

# Chapter 1

# Real Analysis and Topology

Lectured by Someone  
Typed by Yu Coughlin  
Season Year

## Introduction

The following are complementary reading for the course.

- G. Grimmett and D. J. A. Welsh, Probability: An Introduction, 1986
- J. K. Blitzstein and J. Hwang, Introduction to Probability, 2019
- D. F. Anderson et al, Introduction to Probability, 2018
- S. M. Ross, Introduction to Probability Models, 2014
- G. Grimmett and D. Stirzaker, Probability and Random Processes, 2001
- G. Grimmett and D. Stirzaker, One Thousand Exercises in Probability, 2009

# Contents

	<b>1</b>	<b>Real Analysis and Topology</b>	<b>1</b>
L1	1	Euclidean spaces . . . . .	3
	1.1	Euclidean norm . . . . .	3
	1.2	Convergence in $\mathbb{R}^n$ . . . . .	5
	2	Continuity and limits of functions . . . . .	5
	2.1	Open sets . . . . .	5
	2.2	Continuity . . . . .	5
	3	Derivative of maps of Euclidean spaces . . . . .	5
	3.1	Total derivatives . . . . .	5
	3.2	Directional and partial derivatives . . . . .	5
	3.3	Higher order derivatives . . . . .	5
	4	Inverse and implicit function theorems . . . . .	5
	4.1	Inverse function theorem . . . . .	5
	4.2	Implicit function theorem . . . . .	5
	5	Metric spaces . . . . .	5
	5.1	Introduction . . . . .	5
	5.2	Normed vector spaces . . . . .	5
	5.3	Sets in metric spaces . . . . .	5
	5.4	Continuous maps of metric spaces . . . . .	5
	6	Topological spaces . . . . .	5
	6.1	Topologies and their spaces . . . . .	5
	6.2	Convergence and Hausdorff property . . . . .	5
	6.3	Closed sets . . . . .	5
	6.4	Continuous maps . . . . .	5
	7	Connectedness . . . . .	5
	7.1	Definition . . . . .	5
	7.2	Continuous maps . . . . .	5
	7.3	Path connected sets . . . . .	5
	8	Compactness . . . . .	5
	8.1	Covers . . . . .	5
	8.2	Sequential compactness . . . . .	5
	8.3	Continuous maps . . . . .	5
	8.4	Arzelá-Ascoli theorem . . . . .	5
	9	Completeness . . . . .	5
	9.1	Banach spaces . . . . .	5
	9.2	Fixed point theorem . . . . .	5

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# **1 Euclidean spaces**

## **1.1 Euclidean norm**

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## 1.2 Convergence in $\mathbb{R}^n$

# 2 Continuity and limits of functions

## 2.1 Open sets

## 2.2 Continuity

# 3 Derivative of maps of Euclidean spaces

## 3.1 Total derivatives

## 3.2 Directional and partial derivatives

## 3.3 Higher order derivatives

# 4 Inverse and implicit function theorems

## 4.1 Inverse function theorem

## 4.2 Implicit function theorem

# 5 Metric spaces

## 5.1 Introduction

## 5.2 Normed vector spaces

## 5.3 Sets in metric spaces

## 5.4 Continuous maps of metric spaces

# 6 Topological spaces

## 6.1 Topologies and their spaces

## 6.2 Convergence and Hausdorff property

## 6.3 Closed sets

## 6.4 Continuous maps

# 7 Connectedness

## 7.1 Definition

## 7.2 Continuous maps

## 7.3 Path connected sets

# 8 Compactness

## 8.1 Covers

## 8.2 Sequential compactness

## 8.3 Continuous maps

## 8.4 Arzelá-Ascoli theorem

# 9 Completeness

## 9.1 Banach spaces

## 9.2 Fixed point theorem