Week 2

A+

2.1

2. Suppose that vehicles taking a particular freeway exit can turn right (R), turn left (L), or go straight (S). Consider observing the direction for each of three successive vehicles.

a. List all outcomes in the event A that all three vehicles go in the same direction.

b. List all outcomes in the event B that all three vehicles take different directions.

c. List all outcomes in the event C that exactly two of the three vehicles turn right.

d. List all outcomes in the event D that exactly two vehicles go in the same direction.

e. List outcomes in D', $C \cup D$, and $C \cap D$.

a) A = {RRR, LLL, SSS}

b) B = {RLS, RSL, LRS, LSR, SRL, SLR}

c) C = {RRL, RRS, RLR, RSR, LRR, SRR}

d) D = {RRL, RRS, RLR, RSR, LRR, SRR, LLS, LLR, LSL, LRL, SLL, RLL, SSL, SSR, SLS, SRS, LSS, RSS}

e) $D' = A \cup B$ = {RRR, LLL, SSS, RLS, RSL, LRS, LSR, SRL, SLR}

$C \cup D$ = {RRL, RRS, RLR, RSR, LRR, SRR, LLS, LLR, LSL, LRL, SLL, RLL, SSL, SSR, SLS, SRS, LSS, RSS}

$C \cap D$ = {RRL, RRS, RLR, RSR, LRR, SRR}

4. Each of a sample of four home mortgages is classified as fixed rate (F) or variable rate (V).

a. What are the 16 outcomes in $\mathcal{S}$?

b. Which outcomes are in the event that exactly three of the selected mortgages are fixed rate?

c. Which outcomes are in the event that all four mortgages are of the same type?

d. Which outcomes are in the event that at most one of the four is a variable-rate mortgage?

e. What is the union of the events in parts (c) and (d), and what is the intersection of these two events?

f. What are the union and intersection of the two events in parts (b) and (c)?

a) $2^4 = 16$

	1	2	3	4
1	F	F	F	F
2	F	F	F	V
3	F	F	V	F
4	F	F	V	V
5	F	V	F	F
6	F	V	F	V
7	F	V	V	F
8	F	V	V	V
9	V	F	F	F
10	V	F	F	V
11	V	F	V	F
12	V	F	V	V
13	V	V	F	F
14	V	V	F	V
15	V	V	V	F
16	V	V	V	V

b) $\{2, 3, 5, 9\}$

c) $\{1, 16\}$

d) $\{1, 2, 3, 5, 9\}$

e) $c \cup d = \{1, 2, 3, 5, 9, 16\}$

$c \cap d = \{1\}$

f) $b \cup c = \{1, 2, 3, 5, 9, 16\}$

$b \cap c = \{\phi\}$

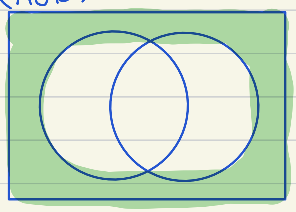
9. Use Venn diagrams to verify the following two relationships for any events A and B (these are called De Morgan's laws):

a. $(A \cup B)' = A' \cap B'$

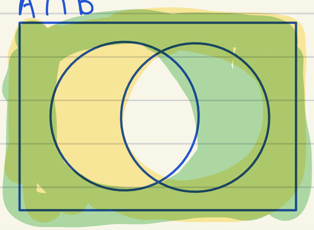
b. $(A \cap B)' = A' \cup B'$

[*Hint:* In each part, draw a diagram corresponding to the left side and another corresponding to the right side.]

a) $(A \cup B)'$

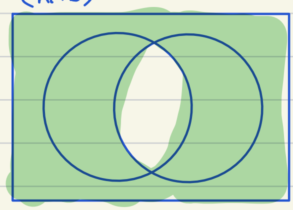


$A' \cap B'$

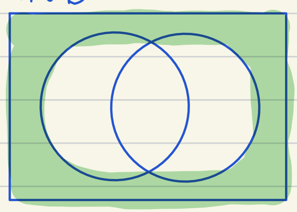


they are the same

b) $(A \cap B)'$



$A' \cup B'$



they're the same

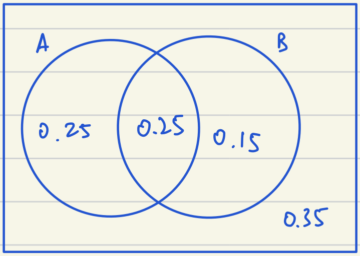
2.2

12. Consider randomly selecting a student at a certain university, and let A denote the event that the selected individual has a Visa credit card and B be the analogous event for a MasterCard. Suppose that $P(A) = .5$, $P(B) = .4$, and $P(A \cap B) = .25$.

a. Compute the probability that the selected individual has at least one of the two types of cards (i.e., the probability of the event $A \cup B$).

b. What is the probability that the selected individual has neither type of card?

c. Describe, in terms of A and B, the event that the selected student has a Visa card but not a MasterCard, and then calculate the probability of this event.



a) $P(A \cup B) = 0.65$

b) $P(A \cup B)' = 0.35$

c) $P(A \cap B') = 0.25$

18. A box contains six 40-W bulbs, five 60-W bulbs, and four 75-W bulbs. If bulbs are selected one by one in random order, what is the probability that at least two bulbs must be selected to obtain one that is rated 75 W?

$6+5+4=15$

$P(75W) = \frac{4}{15}$

$1 - \frac{4}{15} = \frac{11}{15}$

27. An academic department with five faculty members—Anderson, Box, Cox, Cramer, and Fisher—must select two of its members to serve on a personnel review committee. Because the work will be time-consuming, no one is anxious to serve, so it is decided that the representative will be selected by putting the names on identical pieces of paper and then randomly selecting two.

a. What is the probability that both Anderson and Box will be selected? [*Hint:* List the equally likely outcomes.]

b. What is the probability that at least one of the two members whose name begins with *C* is selected?

c. If the five faculty members have taught for 3, 6, 7, 10, and 14 years, respectively, at the university, what is the probability that the two chosen representatives have a total of at least 15 years' teaching experience there?

a) {A, B}, {A, Co}, {A, Cr}, {A, F}, {B, Co}, {B, Cr}, {B, F}
{Co, Cr}, {Co, F}, {Cr, F}

$P = \frac{1}{10}$

b) $\frac{7}{10}$

c) {3, 14}, {6, 10}, {6, 14}, {7, 10}, {7, 14}, {10, 14}

$\frac{6}{10} = 0.6$.

30. A friend of mine is giving a dinner party. His current wine supply includes 8 bottles of zinfandel, 10 of merlot, and 12 of cabernet (he only drinks red wine), all from different wineries.

a. If he wants to serve 3 bottles of zinfandel and serving order is important, how many ways are there to do this?

b. If 6 bottles of wine are to be randomly selected from the 30 for serving, how many ways are there to do this?

c. If 6 bottles are randomly selected, how many ways are there to obtain two bottles of each variety?

d. If 6 bottles are randomly selected, what is the probability that this results in two bottles of each variety being chosen?

e. If 6 bottles are randomly selected, what is the probability that all of them are the same variety?

a) $P_{3,8} = 8 \cdot 7 \cdot 6 = 336$

b) $\binom{30}{6} = 593775$

c) $\binom{8}{2}\binom{10}{2}\binom{12}{2} = 83160$

d) $\frac{c}{b} = \frac{83160}{593775} = 0.14$

e) $\frac{\binom{8}{6} + \binom{10}{6} + \binom{12}{6}}{\binom{30}{6}} = \frac{1162}{593775} = 0.002$

38. A box in a certain supply room contains four 40-W lightbulbs, five 60-W bulbs, and six 75-W bulbs. Suppose that three bulbs are randomly selected.

a. What is the probability that exactly two of the selected bulbs are rated 75-W?

b. What is the probability that all three of the selected bulbs have the same rating?

c. What is the probability that one bulb of each type is selected?

d. Suppose now that bulbs are to be selected one by one until a 75-W bulb is found. What is the probability that it is necessary to examine at least six bulbs?

a) $\dfrac{\binom{6}{2}\binom{9}{1}}{\binom{15}{3}} = \dfrac{(15)(9)}{455} = 0.2967$

b) $\dfrac{\binom{4}{3}+\binom{5}{3}+\binom{6}{3}}{\binom{15}{3}} = \dfrac{4+10+20}{455} = 0.0747$

c) $\dfrac{\binom{4}{1}\binom{5}{1}\binom{6}{1}}{\binom{15}{3}} = \dfrac{120}{455} = 0.2637$

d) $\dfrac{\binom{9}{5}}{\binom{15}{9}} = \dfrac{126}{3003} = 0.042$

40. Three molecules of type A, three of type B, three of type C, and three of type D are to be linked together to form a chain molecule. One such chain molecule is $ABCDABCDABCD$, and another is $BCDDAAABDBCC$.

a. How many such chain molecules are there? [*Hint:* If the three A's were distinguishable from one another—A_1, A_2, A_3—and the B's, C's, and D's were also, how many molecules would there be? How is this number reduced when the subscripts are removed from the A's?]

b. Suppose a chain molecule of the type described is randomly selected. What is the probability that all three molecules of each type end up next to one another (such as in $BBBAAADDDCCC$)?

a) $\frac{12!}{(3!)^4} = 369600$

b) $4! = 24$

$\frac{24}{369600} = 0.00006494$