IMPERIAL

IMPERIAL COLLEGE LONDON

DEPARTMENT OF MATHEMATICS

SECOND-YEAR GROUP RESEARCH PROJECT

Title

Author:
Student name 1 (CID: ———————-)
Student name 2 (CID: ———————-)
Student name 3 (CID: ———————-)
Student name 4 (CID: ———————-)
Student name 5 (CID: ———————-)

Supervisor(s):
Alessio Corti

June 3, 2025

Abstract

Type your abstract here. The abstract is a summary of the contents of the project. It should be brief but informative, and should avoid technicalities as far as possible.

Contents

1 Introduction

The introduction should attempt to set your work in the context of other work done in the field. It should demonstrate that you are aware of what you are doing, and how it relates to other work (with references). It should also provide an overview of the contents of the project. You should highlight your individual contributions and any novel result: which of the calculations, theorems, examples, proofs, conjectures, codes etc. are your own? This is how you cite a reference in the bibliography[1]. All of the commands and formatting are in ./style/header.sty

2 Polytopes

polytopes

3 Classification of finite reflection groups

The plan is to show all finitely generated reflection groups are in fact Coxeter groups, which admit a nice geometric classification. This follows[1]

3.1 Coxeter groups

Definition 3.1.1. We call a group W **Coxeter** if it admits a presentation of the form:

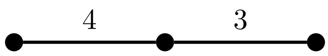
$$\langle r_1, \ldots, r_n \mid (r_i r_j)^{m_{ij}} \text{ for all } i, j \rangle$$

where each $m_{ij} \in \mathbb{N} \cup \{\infty\}$, and $m_{ii} = 2$ for all i. For formal reasons, we will consider the pair (W, R), where R is the set of generators in the presentation, and call this a **Coxeter system**. We call a Coxeter system finite if R is finite.

To any finite Coxeter system (W, R) we can associate an undirected graph called its **Coxeter diagram** by the following rules:

- Draw a node i for each $r_i \in R$;
- For each relation $(r_i r_j)^{m_{ij}}$ with $m_{ij} > 2$ draw an edge between i and j and label it with m_{ij}.

This process can be reversed to obtain a Coxeter system from any Coxeter diagram. This correspondence will associate the graph:



to the group presentation:

$$\langle r_1, r_2, r_3 \mid r_1^2 = r_2^2 = r_3^2 = e,\ (r_1 r_2)^4 = (r_2 r_3)^3 = (r_1 r_3)^2 = e \rangle$$

which happens to correspond to the symmetry group of the octahedron.

3.2 The fundamental domain

3.3 Words

3.4 Classification

4 Uniform polytopes

uniform polytopes

5 Tits' word problem

words

6 Decidability

decidability

References

[1] J. E. Humphreys. *Reflection groups and Coxeter groups*, volume 29 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1990.