zad.3

$f(x) = \sin x$

$\sin(0,\pi) = (0,1]$

$g(x) = \operatorname{arctg} x$

$g(\mathbb{R}) = (-\frac{\pi}{2}, \frac{\pi}{2})$

$\overline{A \times B} = \overline{A} \times \overline{B}$

$\operatorname{Int}(A \times B) = \operatorname{Int}(A) \times \operatorname{Int}(B)$

Pokazujemy, że $\overline{A} \times \overline{B}$ jest najmniejszym zbiorem domkniętym zawierającym $A \times B$.

$X \times Y \setminus \overline{A} \times \overline{B} = \bigcup_{i \in I} C_i \times D_i$

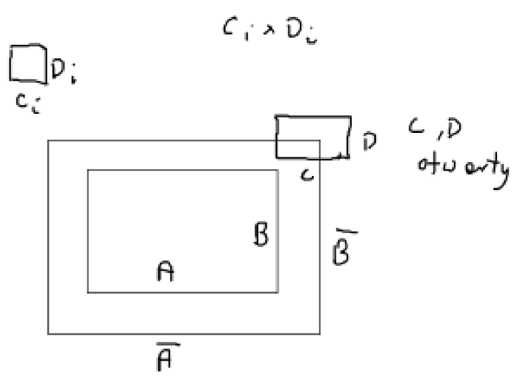
$C_j \cap A \neq \emptyset$

$D_j \cap B \neq \emptyset$

$(C_j \cap A) \times (D_j \cap B)$ ↯

$A \times B \subseteq \overline{A} \times \overline{B}$

$C_i \times D_i$



$\bigcup_{C \times D \in T, (C \cap A = \emptyset \vee D \cap B = \emptyset)} C \times D \in X \times Y \setminus \overline{A} \times \overline{B}$

$C \subseteq X \setminus \overline{A}$

Chcemy pokazać, że nie ma takich otwarte C, D $C \cap A = \emptyset$ lub $D \cap B = \emptyset$. Gdyby były to $\overline{A} \times \overline{B} \setminus C \times D$ $\not\subseteq \overline{A} \times \overline{B}$ byłby zbiorem domkniętym $\supseteq A \times B$

Powiedzmy $C \cap A = \emptyset$

$\Rightarrow C \cap \overline{A} = \emptyset$

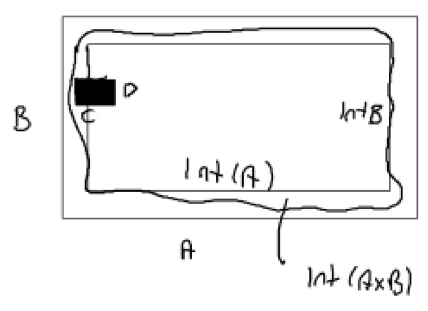
$\Rightarrow (\overline{A} \times \overline{B}) \setminus (C \times D) = \overline{A} \times \overline{B}$

b) $\operatorname{Int}(A \times B) \supseteq \operatorname{Int}(A) \times \operatorname{Int}(B)$

$\bigcup C_i \times D_i = \operatorname{Int}(A \times B)$

$\underbrace{C \setminus \operatorname{Int}(A) \neq \emptyset} \vee D \setminus \operatorname{Int}(B) \neq \emptyset$

$\operatorname{Int}(A) \cup C \subseteq A$



c) $\operatorname{Bd}(A \times B) = \overline{A \times B} \setminus \operatorname{Int}(A \times B)$

zad. 6

$x \in \overline{A}^{d_1} \Longleftrightarrow \exists x_n \in A \; d_1(x_n, x) \to 0$

$\Longleftrightarrow \exists x_n \in A \; d_2(x_2, x) \to 0 \quad \Longleftrightarrow x \in \overline{A}^{d_2}$

zad. 7

Zauważmy $\frac{g(x,y)}{1+g(x,y)} \leq 1$

$$\frac{p(x,y)}{1 + p(x,y)} < R$$

$(\mathbb{R}, d_e)$

$g = d_e$

$g(x_n, y_n) \to \infty$

$\quad = a_n$

$\frac{a_n}{1+a_n} \to 1$

$$p(x,y) < R(1 + p(x,y))$$

$$p(x,y)(1-R) < R$$

$$p(x,y) < \frac{R}{1-R}$$

$(x_n) \quad x$

$\lim_n d_2(x_n, x) = 0 \quad \Leftrightarrow \quad \lim_n g(x_n, x) = 0$

$\Leftarrow$

$\lim_n (p(x_n, x)) = 0 \Rightarrow$

$\lim_n \frac{p(x_n, x)}{1} = 0$

$\lim_n \frac{p(x_n, x)}{1} > \lim_n \frac{p(x_n, x)}{1 + p(x_n, x)} =$

$= d_2(x_n, x)$

$g(x_n, x) = a_n$

Pokażmy

$\lim_{n \to \infty} \frac{p(x_n, x)}{1 + p(x_n, x)} = 0$

$\lim_{n \to \infty} \frac{a_n}{1 + a_n} = 0$

$p(x_n$

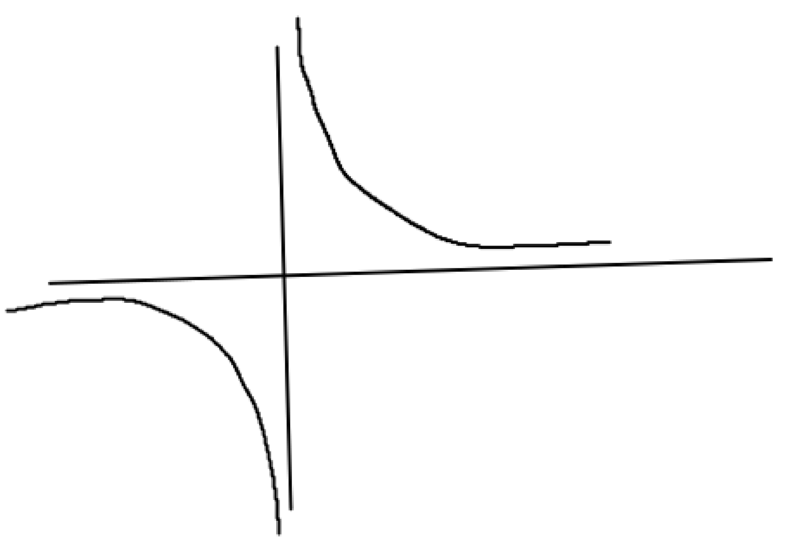
$\Rightarrow \lim a_n = 0$

$\frac{(a_n + 1) - 1}{1 + a_n} = 1 - \frac{1}{1 + a_n}$

$\frac{1}{1+a_n} \to 1 \Rightarrow 1 + a_n \to 1 \Rightarrow$

$\Rightarrow a_n \to 0$

11) $\pi_1 : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$



$f(x) = \frac{1}{x}$

$W(f)$ = wykres $f = \{(x, f(x)) : x \in \mathbb{R} \setminus \{0\}\}$

domknięty

$\pi_1(W(f)) = \mathbb{R} \setminus \{0\}$