Assignment MATH-S401 2023

Consider the following labor demand model. There is a function $R(e,h)$ that represents the revenues of a firm, depending on hours worked (h) and the number of workers (e). Another function $\omega(e,h)$ gives the total cost of hiring e workers when each supplies h units of labor. Finally, $C(e',e)$ is an adjustment cost when the number of workers in the firm go from e to e'.
The firm tries to solve the following problem:

$$\max_{\substack{e_1,\ldots,e_t,\ldots \\ h_1,\ldots,h_t,\ldots}} \sum_{t=0}^{\infty} \beta^t \left(R(e_t,h_t) - \omega(e_t,h_t) - C(e_{t+1},e_t)\right)$$

We assume that costs are quadratic

$$C(e',e) = \frac{\eta}{2}(e' - (1-q)e)^2.$$

Here η is an adjustment cost and q is an exogenous quit rate. We further assume that R takes on a Cobb-Douglas form

$$R(e,h) = A(eh)^{\alpha},$$

where A and α are parameters and we assume that

$$\omega(e,h) = we\left[w_0 + h + w_1(h-40) + w_2(h-40)^2\right].$$

- Write down the Bellman equation. What are the state variables, decision variables, etc.
- Consider the following parameter values

parameter	value
A	1.1
α	0.75
w_0	1.50
w_1	0.19
w_2	0.03
w	0.05
η	1.00
q	0.05
β	0.95

  Compute the Bellman fixed point using value function iteration on a grid for e of 200 values between 1 and 500 and for possible values of $h = 1, 2, \ldots, 70$. Plot the optimal value function and the optimal choice of e in the next period versus the value of e today (also plot the 45 degree line) and interpret the dynamics.

- Compute the Bellman fixed point using interpolation and a grid of e on 20 values.
- Compute the Bellman fixed point using Howard improvement (policy function iteration), once computing the Howard fixed point via the contraction mapping iteration, and once using matrix inversion.

So in total I need a small note and 4 Julia or Python files. Due date is Friday May, 12, 2023.