群公告
Assignments for fifth week
Finish the homework:
Section 3.3 29, 33, 38, 41

A+

Section 3.3

. The pmf of the amount of memory X (GB) in a purchased flash drive was given in Example 3.13 as

x	1	2	4	8	16
$p(x)$	.05	.10	.35	.40	.10

Compute the following:
a. $E(X)$
b. $V(X)$ directly from the definition
c. The standard deviation of X
d. $V(X)$ using the shortcut formula

a. $E(X) = x_1 p_1 + x_2 p_2 + \cdots + x_5 p_5$
$= 1\times 0.05 + 2\times 0.1 + 4\times 0.35 + 8\times 0.4 + 16\times 0.1$
$= 6.45$

b $V(X) = \sum_{i=1}^{5} (x_i - E(X))^2 \cdot p(x)$
$= (1-6.45)^2\times 0.05 + (2-6.45)^2\times 0.1 + \cdots + (16-6.45)^2\times 0.1$
$= 15.6475$

c the standard deviation $\sigma = \sqrt{V(X)} \approx 3.9557$

d $V(X) = E(X^2) - [E(X)]^2$
$= \sum_{i=1}^{5} x_i^2 p_i - [E(X)]^2$
$= 15.675$

33. Let X be a Bernoulli rv with pmf as in Example 3.18.
a. Compute $E(X^2)$.
b. Show that $V(X) = p(1 - p)$.
c. Compute $E(X^{79})$.

a $E(X^2) = 1^2 \times p + 0^2 \times (1-p) = p$

b $V(X) = (1-p)^2 \cdot p + (0-p)^2 \cdot (1-p)$
$= (1-p)p(1-p+p) = (1-p)p$

c. $E(X^{79}) = 1^{79} \times p + 0^{79} \times (1-p) = p$

38. Let X = the outcome when a fair die is rolled once. If before the die is rolled you are offered either (1/3.5) dollars or $h(X) = 1/X$ dollars, would you accept the guaranteed amount or would you gamble? [*Note:* It is not generally true that $1/E(X) = E(1/X)$.]

X	1	2	3	4	5	6
$h(X)$	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$
p	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$E(X) = (1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}) \times \frac{1}{6}$
$= \frac{60+30+20+15+12+10}{60} \times \frac{1}{6}$
$= \frac{147}{360} \approx 0.408 > \frac{1}{3.5}$

I will gamble.

41. Use the definition in Expression (3.13) to prove that $V(aX + b) = a^2 \cdot \sigma_X^2$ [*Hint:* With $h(X) = aX + b$, $E[h(X)] = a\mu + b$ where $\mu = E(X)$.]

$E[h(X)] = a\mu + b \quad E(X) = \mu$

$V(ax+b) = \sum_x (ax+b-\mu)^2 \cdot p$
$= \sum_x (ax+b-a\mu-b)^2 \cdot p$
$= \sum_x a^2 (x-\mu)^2 \cdot p$
$= a^2 V(x)$
$= a^2 \sigma_x^2$

46. Compute the following binomial probabilities directly from the formula for $b(x; n, p)$:
a. $b(3; 8, .35)$
b. $b(5; 8, .6)$
c. $P(3 \le X \le 5)$ when $n = 7$ and $p = .6$
d. $P(1 \le X)$ when $n = 9$ and $p = .1$

B+

46

a. $b(3; 8, 0.35)$

$P = \binom{8}{3}(0.35)^3(0.65)^5$

≈ 0.279

b. $b(5; 8; 0.6)$

$P = \binom{8}{5}(0.6)^5 \times (0.4)^3$

≈ 0.279

c. $P(3 \le X \le 5)$ $n = 7$ $p = 0.6$

$P(3 \le X \le 5) = P(3, 7, 0.6) + P(4, 7, 0.6) + P(5, 7, 0.6)$

$= \binom{7}{3} 0.6^3\, 0.4^4 + \binom{7}{4} 0.6^4\, 0.4^3 + \binom{7}{5} 0.6^5\, 0.4^2$

≈ 0.745

d. $P(1 \le X) = 1 - (0, 9, 0.1)$

$= 1 - \binom{9}{0}(0.1)^0(0.9)^9$

≈ 0.613

47. Use Appendix Table A.1 to obtain the following probabilities:

a. $B(4; 15, .3)$
b. $b(4; 15, .3)$
c. $b(6; 15, .7)$
d. $P(2 \le X \le 4)$ when $X \sim \text{Bin}(15, .3)$
e. $P(2 \le X)$ when $X \sim \text{Bin}(15, .3)$
f. $P(X \le 1)$ when $X \sim \text{Bin}(15, .7)$
g. $P(2 < X < 6)$ when $X \sim \text{Bin}(15, .3)$

48. When circuit boards used in the manufacture of compact disc players are tested, the long-run percentage of defectives is 5%. Let X = the number of defective boards in a random sample of size $n = 25$, so $X \sim \text{Bin}(25, .05)$.

a. Determine $P(X \le 2)$.
b. Determine $P(X \ge 5)$.
c. Determine $P(1 \le X \le 4)$.
d. What is the probability that none of the 25 boards is defective?
e. Calculate the expected value and standard deviation of X.

47

a. $B(4, 15, 0.3) = P(X \le 4)$

$= \binom{15}{4}(0.3)^4(0.7)^{11} + \cdots + \binom{15}{1}(0.3)^1(0.7)^{14} + \binom{15}{0}(0.7)^{15}$

≈ 0.515

b. $b(4, 15, 0.3) = \binom{15}{4}(0.3)^4(0.7)^{11}$

$= 0.515 - 0.297$

≈ 0.218

c. $b(6, 15, 0.7) = P(X \le 6) - P(X \le 5)$

$= 0.15 - 0.04$

$= 0.11$

d $P(2 \le X \le 4) = P(X \le 4) - P(X \le 1)$

$= 0.515 - 0.035$

$= 0.48$

e $P(2 \le X) = 1 - P(X \le 1)$

$= 1 - 0.035 = 0.965$

f $P(1 \le X) = 1 - P(X \le 0)$

≈ 1

g $P(2 < X < 6) = P(3 \le X \le 5)$

$= 0.722 - 0.127$

$= 0.595$

48

a $P(X \le 2) = \binom{25}{0}(0.05)^0(0.95)^{25} + \cdots + \binom{25}{2}(0.05)^2(0.95$

$= 0.873$

b $P(X \ge 5) = 1 - P(X \le 4)$

$= 1 - 0.966$

$= 0.034$

c $P(1 \le X \le 4) = P(X \le 4) - P(X < 1)$

$= 0.993 - 0.277$

$= 0.716$

d $P(X = 0) = 0.277$

e $E(X) = np = 25 \times 0.05 = 1.25$

$6 = np(1-p) = 25 \times 0.05 \times 0.95 = 1.1875$

34. A particular type of tennis racket comes in a midsize version and an oversize version. Sixty percent of all customers at a certain store want the oversize version.

a. Among ten randomly selected customers who want this type of racket, what is the probability that at least six want the oversize version?

b. Among ten randomly selected customers, what is the probability that the number who want the oversize version is within 1 standard deviation of the mean value?

c. The store currently has seven rackets of each version. What is the probability that all of the next ten customers who want this racket can get the version they want from current stock?

$X \sim Bin(10, 0.6)$

a $P(X \geq 6) = 1 - P(X \leq 5)$

$= 0.633$

b $E(X) = np = 10 \times 0.6 = 6$

$V(X) = np(1-p) = 2.4$ $\quad \sigma = \sqrt{V(X)} = 1.55$

$E(X) \pm \sigma = (4.45, 7.55)$

$P(5 \leq X \leq 7) = P(X \leq 7) - P(X \leq 4) = 0.667$

c

X

Section 3.5

68. An electronics store has received a shipment of 20 table radios that have connections for an iPod or iPhone. Twelve of these have two slots (so they can accommodate both devices), and the other eight have a single slot. Suppose that six of the 20 radios are randomly selected to be stored under a shelf where the radios are displayed, and the remaining ones are placed in a storeroom. Let X = the number among the radios stored under the display shelf that have two slots.

a. What kind of a distribution does X have (name and values of all parameters)?

b. Compute $P(X = 2)$, $P(X \leq 2)$, and $P(X \geq 2)$.

c. Calculate the mean value and standard deviation of X.

a $X \sim$ Hypergeometric

$N = 20, \quad n = 6, \quad M = 12$

b $P(X=2) = \dfrac{\binom{12}{2}\binom{8}{4}}{\binom{20}{6}} \approx 0.119$

$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= \dfrac{\binom{12}{0}\binom{8}{6} + \binom{12}{1}\binom{8}{5} + \binom{12}{2}\binom{8}{4}}{\binom{20}{6}}$

≈ 0.137

$P(X \geq 2) = 1 - P(X \leq 2) + P(X=2)$

≈ 0.982

c $E(X) = 6 \times \frac{12}{20} = 3.6$

$\sigma = \sqrt{V(X)} = \sqrt{\left(\frac{20-6}{20-1}\right) \times 6 \times \frac{12}{20} \times \left(1 - \frac{12}{20}\right)}$

≈ 4.607

69. Each of 12 refrigerators of a certain type has been returned to a dealer because of an audible, high-pitched, oscillating noise when the refrigerators are running. Suppose that 7 of these refrigerators have a defective compressor and the other 5 have less serious problems. If the refrigerators are examined in random order, let X be the number among the first 6 examined that have a defective compressor. Compute the following:

a. $P(X = 5)$

b. $P(X \leq 4)$

c. The probability that X exceeds its mean value by more than 1 standard deviation.

d. Consider a large shipment of 400 refrigerators, of which 40 have defective compressors. If X is the number among 15 randomly selected refrigerators that have defective compressors, describe a less tedious way to calculate (at least approximately) $P(X \leq 5)$ than to use the hypergeometric pmf.

70. An instructor who taught two sections of engineering statis...

a $p(X=5) = \frac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}} = \frac{5}{44} \approx 0.114$

b $p(X \leq 4) = p(X=1) + \cdots + p(X=4)$

≈ 0.879

c $E(X) = 6 \times \frac{7}{12} = 3.5$

$\sigma = \sqrt{\frac{6}{11} \times 6 \times \frac{7}{12} \times \frac{5}{12}} \approx 0.892$

$P(X > 3.5 + 0.892) = P(X > 4.392) = P(X \geq 5)$

$= 0.121$

d

X

72. A personnel director interviewing 11 senior engineers for four job openings has scheduled six interviews for the first day and five for the second day of interviewing. Assume that the candidates are interviewed in random order.

a. What is the probability that x of the top four candidates are interviewed on the first day?

b. How many of the top four candidates can be expected to be interviewed on the first day?

72

a It's a Hypergeometric distribution

$(x, 6, 4, 11)$

b $E(X) = 6 \times \frac{4}{11} \approx 2.18$

75. Suppose that $p = P(\text{male birth}) = .5$. A couple wishes to have exactly two female children in their family. They will have children until this condition is fulfilled.

a. What is the probability that the family has x male children?

b. What is the probability that the family has four children?

c. What is the probability that the family has at most four children?

d. How many male children would you expect this family to have? How many children would you expect this family to have?

a $nb(2, x, 0.5)$

b $nb(2, 2, 0.5)$

$= \binom{2+2-1}{2-1} (0.5)^2 \times (0.5)^2$

$= 3 \times (\frac{1}{2})^4 = \frac{3}{16}$

c $P = \sum_{x=0}^{2} (x, 2, 0.5)$

$= \frac{1}{4} + \frac{1}{4} + \frac{3}{16} = \frac{11}{16}$

d $E(X) = 2$

expect have four children

79. Let X, the number of flaws on the surface of a randomly selected boiler of a certain type, have a Poisson distribution with parameter $\mu = 5$. Use Appendix Table A.2 to compute the following probabilities:

a. $P(X \le 8)$ **b.** $P(X = 8)$ **c.** $P(9 \le X)$
d. $P(5 \le X \le 8)$ **e.** $P(5 < X < 8)$

A discrete random variable X is said to have a **Poisson distribution** with parameter μ ($\mu > 0$) if the pmf of X is

$$p(x; \mu) = \frac{e^{-\mu} \cdot \mu^x}{x!} \qquad x = 0, 1, 2, 3, \ldots$$

a $P(X \le 8) = F(8;5) = 0.932$

b $P(X = 8) = F(8;5) - F(7;5)$
$= 0.065$

c $P(X \ge 9) = 1 - P(X \le 8) = 0.068$

d $P(5 \le X \le 8) = F(8;5) - F(4;5)$
$= 0.492$

e $P(5 < X < 8) = F(7;5) - F(5;5)$
$= 0.251$

84. Suppose that only .10% of all computers of a certain type experience CPU failure during the warranty period. Consider a sample of 10,000 computers.

a. What are the expected value and standard deviation of the number of computers in the sample that have the defect?
b. What is the (approximate) probability that more than 10 sampled computers have the defect?
c. What is the (approximate) probability that no sampled computers have the defect?

a $E(X) = np = 10000 \times 0.1\% = 10$
$\sigma = \sqrt{np(1-p)} = 3.161$

b $\lambda = np = 10$
$P(X > 10) = 1 - F(10;10) = 0.417$

c $P(X=0) \approx 0$

86. The number of people arriving for treatment at an emergency room can be modeled by a Poisson process with a rate parameter of five per hour.
a. What is the probability that exactly four arrivals occur during a particular hour?
b. What is the probability that at least four people arrive during a particular hour?
c. How many people do you expect to arrive during a 45-min period?

87. The number of requests for assistance received by a towing service is a Poisson process with rate $\alpha = 4$ per hour.
a. Compute the probability that exactly ten requests are received during a particular 2-hour period.
b. If the operators of the towing service take a 30-min break for lunch, what is the probability that they do not miss any calls for assistance?
c. How many calls would you expect during their break?

86

a

$P(X=4) = F(4;5) - F(3;5)$
$= 0.175$

b $P(X \ge 4) = 1 - P(X \le 3)$
$= 0.735$

c $\lambda = np = \frac{3}{4} \times 5 = 3.75$

87.

a $P(X=10) = F(10;8) - F(9;8) = 0.099$

b $\lambda t = 0.5 \times 4 = 2$ $P(X=0) = F(0;2) = 0.135$

c $E(X) = \lambda t = 2$