5.1 (9, 12, 18, 19)

9.

a. $\int_{20}^{30}\int_{20}^{30} k(x^2+y^2)\,dx\,dy = 1$

$k\int_{20}^{30} x^2\,dx + k\int_{20}^{30} y^2\,dy = 1$

$k = \frac{3}{380000}$

b.

$P(x<26, y<26) =$

$\int_{20}^{26}\int_{20}^{26} k(x^2+y^2)\,dx\,dy$

$= k\int_{20}^{26} \frac{1}{3}x^3 + xy^2 \Big|_{20}^{26}\, dy$

$= k\cdot 3192y+2y^3 \Big|_{20}^{26}$

$= 0.3024$

c. $|x-y| \le 2$

$P(|x-y|\le 2) =$

$1 - k\int_{20}^{28}\int_{x+2}^{30} x^2+y^2\,dy\,dx - k\int_{22}^{30}\int_{20}^{x-2} x^2+y^2\,dy\,dx$

$=$

d. $f_X(x) = \int_{20}^{30} k(x^2+y^2)\,dy = 10kx^2 + \frac{19000}{3}k$

e. $f_X(x)\cdot f_Y(y) = (10kx^2+\frac{19000}{3}k)\cdot(10ky^2+\frac{19000}{3}k)$

$\neq k(x^2+y^2)$

so x and y are independent

12.

a. $P(X \geq 3) = \int_3^\infty \int_0^\infty x e^{-x(1+y)}\,dy\,dx$

$= \int_3^\infty e^{-x}\,dx = e^{-3}$

b. $f_X(x) = \int_0^\infty x e^{-x(1+y)}\,dy = e^{-x}$

$f_Y(y) = \int_0^\infty x e^{-x(1+y)}\,dx = \frac{1}{(1+y)^2}$

~~c. $P(X>3 \text{ or } Y>3) = 1 - P(X\leq 3 \text{ and } Y \leq 3)$~~

~~$= 1 - \int_0^3 \int_0^3 x e^{-x(1+y)}\,dy\,dx$~~

~~$= 1-(\frac{1}{4}e^{-\frac{4}{3}} - e^{-3} + \frac{3}{4}) = \frac{1}{4} + e^{-3} - \frac{1}{4}e^{-\frac{4}{3}}$~~

c. $P(x>3 \text{ or } y>3)$

$= 1 - P(x\le 3 \text{ and } y\le 3)$

$= 1 - \int_0^3\int_0^3 x e^{-x(y+1)}$

$= 1 - \int_0^3 e^{-x} - e^{-4x}\,dx$

$= 1 - (\frac{1}{4}e^{-12} - e^{-3} + \frac{3}{4})$

$= \frac{1}{4} + \frac{1}{e^3} - \frac{1}{4e^{12}}$

18.

$P_X(1) = 0.08 + 0.2 + 0.06 = 0.34$

a. $P_{Y|X}(0|1) = \frac{0.08}{0.34} = 0.235$

$P_{Y|X}(1|1) = \frac{0.2}{0.34} = 0.588$

$P_{Y|X}(2|1) = \frac{0.06}{0.34} = 0.176$

b. $P_{Y|X}(y|2) = 0.06 + 0.14 + 0.3 = 0.5$

~~P(Y|X(y|2) = 0.5~~

y	0	1	2
$P_{Y|X}(y|2)$	0.12	0.28	0.60

d. $P(Y\le 1 \mid X=2) = P_{Y|X}(0|2) + P_{Y|X}(1|2) = \frac{0.06}{0.5} + \frac{0.14}{0.5}$

$= 0.4$

d.

x	0	1	2
$P_{X\mid Y}(x\mid 2)$	0.0526	0.1579	0.7895

19.

a. $f_{Y\mid X}(y\mid x) = \dfrac{k(x^2+y^2)}{10kx^2 + \frac{19000}{3}k}$

$f_{X\mid Y}(x\mid y) = \dfrac{k(x^2+y^2)}{10ky^2 + \frac{19000}{3}k}$

b. $P(Y \geq 25 \mid X=22) = \int_{25}^{30} f_{Y\mid X}(y\mid 22)\,dy$

$= \int_{20}^{30} \dfrac{k(22^2+y^2)}{10k(22)^2 + \frac{19000}{3}k} = 0.556$

$P(Y \geq 25) = \int_{25}^{30}\int_{20}^{30} k(x^2+y^2)\,dx\,dy = 0.549$

c. $E(Y\mid X=22) = \int_{20}^{30} y \cdot \dfrac{k(22^2+y^2)}{10k22^2 + \frac{19000}{3}k}\,dy$

$E(Y^2\mid X=22) = \int_{20}^{30} y^2 \cdot \dfrac{k(22^2+y^2)}{10k22^2 + \frac{19000}{3}k}\,dy = 652.03$

$V(Y\mid X=22) = E(Y^2\mid X=22) - (E(Y\mid X=22))^2 = 8.24$

5.2 (24, 26, 33, 35)

24.

$p(x,y) = \frac{1}{30}$

$$E[h(x,y)] = \sum_x \sum_y h(x,y)\cdot\frac{1}{30}$$

$$= (14+14+14+14+14+14) \div 30$$

$$= \frac{84}{30}$$

26.

~~$p(x,y)$~~ =

$h(x,y) = 3x+10y$

$$E[h(x,y)] = \sum_0^{?} \sum_0^{2} 3x+10y \cdot P(x,y)$$

$$= 15.4$$

33.

since X, Y are independent rv

so. $E(XY) = E(X)\cdot E(Y)$

$$Cov(X,Y) = E(XY) - E(X)\cdot E(Y) = 0$$

$$corr(X,Y) = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} = 0$$

so $Cov(X,Y) = corr(X,Y) = 0$

35.

a.

$cov(aX+b, cY+d) = E[(aX+b)(cY+d)] - E(aX+b)E(cY+d)$

$= E(acXY + adX + bcY + bd) - E(aX+b) \cdot E(cY+d)$

$= acE(XY) + adE(X) + bcE(Y) + bd$

$- acE(X)E(Y) - adE(X) - bcE(Y) - bd$

$= ac[E(XY) - E(X) \cdot E(Y)]$

$= ac\,Cov(X,Y)$

b.

$$corr(aX+b, cY+d) = \frac{cov(aX+b, cY+d)}{\sqrt{V(aX+b) \cdot V(cY+d)}} = \frac{ac\,Cov(X,Y)}{|ac|\sigma_X \sigma_Y}$$

$= corr(X,Y)$

c. $|ac|$ will be "$-ac$"

so $corr(aX+b, cY+d) = -corr(X,Y)$