2.4

Ex 46: Let event A be that the individual is more than 6 feet tall. B is the selected BB player

$P(A|B) = \frac{P(A\cap B)}{P(B)}$, the probability of the even A, knowing that B happened.

$P(B|A) = \frac{P(A\cap B)}{P(A)}$, the probability of even B, knowing that A happended

Because most of Americans are tall and the most popular sport in the USA is Rugby

So $P(A) > P(B)$, that is: $P(B|A)$ is bigger than $P(A|B)$

Ex 50

A

a. $P(\text{medium, long sleeved, print shirt}) = 0.05$

b. $P(\text{medium, print}) = 0.07 + 0.05 = 0.12$

c. $P(\text{short}) = 0.04 + 0.02 + 0.05 + 0.08 + 0.07 + 0.12 + 0.03 + 0.07 + 0.08 = 0.56$

$P(\text{long}) = 1 - P(\text{short}) = 0.44$

d. $P(\text{medium}) = 0.08 + 0.07 + 0.12 + 0.1 + 0.05 + 0.07 = 0.49$

$P(\text{print}) = 0.02 + 0.07 + 0.07 + 0.02 + 0.05 + 0.02 = 0.25$

e. ~~P(medium)~~ $P(\text{short, plaid}) = 0.04 + 0.08 + 0.03 = 0.15$, $P(\text{medium, short, plaid}) = 0.08$

$P(\text{medium}|\text{short, plaid}) = \frac{P(\text{medium, short, plaid})}{P(\text{short, plaid})} = \frac{0.08}{0.15} = \frac{8}{15}$

f. $P(\text{medium, plaid}) = 0.1 + 0.08 = 0.18$

$P(\text{medium, plaid, long}) = 0.1$

$P(\text{short}|\text{medium plaid}) = \frac{P(\text{medium, short, plaid})}{P(\text{medium, plaid})} = \frac{0.08}{0.18} = \frac{4}{9}$

$P(\text{long}|\text{medium plaid}) = \frac{P(\text{medium, plaid, long})}{P(\text{medium, plaid})} = \frac{0.1}{0.18} = \frac{5}{9}$

Ex 58.

that is: $\frac{P((A\cup B)\cap C)}{P(C)} = \frac{P(A\cap C)}{P(C)} + \frac{P(B\cap C)}{P(C)} - \frac{P(A\cap B\cap C)}{P(C)}$, (1)

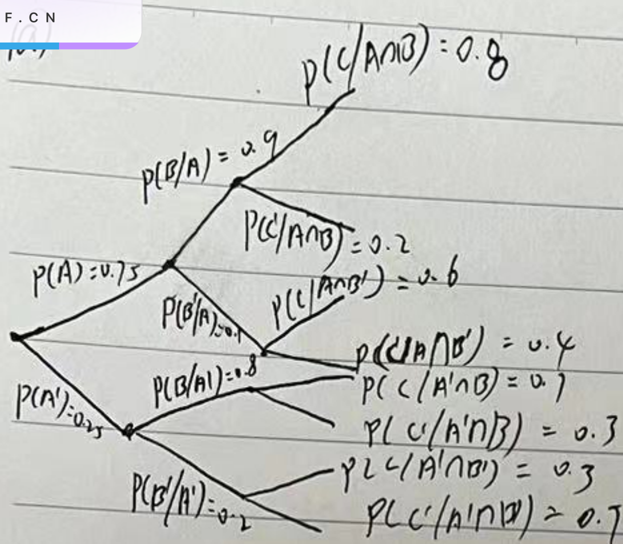
So: $P((A\cup B)\cap C) = P(A\cap C) + P(B\cap C) - P(A\cap B\cap C)$ (2)

given that: $(A\cup B)\cap C \Rightarrow (A\cap C)\cup(B\cap C)$, So $P(A\cup B\cap C) = P(A\cap C) + P(B\cap C) - P(A\cap B\cap C)$, which equals to the right side of (2)

Ex 63.

(a)

(a)



~~(b). $P(A\cap B\cap C) = \frac{P(A\cap B\cap C)}{P(A)}\cdot P(A) = P$~~

(b) $P(A\cap B\cap C) = \frac{P(A\cap B\cap C)}{P(A\cap B)}\cdot P(A\cap B) = P(C/A\cap B)\cdot P(A\cap B) = P(C/A\cap B)\cdot \frac{P(A\cap B)}{P(A)}\cdot P(A) = P(C/A\cap B)\cdot P(B/A)\cdot P(A) = 0.8\times0.9\times0.75 = 0.54$

(c) $P(B\cap C) = P(A\cap B\cap C) + P(A'\cap B\cap C) = 0.54 + P(C/A'\cap B)\cdot P(A'\cap B) = 0.54 + P(C/A'\cap B)\cdot P(B/A')\cdot P(A') = 0.54 + 0.7\times0.8\times0.25 = 0.68$

(d) $P(C) = P(B\cap C) + P(B'\cap C) = P(B\cap C) + P(A\cap B'\cap C) + P(A'\cap B'\cap C) = P(B\cap C) + P(C/A\cap B')\cdot P(A\cap B') + P(C/A'\cap B')\cdot P(A'\cap B')$

$= P(B\cap C) + P(C/A\cap B')\cdot P(B'/A)\cdot P(A) + P(C/A'\cap B')\cdot P(B'/A')\cdot P(A') = 0.68 + 0.3\times(1-0.9)\times0.75 + 0.3\times(1-0.8)\times(1-0.75) = 0.74$

(e) $P(A/B\cap C) = \frac{P(A\cap B\cap C)}{P(B\cap C)} = \frac{0.54}{0.68} = 0.794$

Section 2.5

B

Ex71. $P(A) = 0.4$ $P(B) = 0.7$

a. $P(A\cap B) = P(A)\cdot P(B) = 0.28$

$P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{0.28}{0.4} = 0.7$

b. $P(A'\cap B') = P(A')\cdot P(B') = [1-P(A)][1-P(B)] = 0.6\times 0.3 = 0.18$

$P(A\cup B) = 1 - P(A'\cap B') = 0.82$

c. $P[A\cap(A\cup B)] = P[A\cup(A\cap B)] = P(A) + P(A\cap B) - \cancel{P(A\cap B)}\ P(A\cap A\cap B) = P(A) = 0.4$

$P(A|A\cup B) = \frac{P[A\cap(A\cup B)]}{P(A\cup B)} = \frac{0.4+0.28-0.4\times 0.28}{0.82} = 0.693$

Ex 72.

$P(A_1)\cdot P(A_2) = 0.22\times 0.25 = 0.05 \neq P(A_1\cap A_2)$, ~~so A_1 and A_2~~

$P(A_2)\cdot P(A_3) = 0.25\times 0.28 = 0.07 = P(A_2\cap A_3)$,

$P(A_1)\cdot P(A_3) = 0.22\times 0.28 = 0.06 \neq P(A_2\cap A_3)$,

So only A_2 is independent of A_3.

Ex 80.

let A_i $(i=1,2,3,4)$ denotes the 1, 2, 3, 4 components, $P(A_i) = 0.9$,

$P(A_1\cup A_2) = P(A_1) + P(A_2) - P(A_1)\cdot P(A_2) = 0.9 + 0.9 - 0.9\times 0.9 = 0.99$

$P(\text{system works}) = P((A_1\cup A_2)\cup(A_3\cap A_4)) = P(A_1\cup A_2) + P(A_3\cap A_4) - P((A_1\cup A_2)\cap(A_3\cap A_4)) =$

$0.99 + 0.9\times 0.9 - 0.99\times(0.9\times 0.9) = 0.9981$

Ex84: assume that the probability for the vehicles to pass is 70%

a. $P(\text{all of the three}) = 0.7^3 = 0.343$

b. $P(\text{at least one}) = 1 - P(\text{none of them}) = 1 - (1-0.7)^3 = 1 - 0.027 = 0.973$

c. $P(\text{exactly one}) = C_{3,1} \times 0.7 \times (1-0.7)^2 = 0.189$

d. $P(\text{at most one}) = P(\text{none of them}) + P(\text{exactly one}) = (1-0.7)^3 + 0.189 = 0.216$

Section 3.1

Ex4.

~~So the possible outcome is X=2,3,4~~

Here are some examples of American ZIP code: 90210 (Bevely Hills, california)
10001 (New York)
60601 (Chicago, illinonia)
78701 (Austin, Texas)
85255 (Scottsdale, Arizona)

Basically, there is no 00000 nor any zip code with four zero.

So $X = 2, 3, 4, 5$ for the zip code 10001, 60601, 78701, 85225 respectively.

Ex 5

No. For example, if a coin is tossed ~~repeadedly~~ repeatedly until its head occurs, then this ~~expriment~~ experiment terminates. In this ~~exprim~~ experiment, if the experiment terminates within 3 tosses, then $X=0$, otherwise $X=1$. The sample space is infinite but the X ~~only~~ has only two value.

Ex8.

$Y=3$: SSS

$Y=4$: FSSS

$Y=5$: FFSSS, SFSSS

$Y=6$: FFFSSS, SFFSSS, FSFSSS, ~~FF~~ SSFSSS

$Y=7$: FFFFSSS, SFFFSSS, FSFFSSS, FFSFSSS, SSFFSSS, FSSFSSS, SFSFSSS

Ex10.

a. $T = 0,1,2,3,4,5,6,7,8,9,10$

b. the possible value of X is $-4\ -3\ -2\ -1\ 0\ 1\ 2\ 3\ 4\ 5\ 6$

c. $V = 0,1,2,3,4,5,6$

d. $Z = 0,1,2$

Section 3.2

Ex 12.

a. ~~$P(y \le 49)$~~ $P(y \le 50) = 0.05 + 0.1 + 0.12 + 0.14 + 0.25 + 0.17 = 0.82$

b. ~~$P(y > 50)$~~ $P(50 < y \le 55) = 1 - P(y \le 50) = 0.18$

c. if is the first person on the standby list, that means $y \le 49$,

$P(y \le 49) = 0.05 + 0.1 + 0.12 + 0.14 + 0.25 = 0.82$

if is the third person on the standby list, that means $y \le 47$

$P(y \le 47) = 0.05 + 0.1 + 0.12 = 0.27$

Ex 23

~~a. $p(X=2) = F(2) - F(1) = 0.19 - 0.06 = 0.13$ $0.67 - 0.39$~~

~~b. $p(X > 3) = F(6) - F(3) = 1 - 0.39 = 0.61$~~

~~c. $P(2 \le X \le 5) = F(5) - F($~~

a. $p(X=2) = F(2 \le X < 3) - F(1 \le X < 2) =$ ~~$0.67 - 0.39 =$~~ $0.39 - 0.19 = 0.2$

b. $P(X > 3) = F(6 \le X) - F(3 \le X < 4) = 1 - 0.67 = 0.33$

c. $P(2 \le X \le 5) = F($~~$4 \le X < 5$~~$\overset{5 \le X < 6}{}) - F(1 \le X < 2) = 0.97 - 0.19 = 0.78$

d. $P(2 < X < 5) =$ ~~$F(4 \le X < 5) - F(3 \le X < 4)$~~ $P(2 \le X \le 5) - P(X=2) - P(X=5) =$ ~~$0.97 - 0.92$~~

$0.78 - 0.2 - (0.97 - 0.92) = 0.53$

Ex 25.

$Y: 0, 1, 2, 3, \ldots$

$p(y) = P(Y = y)$

$p(0) = P(Y=0) = p$

$p(1) = P(Y=1) = (1-p)p$

$p(2) = P(Y=2) = (1-p)^2 p$

$p(n) = P(Y=n) = (1-p)^n p$

$$\Rightarrow p(y) = \begin{cases} (1-p)^y p, & y = 0, 1, 2 \ldots \\ 0, & \text{otherwise.} \end{cases}$$