Key Terms and Results

TERMS

combinatorics组合论: the study of arrangements of objects
enumeration枚举: the counting of arrangements of objects
tree diagram树图: a diagram made up of a root, branches leaving the root, and other branches leaving some of the endpoints of branches
permutation排列: an ordered arrangement of the elements of a set
***r*-permutation r-排列:** an ordered arrangement of r elements of a set
$P(n,r)$: the number of r-permutations of a set with n elements
***r*-combination r-组合:** an unordered selection of r elements of a set
$C(n,r)$: the number of r-combinations of a set with n elements
binomial coefficient $\binom{n}{r}$ 二项式系数: also the number of r-combinations of a set with n elements
combinatorial proof 组合证明: a proof that uses counting arguments rather than algebraic manipulation to prove a result
Pascal's triangle 帕斯卡三角形: a representation of the binomial coefficients where the ith row of the triangle contains $\binom{i}{j}$ for j = 0, 1, 2,...,i

$S(n, j)$: the Stirling number of the second kind denoting the number of ways to distribute n distinguishable objects into j indistinguishable boxes so that no box is empty

RESULTS

product rule for counting计数乘法规则: The number of ways to do a procedure that consists of two tasks is the product of the number of ways to do the first task and the number of ways to do the second task after the first task has been done.

product rule for sets集合乘法规则: The number of elements in the Cartesian product of finite sets is the product of the number of elements in each set.
sum rule for counting计数加法规则: The number of ways to do a task in one of two ways is the sum of the number of ways to do these tasks if they cannot be done simultaneously.

sum rule for sets集合加法规则: The number of elements in the union of pairwise disjoint finite sets is the sum of the numbers of elements in these sets.

subtraction rule for counting计数减法规则 or **inclusion–exclusion for counting计数容斥原理:** If a task can be done in either n_1 ways or n_2 ways, then the number of ways to do the task is n_1+n_2 minus the number of ways to do the task that are common to the two different ways.
subtraction rule or **inclusion–exclusion for sets集合容斥原理:** The number of elements in the union of two sets is the sum of the number of elements in these sets minus the number of elements in their intersection.
division rule for counting计数除法规则: There are n/d ways to do a task if it can be done using a procedure that can be carried out in n ways, and for every way w, exactly d of the n ways correspond to way w.
division rule for sets集合除法规则: Suppose that a finite set A is the union of n disjoint subsets each with d elements. Then $n=|A|\ /d$.

the pigeonhole principle鸽巢原理: When more than k objects are placed in k boxes, there must be

a box containing more than one object.
the generalized pigeonhole principle广义鸽巢原理: When N objects are placed in k boxes, there must be a box containing at least ceiling(N/k) objects.

$$P(n, r) = \frac{n!}{(n-r)!}$$

$$C(n, r) = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Pascal's identity帕斯卡恒等式: $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$

the binomial theorem二项式定理: $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$

There are n^r r-permutations of a set with n elements when repetition is allowed.
There are C(n + r - 1, r) r-combinations of a set with n elements when repetition is allowed.
There are $n!/(n_1!\ n_2!\ \ldots\ n_k!)$ permutations of n objects of k types where there are n_i indistinguishable objects of type i for i =1, 2, 3,..., k.
the algorithm for generating the permutations of the set{1, 2,..., n}