No.

Date

22CST 蒋云翔

homework 05 and homework 06

A

Section 3.3

Ex. 29.

(a) $E(X)=\sum_{x\in D} xp(x)=1\times0.05+2\times0.10+4\times0.35+8\times0.40+16\times0.10=6.45$ GB

(b) $V(X)=\sum_{x\in D}(x-\mu)^2p(x)=(1-6.45)^2\times0.05+(2-6.45)^2\times0.05+\cdots+(16-6.45)^2\times0.10=15.6475$ GB

(c) $\sigma=\sqrt{V(X)}=3.956$ GB

(d) $E(X^2)=\sum_{x\in D}x^2p(x)=57.25$; $E(X)^2=6.45^2$

$V(X)=E(X^2)-E(X)^2=57.25-(6.45)^2=15.6475$

Ex. 33

(a) $E(X^2)=0^2(1-p)+1^2\cdot p=p$

(b) $V(X)=(0-p)^2(1-p)+(1-p)^2\cdot p=p(1-p)$

(c) $E(X^{19})=p$, it is the same as question (a)

Ex (38.)

Solution: $E(h(x))=E(\frac{1}{x})=\sum_{x=1}^{6}\frac{1}{x}p(x)=0.408$

as for: $1/3.5=0.286<E(\frac{1}{x})$, so if u gamble, u will win more as expected

Ex (41.)

proof: $V(ax+b)=\sum(ax+b-E(ax+b))^2p(x)$

$=\sum(ax+b-aE(x)-b)\cdot p(x)$

$=\sum(ax-aE(x))^2\cdot p(x)$

$=a^2\cdot\sum(x-E(x))^2\cdot p(x)$

$=a^2\cdot V(x)$

$=a^2\sigma_x^2$

Section 3.4

Ex.(46.)

(a). $b(3;8,0.35) = C_8^3 \cdot (0.35)^3 (1-0.35)^5 = 0.279$

(b) $b(5;8,0.6) = C_8^5 (0.6)^5 \cdot (1-0.6)^3 = 0.279$

~~(c) $P(3 \le X \le 5) = b(3;7,0.6) + b(4;7,0.6) + b(5;7,0.6) = 0.745$~~

(c) $P(3 \le X \le 5) = b(3;7,0.6) + b(4;7,0.6) + b(5;7,0.6) = 0.745$

(d) $P(1 \le X) = 1 - P(X=0) = 1 - C_9^0 \cdot (0.1)^0 (1-0.1)^9 = 0.613$

Ex.47

(a) $B(4;15,0.3) = 0.515$

(b) $b(4;15,0.3) = B(4;15,0.3) - B(3;15,0.3) = 0.219$

(c) $b(6;15,0.7) = B(6;15,0.7) - B(5;15,0.7) = 0.012$

(d) $P(2 \le X \le 4) = B(4;15,0.3) - B(1;15,0.3) = 0.480$

(e) $P(2 \le X) = 1 - B(1;15,0.3) = 0.965$

(f) $P(X \le 1) = B(1;15,0.7) = 0.000$

(g) $P(2 < X < 6) = B(5;15,0.3) - B(2;15,0.3) = 0.595$

Ex.48

(a) $P(X \le 2) = B(2;25,0.05) = 0.873$

(b) $P(X \le 5) = 1 - P(X < 5) = 1 - B(4;25,0.05) = 1 - 0.993 = 0.007$

(c) $P(1 \le X \le 4) = B(4;25,0.05) - B(0;25,0.05) = 0.716$

(d) $P(X=0) = P(0;25,0.05) = 0.277$

(e) $E(X) = np = 25 \times 0.05 = 1.25$

$\sigma_X = \sqrt{np(1-p)} \approx 1.09$

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Ex.54 Let X denote the number of people who buys an oversized racket.

(a) $P(X \geq 6) = 1 - P(X < 6) = 1 - B(5; 10, 0.6) = 1 - 0.367 = 0.633$

(b) $E(X) = np = 10 \times 0.6 = 6$, $\sigma = \sqrt{np(1-p)} = 1.55$ so the range is $(4.45, 7.55)$

$P(4.45 < X < 7.75) = B(7; 10, 0.6) - B(4; 10, 0.6) = 0.667$

(c) This situation can be expressed like: $P(3 \leq X \leq 7) = B(7; 10, 0.6) - B(2; 10, 0.6) = 0.667$

Section 3.5

Ex.68.

(a) It is clear that it is a hypergeomatic distribution. $X \sim h(x; 6, 12, 20)$

(b) $P(X=2) = \frac{C_{12}^2 \cdot C_8^4}{C_{20}^6} = \frac{66 \times 70}{38760} = 0.119$

$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) = \frac{C_{12}^0 \cdot C_8^6}{C_{20}^6} + \frac{C_{12}^1 \cdot C_8^5}{C_{20}^6} + P(X=2) = 0.137$

$P(X \geq 2) = 1 - P(X \leq 1) = 0.982$

(c) $E(X) = n \cdot \frac{M}{N} = 6 \times \frac{12}{20} = 3.6$; $V(X) = \frac{6 \times 12(20-6)(20-12)}{20^2(20-1)} = 1.06$

$\sigma(X) = \sqrt{V(X)} = 1.03$.

Ex 69.

(a) $P(X=5) = \frac{C_7^5 \cdot C_5^1}{C_{12}^6} = 0.114$

(b) $P(X \leq 4) = 1 - P(X=5) - P(X=6) = 1 - 0.114 - \frac{C_7^6 \cdot C_5^0}{C_{12}^6} = 0.879$

(c) $E(X) = 6 \times \frac{7}{12} = 3.5$; $V(X) = \frac{6(12-6)(12-7)}{12^2(12-1)} = 0.795$; $\sigma_X = \sqrt{V(X)} = 0.892$

$P(X > \mu + \sigma) = P(X > 4.392) = P(X=5) + P(X=6) = 0.121$

(d) View it as a binomial distribution

$P(X \leq 5) = B(5; 15, 0.1) = 0.998$

Ex.72.

(a). $\frac{C_4^x \cdot C_7^{6-x}}{C_{11}^6}$

(b) $E(X) = 6 \times \frac{4}{11} = 2.18$, 2.18 as expected will be interviewed at 1st day.

Ex.1). It is a negative Binomial distribution

(a) $P(X=x) = C_{x+2-1}^{2-1}(0.5)^2\cdot(1-0.5)^x = (0.5)^{x+2}\cdot(x+1)$

(b) $P(X=2) = (2+1)\times(0.5)^{2+2} = 0.188$

(c) $P(X\le 2) = 0.5^2 + 2\cdot(0.5)^3 + 0.188 = 0.688$

(d) $E(X) = \frac{2(1-0.5)}{0.5} = 2$, so the family is expected to have 2+2=4 children.

Ex. Section 3.6

Ex.79

(a). $P(X\le 8) = F(8.5) = 0.932$

(b) $P(X=8) = F(8.5) - F(7.5) = 0.065$

(c). $P(X\ge 9) = 1 - F(8.5) = 0.068$

(d) $P(5\le X\le 8) = F(8.5) - F(4.5) = 0.492$

(e) $P(5< X<8) = F(7.5) - F(5.5) = 0.251$

Ex. 84.

(a) $\mu = np = 10000\times 0.001 = 10$, $\sigma = \sqrt{V(X)} = \sqrt{10000\times 0.001\times 0.999} = 3.16$

(b) $P(X>10) = 1 - P(X\le 10) = 1 - F(10;10) = 1 - 0.583 = 0.417$ (泊松分布中 $\mu=\lambda$, $V(X)=\lambda$)

(c) $P(X=0) = F(0;10) = 0.0000045$

Ex.86

(a) $F(4;5) = 0.175$

(b) $P(X\ge 4) = 1 - P(X\le 3) = 1 - F(3;5) = 0.735$

(c) expect: $\mu\times 0.75 = 5\times 0.75 = 3.75$

Ex.87

(a) $F(10;8) - F(9;8) = 0.99$

(b) $F(0,2) = 0.135$

(c) $E(X)$ $\mu = 2$