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$$X = \begin{array}{c} \begin{bmatrix} 2 & 0 & 1 \\ 3 & 1 & 2 \\ 4 & 0 & 3 \end{bmatrix} \\ \mu_1 \quad \mu_2 \quad \mu_3 \end{array} \qquad \begin{aligned} \mu_1 &= 3 \\ \mu_2 &= 1/3 \\ \mu_3 &= 2 \end{aligned}$$

$$X_c = \begin{bmatrix} 2-3 & 0-1/3 & 1-2 \\ 3-3 & 1-1/3 & 2-2 \\ 4-3 & 0-1/3 & 3-2 \end{bmatrix} = \begin{bmatrix} -1 & -1/3 & -1 \\ 0 & 2/3 & 0 \\ 1 & -1/3 & 1 \end{bmatrix}$$

Matriz de Covarianza

$$C = \frac{1}{n-1} X_c^T X_c \qquad n=3$$

$$X_c^T = \begin{bmatrix} -1 & 0 & 1 \\ -1/3 & 2/3 & -1/3 \\ -1 & 0 & 1 \end{bmatrix}$$

$$C = \frac{1}{2} \begin{bmatrix} -1 & 0 & 1 \\ -1/3 & 2/3 & -1/3 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & -1/3 & -1 \\ 0 & 2/3 & 0 \\ 1 & -1/3 & 1 \end{bmatrix}$$

$$C = \frac{1}{2} \begin{bmatrix} 2 & 0 & 2 \\ 0 & 2/3 & 0 \\ 2 & 0 & 2 \end{bmatrix} \rightarrow C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1/3 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Autovalores y autovectores

$$\det(C-\lambda I)=0$$

$$\det\begin{bmatrix} 1-\lambda & 0 & 1 \\ 0 & 1/3-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{bmatrix}$$

$$((1-\lambda)^2(1/3-\lambda))-(1/3-\lambda)$$

$$(1/3-\lambda)(\lambda^2-2\lambda)$$

$\lambda_1=\frac{1}{3}$ $\lambda_2=0$ $\lambda_3=2$

$-x+z=0$
$-5/3\,y=0$
$x-z=0$

Autovectores

$\lambda_3=2$

$$(C-2I)v=0\rightarrow\begin{bmatrix} 1-2 & 0 & 1 \\ 0 & 1/3-2 & 0 \\ 1 & 0 & 1-2 \end{bmatrix}=\begin{bmatrix} -1 & 0 & 1 \\ 0 & -5/3 & 0 \\ 1 & 0 & -1 \end{bmatrix}v=0$$

solución $\Rightarrow v_1=(1,0,1)^T$

$$\lambda_1=\frac{1}{3}\quad(C-\frac{1}{3}I)v=0\rightarrow\begin{bmatrix} 1-1/3 & 0 & 1 \\ 0 & 1/3-1/3 & 0 \\ 1 & 0 & 1-1/3 \end{bmatrix}=\begin{bmatrix} 2/3 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 2/3 \end{bmatrix}v$$

$\frac{2}{3}x+z=0\rightarrow z=-\frac{2}{3}x$

$0=0$

$x+\frac{2}{3}z=0$

sustituyo en 3

$v_2=(0,1,0)^T$

$$x+\frac{2}{3}\left(-\frac{2}{3}\right)x=\frac{9x}{9}-\frac{4}{9}x=\frac{5}{9}x=0$$

$\lambda_2 = 0 \rightarrow (C - 0I)\,a = 0 \rightarrow \begin{bmatrix} 1-0 & 0 & 1 \\ 0 & 1/3-0 & 0 \\ 1 & 0 & 1-0 \end{bmatrix} \begin{pmatrix} x \\ y \\ o \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$x + z = 0 \rightarrow z = -x$

$1/3\, y = 0 \rightarrow y = 0 \qquad v_3 = (1, 0, -1)^T$

$x + z = 0 \rightarrow z = -x$

El primer componente Principal corresponde a $\lambda = 2$, vector $(1,0,1)$

2^{do} $\lambda = 1/3 \rightarrow$ explica 14,3%

3^{do} $\lambda = 0 \rightarrow$ no dice nada

1^{o} $\lambda = 2 \rightarrow$ explica [illegible] $= 85,7\%$ de la varianza.