Physics CST (2022-23) Homework 2

Please send the completed file to my mailbox yy.lam@qq.com by the 5th October, with using the filename format:

student_number-name-cst-hw2

Please answer the questions by filling on these sheets. Or alternatively, do the homework as usual by using papers, then take the pictures and paste them onto these question sheets.

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1. Friction is needed to keep a car from sliding toward the inside of a banked curved if the car moves too slow. (a) Express the minimum coefficient of friction μ needed for a driver to take a curve at speed v banked at θ. (b) Find μ for taking a 100 m radius curve at 28 km/h banked at 13°. (c) Calculate the **ideal** speed to take the banked curve in (b).

   Solution. *(a) Set up the equations for the horizontal and the vertical components*

   $$F_c = N\sin\theta - f\cos\theta = N(\sin\theta - \mu\cos\theta),$$
   $$mg = N\cos\theta + f\sin\theta = N(\cos\theta + \mu\sin\theta)$$

   where F_c, N, f and μ are the centripetal force, normal reaction against the surface, frictional force and the coefficient of kinetic friction. Eliminating N gives

   $$\mu = \frac{mg\sin\theta - F_c\cos\theta}{F_c\sin\theta + mg\cos\theta} = \frac{mg\tan\theta - F_c}{F_c\tan\theta + mg}.$$

   Since $F_c = \frac{mv^2}{r}$, putting it in we get

   $$\mu = \frac{rg\tan\theta - v^2}{v^2\tan\theta + rg}.$$

   (b) Since the velocity is $28 \times 10^3/60^2 = 7.77$ m/s, inserting into the equation we obtain the coefficient of the kinetic friction for avoiding skidding

   $$\mu = \frac{100 \times 9.8\tan 13 - 7.77^2}{7.77^2\tan 13 + 100 \times 9.8} = 0.167$$

   (c) The ideal speed is supposed no friction at all but the car manages to round corner without skidding. Setting $\mu = 0$ above equation gives

   $$v^2 = rg\tan\theta.$$

   Inserting $r = 100$ and $\theta = 13°$ we get $v = 15.0$ ms^{-1}.

2. A 1100 kg car pulls a boat on a trailer. (a) What total force resists the motion of the car, boat, and trailer, if the car exerts a 1900 N force on the road and produces an acceleration of $0.55\,\mathrm{ms}^{-2}$? The mass of the boat plus trailer is 700 kg. (b) What is the force in the hitch between the car and the trailer if 80% of the resisting forces are experienced by the boat and trailer?

Solution. *(a) Let f be the resistance force against the motion, the equation of motion gives*

$$1900 - f = (1100 + 700)(0.55) \quad \Rightarrow \quad f = 910 \ N.$$

(b) Let T be the force in the hitch. It is obtained by the equation of motion of the boat and the trailer subsystem

$$T - 0.8 \times 910 = 700 \times 0.55 \quad \Rightarrow \quad T = 1113 \ N.$$

3. A 50 kg crate rests on the bed of a truck. The coefficients of friction between the surfaces are $\mu_k = 0.3$ and $\mu_s = 0.4$. Find the frictional force on the crate when the truck is accelerating forward relative to the ground at (a) 2.0 ms^{-2}, and (b) 5.0 ms^{-2}.

   Solution. *Let F and f be the force due to the acceleration and the frictional force between the crate and the bed of the truck. Note that F is opposite to the direction of the movement of the truck and f is the same direction to the truck's motion. In equilibrium,*

   $$F = f \quad \Rightarrow \quad a = \mu g$$

   where $\mu \leq \mu_s$. (a) When $a = 2 \ ms^{-1}$ it is smaller than the maximum static frictional force per unit mass $0.4 \times 9.8 = 3.92 \ N/kg$. The frictional force is just $F = ma = 50 \times 2 = 100 \ N$. (b) When $a = 5 \ ms^{-1}$, $F = 50 \times 5 = 250 \ N$ which is bigger than the maximum static friction

   $$f_s = 0.4 \times 50 \times 9.8 = 196 \ N$$

   in this case the static friction no longer holds. The frictional force becomes kinetic

   $$f_k = 0.3 \times 50 \times 9.8 = 147 \ N$$

   As $F > f_s > f_k$, we expect the crate will slide along the backward direction of the truck.

4. A small diamond of mass 10 g drops from a swimmer's earring and falls through the water, reaching a terminal velocity of 2 ms^{-1}. (a) Assuming the frictional force on the diamond obeys $f = -bv$, what is b? (b) How far does the diamond fall before it reaches 90 percent of its terminal speed?

   Solution. *The equation of motion is $mg - bv = ma$ which gives $b = \dfrac{m(g-a)}{v}$. When $a = 0$, $v = 2 \ ms^{-1}$, we have $b = 0.049 \ N(ms)^{-1}$. We had better write it in term of kilogram, i.e., $b = 0.049 \ kgs^{-1}$. In order to find the distance travel while the velocity is $0.9v$, we write the equation*

   $$mg - bv = ma = m\frac{\mathrm{d}v}{\mathrm{d}t} = m\frac{\mathrm{d}v}{\mathrm{d}s}\frac{\mathrm{d}s}{\mathrm{d}t} = mv\frac{\mathrm{d}v}{\mathrm{d}s}$$

   For clarity, let $B = b/m$. We have the ordinary first order differential equation

   $$g - Bv = v\frac{\mathrm{d}v}{\mathrm{d}s}.$$

   Integrating the expression for solving s we proceed

   $$\begin{aligned} \int_0^{s'} \mathrm{d}s &= \int_0^{v'} \frac{v}{g - Bv}\mathrm{d}v \\ &= -\frac{1}{B}\int_0^{v'} \left(1 - \frac{g}{g - Bv}\right)\mathrm{d}v \\ s' &= -\frac{v'}{B} - \frac{g}{B^2}\ln\left|\frac{g - Bv'}{g}\right| \end{aligned}$$

Inserting $B = b/m = 0.049/0.01 = 4.9$, $v' = 0.9 \times 2 = 1.8$ *and* g *into the equation we get* $s' = 0.57\ ms^{-1}$.

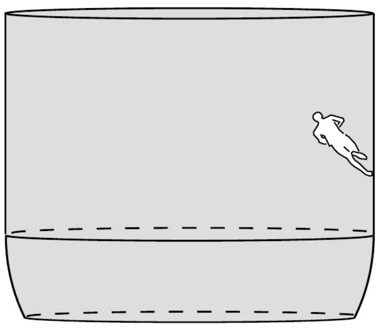
5. Grains from a hopper falls at a rate of 10 kgs^1 vertically onto a conveyor belt that is moving horizontally at a constant speed of 2 ms^{-1}. (a) What force is needed to keep the conveyor belt moving at the constant velocity? (b) What is the minimum power of the motor driving the conveyor belt?

   Solution. *(a) The kinetic energy per second (power) is*

$$\frac{1}{2}(\Delta m)v^2 = \frac{1}{2} \times 10 \times 2^2 = 20\ Js^{-1}.$$

   In one second, the belt translates for 2 m. Thus, the force is $20/2 = 10$ *N. (b) The minimum power is just 20 W.*

6. A person with mass m runs around inside a big bowl, gradually speeding up and rising higher. Eventually she can get up to where the walls of the bowl are vertical as shown in the figure. If the maximum radius of the bowl (cylinder) is R, and her shoes have a coefficient of static friction μ_s, at what the minimum speed v does she run to reach a higher position?



   Solution. *Similar to the classwork problem, we assume the person as a rigid moving object. Set the vertical and horizontal equations:*

$$N\mu_s = mg, \qquad N = \frac{mv^2}{R}$$

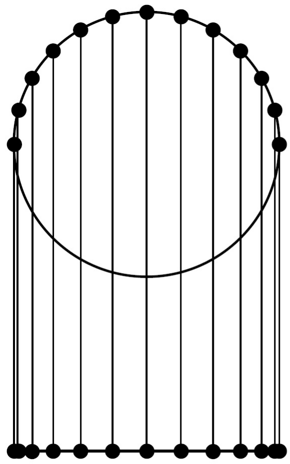
   where N *is the normal reaction against her shoes from the wall. Solving them for the radius*

$$v = \sqrt{\frac{Rg}{\mu_s}}.$$

7. One-dimensional *harmonic motion* of a small particle with mass m is regarded as an uniform circular motion projected onto a line. Please use the projection to show the period

$$T = 2\pi\sqrt{\frac{m}{k}}$$

   of the motion where k is the coefficient of the material defined in Hook's law $F = -kx$.



Solution. *The usual uniform circular motion is given by*

$$F = -mr\omega^2$$

where F is the centripetal force, ω the angular velocity, the negative sign indicating the acceleration always pointing to the centre. Using the standard projection of r on x-axis, we have $x = r\cos\theta$. Thus,

$$\begin{aligned} F &= -mr\omega^2 \\ F &= -m\frac{x}{\cos\theta}\omega^2 \\ F_x = F\cos\theta &= -mx\omega^2 \end{aligned}$$

Using Hook's law $F_x = -kx$ above equation simply gives

$$\omega^2 = \frac{k}{m} \quad \Rightarrow \quad T = 2\pi\sqrt{\frac{m}{k}}$$

where we have used $\omega = 2\pi/T$.

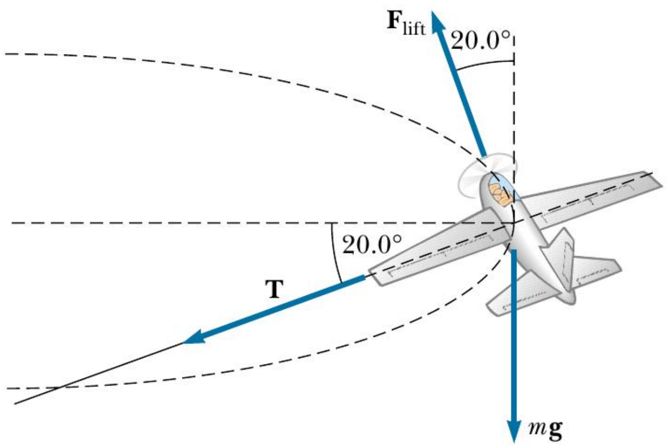
8. A model airplane of mass 0.75 kg flies in a horizontal circle at the end of a 60 m control wire, with a speed of 35 m/s. Compute the tension in the wire if it makes a constant angle of 20° with the horizontal. The forces exerted on the airplane are the pull of the control wire, its own weight, and aerodynamic lift, which acts at 20° inward from the vertical as shown in the diagram.

   Solution. *We may write down the vertical and horizontal components of the forces for the instant static equilibrium as given in the diagram:*

$$F_l\cos 20 = mg + T\sin 20, \qquad F_l\sin 20 + T\cos 20 = \frac{mv^2}{r}$$

   where mv^2/r is the centripetal force for the plane doing circular motion. We are not interested in to know about F_l which follows being eliminated for the tension T by

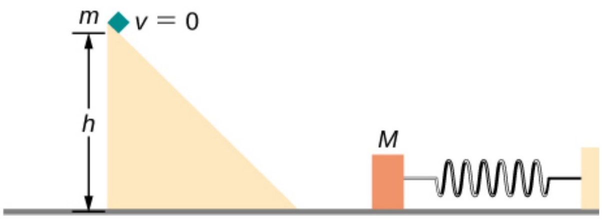
$$mg\sin 20 + T\sin^2 20 = \frac{mv^2}{r}\cos 20 - T\cos^2 20.$$



That is,

$$\begin{aligned} T(\sin^2 20 + \cos^2 20) &= \frac{mv^2}{r}\cos 20 - mg\sin 20 \\ T &= m\left(\frac{v^2}{r}\cos 20 - g\sin 20\right) \\ &= 0.75\left(\frac{35^2}{60\cos 20}\cos 20 - 9.8\sin 20\right) \\ &= 12.80\ N \end{aligned}$$

9. A block of mass m, after sliding down a frictionless incline, strikes another block of mass m that is attached to a spring of spring constant k. The blocks stick together upon impact and travel together. (a) Find the compression of the spring in terms of m, M, g and k when the combination comes to rest. (b) The loss of kinetic energy as a result of the bonding of the two masses upon impact is stored in the so-called binding energy of the two masses. Calculate the binding energy.



Solution. *(a) The potential energy converts to kinetic energy of the mass m, then after collision sticking together the kinetic energy equation is*

$$mgh = \frac{1}{2}mv_i^2 = \frac{1}{2}(M+m)v^2$$

where v_i, v are the velocities before and after the collision. Momentum must be conserved during collision, we have $mv_i = (M+m)v$. This gives

$$\begin{aligned} \frac{1}{2}(M+m)v^2 &= \frac{1}{2}\frac{m^2}{M+m}v_i^2 \\ &= \frac{1}{2}\frac{m^2}{M+m}(2gh) \\ &= \frac{m^2gh}{M+m} \end{aligned}$$

where we have used the first part of the first equation for the second step. And this should equal to the potential energy stored by the spring

$$\frac{m^2 gh}{M+m} = \frac{1}{2}kx^2 \quad \Rightarrow \quad x = \sqrt{\frac{2m^2 gh}{k(M+m)}}.$$

(b) The binding energy is the difference of the original potential energy and the $kx^2/2$ *term, i.e.,*

$$mgh - \frac{m^2 gh}{M+m} = \frac{mgh(M+m) - m^2 gh}{M+m} = \frac{mMgh}{M+m}.$$