Zad 12

$m(t_0) = m_0$ Jeśli $m_0 = 0$ wtedy $m'(t_0) = 0$ więc $m \equiv 0$

$m'(t) = a\,m(t) - b\,m^2(t) \to m \equiv 0$

dla uproszczenia $m = m(t)$

$\cancel{u(t)}$

$\boxed{\begin{array}{l} u = m^{-1} \\ \cancel{du = -dm \frac{1}{m^2}} \\ \cancel{-du\, m^2 = dm} \end{array}}$ $du = -dm \frac{1}{m^2}$

$-du\, m^2 = \cancel{dm}\, dm$

$$\frac{dm}{dt} = am - bm^2$$

$$\frac{-du\, m^2}{dt} = a u m^2 - b m^2$$

$$-\frac{du}{dt} = au - b$$

$$b = \frac{du}{dt} + au$$

$$b = u' + au$$

$$b \cdot e^{at} = u' \cdot e^{a \cdot t} + au \cdot e^{at}$$

$$b \cdot e^{at} = (u \cdot e^{a \cdot t})'$$

$$\int b e^{at} dt = u \cdot e^{at}$$

$$\frac{b}{a} e^{at} = u \cdot e^{at}$$

$$u = \frac{b}{a} \qquad m = \frac{1}{u} = \frac{a}{b}$$

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Zad 5

$$x' + a(t)x = b(t)x^m \qquad m \in \mathbb{R}$$

1° $m=1$

z metody czynnika całkującego

$$x' + (a(t)-b(t))x = 0 \longrightarrow x \equiv 0$$

$$\left(x e^{\int a(t)-b(t)\,dt}\right)' = 0$$

$\cancel{2^\circ\ m}$ $x = c \cdot e^{-\int a(t)-b(t)\,dt}$

2° $m \neq 1$

$$x' + a(t)x = b(t)x^m \nearrow x \equiv 0$$

$$\boxed{\begin{aligned} &u = x^{1-m} \\ &du = dx\,(1-m)\,x^{-m} \\ &\frac{du\,x^m}{1-m} \end{aligned}}$$

$$\frac{dx}{dt} + a(t)x = b(t)x^m$$

$$\frac{du\,x^m}{dt\,(1-m)} + a(t)x^m \cdot u = b(t)x^m$$

$$\frac{u'}{1-m} + a(t)\,u = b(t)$$

$$u' + (1-m)a(t)u = (1-m)b(t) \quad / \cdot e^{\int(1-m)a(t)\,dt}$$

$$\left(u' e^{\int(1-m)a(t)\,dt} + (1-m)a(t)u\,e^{\int(1-m)a(t)\,dt}\right) = (1-m)b(t)\,e^{\int(1-m)a(t)\,dt}$$

$$\left(u e^{\int(1-m)a(t)\,dt}\right)' = (1-m)b(t)e^{\int(1-m)a(t)}$$

$$u e^{\int(1-m)a(t)\,dt} = \int (1-m)b(t)e^{\int(1-m)a(t)\,dt}\,dt$$

$$u = \frac{1}{e^{\int(1-m)a(t)\,dt}} \int (1-m)b(t)e^{\int(1-m)a(t)\,dt}\,dt$$

Zad 9)

$$y' = 2y^{1/2} \to y \equiv 0$$

$$\frac{dy}{dt} = 2\sqrt{y}$$

$$\frac{dy}{2\sqrt{y}} = dt$$

$$\sqrt{y} = t + c$$

$$y = t^2 + 2ct + c^2$$

Zad 12

Jeśli $M_0 = 0$ wtedy $M'(t_0) = 0$ więc $M \equiv 0$

$$M(t_0) = M_0$$

$$M'(t) = aM(t) - bM^2(t) \to M \equiv 0$$

dla uproszczenia $M = M(t)$

$$\frac{dM}{dt} = aM - bM^2$$

$$u = M^{-1}$$

$$\cancel{du} = -dM\frac{1}{M^2} \quad du = -dM\frac{1}{M^2}$$

$$\cancel{-du\, M^2 = bM} \quad -du\, M^2 = dM$$

$$\frac{-du\, M^2}{dt} = auM^2 - bM^2$$

$$-\frac{du}{dt} = au - b$$

$$b = \frac{du}{dt} + au$$

$$b = u' + au$$

$$b \cdot e^{at} = u' \cdot e^{a \cdot t} + au \cdot e^{at}$$

$$b \cdot e^{at} = (u \cdot e^{a \cdot t})'$$

$$\int b e^{at} dt = u \cdot e^{at}$$

$$\frac{b}{a} e^{at} = u \cdot e^{at}$$

$$u = \frac{b}{a} \quad M = \frac{1}{u} = \frac{a}{b}$$

Zad 12 c.d.

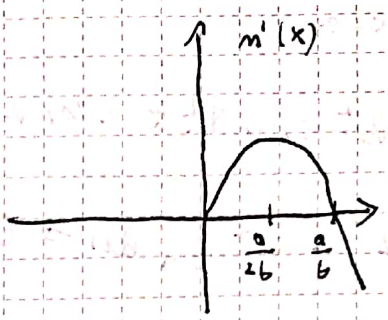
Kiedy $M'(t)$ osiąga maksimum?

Rozwiążmy równanie kwadratowe

~~$M'(t)$~~

$$aX - bX^2 = 0$$

$$X = \frac{a}{b} \lor X = 0$$



~~Czyli im $|M(t) - \frac{a}{b}|$ większe tym M~~

Czyli im $|M(t) - \frac{a}{2b}|$ mniejsze tym $M'(t)$ większe

Jeśli $M_0 < \frac{a}{b}$ wtedy $M(t)$ nie będzie większe od $\frac{a}{b}$ ponieważ osiągając punkt $\frac{a}{b}$ $M'(t) = 0$

Podobnie dla $M_0 > \frac{a}{b}$ wtedy $M(t)$ nie będzie mniejsze od $\frac{a}{b}$.

1° $M_0 \in \langle \frac{a}{2b}, a/b \rangle$

wtedy ~~m~~ $M'(t)$ będzie największe w t_0

2° $M_0 \in (0, \frac{a}{2b})$

3° $M_0 \in (\frac{a}{b}, \infty)$

wtedy $M'(t)$ będzie największe dla $t \to \infty$

Zad 14

a) $\left(t - x\cos\frac{x}{t}\right)dt + t\cos\frac{x}{t}\,dx = 0$ Wiadomo, że $t \neq 0$

$u = \frac{x}{t}$ $\quad ut = x$

$t\,du + u\,dt = dx$

~~$t - x\cos$~~

$(t - ut\cos u)dt + t\cos u\,(t\,du + u\,dt) = 0$

$t\,dt + t^2\cos u\,du = 0$

$t^2\cos u\,du = -t\,dt$

$-t^2\sin u = -\frac{1}{2}t^2 + c$

$\sin u = \frac{1}{2} + C$

$u = \arcsin\left(\frac{1}{2} + c\right)$

b)

$\boxed{1^\circ \; t \neq -1 \qquad \frac{x-2t}{t+1} \neq \frac{\pi}{2} + k\pi \quad k \in \mathbb{Z}}$

$\frac{dx}{dt} = \frac{x+2}{t+1} + tg\,\frac{x-2t}{t+1}$

$u = x+2 \qquad z = t+1$

$du = dx \qquad dz = dt$

$\frac{du}{dz} = \frac{u}{z} + tg\,\frac{u-2z}{z}$

$\frac{du}{dz} = \frac{u}{z} + tg\left(\frac{u}{z} - 2\right)$

$yz = u$

$z\,dy + y\,dz = du$

$\frac{z\,dy + y\,dz}{dz} = y + tg(y-2)$

$\frac{z\,dy}{dz} = tg(y-2)$

$V = y-2$

$dV = dy$

$\frac{z\,dV}{dz} = tg\,V$

$\frac{dV}{tg\,V} = \frac{1}{z}\,dz$

$\ln|\sin V| = \ln z + c$

$\ln\frac{\sin V}{z} = C$

$\ln\frac{\sin(y-2)}{z} = C$

$\ln\frac{\sin\left(\frac{u}{z} - 2\right)}{z} = C$

~~$\ln \sin\left(\frac{x+2}{t+1} - 2\right)$~~

$\ln\frac{\sin\left(\frac{x+2}{t+1} - 2\right)}{t+1} = C$

$\frac{\sin\left(\frac{x+2}{t+1} - 2\right)}{t+1} = e^C$

$\sin\left(\frac{x+2}{t+1} - 2\right) = (t+1)e^C$

$\frac{x+2}{t+1} - 2 = \arcsin\left[(t+1)e^C\right]$

$x = (t+1)\left(\arcsin\left[(t+1)e^C\right] + 2\right) - 2$