3. Use the Bisection method to find solutions accurate to within 10^{-2} for $x^3 - 7x^2 + 14x - 6 = 0$ on each interval.

   a. $[0, 1]$ b. $[1, 3.2]$ c. $[3.2, 4]$

4. Use the Bisection method to find solutions accurate to within 10^{-2} for $x^4 - 2x^3 - 4x^2 + 4x + 4 = 0$ on each interval.

   a. $[-2, -1]$ b. $[0, 2]$ c. $[2, 3]$ d. $[-1, 0]$

7. Use the Bisection method to find solutions accurate to within 10^{-5} for the following problems.

   a. $x - 2^{-x} = 0$ for $0 \le x \le 1$

   b. $e^x - x^2 + 3x - 2 = 0$ for $0 \le x \le 1$

   c. $2x\cos(2x) - (x+1)^2 = 0$ for $-3 \le x \le -2$ and $-1 \le x \le 0$

   d. $x\cos x - 2x^2 + 3x - 1 = 0$ for $0.2 \le x \le 0.3$ and $1.2 \le x \le 1.3$

8. Let $f(x) = (x+2)(x+1)^2 x(x-1)^3(x-2)$. To which zero of f does the Bisection method converge when applied on the following intervals?

   a. $[-1.5, 2.5]$ b. $[-0.5, 2.4]$ c. $[-0.5, 3]$ d. $[-3, -0.5]$

10. Find an approximation to $\sqrt{3}$ correct to within 10^{-4} using the Bisection Algorithm. [*Hint:* Consider $f(x) = x^2 - 3$.]

11. Find an approximation to $\sqrt[3]{25}$ correct to within 10^{-4} using the Bisection Algorithm.

3. Se no método da bissecção tomarmos sistematicamente $x = (a_k + b_k)/2$, teremos que $|\bar{x} - \xi| \leq (b_k - a_k)/2$.

   Considerando este fato:

   a) estime o número de iterações que o método efetuará;

   b) escreva um novo algoritmo.

1. Use algebraic manipulation to show that each of the following functions has a fixed point at p precisely when $f(p) = 0$, where $f(x) = x^4 + 2x^2 - x - 3$.

   a. $g_1(x) = \left(3 + x - 2x^2\right)^{1/4}$ b. $g_2(x) = \left(\dfrac{x + 3 - x^4}{2}\right)^{1/2}$

2. a. Perform four iterations, if possible, on each of the functions g defined in Exercise 1. Let $p_0 = 1$ and $p_{n+1} = g(p_n)$, for $n = 0, 1, 2, 3$.

   b. Which function do you think gives the best approximation to the solution?

x_{n-1}

5. Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^4 - 3x^2 - 3 = 0$ on $[1, 2]$. Use $p_0 = 1$.

6. Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^3 - x - 1 = 0$ on $[1, 2]$. Use $p_0 = 1$.

11. For each of the following equations, determine an interval $[a, b]$ on which fixed-point iteration will converge. Estimate the number of iterations necessary to obtain approximations accurate to within 10^{-5}, and perform the calculations.

a. $x = \dfrac{2 - e^x + x^2}{3}$ **b.** $x = \dfrac{5}{x^2} + 2$

c. $x = (e^x/3)^{1/2}$ **d.** $x = 5^{-x}$

e. $x = 6^{-x}$ **f.** $x = 0.5(\sin x + \cos x)$

12. For each of the following equations, determine a function g and an interval $[a, b]$ on which fixed-point iteration will converge to a positive solution of the equation.

a. $3x^2 - e^x = 0$ **b.** $x - \cos x = 0$

Find the solutions to within 10^{-5}.

19. **a.** Use Theorem 2.3 to show that the sequence defined by

$$x_n = \frac{1}{2}x_{n-1} + \frac{1}{x_{n-1}}, \quad \text{for } n \geq 1,$$

converges to $\sqrt{2}$ whenever $x_0 > \sqrt{2}$.

b. Use the fact that $0 < (x_0 - \sqrt{2})^2$ whenever $x_0 \neq \sqrt{2}$ to show that if $0 < x_0 < \sqrt{2}$, then $x_1 > \sqrt{2}$.

c. Use the results of parts (a) and (b) to show that the sequence in (a) converges to $\sqrt{2}$ whenever $x_0 > 0$.

20. **a.** Show that if A is any positive number, then the sequence defined by

$$x_n = \frac{1}{2}x_{n-1} + \frac{A}{2x_{n-1}}, \quad \text{for } n \geq 1,$$

converges to $\sqrt{A}$ whenever $x_0 > 0$.

b. What happens if $x_0 < 0$?

19. O polinômio $p(x) = x^5 - \frac{10}{9}x^3 + \frac{5}{21}x$ tem seus cinco zeros reais, todos no intervalo $(-1, 1)$.

a) Verifique que $x_1 \in (-1, -0.75)$, $x_2 \in (-0.75, -0.25)$, $x_4 \in (0.3, 0.8)$ e $x_5 \in (0.8, 1)$.

b) Encontre, pelo respectivo método, usando $\varepsilon = 10^{-5}$

x_1: Newton ($x_0 = -0.8$)

x_2: bissecção ($[a, b] = [-0.75, -0.25]$)

x_3: posição falsa ($[a, b] = [-0.25, 0.25]$)

x_4: MPF ($I = [0.2, 0.6]$, $x_0 = 0.4$)

22. Suppose that g is continuously differentiable on some interval (c, d) that contains the fixed point p of g. Show that if $|g'(p)| < 1$, then there exists a $\delta > 0$ such that if $|p_0 - p| \leq \delta$, then the fixed-point iteration converges.

23. An object falling vertically through the air is subjected to viscous resistance as well as to the force of gravity. Assume that an object with mass m is dropped from a height s_0 and that the height of the object after t seconds is

$$s(t) = s_0 - \frac{mg}{k}t + \frac{m^2 g}{k^2}(1 - e^{-kt/m}),$$

where $g = 32.17$ ft/s^2 and k represents the coefficient of air resistance in lb-s/ft. Suppose $s_0 = 300$ ft, $m = 0.25$ lb, and $k = 0.1$ lb-s/ft. Find, to within 0.01 s, the time it takes this quarter-pounder to hit the ground.

24. Let $g \in C^1[a, b]$ and p be in (a, b) with $g(p) = p$ and $|g'(p)| > 1$. Show that there exists a $\delta > 0$ such that if $0 < |p_0 - p| < \delta$, then $|p_0 - p| < |p_1 - p|$. Thus, no matter how close the initial approximation p_0 is to p, the next iterate p_1 is farther away, so the fixed-point iteration does not converge if $p_0 \neq p$.

6. *a)* Calcule b/a em uma calculadora que só soma, subtrai e multiplica.

   b) Calcule 3/13 nessa calculadora.

- Encontre a raiz quadrada de 18, com erro de 0,5%, usando o método das cordas.