Optimization of Model Rocket Fuel Consumption

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In this project the fuel and launch angle of a model rocket will be optimized to achieve the best performance of in a variety of locations and launch conditions. The desired output is an optimization algorithm which considers several ideal launch goals and launch conditions. After completing simulations of the rocket motion, 0.0093 kg of fuel is required to reach an altitude of 50 m, and 0.0291 kg of fuel is needed to reach an altitude of 250 m, with a probability of failure of 4.74×10^{-5}. Under target altitude of 150 meters and wind speed of 6 m/s, 0.0194 kg of fuel is required at a launch angle of 0.2443 radians. While these simulations accurately describe model rocket motion and optimal fuel consumption at given design constraints and environmental conditions, further simulations with more diverse data points need to be conducted to improve the simulation results.

Nomenclature

A	=	cross sectional area
A_e	=	exit cross section
P	=	peak altitude
C	=	drag coefficient
Yb	=	boost phase height
Yc	=	coasting height
Xb	=	boost phase horizontal travel
Xc	=	coasting horizontal travel
m_t	=	total mass of rocket
m_c	=	mass of rocket without propellant
m	=	mass of propellant
T	=	rocket thrust
I	=	impulse
P_f	=	probability of failure
v	=	maximum velocity
k	=	wind resistance factor
ρ	=	air density
P_0	=	atmospheric pressure
P_e	=	internal engine pressure
g	=	gravitational constant
v_w	=	ambient wind speeds

I. Introduction

THIS study aims to determine the minimum fuel needed and optimal launch angle for a model rocket to achieve a desired peak altitude while adhering to a series of design constraints. In order to compute the minimum necessary fuel required, the peak altitude (P) and the maximum velocity (v) need to be considered. These design goals will take the form of inequality constraints. Environmental conditions such as ambient wind speeds (W) and atmospheric pressure (P_0) will also be considered as such factor impact the maximum thrust of the rocket further impacting the goal parameters. To best correct for environmental effects, launch angle will also be considered.

While a long-term goal of these simulations is to develop the ability to compute the optimal amount of fuel required for a full-scale rocket to reach a set altitude (i.e. outer atmosphere/space), the complexity of such

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a project is beyond the available computing power and scope of this project. To simplify the analysis, the rocket will be scaled down to a model rocket of approximately 1 meter to reduce the model complexity and number of design variables. Using typical design constraints for a model rocket, a theoretical simulation of the rocket motion can be achieved.

The objective of this project is to create a standard optimization statement to minimize the amount of fuel and determine the optimal launch angle, while considering design constraints of a typical model rocket. The optimization statement will be used to develop an algorithm that efficiently computes minimum necessary liquid fuel and ideal launch angle for a model rocket in any given environment and with a desired altitude requirement. The algorithm will also yield a probability of success given any uncertainty in the values. By simulating and optimizing the motion of a model rocket, this design project can illustrate the basic theory and process behind the optimization of the required fuel to reach a desired heigh altitude in a full scale rocket, despite lacking the same complexity in analysis.

II. Theory

This problem will be solved using the Exterior Penalty Function method and KT conditions with the steepest descent search algorithm as the optimization method to minimize the rocket fuel mass and determine the optimal launch angle while satisfying the inequality constraints defined in the problem statement. A common regression of rocket thrust data will also be performed to determine the thrust duration of the rocket motors based on data sheets of varying rocket motors. After simulations are conducted to determine the optimal fuel usage and launch angle given a target peak altitude, Monte Carlo simulations will be conducted to determine the probability of failure P_f to account for uncertainty and to analyze the accuracy and validity of the simulations.

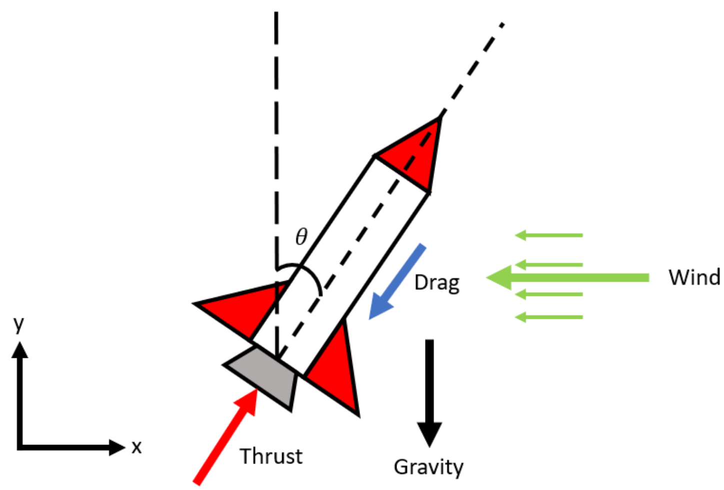

Figure 1. Sum of Forces on Rocket[1]

In order to compute the final height and velocity of the rocket, all of the forces acting on the rocket need to be considered. However, since majority of the forces acting on the rocket are functions of velocity, an inverse analysis must be performed. First, an equation calculating the peak rocket altitude must be determined. Where the boost phase distance (Yb) can be calculated as[3]:

$$Yb = -\frac{m_B}{2k} ln\left(\frac{T - m_B g - kv^2}{T - m_B g}\right) \tag{1}$$

And the coasting phase distance (Yc) can be calculated as:

$$Yc = \frac{m_C}{2k} ln\left(\frac{m_C g + kv^2}{m_c g}\right) \tag{2}$$

Therefore, the peak altitude of the rocket is equivalent to the total vertical distance traveled[3]:

$$P = Yb + Yc = \frac{m_C}{2k}\ln\left(\frac{m_C g + kv^2}{m_C g}\right) - \frac{m_B}{2k}\ln\left(\frac{T - m_B g - kv^2}{T - m_B g}\right) \tag{3}$$

Where the total mass of the rocket (m), the calculated wind resistance factor (k), the gravitational force (g), the thrust generated by the rocket motor (T), the motor impulse (I), and velocity (v) can be used to determine the peak altitude. The velocity (v) and wind resistance factor (k) can be calculated by:

$$v = \frac{\frac{\sqrt{T - m_B g}}{k}\left(1 - e^{-\frac{2kqI}{m_B T}}\right)}{\left(1 + e^{-\frac{2kqI}{m_B T}}\right)} \tag{4}$$

$$k = \frac{1}{2}\rho CA \tag{5}$$

Where the density of air (ρ), and the drag coefficient of the rocket (C) are used in calculating the wind resistance factor. The thrust of the rocket, T can be computed as[5]:

$$T = \dot{m}V_e + (P_e - P_0)A_e \tag{6}$$

Where $\dot{m}$ is the mass flow rate of the fuel leaving the liquid fuel engine, V_e is the exit velocity, P_e is the pressure inside the engine, P_0 is the atmospheric pressure and A_e is the exit cross-section. Finally, the mass of the rocket fuel can be calculated using[8]:

$$m = m_t - m_c \tag{7}$$

Where the mass of the propellant (m_p) can be derived from the total mass of the rocket and the mass of the rocket without propellant (m_e). Therefore, the peak altitude of the rocket can be determined by the rocket design constraints: the total rocket weight (m), the mass of the rocket without propellant (m_e), the peak altitude (P), thrust (T), impulse (I), rocket drag coefficient (C), and cross-sectional area of the rocket (A). Using these design variables, constraints can be defined as acceptable ranges of values for each design variable based on literature, and the minimum required propellant amount to achieve peak altitude (P) can be determined.

However, since this problem is considering forces in 2D dimensions, as depicted in figure 1, the equations must be re-written. The most notable consequence of this is that the thrust, T and drag on the rocket must be broken into their respective components.

$$P = Yb + Yc = -\frac{m_B}{2k} ln\left(\frac{T\cos\theta - m_B g - kv_y^2}{T\cos\theta - mg}\right) + \frac{m_C}{2k} ln\left(\frac{m_C g + kv_y^2}{m_C g}\right) \tag{8}$$

$$\Delta x = Xb + Xc = -\frac{m_B}{2k}\ln\left(\frac{T\sin\theta - k_w v_w^2 - kv_x^2}{T\sin\theta - k_w v_w^2}\right) + \left(v_x t_c - \frac{k_w v_w^2}{2m_C} t_c^2\right) \tag{9}$$

Where

$$t_c = \sqrt{\frac{m_C}{kg}}\tan^{-1}\left(v_y\sqrt{\frac{k}{m_C g}}\right) \tag{10}$$

$$v_{x,y} = q_{x,y}\frac{1 - e^{-p_{x,y}\frac{I}{T}}}{1 + e^{-p_{x,y}\frac{I}{T}}} \tag{11}$$

$$p_{x,y} = \frac{2kq_{x,y}}{m_B} \tag{12}$$

$$q_x^2 = \frac{T\sin\theta - k_w v_w^2}{k} \tag{13}$$

$$q_y^2 = \frac{T\cos\theta - m_B g}{k} \tag{14}$$

$$m_B = m_c + \frac{1}{2}m \tag{15}$$

In order to solve possible solutions to this series of equations, the constrained variables must be assigned ranges of acceptable values (Table I). The cross-sectional area, drag coefficient, and total weight of the rocket are constrained by the average size of model rockets. The rocket thrust, impulse, and mass of rocket propellant are defined by the available range of rocket motors that can fit the rocket dimension constraints (i.e., motor impulse classes A-D). The peak altitude is set at multiple different heights to gauge the rocket dimensions and fuel composition required to achieve a designated height.

Table I
Constrained Variable Ranges

Variable	Unit	Range
Cross sectional area (A)	m^2	0.005 < A < 0.1
Drag coefficient (C)	N/A	0.5 < C < 1.0
Total rocket weight (m_t)	kg	0 < m_t < 10
Mass of rocket propellant (m)	kg	0.001 < m_p < 0.03
Peak altitude (P)	m	P = 50-250
Thrust (T)	N	7.8 < T < 30
Impulse (I)	N*s	0.5 < I < 20
Gravitational constant (g)	m/s^2	g = 9.81
Rocket length (L)	m	L = 1

As a result, the optimization statement for this design problem can be described in standard form as:

$$f(m) = m \tag{16}$$

$$h_1(x): 0 = x_b + x_c \tag{17}$$

$$g_1(x): 0 \geq P - (Y_b + Y_c) \tag{18}$$

$$g_2(x): 0 \geq -x_b \tag{19}$$

$$g_3(x): 0 \geq -a_1\theta \tag{20}$$

$$g_4(x): 0 \geq -a_2 m \tag{21}$$

$$g_5(x): 0 \geq a_3(m - m_{max}) \tag{22}$$

$$g_6(x): 0 \geq a_4(\theta - \theta_{max}) \tag{23}$$

With a_1 to a_4 being weights (that must be strictly greater than 0) to improve the effectiveness of their respective gradients.

$$\phi(\bar{\bar{x}}) = f(\bar{\bar{x}}) + r\left[\sum h_i(\bar{\bar{x}})^2 + \sum g_j^+(\bar{\bar{x}})^2\right] \tag{24}$$

And

$$\nabla\phi(\bar{\bar{x}}) = \nabla f(\bar{\bar{x}}) + r\left[2\sum h_i(\bar{\bar{x}}) * \nabla h_i(\bar{\bar{x}}) + 2\sum g_j^+(\bar{\bar{x}}) * \nabla g_j(\bar{\bar{x}})\right] \tag{25}$$

Where

$$g_j^+(\bar{\bar{x}}) = \max\left(g_j(\bar{\bar{x}}), 0\right) \tag{26}$$

Using the steepest descent method to update the current value to find the optimal conditions,

$$x^{k+1} = x^k + \alpha\bar{d} \tag{27}$$

Where

$$\bar{d} = -\nabla\phi \tag{28}$$

The updating parameter, α was initially selected via the golden section search algorithm, however, it was found that the gradient of ϕ was exceedingly large while the values of x^k were many, many orders of magnitude smaller. The equations defining ϕ often exploded in one way or the other in unsalvageable ways when supplied with unexpectedly large input values. So it was decided to use a manual updating rule to allow the engineer to offset the enormous gradients, which is also defined separately for either change in mass and change in angle. Once the mass and angle reach a sufficient convergence criteria define as

$$\sum |d_i| < \varepsilon \tag{29}$$

The optimized mass, m_{ideal} and optimized angle, θ_{ideal} are further updated through an uncertainty analysis, these two parameters are considered ideal. The uncertainty analysis is performed via a Monte Carlo simulation in which three key parameters are considered. Potential variations in the mass of the fuel, the launch angle and the thrust are considered using separate gaussian distributions, which are then fed into equation 8 to determine the variation in the peak height of the rocket. The standard deviation of the peak height is then obtained from the simulations. The whole optimization process is then repeated, this time the peak height goal is increased using,

$$P^{updated} = P^{original} + 3\sigma_P \tag{30}$$

With the new optimized mass and angle, a second Monte Carlo simulation is performed to compute the probability of failure, which in this case, failure refers to the case in where the rocket does not reach the original specified goal height. The probability of failure is computed as,

$$P_f = \frac{\Sigma\left(\left(Y_{b,updated} + Y_{c,updated}\right) < P^{Original}\right)}{N} \tag{31}$$

Where N is the number of simulations, in most cases N ~ 10^6

III. Example Problem/ Algorithm Iteration

The first step of the developed algorithm is to construct the modified exterior penalty function which constrains the optimization of the mass of the fuel as defined in equation 8. For this example problem, the values in table II will be applied.

Table II
Variable Values Used in Optimization Algorithm

Variable	Unit	Value
Diameter of Rocket	m	$d = 0.1$
Drag coefficient on rocket nose	N/A	$C_{nose} = 0.5$
Drag coefficient on rocket side	N/A	$C_{side} = 1.0$
Total rocket weight	kg	$m_R = 0.4$
Mass of rocket propellant	kg	$m_p < 0.035$
Deviation in rocket propellent	kg	$m_p \pm 0.001$
Launch Angle	rad	$\theta < \pi / 4$
Deviation in Launch Angle	rad	$\theta \pm 0.01$
Peak altitude	m	$P = 100$
Thrust	N	$T = 13.3 \pm 0.2$
Gravitational constant	m/s^2	$g = 9.81$
Air Density	kg/m^3	$\rho = 1.223$
Rocket length	m	$L = 1$
Wind Speed	m/s	$v_w = 5$
Penalty Weight	N/A	$r_0 = 0.001$
g_i Weights	N/A	$a_{1, 2, 3, 4} = 1e3$

Values primarily obtained from [5]

Additionally, a guess of the desired parameters in required to begin,

$$\bar{x}^0 = \begin{bmatrix} m \\ \theta \end{bmatrix} = \begin{bmatrix} 0.01\ kg \\ 0.2\ rad \end{bmatrix}$$

However, the calculation cannot be performed as is because the impulse of the rocket engine is unknown, though is fortunately a function of the mass of the fuel, m. The issue is that the precise function is also unknown, but using the common regression method, it is possible to turn a set of known data points into a linear function. For convenience, the impulse of the rocket will instead be calculated as the burn time, τ, where,

$$\tau = \frac{I}{T} \tag{32}$$

And computed from the datapoints[4],

Table III
Fuel and Mass Data

	Fuel Mass, m_i (kg)	Burn Time, τ_i (sec)
1	0.00156	0.2
2	0.00312	0.32
3	0.00833	1.2
4	0.00833	1.2
5	0.00624	0.83
6	0.00624	0.83
7	0.00624	0.6
8	0.0127	2.1
9	0.01248	1.7
10	0.01248	1.7

Constructing the system of equations,

$$\bar{\tau} = [M]\,\bar{w} \tag{32}$$

Where:

$$[M] = \begin{bmatrix} 1 & m_1 \\ 1 & m_2 \\ & \vdots \\ 1 & m_n \end{bmatrix} \tag{33}$$

$$\bar{w} = \begin{bmatrix} w_0 \\ w_1 \end{bmatrix} \tag{34}$$

Then applying the Moore-Penrose Pseudo Inverse to solve for $\bar{w}$,

$$\bar{w} = (M^T M)^{-1} M \cdot \tau \tag{35}$$

Yields the equation,

$$\tau = -0.1562 + 157.5116\, m \tag{36}$$

Now it is possible to perform the first iteration of optimization. To begin, the search direction from equation 28 is computed,

$$d = -\nabla\phi\left(\begin{bmatrix} 0.01 \\ 0.2 \end{bmatrix}\right) \tag{37}$$

Table IV
Search Direction Computation

				$\frac{\partial}{\partial m}$	$\frac{\partial}{\partial \theta}$
f		0.01		1	0
h_1		0.0055		595.8139	150.3755
g_1	g_1^+	43.4525	43.4525	-9811.08	21.37247
g_2	g_2^+	-2.7529	0	-604.514	-32.0503
g_3	g_3^+	-0.2	0	0	-1e3
g_4	g_4^+	-0.01	0	-1e3	0
g_5	g_5^+	-0.02	0	1e3	0
g_6	g_6^+	-0.5854	0	0	1e3

Thus,

$$d = -\nabla\phi\left(\begin{bmatrix} 0.01 \\ 0.2 \end{bmatrix}\right) = \begin{bmatrix} 851.62 \\ -1.859 \end{bmatrix} \tag{38}$$

Then plugging into equation 27, yields

$$x^1 = \begin{bmatrix} 0.01 \\ 0.2 \end{bmatrix} + \begin{bmatrix} 1 \times 10^{-6} & 0 \\ 0 & 5 \times 10^{-4} \end{bmatrix} \begin{bmatrix} 851.62 \\ -1.859 \end{bmatrix} = \begin{bmatrix} 0.0109 \\ 0.1991 \end{bmatrix} \tag{39}$$

IV. Results

Table V
Optimized Mass and P_f for Specified Peak Altitude (for v_w = 5 m/s)

Peak altitude (m)	Ideal θ (rad)	Ideal m (kg)	Updated θ (rad)	Updated m (kg)	Probability of Failure
50	0.2026	0.0093	0.1895	0.0142	0
75	0.1936	0.0118	0.1854	0.0156	9.6e-5
100	0.1847	0.0142	0.1779	0.0182	6.1e-5
125	0.1766	0.0166	0.1715	0.0207	4.0e-5
150	0.1697	0.0191	0.1660	0.0233	3.3e-5
200	0.1586	0.0240	0.1571	0.0286	1.2e-5
250	0.1503	0.0291	0.1503	0.0339	9.0e-6

Table VI
Ideal Launch Angle and Fuel Mass Based on Wind Speed (for P = 150 m)

Wind Speed (m/s)	Ideal θ (rad)	Ideal m (kg)	Updated θ (rad)	Updated m (kg)	Probability of Failure
2.0	0.0271	0.0188	0.0264	0.0230	2.7e-5
2.5	0.0423	0.0188	0.0413	0.0230	2.3e-5
3.0	0.0610	0.0188	0.0596	0.0231	3.7e-5
3.5	0.0831	0.0189	0.0811	0.0231	3.2e-5
4.0	0.1086	0.0189	0.1061	0.0231	2.8e-5
5.0	0.1697	0.0191	0.1660	0.0233	3.3e-5
6.0	0.2443	0.0194	0.2393	0.0237	1.0e-5

Table VII
Rocket Design Variable Values Used in Algorithm

Variable	Unit	Value
Diameter of Rocket	m	d = 0.1
Drag coefficient on rocket nose	N/A	C_{nose} = 0.5
Drag coefficient on rocket side	N/A	C_{side} = 1.0
Total rocket weight	kg	m_R = 0.4
Thrust	N	T = 13.3 ± 0.2
Rocket length	m	L = 1
Deviation in rocket propellent	kg	m_p ± 0.001
Deviation in Launch Angle	rad	θ ± 0.01
Sample Points	N/A	N = 10^6
Wind Speed (only constant for Table 5)	m/s	v_w = 5
Peak Height (only constant for Table 6)	m	P = 150
Penalty Weight	N/A	r_0 = 0.001
Weights (g_i)	N/A	$a_{1, 2, 3, 4}$ = 1e3

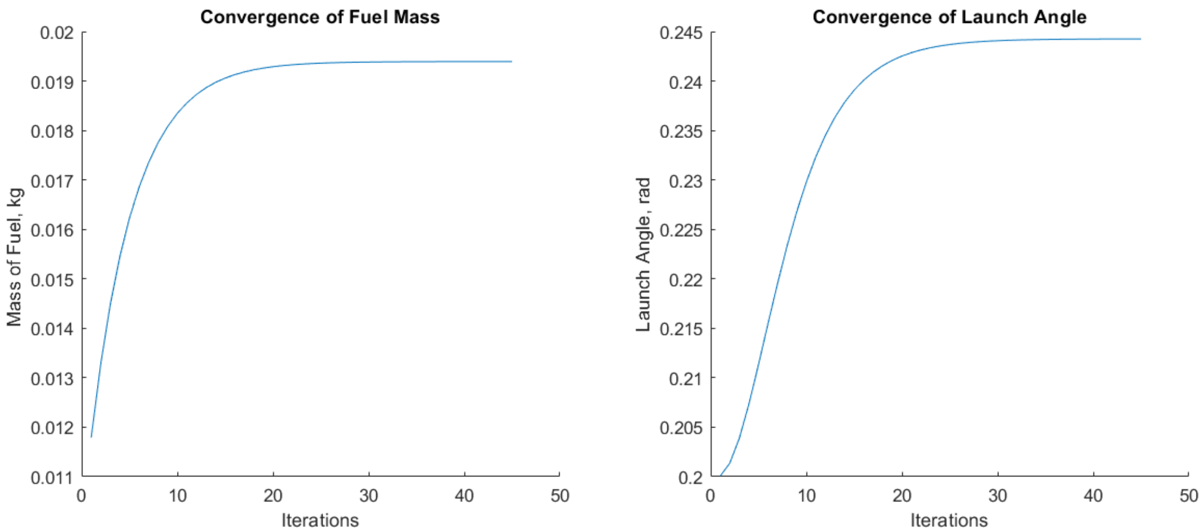

Figure 2: Convergence of fuel and launch angle for a rocket assuming a peak launch height of 150 meters and experiencing 6 m/s winds

V. Discussion

This design study aimed to optimize the fuel usage and launch angle of a model rocket constrained by various design variables (Table VII) with set wind conditions and a target peak altitude. Given these conditions, a standard optimization statement and resulting algorithm were developed to portray the model rocket system. For peak altitudes ranging between 50 m and 250 m, the quantity of fuel required to reach the desired heights ranged between 0.0142 and 0.0339 kg of fuel, and the optimal launch angle between 0.1503 and 0.1895 radians (Table V). As expected, increasing the peak altitude requirement for the model rocket proportionally increased the fuel required. Using a fixed altitude of 150 m and testing wind speeds from 2 m/s to 6 m/s, the required fuel ranged between 0.0188 and 0.0194 kg, with optimal launch angle between 0.0271 and 0.2443 (Table VI). While the varying wind conditions caused fluctuations in launch angle, the fuel consumption necessary to reach 150 m was not affected.

The optimization algorithm yielded results that were expected, with the mass of fuel increasing proportionally to the increase in target altitude. As the target peak altitude increased from 50 m to 250 m, the fuel required increased by 68.1% overall, with a 15% to 25% increase in fuel requirement for an altitude increase between 25 m to 50 m. Using the Exterior Penalty Function method in minimizing the mass proved effective, with a mean probability of failure of 4.74×10^{-5}, with convergence criterion of $\epsilon = 0.001$. When testing a constant target altitude of 150 m and incrementing the wind speed, the fuel usage at lower wind speeds remains at 0.0188 kg. As the wind speed increases past 5 m/s, the fuel usage begins to increase by 3.1%. While this increase is insignificant in cheaper model rockets, a 3.1 % increase in fuel of a full scale rocket that uses approximately 400 tons of fuel to exit the atmosphere is approximately 12.4 tons. The fuel increase based on wind conditions emphasizes the importance of using optimization methods to determine a suitable range of allowable windspeeds on launch day.

To further analyze the efficiency of using the Exterior Penalty Function method in solving this optimization problem, the number of iterations required for convergence was graphed for wind speeds of 6 m/s and altitude of 150 m (Figure 2). Convergence of the algorithm occurred in 47 iterations. While the algorithm is not so simple that it converges in less than 5 iterations, it is not computationally intensive to conduct, whereas in a more realistic rocket model, a supercomputer will most likely be required to compute all of the design parameters. In conducting the Monte Carlo simulations to determine the probability of failure, 1,000,000 iterations were performed per data point. While Monte Carlo simulations can be very computationally intensive to perform, the simplicity of the design equations kept the complexity of the simulations less intensive and reasonable given the available hardware (See Appendix).

While the design study of this model rocket illustrates the process of using optimization methods to determine fuel requirements and launch angle given a set of environmental conditions, the study is limited by the complexity of the system. Although the system was simulated with varying wind speeds, fuel burn rates, and slight deviations in the rocket design parameters (Table VII), the rocket motion is modeled by theory, and further simulations including greater uncertainties must be conducted. This simulation only considers model rockets with a single 1-phase booster motor. To further improve this analysis, expanding the model to consider a rocket with multiple booster phases can better simulate motion of a much larger rocket. These simulations also consider the absolute minimum fuel mass required to reach the desired peak altitude. Further uncertainty analysis of the fuel consumption, such as variation in burn rate, motor efficiency, and environmental factors need to be conducted to determine a more reliable result.

VI. Conclusion

This optimization study aimed to model the motion of a model rocket to determine the optimal fuel amount and launch angle, given a set of design constraints. Using target peak altitudes and varying wind conditions, a standard optimization statement and algorithm were developed to simulate the system. The developed algorithm accurately predicted the minimum fuel requirement based on the given conditions, where 0.0093 kg of fuel is required to reach an altitude of 50 m, and 0.0291 kg of fuel needed to reach an altitude of 250 m (given wind speed does not exceed 5 m/s). Conducting Monte Carlo simulations yields an average probability of failure of 4.74×10^{-5}. While these simulations model a model rocket, further simulations using other data sets should be conducted to diversify the data and gain more accurate results. While these simulations are conducted for a model rocket motion, they illustrate a simplified version of the process necessary in determining the optimal fuel and launch conditions for a full scale rocket aimed to reach outer space.

VII. References

[1]Culp, R. (n.d.). *Rocket Equations*. Rocket equations. Retrieved November 8, 2021, from http://www.rocketmime.com/rockets/rckt_eqn.html.

[2]Culp, R. (n.d.). *Rocket Equations Quick Reference*. Rocket equations quick reference. Retrieved November 8, 2021, from http://www.rocketmime.com/rockets/qref.html.

[3]*DESIGN EQUATIONS*. Design equations. (n.d.). Retrieved November 8, 2021, from https://risacher.org/rocket/eqns.html.

[4]Estes. (n.d.). *Estes Model Rocket Specs*. East Coast Model Center Engine Specifications. Retrieved November 8, 2021, from http://www.rocketmime.com/rockets/estes_spec.html.

[5]Estes. (n.d.). *Saturn v Skylab*. Estes Rockets. Retrieved November 8, 2021, from https://estesrockets.com/product/001973-saturn-v-skylab/.

[6]NASA. (n.d.). *Ideal rocket equation*. NASA. Retrieved November 8, 2021, from https://www.grc.nasa.gov/www/k-12/rocket/rktpow.html.

[7]NASA. (n.d.). *Liquid rocket engine*. NASA. Retrieved November 8, 2021, from https://www.grc.nasa.gov/www/k-12/airplane/lrockth.html.

[8]NASA. (n.d.). *Mass Ratios*. NASA. Retrieved November 8, 2021, from https://www.grc.nasa.gov/www/k-12/rocket/rktpow.html.

VIII. Appendix

```
Main Script Tutorial
{
  Define variables {

    Define Physical Constants, gravity, air density, etc.
    Define Rocket parameters, size, weight, etc.
    Define Convergence Criteria, α, ε, weights, etc.

  }

  Compute launch thrust duration, τ {

    Equations 32 to 36

  }

  Define equations using Matlab’s Symbolic Toolbox {

    Equations 8 to 15

  }

  Define Optimization Statement {

    Equations 16 to 26

  }

  Perform Optimization {

    Same Process defined in section III, equations 27, 28, and 37 to 39

  }

  Uncertainty Analysis {

    First Monte Carlo Simulation
    Update Goal Height, equation 30

  }

  Update Optimization {

    Redefine equations 16 to 26 for updated parameters

  }

  Probability of Failure Analysis {

    Second Monte Carlo Simulation
    Compute probability of failure, equation 31

  }
}
```