Quiz 2

1. Suppose the pdf of the magnitude X of a dynamic load on a bridge is given by

$$f(x)=\begin{cases}\frac{1}{8}+\frac{3}{8}x, & 0\le x\le 2\\ 0, & otherwise\end{cases}$$

Find F(x), P(1⩽X⩽1.5) and P(X>1).

2 The following pdf of *X* is essentially the one suggested in "The Statistical Properties of Freeway Traffic".

$$f(x)=\begin{cases}0.15e^{-0.15(x-0.5)} & x\ge 0.5\\ 0 & otherwise\end{cases} \qquad (1)$$

(A) The formula (1) satisfy the pdf conditions?

(B) The probability that headway time is at most 5 sec is ?

3. Let *X* denote the number of creatures of a particular type captured in a trap during a given time period. Suppose that *X* has a Poisson distribution with λ=4.5, so on average traps will contain 4.5 creatures.

(A) What is the probability that a trap contains exactly five creatures?

(B) What is the probability that a trap has at most five creatures?

4. The breakdown voltage of a randomly chosen diode of a particular type is known to be normally distributed. What is the probability that a diode's breakdown voltage is within 1 standard deviation of its mean value?

5. A bank operates both a drive-up facility and a walk-up window. On a randomly selected day, let *X* = the proportion of time that the drive-up facility is in use, *Y* = the proportion of time that the walk-up window is in use. Let the joint pdf of (*X*,*Y*) be

$$f(x,y)=\begin{cases}\frac{6}{5}(x+y^2) & 0\le x\le 1, 0\le y\le 1\\ 0 & otherwise\end{cases}$$

(A). Determine the probability $P(0 \le X \le \frac{1}{4}, 0 \le Y \le \frac{1}{4})$

(B) Find $f_x(x)$ and $f_y(y)$

(C) X and Y are independent or not independent? why?

6. The joint p.d.f. of (X, Y) is

$$f(x,y) = \begin{cases} Axy & 0 \le x \le 1, 0 \le y \le 1; \\ 0 & otherwise \end{cases}$$

(A) Find A

(B) Find the marginal pdf of X and Y

(C) Whether X and Y are independent?

7. If X_1, …,X_n represent the lifetime of *n* components, the components operate independently of one another, and each lifetime is exponentially distributed with parameter λ.

(A) Joint pdf is?

(B) If there n components constitute a system that will fail as soon as a single component fails, then the probability that the system lasts past t time is?

8. When a batch of a certain chemical product is prepared, the amount of a particular impurity in the batch is a random variable with mean value 4.0g and standard deviation 1.5g. If 50 batches are independently prepared, what is the (approximate) probability that the sample average amount of impurity X is between 3.5 and 3.8g?