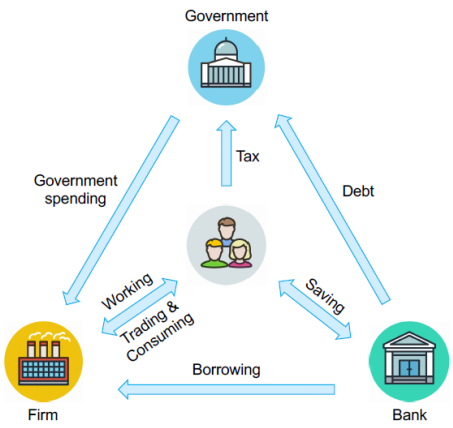
Complementary material

This is the complementary material for TaxingAI, A Dynamic Economic Simulator involving N households, government, firms, and financial intermediaries based on the Bewley-Aiyagari economics model.

Content

1.Roles



There are four roles in the game:

a. Multiple households
b. A government
c. A firm
d. A bank

2.Notations

Figure 1 gives the notation we used in the following discussion.

VARIABLE	MEANING
W_t	WAGE RATE
e_t	INDIVIDUAL LABOR PRODUCTIVITY LEVELS
h_t	WORKING HOURS
a_t	HOUSEHOLD'S ASSET(WEALTH)
a_0	HOUSEHOLD'S INITIAL ASSET(WEALTH)
r_t	INTEREST RATE
i_t	HOUSEHOLD'S INCOME
N	HOUSEHOLDS' NUMBER
ρ_e	PERSISTENCE
u_t	STANDARD NORMAL SHOCKS
σ_e	VOLATILITY OF THE STANDARD NORMAL SHOCKS
$\bar{e}$	LABOR MARKET ABILITY OF SUPER-STAR STATE
p	TRANSITION PROBABILITY FROM NORMAL STATE TO SUPER-STAR STATE
q	TRANSITION PROBABILITY OF REMAINING IN SUPER-STAR STATE
β	DISCOUNT FACTOR
θ	COEFFICIENT OF RELATIVE RISK AVERSION (CRRA)
γ	INVERSE FRISCH ELASTICITY
τ_s	CONSUMPTION TAXES
$T(i_t)$	INCOME TAXES
$T^a(a_t)$	ASSET TAXES
τ, τ_a	AVERAGE LEVEL OF MARGINAL INCOME AND ASSET TAX
ξ, ξ_a	SLOPE OF MARGINAL INCOME AND ASSET TAX SCHEDULE
Y_t	FIRM'S OUTPUT
K_t	CAPITAL FOR PRODUCTION
L_t	LABOR FOR PRODUCTION
α	CAPITAL ELASTICITY
C_t	HOUSEHOLDS' GROSS CONSUMPTION
G_t	GOVERNMENT SPENDING
X_t	PHYSICAL CAPITAL INVESTMENT
δ	DEPRECIATION RATE
R_t	BORROWING RATE OF FIRM
G_t	GOVERNMENT SPENDING
T_t	HOUSEHOLDS' GROSS TAXATION
B_t	DEBT
$\mathcal{I}_t$	HOUSEHOLDS' GROSS POST-TAX INCOME
$\mathcal{W}_t$	HOUSEHOLDS' GROSS POST-TAX ASSETS
A_t	HOUSEHOLDS' GROSS DEPOSITS
ω_1, ω_2	RELATIVE WEIGHT OF SOCIAL EQUALITY AND SOCIAL WELFARE

Figure 1 Notations

3.Households

- Households' income consists of labor income and asset income(saving):

$$i_t = W_t e_t h_t + r_{t-1} a_t \qquad (1)$$

- N households are heterogeneous in terms of initial wealth and labor productivity levels e_t, the latter follows:

$$\log e_t = \rho_e \log e_{t-1} + \sigma_e u_t \qquad (2)$$

	Normal	Super-star
Normal	$1-p$	p
Super-star	$1-q$	q

$$(p = 2.2e^{-6}, q = 0.990)$$

- Each household seeks to maximize lifetime utility depends on consumption c_t and working hours h_t,

$$\mathbb{E}_0 \sum_{t=0}^{T_N} \beta^t \left(\frac{c_t^{1-\theta}}{1-\theta} - \frac{h_t^{1+\gamma}}{1+\gamma} \right) \qquad (3)$$

subject to budget constraint

$$(1+\tau_s)c_t + a_{t+1} = i_t - T(i_t) + a_t - T^a(a_t) \qquad (4)$$

- Households are required to pay taxes to the government, including consumption taxes τ_s, income taxes $T(i_t)$, and asset taxes $T(a_t)$,

$$T(i_t) = i_t - (1-\tau)\frac{i_t^{1-\xi}}{1-\xi}, \quad T^a(a_t) = a_t - \frac{1-\tau_a}{1-\xi_a} a_t^{1-\xi_a} \qquad (5)$$

4.Government

- The government typically has three deals, including the government spending G_t, the taxation T_t, and the debt B_t with the interest rate r_{t-1}, following the budget constraint:

$$(1 + r_{t-1})\, B_t + G_t = B_{t+1} + T_t \tag{6}$$

$$T_t = \sum_{i}^{N} \left(T(i_t^i) + T(a_t^i) + \tau_s c_t^i \right)$$

- Objective of the government:
  - Maximizing GDP Growth Rate:

$$\max \quad J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{Y_t - Y_{t-1}}{Y_{t-1}} \right) \tag{7}$$

  - Minimizing Social Inequality:

$$\min \quad J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\text{Gini}(\mathcal{I}_t) \text{Gini}(\mathcal{W}_t) \right) \tag{8}$$

  - Hybrid Objective

$$\max \quad J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{Y_t - Y_{t-1}}{Y_t} - \omega_1 \cdot \text{Gini}(\mathcal{I}_t) \text{Gini}(\mathcal{W}_t) \right] \tag{9}$$

5.Firm

- The firm produces a homogeneous good, following the Cobb–Douglas production function,

$$Y_t = K_t^{\alpha} L_t^{1-\alpha} \tag{10}$$

- The produced output is for households' gross consumption C_t, government spending G_t, and physical capital investment X_t

$$Y_t = C_t + X_t + G_t \tag{11}$$

- The firm rent capital at a rental rate R_t and hires labor at a wage rate W_t based on market clearing assumption:

$$W_t = \frac{\partial Y_t}{\partial L_t} = (1-\alpha)(\frac{K_t}{L_t})^{\alpha}, \quad R_t = \frac{\partial Y_t}{\partial K_t} = \alpha(\frac{K_t}{L_t})^{\alpha-1} \tag{12}$$

- The labor market clearing condition is:

$$L_t = \sum_i^N e_t^i h_t^i \tag{13}$$

6.Financial Intermediary

- The financial intermediary provides a platform where households can deposit their savings and the intermediary can use these funds to purchase capital and government bonds:

$$K_{t+1} + B_{t+1} - A_{t+1} = (R_t + 1 - \delta) K_t + (1 + r_{t-1}) (B_t - A_t) \tag{14}$$

No-arbitrage implies that $R_{t+1} = r_t + \delta$

7.Markov Game

- Action space

  Action Space:

  - Government agent: $\mathcal{A}_g = \{\tau_t, \xi_t, \tau_{a,t}, \xi_{a,t}, r_t^G\}$
  - Household agent i: $\mathcal{A}_h^i = \{p_t^i, h_t^i\}, , i \in \{1, ..., N\}.$

  Observation Space:

  - Government agent: $\mathcal{O}_g = \{W_t, \bar{a}_t^r, \bar{i}_t^r, \bar{e}_t^r, \bar{a}_t^p, \bar{i}_t^p, \bar{e}_t^p\}$
  - Household agent i: $\mathcal{O}_h^i = \{W_t, \bar{a}_t^r, \bar{i}_t^r, \bar{e}_t^r, \bar{a}_t^p, \bar{i}_t^p, \bar{e}_t^p, a_t^i, e_t^i\}.$

- Reward of household agents:

$$\text{Household agent } i\text{: } r_{h,t}(s_t, a_{h,t}^i) = \frac{{c_t^i}^{1-\theta}}{1-\theta} - \frac{{h_t^i}^{1+\gamma}}{1+\gamma}$$

- Reward of the government agent:

$$r_{g,t}(s_t, a_{g,t}) = \text{sigmoid}\left(\frac{Y_t - Y_{t-1}}{Y_t} - \omega_1 \cdot \text{Gini}(\mathcal{I}_t)\text{Gini}(\mathcal{W}_t)\right)$$

- Evaluation of household agents:

$$\sum_{t=0}^{T} r_{h,t}(s_t, a_{h,t}^i)$$

- Evaluation of the government agent:

$$\sum_{t=0}^{T} r_{g,t}(s_t, a_{g,t})$$

  So the government task encourage longer episode(T is set at 300), and better GDP and Gini index afterwards.