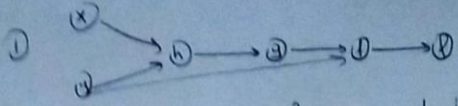


Machine Learning HW3

Question 1 – Neural Networks

I made my calculations on paper, so that I am putting the scanned versions.

Given:



- ① $l(x,y) = (f(x,y) - t)^2$
- ② $f(x,y) = g(x,y) + y \cdot c$
- ③ $g(x,y) = \sigma(h(x,y))$
- ④ $h(x,y) = ax + by$

L is good truth table
 $\sigma(s)$ is sigmoid activation function
 $\sigma(s) = \frac{1}{1 + e^{-s}}$

Wanted:

- ① $\frac{\partial l(x,y)}{\partial a}$
- ② $\frac{\partial l(x,y)}{\partial b}$
- ③ $\frac{\partial l(x,y)}{\partial c}$

① $\frac{\partial l(x,y)}{\partial a} = \frac{\partial l(x,y)}{\partial f(x,y)} \cdot \frac{\partial f(x,y)}{\partial g(x,y)} \cdot \frac{\partial g(x,y)}{\partial h(x,y)} \cdot \frac{\partial h(x,y)}{\partial a}$

$= 2(f(x,y) - t) \cdot 1 \cdot \sigma(h(x,y))(1 - \sigma(h(x,y))) \cdot x$

② $\frac{\partial l(x,y)}{\partial b} = \frac{\partial l(x,y)}{\partial f(x,y)} \cdot \frac{\partial f(x,y)}{\partial g(x,y)} \cdot \frac{\partial g(x,y)}{\partial h(x,y)} \cdot \frac{\partial h(x,y)}{\partial b}$

$= 2(f(x,y) - t) \cdot 1 \cdot \sigma(h(x,y))(1 - \sigma(h(x,y))) \cdot y$

③ $\frac{\partial l(x,y)}{\partial c} = \frac{\partial l(x,y)}{\partial f(x,y)} \cdot \frac{\partial f(x,y)}{\partial c}$

$= 2(f(x,y) - t) \cdot y$

For the b part:

b) Values of the loss and Partial derivatives when

$$\begin{aligned} x &= 2 \\ y &= -1 \\ a &= 1 \\ b &= 2 \\ c &= -0.5 \\ t &= 0 \end{aligned}$$

$$\begin{aligned} h(x,y) &= ax + by = 0 \\ g(x,y) &= \sigma(h(x,y)) = \sigma(0) = \frac{1}{1 + \exp(-0)} = \frac{1}{1+1} = 0.5 \\ f(x,y) &= g(x,y) + yc = 0.5 + 0.5 = 1 \\ l(x,y) &= (f(x,y) - t)^2 = (1 - 0)^2 = 1 \end{aligned} \quad \left. \begin{array}{l} \text{forward} \\ \text{pass} \end{array} \right\}$$

JS calculated the partial derivatives of the earlier part:

$$\begin{aligned} \frac{dl(x,y)}{da} &= 2 \left(\overbrace{f(x,y)}^1 - \overbrace{t}^0 \right) \cdot \overbrace{1}^{0.5} \cdot \overbrace{\sigma(h(x,y))}^{0.5} \left(\overbrace{1 - \sigma(h(x,y))}^{0.5} \right) \cdot \overbrace{x}^2 \\ &= 2(1-0) \cdot 1 \cdot 0.5 \cdot (1-0.5) \cdot 2 \\ &= 1 \end{aligned}$$

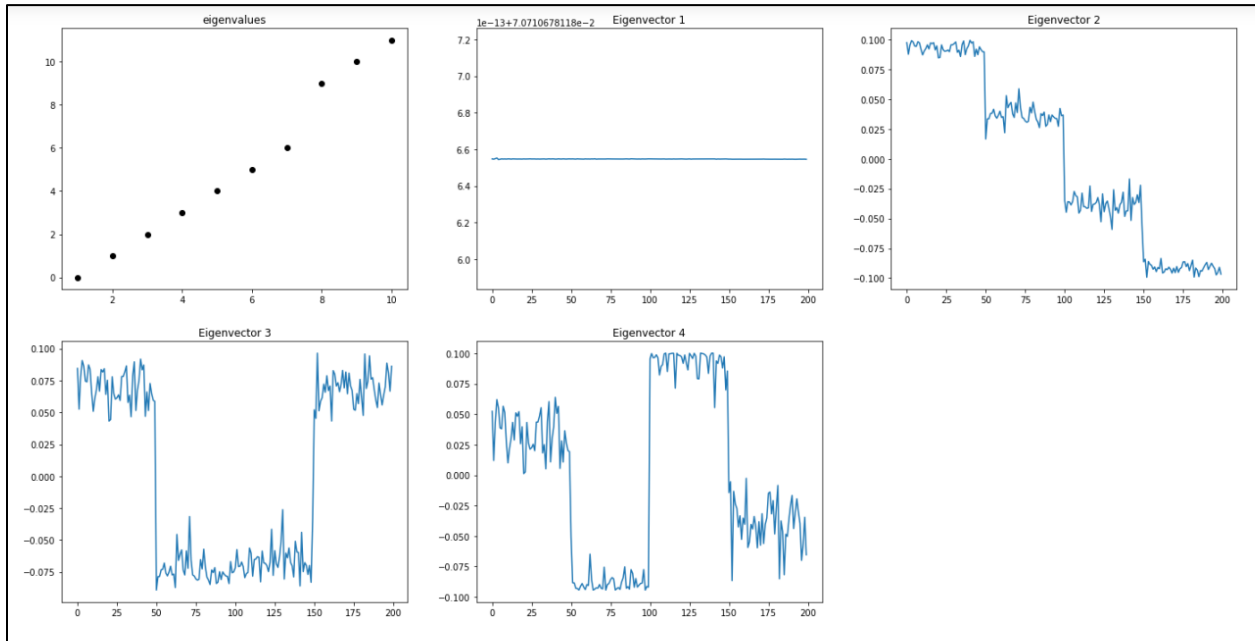
$$\begin{aligned} \frac{dl(x,y)}{db} &= 2 \left(\overbrace{f(x,y)}^1 - \overbrace{t}^0 \right) \cdot \overbrace{1}^{0.5} \cdot \overbrace{\sigma(h(x,y))}^{0.5} \left(\overbrace{1 - \sigma(h(x,y))}^{0.5} \right) \cdot \overbrace{y}^{-1} \\ &= 2(1-0) \cdot 1 \cdot 0.5 \cdot 0.5 \cdot -1 \\ &= -0.5 \end{aligned}$$

$$\begin{aligned} \frac{dl(x,y)}{dc} &= 2 \left(\overbrace{f(x,y)}^1 - \overbrace{t}^0 \right) \cdot \overbrace{y}^{-1} \\ &= -2 \end{aligned}$$

Question 2 - Spectral Clustering

I attach the code I wrote as python notebook with the name **HW3.ipynb**.

For the b part I generated the following graphs:



The code of the graphs is also in the python notebook.