Chapter 1

Calculus and Basic Algebra

1.1 Limits and Continuity

1.1.1 Definition of a Limit

Definition 1.1 : Limit Definition.
A function $f(x)$ approaches the limit L as x approaches c if, for every number $\epsilon > 0$, there exists a number $\delta > 0$ such that whenever $0 < |x - c| < \delta$, we have $|f(x) - L| < \epsilon$.

1.1.2 Theorem: Intermediate Value Theorem

Theorem 1.1 : Intermediate Value Theorem . If a function f is continuous on the interval $[a, b]$, and $f(a)$ and $f(b)$ have opposite signs, then there exists some $x \in [a, b]$ such that $f(x) = 0$.

Exercise 1.1. *Use the Intermediate Value Theorem to show that the equation $x^3 - x - 1 = 0$ has a solution in the interval $[1, 2]$.*

1.1.3 Example: Calculating Limits

Example 1.1 .
Calculate the limit:

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$$

Solution: Factor the numerator:

$$\lim_{x \to 1} \frac{(x-1)(x+1)}{x-1} = \lim_{x \to 1} (x+1) = 2$$