The plan is to show all finitely generated reflection groups are in fact Coxeter groups, which admit a nice geometric classification. This follows[**?**]

0.1 Coxeter groups

Definition 0.1.1. We call a group W **Coxeter** if it admits a presentation of the form:

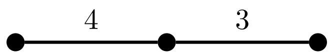
$$\langle r_1, \ldots, r_n \mid (r_i r_j)^{m_{ij}} \text{ for all } i, j \rangle$$

where each $m_{ij} \in \mathbb{N} \cup \{\infty\}$, and $m_{ii} = 2$ for all i. For formal reasons, we will consider the pair (W, R), where R is the set of generators in the presentation, and call this a **Coxeter system**. We call a Coxeter system finite if R is finite.

To any finite Coxeter system (W, R) we can associate an undirected graph called its **Coxeter diagram** by the following rules:

- Draw a node i for each $r_i \in R$;
- For each relation $(r_i r_j)^{m_{ij}}$ with $m_{ij} > 2$ draw an edge between i and j and label it with m_{ij}.

This process can be reversed to obtain a Coxeter system from any Coxeter diagram. This correspondence will associate the graph:



to the group presentation:

$$\langle r_1, r_2, r_3 \mid r_1^2 = r_2^2 = r_3^2 = e,\ (r_1 r_2)^4 = (r_2 r_3)^3 = (r_1 r_3)^2 = e \rangle$$

which happens to correspond to the symmetry group of the octahedron.

0.2 The fundamental domain

0.3 Words

0.4 Classification