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Zad 1

c) $(x-2)\frac{\partial u}{\partial x} + (y-2)\frac{\partial u}{\partial y} + 2z\frac{\partial u}{\partial z} = 0$

$$\begin{cases} x' = x - z \\ y' = y - z \\ z' = 2z \end{cases} \Rightarrow \begin{cases} x' = x - c_1 e^{2t} \\ y' = y - c_1 e^{2t} \\ z' = c_1 e^{2t} \end{cases} \Rightarrow \begin{cases} x = c_3 e^{t} - c_1 e^{2t} \\ y = c_2 e^{t} - c_1 e^{2t} \\ y = c_1 e^{2t} \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x + 2 = c_3 e^{t} \\ y + 2 = c_2 e^{t} \\ z = c_1 e^{2t} \end{cases} \overset{c_1 \neq 0}{\Rightarrow} \begin{cases} \frac{(x+z)^2}{z} = c_4 \\ \frac{(y+z)^2}{z} = c_5 \end{cases}$$

Charakterystyki jest wyznaczona jednoznacznie przez c_4 i c_5

więc

$$u(x, y, z) = \phi\left(\frac{(x+z)^2}{z}, \frac{(y+z)^2}{z}\right)$$

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Zad 3

a) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + 2\frac{\partial u}{\partial z} = 0$ oraz $u = yz$ dla $x = 1$

$\begin{cases} x' = 1 \\ y' = 1 \\ z' = 2 \end{cases}$ więc $\begin{aligned} x &= t + c_1 \\ y &= t + c_2 \\ z &= 2t + c_3 \end{aligned}$ czyli $\begin{aligned} y &= x + c_4 \\ z &= 2x + c_5 \end{aligned}$

Charakterystyka jest wyznaczona jednoznacznie przez c_4 i c_5
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \overset{\shortparallel}{y-x} \quad \overset{\shortparallel}{z-2x}$

więc

$$u(x,y,z) = \phi(y-x,\ z-2x)$$

Chcemy, żeby dla $x = 1$ $u = yz$

czyli $u(1,y,z) = yz = \phi(y-1,\ z-2)$

$\phi(y-1,\ z-2) = (y-1)(z-2) + 2(y-1) + (z-2) + 2$

więc $\phi(a,b) = a \cdot b + 2a + b + 2$

$$u = \phi(y-x,\ z-2x) = yz - 2xy - xz + 2x^2 + 2y - 2x + z - 2x + 2 =$$

$$= 2x^2 - 2xy - xz + yz - 4x + 2y + z + 2$$