1; ///
46.     base.insert(make_pair(st[cur].len, cur));///
47.     int p;
48.     for (p=last; p!=-1 && !st[p].next.count(c); p=st[p].link)
49.         st[p].next[c] = cur;
50.     if (p == -1)
51.         st[cur].link = 0;
52.     else {
53.         int q = st[p].next[c];
54.         if (st[p].len + 1 == st[q].len)
55.             st[cur].link = q;
56.         else {
57.             int clone = sz++;
58.             st[clone] = state();
59.             st[clone].len = st[p].len + 1;
60.             st[clone].next = st[q].next;
61.             st[clone].link = st[q].link;
62.             st[clone].fpos = st[q].fpos;///
63.             cnt[clone]=0;///
64.             base.insert(make_pair(st[clone].len,clone)); ///
65.             for (; p!=-1 && st[p].next[c]==q; p=st[p].link)
66.                 st[p].next[c] = clone;
67.             st[q].link = st[cur].link = clone;
68.         }
69.     }
70.     last = cur;
71. }
72. //6. Longest common substring of two strings s, t.
73. string lcs (string s, string t) {
74.     sa_init();
75.     for (int i=0; i<(int)s.length(); i++)

```

```

76.     sa_extend (s[i]);
77.     int v = 0, l = 0, best = 0, bestpos = 0;
78.     for (int i=0; i<(int)t.length(); i++) {
79.         while (v && !st[v].next.count(t[i])) {
80.             v = st[v].link;
81.             l = st[v].len;
82.         }
83.         if (st[v].next.count(t[i])) {
84.             v = st[v].next[t[i]];
85.             l++;
86.         }
87.         if (l > best) best = l, bestpos = i;
88.     }
89.     return t.substr (bestpos-best+1, best);
90. }

```

6.7. Tandems O(NlogN)

```

1. void output_tandem (const string & s, int shift,
2.     bool left, int cntr, int l, int l1, int l2){
3.     int pos;
4.     if (left) pos = cntr-l1;
5.     else pos = cntr-l1-l2-l1+1;
6.     cout << "[" << shift + pos << ".."; // ini
7.     cout << shift + pos+2*l-1 << "]" = "; // fin
8.     cout << s.substr (pos, 2*l) << endl;
9. }
10. void output_tandems (const string & s, int shift,
11.     bool left, int cntr, int l, int k1, int k2){
12.     for (int l1=1; l1<=l; l1++) {
13.         if (left && l1 == 1) break;
14.         if (l1 <= k1 && l-l1 <= k2)
15.             output_tandem(s,shift,left,cntr, l, l1, l-l1);
16.     }
17. }
18. inline int get_z (const vector<int> & z, int i) {
19.     return 0<=i && i<(int)z.size() ? z[i] : 0;
20. }
21. void find_tandems (string s, int shift = 0) {
22.     int n = (int) s.length();
23.     if (n == 1) return;
24.     int nu = n/2, nv = n-nu;
25.     string u = s.substr (0, nu),
26.           v = s.substr (nu);
27.     string ru = string (u.rbegin(), u.rend()),
28.           rv = string (v.rbegin(), v.rend());
29.     find_tandems (u, shift);
30.     find_tandems (v, shift + nu);
31.     vector<int> z1 = z_function (ru),
32.               z2 = z_function (v + '#' + u),
33.               z3 = z_function (ru + '#' + rv),
34.               z4 = z_function (v);
35.     for (int cntr=0; cntr<n; cntr++) {
36.         int l, k1, k2;
37.         if (cntr < nu) {
38.             l = nu - cntr;

```

```

39.             k1 = get_z (z1, nu-cntr);
40.             k2 = get_z (z2, nv+1+cntr);
41.         }else {
42.             l = cntr - nu + 1;
43.             k1 = get_z (z3, nu+1 + nv-1-(cntr-nu));
44.             k2 = get_z (z4, (cntr-nu)+1);
45.         }
46.         if(k1 + k2 >= 1) // longitud 2*l
47.             output_tandems(s,shift,cntr<nu,cntr,l,k1,k2);
48.     }
49. }

```

6.8. Z Algorithm O(N)

```

1. vector<int> z_function (const string & s){
2.     int n = (int) s.length();
3.     vector<int> z (n);
4.     for (int i=1, l=0, r=0; i<n; i++) {
5.         if (i <= r) z[i] = min (r-i+1, z[i-l]);
6.         while (i+z[i] < n && s[z[i]] == s[i+z[i]])
7.             z[i]++;
8.         if (i+z[i]-1 > r) l = i, r = i+z[i]-1;
9.     }
10.     return z;
11. }

```