Solving Physics Problems on a Computer

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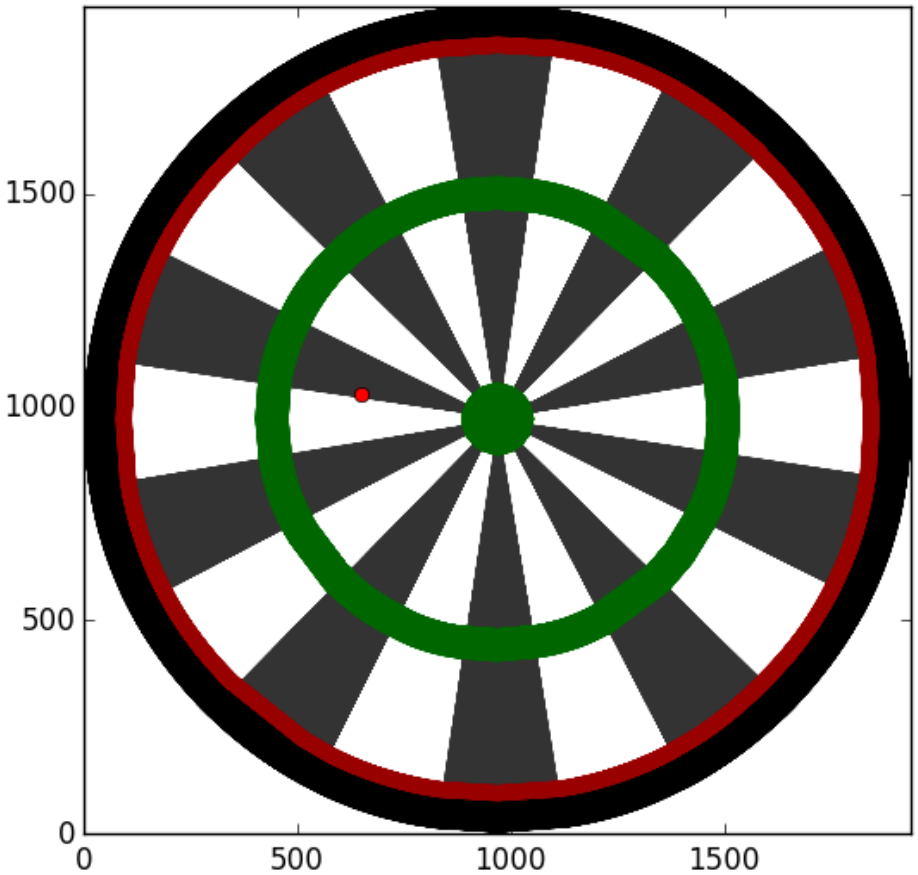
Perimeter Institute for Theoretical Physics

Lunch and Learn
May 5, 2016

Agenda (time permitting)

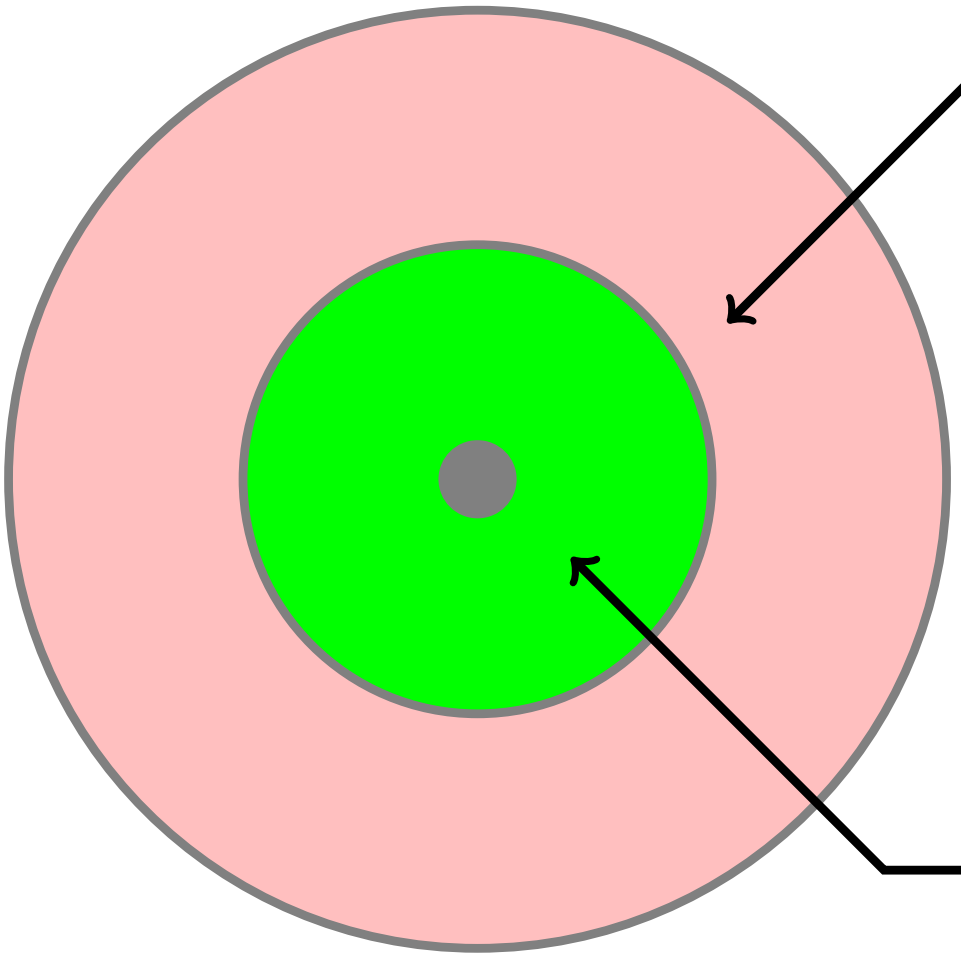
1. Throwing Darts for π
2. Planetary Orbits a la Newton
3. Structure of a Star

The Random Darts Player



Source: Wikimedia Commons

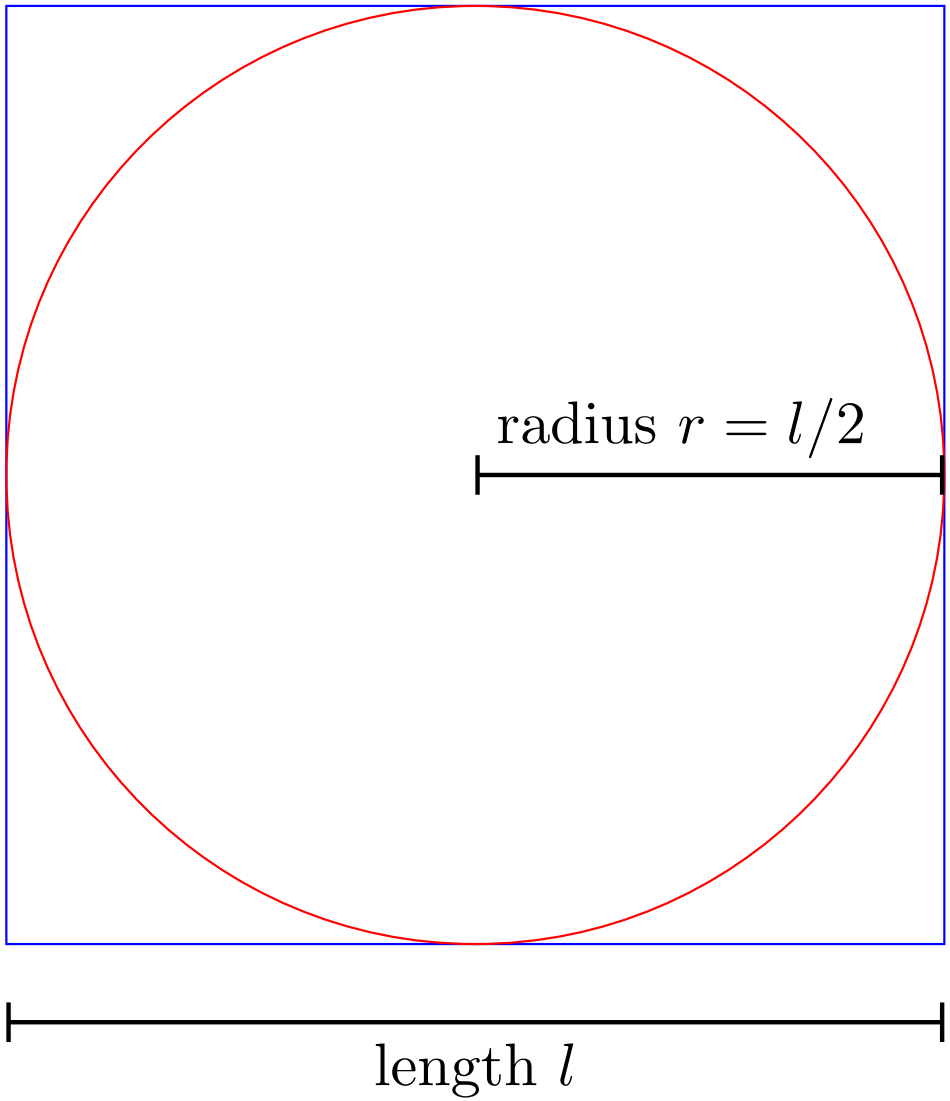
The Darts Tell us the Area of the Ring!



3× as many darts fall here...

...as here.

Areas and π



Area of the circle:

$$A_c = \pi \frac{l^2}{4}$$

Area of the square:

$$A_{sq} = l^2$$

Therefore:

$$\pi = 4 \frac{A_c}{A_{sq}}$$

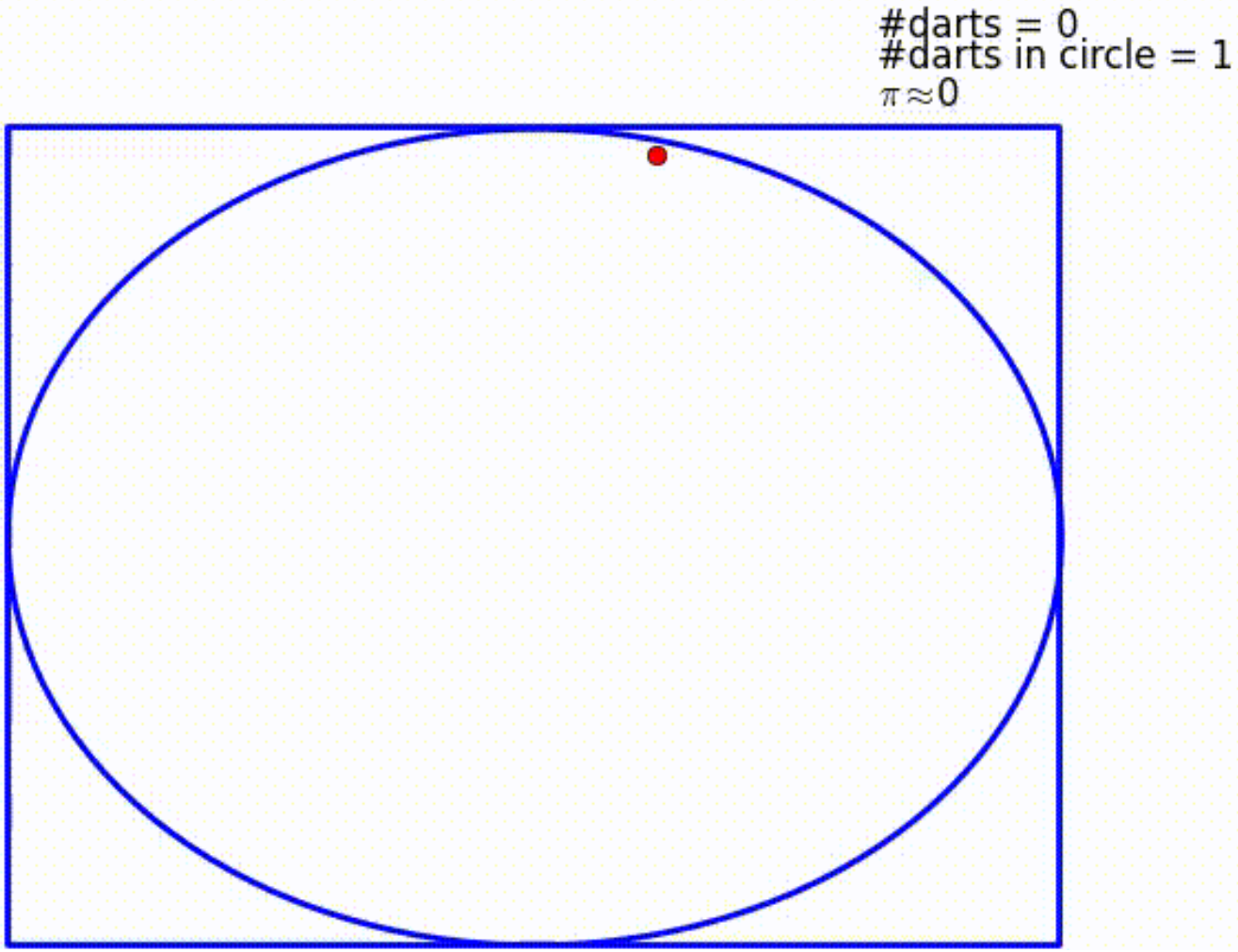
Darts and π

$$\pi \approx 4 \frac{\text{number of darts in circle}}{\text{number of darts in square}}$$

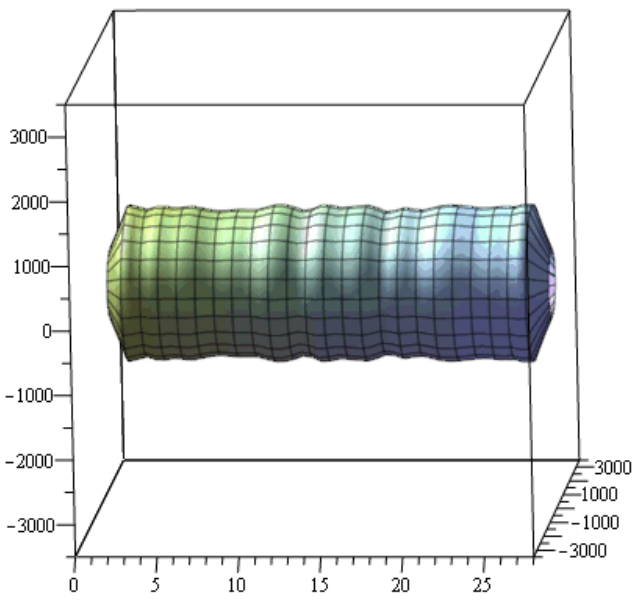
```
def throw_dart():
    x = random.uniform(rgl_min,rgl_max)
    y = random.uniform(rgl_min,rgl_max)
    return x,y
for i in range(darts_max):
    x,y = throw_dart()
    if in_circle(x,y):
        darts_in_circle += 1
    total_darts += 1
    pi = 4*(darts_in_circle/float(total_darts))
```

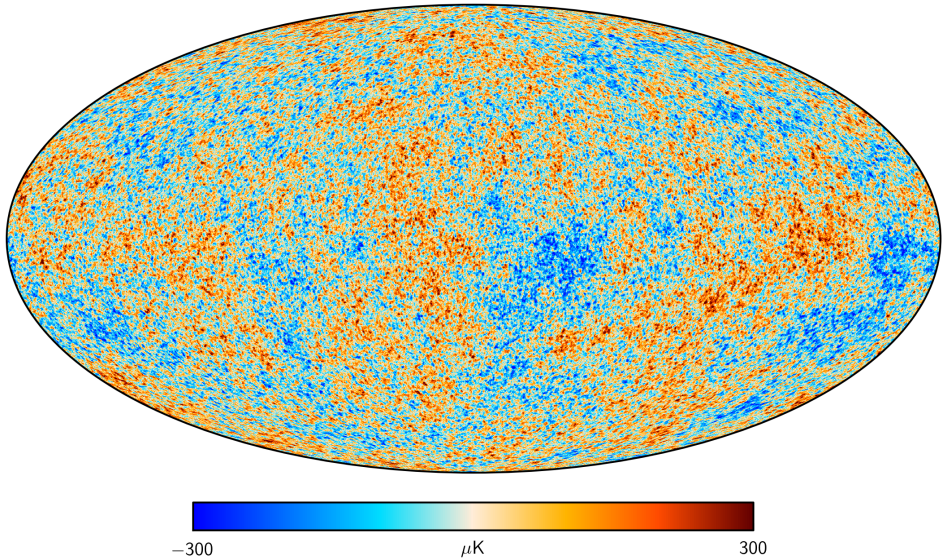
Darts and π

$$\pi \approx 4 \frac{\text{number of darts in circle}}{\text{number of darts in square}}$$

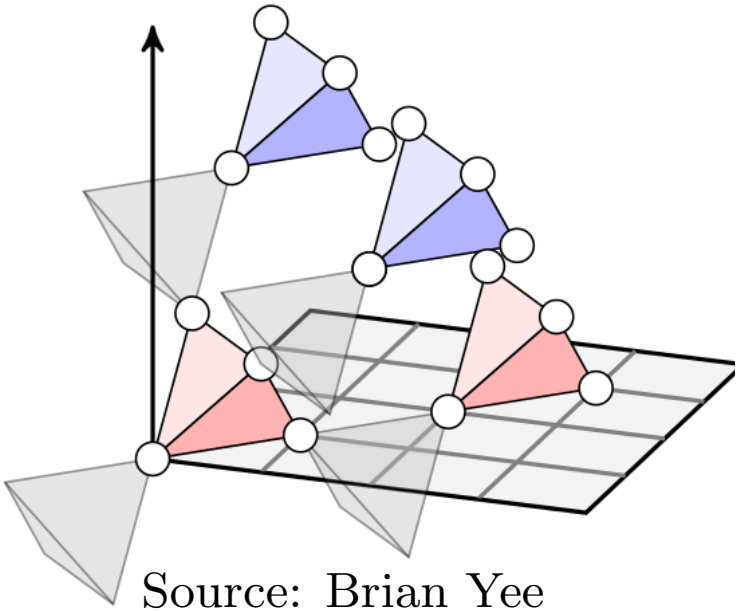


Applications

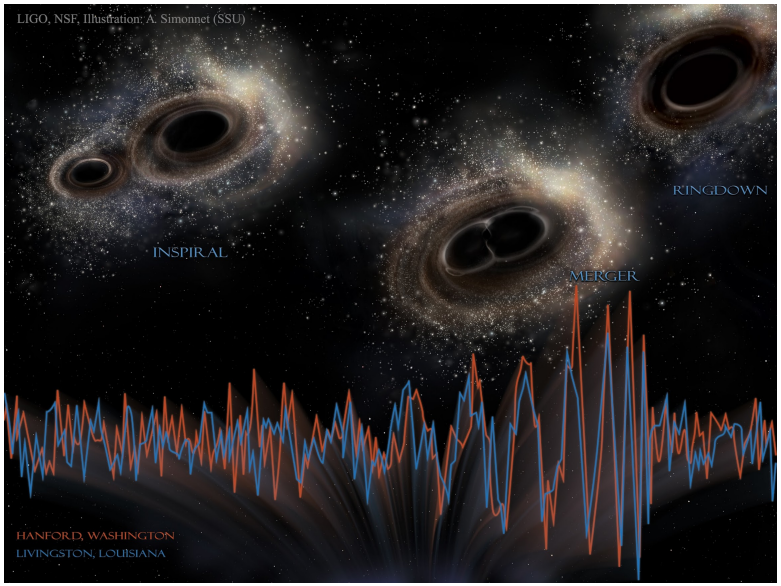




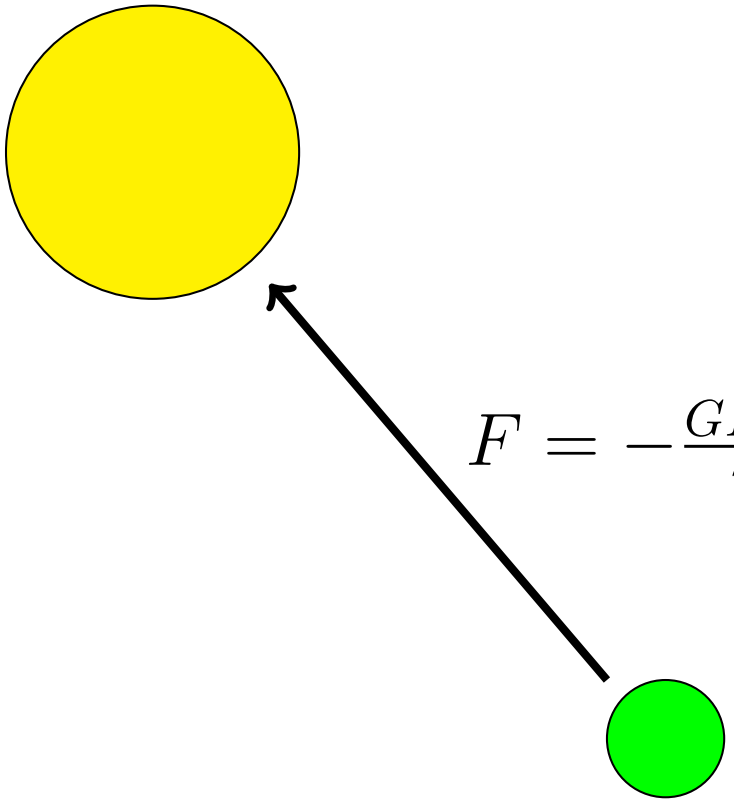
Source: Planck Collaboration



Source: Brian Yee



Newton's Laws



$$F = -\frac{GMm}{r^2}\hat{r}$$

$$F = ma$$

$$\# \approx \text{change in speed}$$

$$\text{change in position} \approx \text{speed}$$

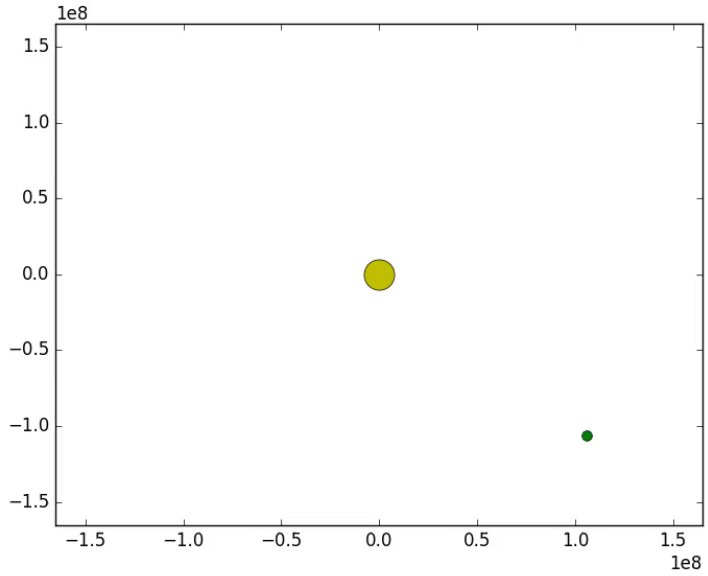
Predict the Future Given Information Now

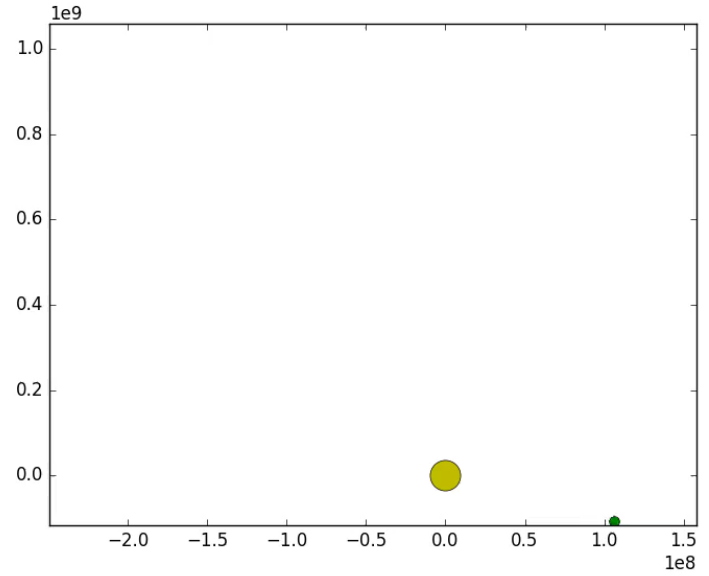
What we have now	What we get in the future
position (x, y)	velocity (v_x, v_y)
velocity (v_x, v_y)	position (x, y)

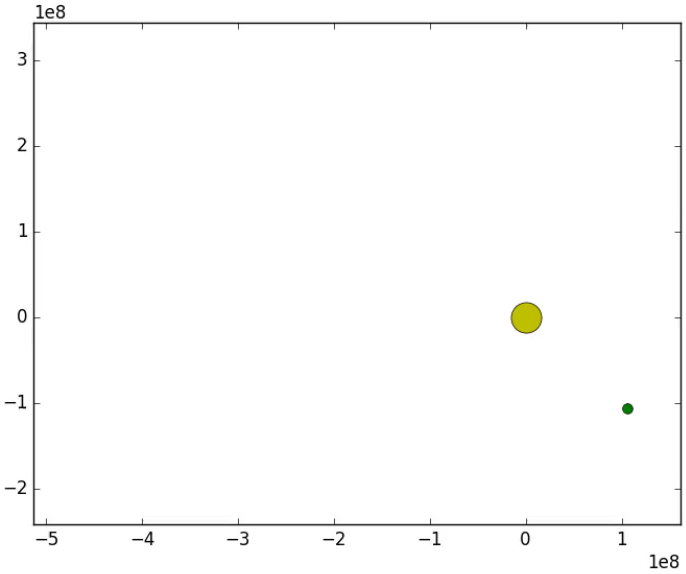
The Iterative Algorithm: Forward Euler

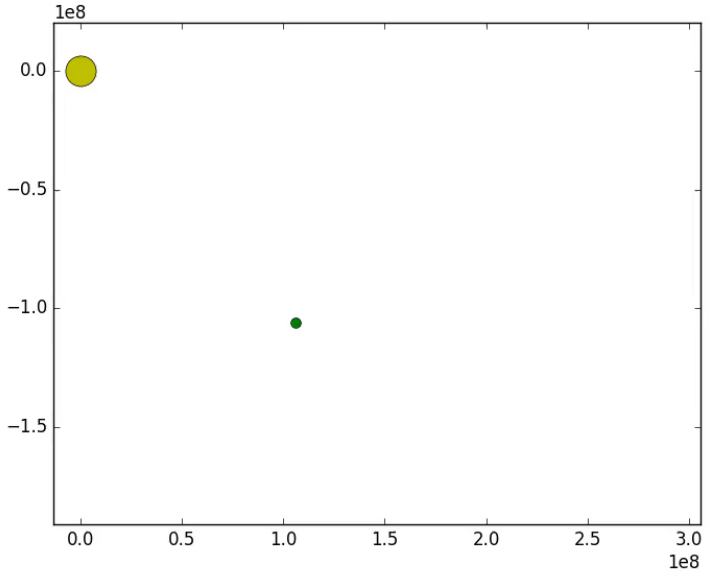
```
for i in range(1,num_times):
    # update positions
    x[i] = x[i-1] + vx[i-1]*dt
    y[i] = y[i-1] + vy[i-1]*dt
    # update velocities
    ax,ay = get_acceleration(x[i-1],y[i-1])
    vx[i] = vx[i-1] + ax*dt
    vy[i] = vy[i-1] + ay*dt
```

Results

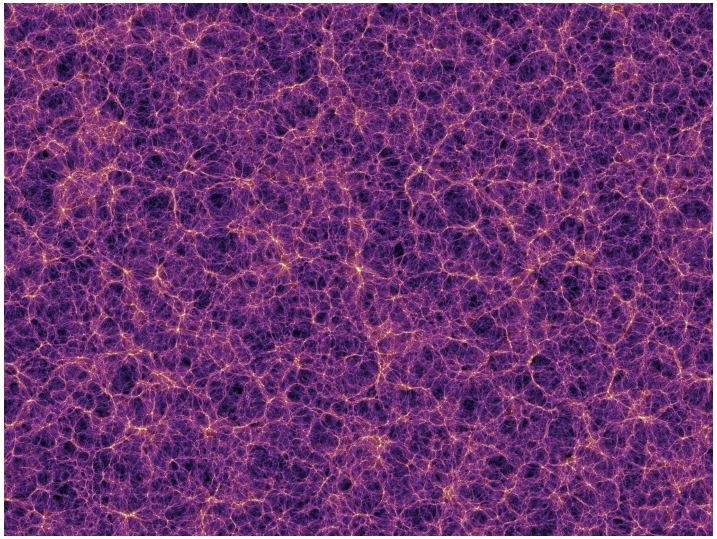

1e8
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
1e8


1e9
1.0
0.8
0.6
0.4
0.2
0.0
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
1e8

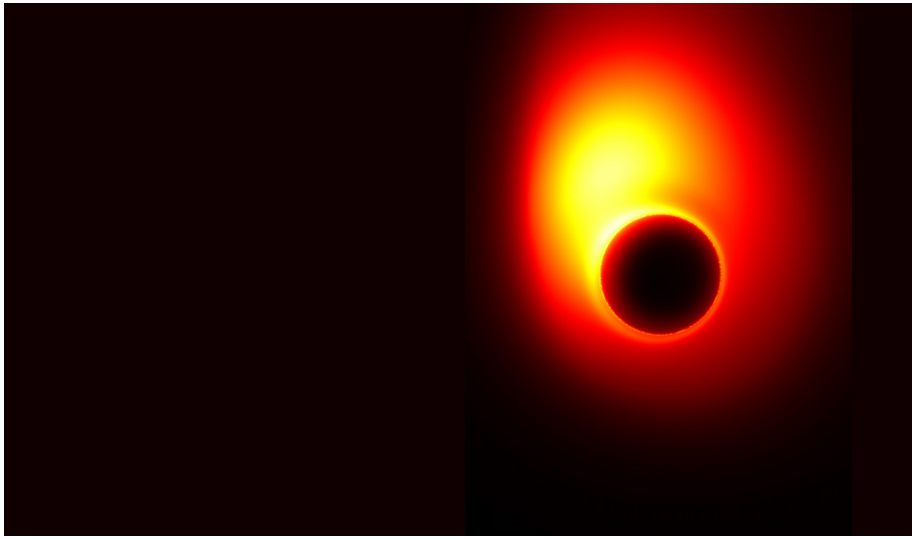

1e8
3
2
1
0
−1
−2
−5
−4
−3
−2
−1
0
1
1e8


1e8
0.0
−0.5
−1.0
−1.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
1e8

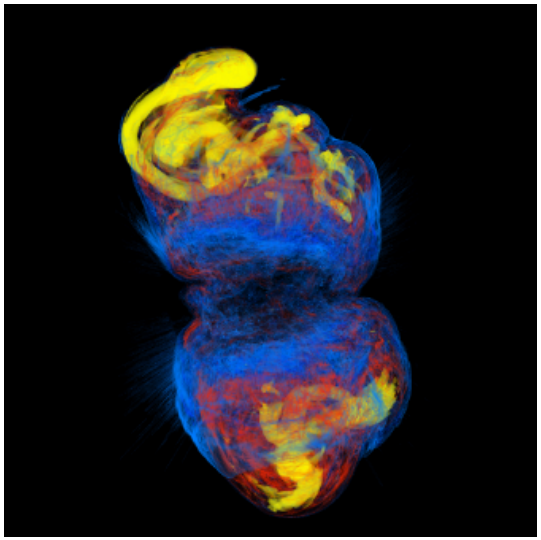
Applications



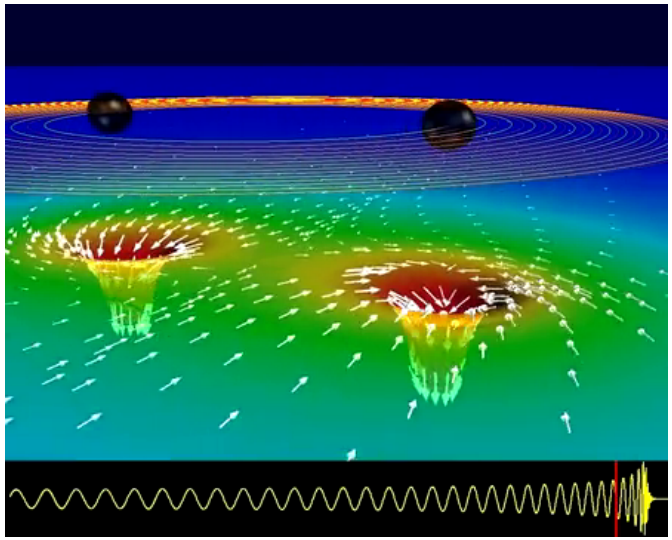
Source: Millennium Simulation



Source: Avery Broderick



Source: Mösta et al.



Source: The SXS Collaboration

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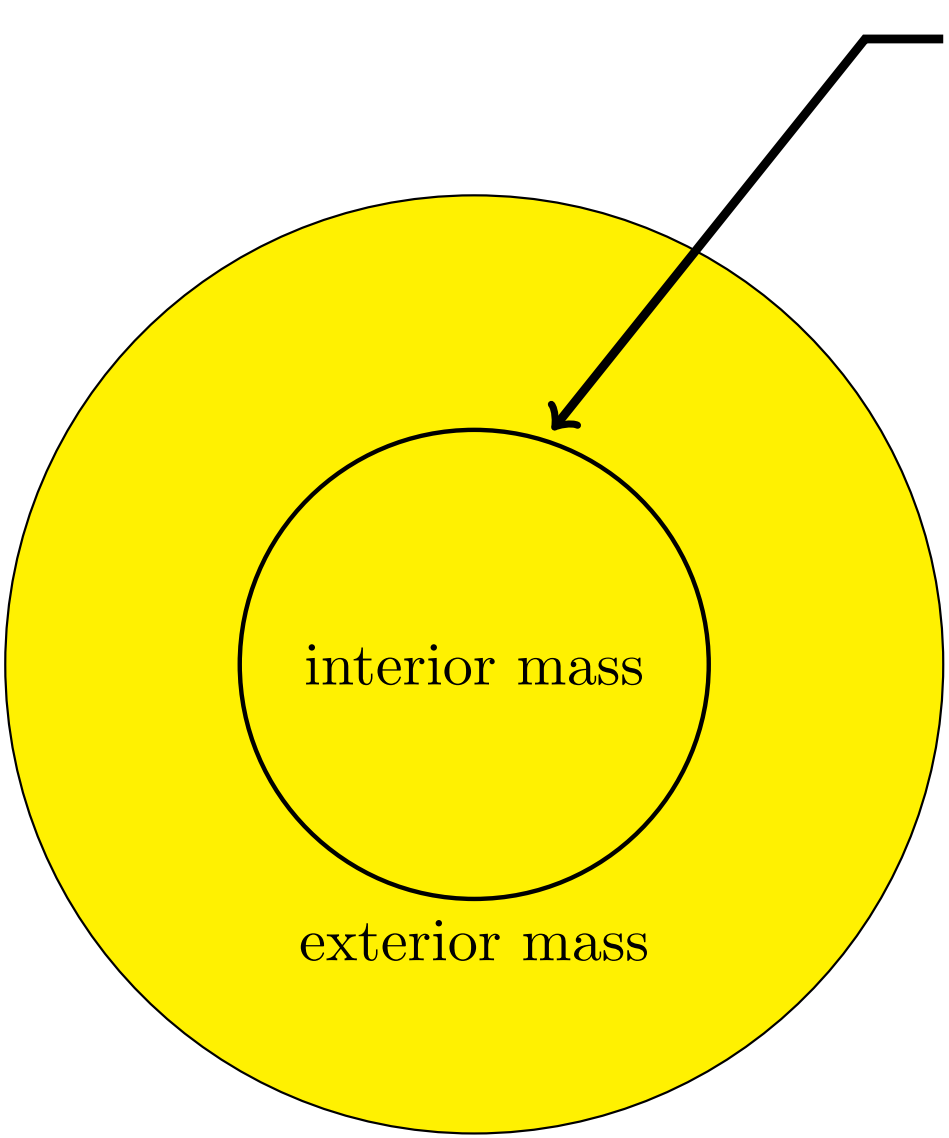
Seeing Inside a Star

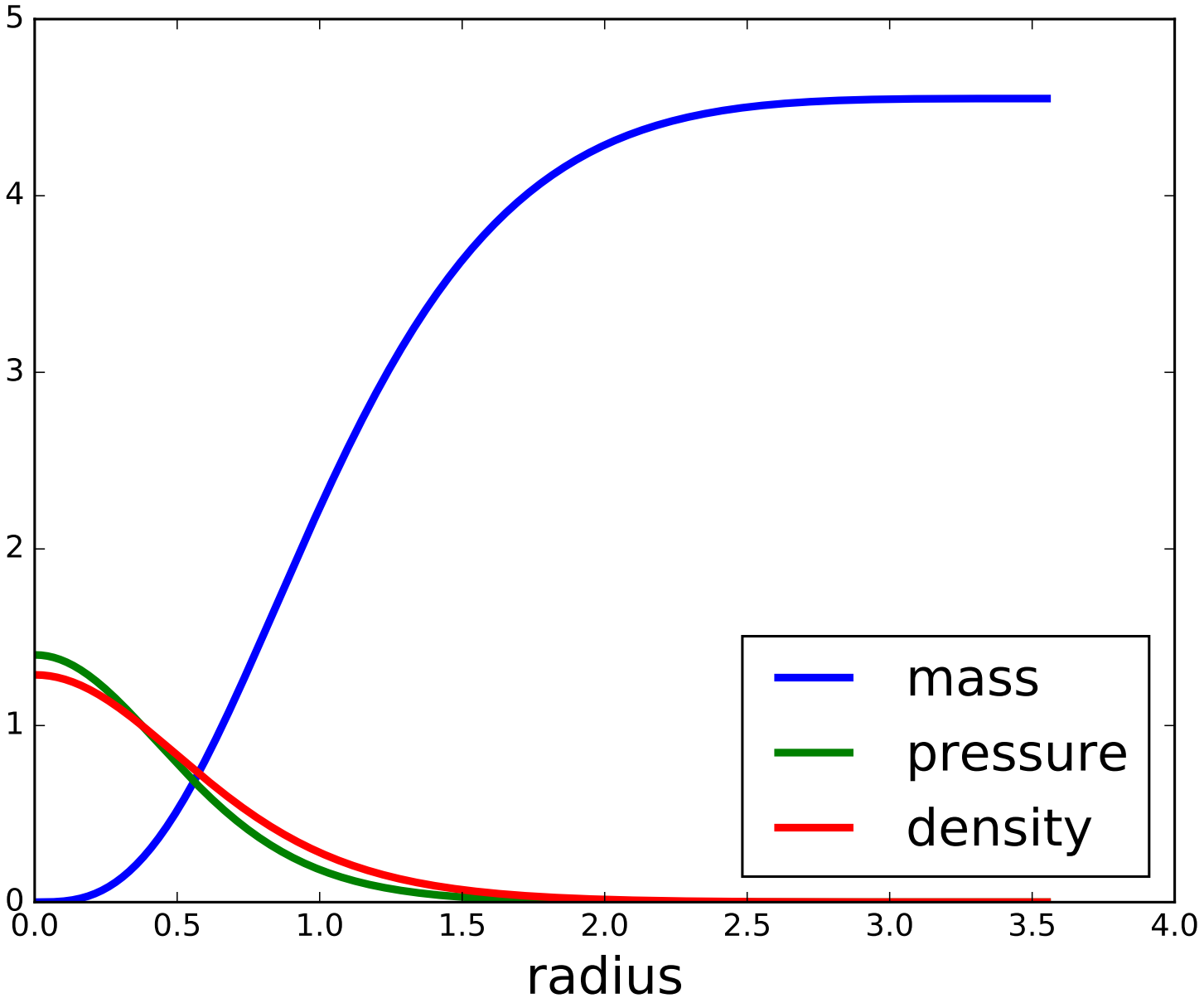


change in force on shell

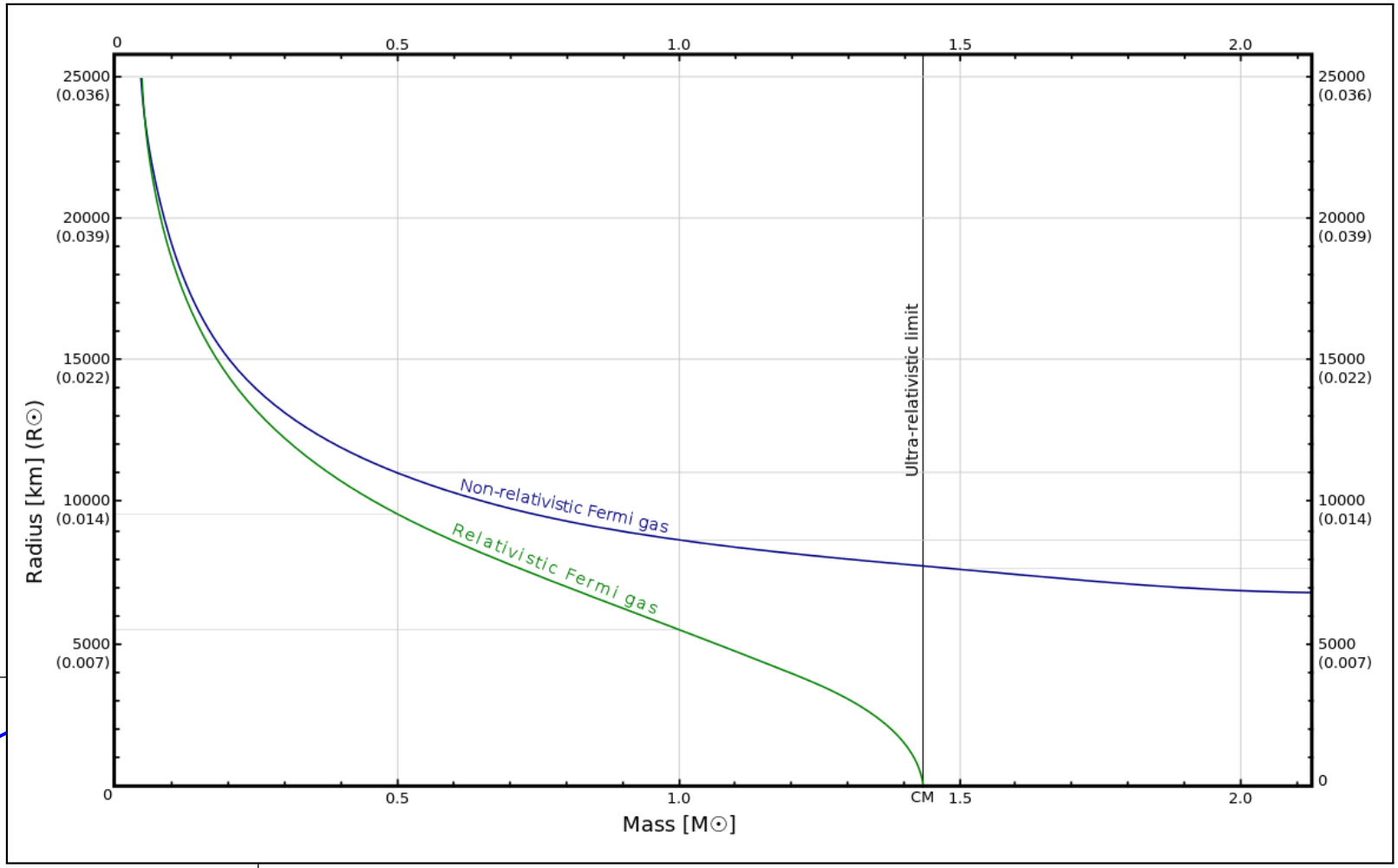
$\approx$

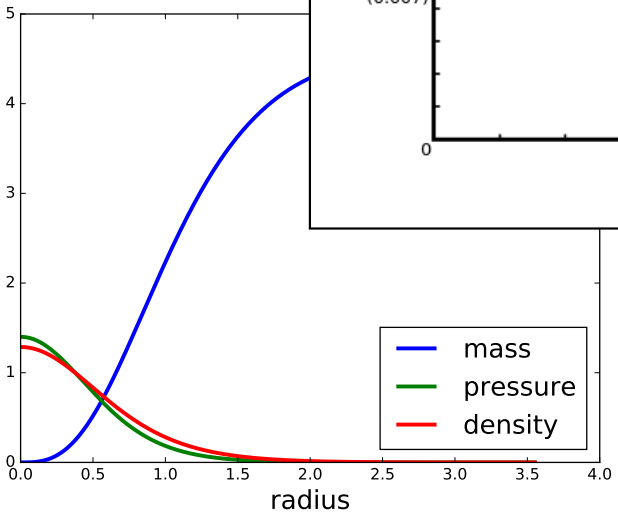
mass inside shell

Mass, Radius



Mass, Radius





Source: Wikimedia Commons

Code

- These slides:
  `https://github.com/Yurlungur/computational-physics-demos`
- Monte Carlo for π:
  `https://bitbucket.org/Yurlungur/pi_monte_carlo`
- Newton's Laws:
  `https://github.com/Yurlungur/forward-euler-demo`
- Stellar Structure:
  `https://github.com/Yurlungur/stellar-structure`