B+

Probability and statistic

Section 3.3

29. a. $E(x) = \sum_{i=1}^{n} x_i P(x_i) = 1\times0.05 + 2\times0.10 + 4\times0.35 + 8\times0.40 + 16\times0.1$
$= 9.65.$

b. $V(x) = \sum_{x\in D} (x-\mu)^2 \cdot p(x) = E[(x-\mu)^2] = 0.05\times(9.65-1)^2 + 0.1\times(9.65-2)^2 + \cdots$
$= 25.887.$

c. $\sigma(x) = \sqrt{V(x)} = 5.08.$

d. $V(x) = E(x^2) - [E(x)]^2 = 25.887.$

33. a. $E(x^3) = 0 \cdot p(0) + 1^2 \cdot p(1) = p$

b. $V(x) = E(x^2) - [E(x)]^2 = p - p^2 = p(1-p)$

c. $E(x^{79}) = 0 \cdot p(0) + 1^{79} \cdot p(1) = p$

38.

x	1	2	3	4	5	6
$h(x)$	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$
$p(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$E(x) = \frac{1}{6}\times1 + \frac{1}{6}\times\frac{1}{2} + \frac{1}{6}\times\frac{1}{3} + \frac{1}{6}\times\frac{1}{4} + \frac{1}{6}\times\frac{1}{5} + \frac{1}{6}\times\frac{1}{6} = \frac{49}{120} > \frac{1}{3.5}$

It seems better to gamble.

41. $V(ax+b) = E[(ax+b)^2] - [E(ax+b)]^2$

$= E(a^2x^2 + 2abx + b^2) - a^2E(x)^2 - 2ab\,E(x) - b^2$

$= a^2E(x^2) + 2ab\,E(x) + b^2 - a^2E(x)^2 - 2ab\,E(x) - b^2$

$= a^2[E(x^2) - E(x)^2]$

$= a^2 \cdot \sigma^2.$

B+

Section 3.4

46. a. $b(3;8,0.35) = \binom{8}{3} 0.35^3 \cdot (1-0.35)^5 = 0.278$

b. $b(5;8,0.6) = \binom{8}{5} 0.6^5 \cdot (1-0.6)^3 = 0.278$

c. $P(3 \le x \le 5) = b(3;7,0.6) + b(4;7,0.6) + b(5;7,0.6)$

$= \binom{7}{3} 0.6^3 (1-0.6)^4 + \binom{7}{4} 0.6^4 (1-0.6)^3 + \binom{7}{5} 0.6^5 (1-0.6)^2$

$= 0.19 + 0.29 + 0.26$

$= 0.74$

d. $P(1 \le x) = 1 - P(x=0) = 1 - b(0;9,0.1) = 1 - \binom{9}{0} \cdot 0.1^0 \cdot (0.9)^9 = 0.61$

47. a. $B(4;15,0.3) = b(0;15,0.3) + b(1;15,0.3) + b(2;15,0.3) + b(3;15,0.3)$

$+ b(4;15,0.3)$

$= \binom{15}{0} 0.3^0 \cdot 0.7^{15} + \binom{15}{1} 0.3^1 \cdot 0.7^{14} + \binom{15}{2} 0.3^2 \cdot 0.7^{13} + \binom{15}{3} 0.3^3 \cdot 0.7^{12}$

$+ \binom{15}{4} 0.3^4 \cdot 0.7^{11}$

$= 0.004 + 0.030 + 0.091 + 0.17 + 0.21 = 0.505$

b. $b(4;15,0.3) = 0.21$

c. $b(6;15,0.7) = \binom{15}{6} 0.7^6 \cdot 0.3^9 = 0.011$

d. $P(2 \le X \le 4) = B(4;15,0.3) - B(1;15,0.3) = 0.471.$

e. $P(2 \le X) = 1 - B(1;15,0.3) = 0.966.$

f. $P(X \le 1) = \binom{15}{1} 0.7^1 \cdot 0.3^{14} + \binom{15}{0} 0.7^0 + 0.3^{15} = 5.1 \times 10^{-7}$

g. $P(2 < x < 6) = \binom{15}{3} 0.3^3 \cdot 0.7^{12} + \binom{15}{4} 0.3^4 \cdot 0.7^{11} + \binom{15}{5} 0.3^5 \cdot 0.7^{10} = 0.2$

48. a. $P(x \le 2) = \binom{25}{0} 0.05^0 \cdot 0.95^{25} + \binom{25}{1} 0.05^1 \cdot 0.95^{24} + \binom{25}{2} 0.05^2 \cdot 0.95^{23}$

$= 0.87$

b. $P(X \ge 5) = 1 - P(X=0) - P(X=1) - \cdots P(X=4) = 1 - B(4;25,0.05) = 0.01$

c. $P(1 \le X \le 4) = 0.68$

d. $P(X=0) = 0.95^{25} = 0.277$

e. $E(x) = np = 1.25.$

54. a. give that the probability of the customer wants to have oversize is $P=0.6$

$1-B(5;10,0.6) = 1-\left(\binom{10}{0}0.6^0\cdot0.4^{10}+\binom{10}{1}0.6^1\cdot0.4^9+\cdots+\binom{10}{5}0.6^5\cdot0.4^5\right)$

$=0.633$

b. $E(x)=np=6$.

$S(x)=\sqrt{V(x)}=np(1-p)=1.5$.

$P(5\le x\le 7)=\binom{10}{5}0.6^5\,0.4^5+\binom{10}{6}0.6^6\,0.4^4+\binom{10}{7}0.6^7\cdot0.4^3$

$=0.67$.

c. $1-(P(x\ge8)+P(x\le2))=P(3\le x\le7)=\binom{10}{3}0.6^3\cdot0.4^7+\binom{10}{4}0.6^4\cdot0.4^6+\cdots$

$=0.82$.

Section 3.5.

68. a. $X\sim(x;6;12,20)$ it's Hypergeometric distribution.

b. $P(X=2)=\dfrac{\binom{12}{2}\cdot\binom{8}{4}}{\binom{20}{6}}=0.12$

$P(X\le2)=P(X=2)+P(X=1)+P(X=0)=\dfrac{\binom{12}{2}\cdot\binom{8}{4}}{\binom{20}{6}}+\dfrac{\binom{12}{1}\binom{8}{5}}{\binom{20}{6}}+\dfrac{\binom{12}{0}\binom{8}{6}}{\binom{20}{6}}$

$=0.137$.

c. $E(x)=6\times\frac{12}{20}=3.6$

$V(x)=6\times\dfrac{12(20-12)(20-6)}{20^2(20-1)}=1.06$.

$S(x)=\sqrt{V(x)}=1.03$

69. a. $P(X=5)=\dfrac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}}=0.11$.

b. $P(X\le4)=1-P(X=5)-P(X=6)=1-\dfrac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}}-\dfrac{\binom{7}{6}\binom{5}{0}}{\binom{12}{6}}=0.88$.

c. $E(x)=6\times\frac{7}{12}=3.5$ $V(x)=6\times\dfrac{7\times5\times6}{12^2\times11}=0.795$

$S(x)=\sqrt{V(x)}=0.89$

$P(X>4.39)=P(X=5)=0.12$.

72. a. $P = \frac{\binom{4}{4}\binom{7}{2}}{\binom{11}{6}} = 0.045$.

b. $E(x) = 4 \times \frac{6}{11} = 2.18$.

75. a. $p = \binom{x+2-1}{1} p (1-p)^x$

b. when $x=2$. $p = \binom{3}{1} p^2 (1-p)^2 = 0.1875$.

c. $P(x \le 2) = P(x=0) + P(x=1) + P(x=2) = 0.6875$.

d. $E(x) = \frac{2(1-0.5)}{0.5} = 2$. They are expected to have 2 male children and 4 children in total.

Section 3.6

79. a. $P(x \le 8) = 0.932$.

b. $P(x=8) = 0.932 - 0.867 = 0.065$

c. $P(x \ge 9) = 1 - P(x \le 8) = 0.068$

d. $P(5 \le x \le 8) = 0.492$

e. $P(5 < x < 8) = 0.251$

84. a. $E(x) = np = 10000 \times 0.001 = 10$

$V(x) = npq = 9.99$ $\sigma(x) = \sqrt{V(x)} = 3.16$.

b. $1 - P(X \le 10) = 1 - P(X=10) - P(X=9) - \cdots = 0.417$.

c. $P(X=0) = P(0; 10) = 4.5 \times 10^{-5}$

86. a. $P(4; 5) = \frac{e^{-5} \cdot 5^4}{4!} = 0.175$.

b. $1 - P(X \le 3) = 1 - P(X=3) - P(X=2) - P(X=1) - P(X=0) = 0.735$.

c. $5 \times \frac{3}{4} = 3.75$

87. a. $P(10;8) = 0.099$.

b. $P(0;2) = 0.135$.

c. $d \times 0.5 = 2$.