Supplementary Material

High-speed Tracking with Multi-kernel Correlation Filters

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Abstract

This supplementary material includes 1) the experimental results on OTB2015, 2) the proof of Theorem 1.

1. Experimental Results on OTB2015

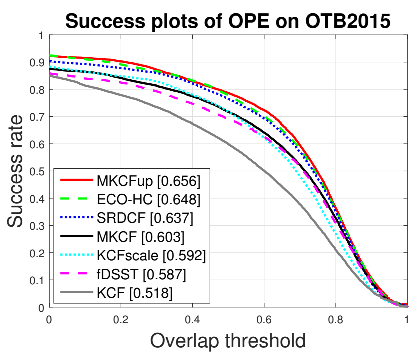

Figure 1. The success plot of MKCFup, KCF, KCFscale, MKCF, SRDCF, fDSST, and ECO_HC on small move sequences of OTB2015. The AUCs of the trackers on the sequences are reported in the legends.

2. Proof of Upper Bound of MKCF Objective Function [2]

Lemma 1 *Suppose* $\mathbf{a}_m$ *is a vector,* $m = 1, \ldots, M$. *Then,*

$$\left\| \sum_{m=1}^{M} \mathbf{a}_m \right\|_2^2 \leq (2M+1) \sum_{m=1}^{M} \|\mathbf{a}_m\|_2^2.$$

The equality holds when $\mathbf{a}_m = \mathbf{a}_n$, where $m = 1, \ldots, M$ and $n = 1, \ldots, M$.

Proof of Lemma 1:

It is true that

$$\left\| \sum_{m=1}^{M} \mathbf{a}_m \right\|_2^2 = \left(\sum_{m=1}^{M} \mathbf{a}_m \right)^\top \left(\sum_{m=1}^{M} \mathbf{a}_m \right) = \sum_{m=1}^{M} \mathbf{a}_m^\top \sum_{m=1}^{M} \mathbf{a}_m = \sum_{m=1}^{M} \mathbf{a}_m^\top \mathbf{a}_m + 2 \sum_{m=1}^{M} \sum_{n=1}^{M} \mathbf{a}_m^\top \mathbf{a}_n.$$

It is also true that

$$\begin{aligned} &\mathbf{a}_m^\top \mathbf{a}_m + \mathbf{a}_n^\top \mathbf{a}_n - 2\mathbf{a}_m^\top \mathbf{a}_n = \\ &\mathbf{a}_m^\top (\mathbf{a}_m - \mathbf{a}_n) - \mathbf{a}_n^\top (\mathbf{a}_m - \mathbf{a}_n) = \\ &(\mathbf{a}_m - \mathbf{a}_n)^\top (\mathbf{a}_m - \mathbf{a}_n) = \|\mathbf{a}_m - \mathbf{a}_n\|_2^2 \geq 0. \end{aligned}$$

Therefore,

$$\begin{aligned} &\mathbf{a}_m^\top \mathbf{a}_m + \mathbf{a}_n^\top \mathbf{a}_n \geq 2\mathbf{a}_m^\top \mathbf{a}_n, \\ &\sum_{m=1}^{M} \mathbf{a}_m^\top \mathbf{a}_m + M\mathbf{a}_n^\top \mathbf{a}_n \geq 2 \sum_{m=1}^{M} \mathbf{a}_m^\top \mathbf{a}_n, \\ &M \sum_{m=1}^{M} \mathbf{a}_m^\top \mathbf{a}_m + M \sum_{n=1}^{M} \mathbf{a}_n^\top \mathbf{a}_n \geq 2 \sum_{n=1}^{M} \sum_{m=1}^{M} \mathbf{a}_m^\top \mathbf{a}_n, \\ &2M \sum_{m=1}^{M} \mathbf{a}_m^\top \mathbf{a}_m \geq 2 \sum_{m=1}^{M} \sum_{n=1}^{M} \mathbf{a}_m^\top \mathbf{a}_n. \end{aligned}$$

Therefore,

$$\begin{aligned} \left\| \sum_{m=1}^{M} \mathbf{a}_m \right\|_2^2 &= \sum_{m=1}^{M} \mathbf{a}_m^\top \mathbf{a}_m + 2 \sum_{m=1}^{M} \sum_{n=1}^{M} \mathbf{a}_m^\top \mathbf{a}_n \\ &\leq \sum_{m=1}^{M} \mathbf{a}_m^\top \mathbf{a}_m + 2M \sum_{m=1}^{M} \mathbf{a}_m^\top \mathbf{a}_m \\ &= (2M+1) \sum_{m=1}^{M} \mathbf{a}_m^\top \mathbf{a}_m. \end{aligned}$$

That is,

$$\left\|\sum_{m=1}^{M} \mathbf{a}_m\right\|_2^2 \le (2M+1)\sum_{m=1}^{M} \|\mathbf{a}_m\|_2^2.$$

It is clear that the equality holds when $\mathbf{a}_m = \mathbf{a}_n$, where $m = 1, \dots, M$ and $n = 1, \dots, M$.

Q.E.D.

2.1. Proof of Upper Bound

According to Lemma 1,

$$\left\|\mathbf{y} - \sum_{m=1}^{M} d_m \mathbf{K}_m \boldsymbol{\alpha}\right\|_2^2 \le (2M+1)\sum_{m=1}^{M} \|\mathbf{y}_c - d_m \mathbf{K}_m \boldsymbol{\alpha}\|_2^2 .$$

Therefore, $U_{F(\boldsymbol{\alpha},\mathbf{d})}$ is the upper bound of $F(\boldsymbol{\alpha}, \mathbf{d})$, and the upper bound is reached when $d_m \mathbf{K}_m \boldsymbol{\alpha} = d_n \mathbf{K}_n \boldsymbol{\alpha}$, where $m = 1, \dots, M$ and $n = 1, \dots, M$.

Q.E.D.

3. Proof of Theorem 1

In the extension of MKCF with upper bound, to optimize the unconstrained problem

$$\min_{\boldsymbol{\alpha}_p, \mathbf{d}_p} F_p(\boldsymbol{\alpha}_p, \mathbf{d}_p), \tag{1}$$

we achieve that

$$\boldsymbol{\alpha}_p = \left(\sum_{j=1}^{p}\sum_{m=1}^{M} \beta_m^j \left((d_{m,p}\mathbf{K}_m^j)^2 + \lambda d_{m,p}\mathbf{K}_m^j\right)\right)^{-1} \cdot \sum_{j=1}^{p}\sum_{m=1}^{M} \beta_m^j d_{m,p}\mathbf{K}_m^j \mathbf{y}_c, \tag{2}$$

and

$$d_{m,p} = \frac{d_{m,p}^N}{d_{m,p}^D}, \tag{3}$$

where

$$\begin{aligned} d_{m,p}^N &= (1-\gamma_m) d_{m,p-1}^N + \gamma_m (\mathbf{K}_m^p \boldsymbol{\alpha}_p)^\top (2\mathbf{y}_c - \lambda\boldsymbol{\alpha}_p), \\ d_{m,p}^D &= (1-\gamma_m) d_{m,p-1}^D + 2\gamma_m (\mathbf{K}_m^p \boldsymbol{\alpha}_p)^\top (\mathbf{K}_m^p \boldsymbol{\alpha}_p), \end{aligned}$$

when $p > 1$. If $p = 1$, then

$$\begin{aligned} d_{m,1}^N &= (\mathbf{K}_m^1 \boldsymbol{\alpha}_1)^\top (2\mathbf{y}_c - \lambda\boldsymbol{\alpha}_1), \\ d_{m,1}^D &= (\mathbf{K}_m^1 \boldsymbol{\alpha}_1)^\top (\mathbf{K}_m^1 \boldsymbol{\alpha}_1). \end{aligned}$$

To simplify the denotation, in the proof, $d_{m,p}$ expresses the kernel weight $d_{m,p}^t$ of the t^{th} iteration of $\boldsymbol{\alpha}_p$ and $\mathbf{d}_p$.

3.1. Proof of First Conclusion

According to Eq. (2), we set $\boldsymbol{\alpha}_p = \mathbf{D}_p^{-1}\mathbf{N}_p\mathbf{y}_c$, where

$$\mathbf{D}_p = \sum_{j=1}^{p}\sum_{m=1}^{M} \beta_m^j \left((d_{m,p}\mathbf{K}_m^j)^2 + \lambda d_{m,p}\mathbf{K}_m^j\right)$$

and

$$\mathbf{N}_p = \sum_{j=1}^{p}\sum_{m=1}^{M} \beta_m^j d_{m,p}\mathbf{K}_m^j.$$

It is clear that both $\mathbf{D}_p$ and $\mathbf{N}_p$ are positive definite because $\beta_m^j > 0$, $d_{m,p} > 0$, $\lambda > 0$, and $\mathbf{K}_m^j$ is positive definite. Because $\mathbf{K}_m^j$ is circulant Gram matrix, we have $\mathbf{K}_m^j = \mathbf{U}\boldsymbol{\Sigma}_m^j\mathbf{U}^H$, where $\mathbf{U} = \frac{1}{\sqrt{l}}\mathbf{F}_l^{-1}$ and $\mathbf{F}_l$ is the 1-D discrete Fourier transform matrix [1]. Because the linear combination of circulant matrices is also circulant, we have

$$\mathbf{D}_p = \mathbf{U}\left(\sum_{j=1}^{p}\sum_{m=1}^{M} \beta_m^j \left(d_{m,p}^2(\boldsymbol{\Sigma}_m^j)^2 + \lambda d_{m,p}\boldsymbol{\Sigma}_m^j\right)\right)\mathbf{U}^H$$

and

$$\mathbf{N}_p = \mathbf{U}\left(\sum_{j=1}^{p}\sum_{m=1}^{M} \beta_m^j d_{m,p}\boldsymbol{\Sigma}_m^j\right)\mathbf{U}^H.$$

Let $\boldsymbol{\Sigma}_m^j = \mathbf{diag}\left(\sigma_{m,1}^j, \dots, \sigma_{m,l}^j\right)$, $\sigma_{m,n}^j > 0$, $n = 1, \dots, l$. Then the n^{th} eigenvalue of $\mathbf{D}_p^{-1}\mathbf{N}_p$ is

$$\begin{aligned} \sigma_{\boldsymbol{\alpha}_p,n} &\equiv \frac{\sum_{j=1}^{p}\sum_{m=1}^{M} \beta_m^j d_{m,p}\sigma_{m,n}^j}{\sum_{j=1}^{p}\sum_{m=1}^{M} \beta_m^j d_{m,p}\sigma_{m,n}^j (d_{m,p}\sigma_{m,n}^j + \lambda)} \\ &= (\lambda + b_n)^{-1}, \end{aligned}$$

where

$$b_n = \frac{\sum_{j=1}^{p}\sum_{m=1}^{M} \beta_m^j d_{m,p}^2 (\sigma_{m,n}^j)^2}{\sum_{j=1}^{p}\sum_{m=1}^{M} \beta_m^j d_{m,p}\sigma_{m,n}^j}.$$

It is clear that $b_n > 0$.

According to Eq. (3), we also have

$$\begin{aligned} d_{m,p}^N &= \sum_{j=1}^{p} \beta_m^j (\mathbf{K}_m^j \boldsymbol{\alpha}_p)^\top (2\mathbf{y}_c - \lambda\boldsymbol{\alpha}_p) \\ &= \mathbf{y}_c^\top \sum_{j=1}^{p} \beta_m^j \mathbf{N}_p\mathbf{D}_p^{-1}\mathbf{K}_m^j (2\mathbf{I} - \lambda\mathbf{D}_p^{-1}\mathbf{N}_p)\mathbf{y}_c \\ &= \mathbf{y}_c^\top \mathbf{D}_{m,p}^N \mathbf{y}_c, \end{aligned}$$

where $\mathbf{D}_{m,p}^N = \mathbf{N}_p\mathbf{D}_p^{-1}\sum_{j=1}^{p}\beta_m^j\mathbf{K}_m^j(2\mathbf{I} - \lambda\mathbf{D}_p^{-1}\mathbf{N}_p)$, and its n^{th} eigenvalue is

$$\sigma_{m,p,n}^N = \sigma_{\boldsymbol{\alpha}_p,n}(2 - \lambda\sigma_{\boldsymbol{\alpha}_p,n})\sum_{j=1}^{p}\beta_m^j\sigma_{m,n}^j.$$

$\because \lambda\sigma_{\boldsymbol{\alpha}_p,n} = \lambda(\lambda+b_n)^{-1} < 1$, $\therefore 2-\lambda\sigma_{\boldsymbol{\alpha}_p,n} > 1$, $\therefore \sigma^N_{m,p,n} > 0$, $n = 1,\ldots,l$, $\therefore \mathbf{D}^N_{m,p}$ is positive definite, and $d^N_{m,p} > 0$. It is obvious that $d^D_{m,p} > 0$. Consequently,

$$d^{t+1}_{m,p} = \frac{d^N_{m,p}}{d^D_{m,p}} > 0,$$

where $m = 1,\ldots,M$.

Q.E.D.

3.2. Proof of Second Conclusion

According to Eq. (2), we have

$$\begin{aligned} d^D_{m,p} &= 2\sum_{j=1}^{p} \beta^j_m (\mathbf{K}^j_m\boldsymbol{\alpha}_p)^\top(\mathbf{K}^j_m\boldsymbol{\alpha}_p) \\ &= \mathbf{y}_c^\top 2\sum_{j=1}^{p} \beta^j_m \mathbf{N}_p\mathbf{D}_p^{-1}(\mathbf{K}^j_m)^2\mathbf{D}_p^{-1}\mathbf{N}_p\mathbf{y}_c \\ &= \mathbf{y}_c^\top \mathbf{D}^D_{m,p}\mathbf{y}_c \end{aligned}$$

where $\mathbf{D}^D_{m,p} = 2\mathbf{N}_p\mathbf{D}_p^{-1}\sum_{j=1}^{p}\beta^j_m(\mathbf{K}^j_m)^2\mathbf{D}_p^{-1}\mathbf{N}_p$, and its n^{th} eigenvalue is

$$\sigma^D_{m,p,n} = 2\sigma^2_{\boldsymbol{\alpha}_p,n}\sum_{j=1}^{p}\beta^j_m(\sigma^j_{m,n})^2.$$

Then, according to Eq. (3),

$$d^{t+1}_{m,p} = \frac{d^N_{m,p}}{d^D_{m,p}} = \frac{\mathbf{y}_c^\top\mathbf{D}^N_{m,p}\mathbf{y}_c}{\mathbf{y}_c^\top\mathbf{D}^D_{m,p}\mathbf{y}_c} = \frac{\mathbf{y}_c^\top\mathbf{U}\boldsymbol{\Sigma}^N_{m,p}\mathbf{U}^H\mathbf{y}_c}{\mathbf{y}_c^\top\mathbf{U}\boldsymbol{\Sigma}^D_{m,p}\mathbf{U}^H\mathbf{y}_c}.$$

Let $\mathbf{U}^H\mathbf{y}_c = (y_{u,1},\ldots,y_{u,l})$, $c^N_n = \sum_{j=1}^{p}\beta^j_m\sigma^j_{m,n}$, and $c^D_n = \sum_{j=1}^{p}\beta^j_m(\sigma^j_{m,n})^2$. Then

$$d^{t+1}_{m,p} = \frac{\sum_{n=1}^{l} y^2_{u,n}c^N_n\sigma_{\boldsymbol{\alpha}_p,n}(2-\lambda\sigma_{\boldsymbol{\alpha}_p,n})}{2\sum_{n=1}^{l} y^2_{u,n}c^D_n\sigma^2_{\boldsymbol{\alpha}_p,n}}.$$

Let $c^N_{\text{max}} = \max_n c^N_n$, $c^N_{\text{min}} = \min_n c^N_n$, $c^D_{\text{max}} = \max_n c^D_n$, $c^D_{\text{min}} = \min_n c^D_n$, $y_{\text{max}} = \max_n y_{u,n}$, $y_{\text{min}} = \min_n y_{u,n}$,

$$c_l = \frac{y^2_{\text{min}}c^N_{\text{min}}}{y^2_{\text{max}}c^N_{\text{max}}},\quad c_u = \frac{y^2_{\text{max}}c^N_{\text{max}}}{y^2_{\text{min}}c^N_{\text{min}}},$$

and

$$\sigma_r = \frac{\sum_{n=1}^{l}\sigma_{\boldsymbol{\alpha}_p,n}(2-\lambda\sigma_{\boldsymbol{\alpha}_p,n})}{2\sum_{n=1}^{l}\sigma^2_{\boldsymbol{\alpha}_p,n}}.$$

Then

$$c_l\cdot\sigma_r < d^{t+1}_{m,p} < c_u\cdot\sigma_r.$$

Furthermore,

$$\begin{aligned} \sigma_r &= \frac{\sum_{n=1}^{l}(\lambda+b_n)^{-1}(2(\lambda+b_n)-\lambda)(\lambda+b_n)^{-1}}{2\sum_{n=1}^{l}(\lambda+b_n)^{-2}} \\ &= \frac{\sum_{n=1}^{l}(\lambda+b_n)^{-2}(\lambda+2b_n)}{2\sum_{n=1}^{l}(\lambda+b_n)^{-2}} \\ &= \frac{1}{2}\lambda + \frac{\sum_{n=1}^{l}b_n(\lambda+b_n)^{-2}}{\sum_{n=1}^{l}(\lambda+b_n)^{-2}}. \end{aligned}$$

Let $\sigma^j_{m,\max} = \max_n \sigma^j_{m,n}$, $\sigma_{m,\min} = \min_n \sigma^j_{m,n}$,

$$\begin{aligned} b^{\max} &= \frac{\sum_{j=1}^{p}\sum_{m=1}^{M}\beta^j_m d^2_{m,p}(\sigma^j_{m,\max})^2}{\sum_{j=1}^{p}\sum_{m=1}^{M},\beta^j_m d_{m,p}\sigma^j_{m,\min}}, \\ b^{\min} &= \frac{\sum_{j=1}^{p}\sum_{m=1}^{M}\beta^j_m d^2_{m,p}(\sigma^j_{m,\min})^2}{\sum_{j=1}^{p}\sum_{m=1}^{M},\beta^j_m d_{m,p}\sigma^j_{m,\max}}. \end{aligned}$$

Then, $b^{\min} \le b_n \le b^{\max}$, and

$$\frac{1}{2}\lambda + b^{\min} \le \sigma_r \le \frac{1}{2}\lambda + b^{\max},$$

$$\frac{c_l}{2}\lambda + c_l\cdot b^{\min} < d^{t+1}_{m,p} < \frac{c_u}{2}\lambda + c_u\cdot b^{\max}.$$

where $m = 1,\ldots,M$.

Q.E.D.

References

[1] P. David. *Circulant Matrices*. Chelsea Publishing Company, 2nd edition, 1994. 2

[2] M. Tang and J. Feng. Multi-kernel correlation filter for visual tracking. In *Proc. International Conference on Computer Vision*, 2015. 1