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A

38. There are two traffic lights on a commuter's route to and from work. Let X_1 be the number of lights at which the commuter must stop on his way to work, and X_2 be the number of lights at which he must stop when returning from work. Suppose these two variables are independent, each with pmf given in the accompanying table (so X_1, X_2 is a random sample of size $n = 2$).

x_1	0	1	2
$p(x_1)$	.2	.5	.3

$\mu = 1.1, \sigma^2 = .49$

a. Determine the pmf of $T_o = X_1 + X_2$.

b. Calculate μ_{T_o}. How does it relate to μ, the population mean?

c. Calculate $\sigma^2_{T_o}$. How does it relate to σ^2, the population variance?

d. Let X_3 and X_4 be the number of lights at which a stop is required when driving to and from work on a second day assumed independent of the first day. With T_o = the sum of all four X_i's, what now are the values of $E(T_o)$ and $V(T_o)$?

e. Referring back to (d), what are the values of $P(T_o = 8)$ and $P(T_o \geq 7)$ [*Hint*: Don't even think of listing all possible outcomes!]

a) $X = 0, 1, 2.$

that $T_o = 0, 1, 2, 3, 4.$

$P(T_o=0) = P(X_1=0 \text{ and } X_2=0) = 0.2\times0.2 = 0.04$

$P(T_o=1) = P(X_1=1 \text{ and } X_2=0, \text{ or } X_1=0 \text{ and } X_2=1)$

$= (0.2\times0.5) + (0.5\times0.2) = 0.20$

$P(T_o=2) = P(X_1=1 \text{ and } X_2=1, \text{ or } X_1=0 \text{ and } X_2=2, \text{ or } X_1=2 \text{ and } X_2=0)$

$= 0.37$

$P(T_o=3) = P(X_1=1 \text{ and } X_2=2, \text{ or } X_1=2 \text{ and } X_2=1)$

$= 0.30$

$P(T_o=4) = 0.09.$

b) $\mu_{T_o} = \sum p_{T_o} \cdot T_o = 0\times(0.04) + 1\times(0.20) + 2\times(0.37) + 3\times(0.30) + 4\times(0.09) = 2.2$

c) $\sigma^2_{T_o} = E(T_o^2) - [E(T_o)]^2$

$E(T_o^2) = 5.82,$

from b), we have $E(T_o) = 2.2$

that $V(T_o) = 0.98.$

d) $T_o = X_1 + X_2 + X_3 + X_4$. Since they're independent.

$E(T_o) = 4\times\mu = 4.4.$ $V(T_o) = 4\times\sigma^2 = 1.96.$

e). $P(T_o=8) = P(X_1=2 \text{ and } X_2=2 \text{ and } X_3=2 \text{ and } X_4=2)$

since they're independent. $P(T_o=8) = P(X_1=2)\cdot \cdots P(X_4=2) = (0.3)^4 = 0.0081$

Since $T_o \leq 8$, $T_o = 7$ can be obtained from three 2s and one 1s.

$P(T_o=7) = 4\cdot(0.3)^3\cdot(0.5) = 0.054.$

$P(T_o \geq 7) = P(T_o=7) + P(T_o=8) = 0.0621$

... number of packages being mailed by a randomly selected customer at a certain shipping facility. Suppose the distribution of X is as follows:

x	1	2	3	4
$p(x)$	.4	.3	.2	.1

a. Consider a random sample of size $n = 2$ (two customers), and let $\bar{X}$ be the sample mean number of packages shipped. Obtain the probability distribution of $\bar{X}$.

b. Refer to part (a) and calculate $P(\bar{X} \leq 2.5)$.

c. Again consider a random sample of size $n = 2$, but now focus on the statistic R = the sample range (difference between the largest and smallest values in the sample). Obtain the distribution of R. [*Hint*: Calculate the value of R for each outcome and use the probabilities from part (a).]

d. If a random sample of size $n = 4$ is selected, what is $P(\bar{X} \leq 1.5)$? [*Hint*: You should not have to list all possible outcomes, only those for which $\bar{x} \leq 1.5$.]

a.) it is a random sample size of 2.

x_1	x_2	$P(x_1, x_2)$	$\bar{x}$
1	1	0.16	1
1	2	0.12	1.5
1	3	0.08	2
1	4	0.04	2.5
2	1	0.12	1.5
2	2	0.09	2
2	3	0.06	2.5
2	4	0.03	3
3	1	0.08	2
3	2	0.06	2.5
3	3	0.04	3
3	4	0.02	3.5
4	1	0.04	2.5
4	2	0.03	3
4	3	0.02	3.5
4	4	0.01	4

$\bar{X}$	1	1.5	2	2.5	3	3.5	4
$P(\bar{X})$	0.16	0.24	0.25	0.20	0.10	0.04	0.01

b) $P(\bar{X} \leq 2.5) = 0.16 + 0.24 + 0.25 + 0.20 = 0.85.$

c). R = the sample range.

R	0	1	2	3
$P(R)$	0.30	0.40	0.22	0.08.

d). $n=4$ $\bar{X} \leq 1.5$ means that the sum of x_i is at most 6.

$$P(\bar{X} \leq 1.5) = P(1,1,1,1) + P(1,1,1,2) + \cdots + P(3,1,1,1) = 0.2400$$

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46. The inside diameter of a randomly selected piston ring is a random variable with mean value 12 cm and standard deviation .04 cm.

a. If $\overline{X}$ is the sample mean diameter for a random sample of $n = 16$ rings, where is the sampling distribution of $\overline{X}$ centered, and what is the standard deviation of the $\overline{X}$ distribution?

b. Answer the questions posed in part (a) for a sample size of $n = 64$ rings.

c. For which of the two random samples, the one of part (a) or the one of part (b), is $\overline{X}$ more likely to be within .01 cm of 12 cm? Explain your reasoning.

a). $\mu = 12$. $\sigma = 0.04$

$E(\overline{X}) = \mu = 12cm$, $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = 0.01 \ cm.$

b) $E(\overline{X}) = \mu = 12cm$, $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{0.04}{\sqrt{64}} = 0.005cm.$

c). when the sample size increase, the $V(\overline{X})$ is decreasing, that the $\overline{X}$ more likely to be within 0.01 cm of 12

51. The time taken by a randomly selected applicant for a mortgage to fill out a certain form has a normal distribution with mean value 10 min and standard deviation 2 min. If five individuals fill out a form on one day and six on another, what is the probability that the sample average amount of time taken on each day is at most 11 min?

$\mu = 10min$, $\sigma = 2min$ $\quad E(\overline{X}) = \mu = 10$

that is for the first day, $n=5$ $\sigma_{\overline{X}_1} = \frac{2}{\sqrt{n}} = \frac{2}{\sqrt{5}}$

$P(\overline{X} \le 11) = P(Z \le \frac{11-10}{2/\sqrt{5}}) = P(Z \le 1.12)$ from the table, we get $P(\overline{X} \le 11) = 0.868$

for the second day, $n=6$

$P(\overline{X} \le 11) = P(Z \le \frac{11-10}{2/\sqrt{6}}) = P(Z \le 1.22) = 0.8888$

since two days are independent, the probability that the sample average is at most 11 min on both days is $P(\overline{X}_1 \le 11) \times P(\overline{X}_2 \le 11) = 0.8686 \times 0.8888$
$= 0.7720$

55. The number of parking tickets issued in a certain city on any given weekday has a Poisson distribution with parameter $\mu = 50$. What is the approximate probability that $\quad P(x;\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$

a. Between 35 and 70 tickets are given out on a particular day? [*Hint*: When μ is large, a Poisson rv has approximately a normal distribution.]

b. The total number of tickets given out during a 5-day week is between 225 and 275?

a. $\mu = 50$. $V(X) = \mu = 50$.

$P(35 \le X \le 70)$

$= P(\frac{35-50}{\sqrt{50}} \le Z \le \frac{70-50}{\sqrt{50}}) = \Phi(2.83) - \Phi(-2.12) = 0.9977 - 0.0170 = 0.9807$

b. since the ticket in each day are independent.

$T_0 = X_1 + X_2 + X_3 + X_4 + X_5$, that $E(T_0) = nE(\overline{X}) = n\mu = 250$, $V(T_0) = nV(\overline{X}) = 250$

$P(225 \le T_0 \le 275) = P(\frac{225-250}{\sqrt{250}} \le Z \le \frac{275-250}{\sqrt{250}}) = \Phi(1.58) - \Phi(-1.58)$

$= 0.9429 - 0.0571 = 0.8858$

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58. A shipping company handles containers in three different sizes: (1) 27 ft³ (3 × 3 × 3), (2) 125 ft³, and (3) 512 ft³. Let X_i ($i = 1, 2, 3$) denote the number of type i containers shipped during a given week. With $\mu_i = E(X_i)$ and $\sigma_i^2 = V(X_i)$, suppose that the mean values and standard deviations are as follows:

$\mu_1 = 200$ $\quad\mu_2 = 250$ $\quad\mu_3 = 100$

$\sigma_1 = 10$ $\quad\sigma_2 = 12$ $\quad\sigma_3 = 8$

a. Assuming that X_1, X_2, X_3 are independent, calculate the expected value and variance of the total volume shipped. [*Hint*: Volume $= 27X_1 + 125X_2 + 512X_3$.]

b. Would your calculations necessarily be correct if the X_i's were not independent? Explain.

a) the total volume $T_0 = 27X_1 + 125X_2 + 512X_3$

$$E(T_0) = 27E(X_1) + 125E(X_2) + 512E(X_3) = 87850$$

$$V(T_0) = 27^2 V(X_1) + 125^2 V(X_2) + 512^2 V(X_3) = 19100116$$

b) the expected value will not be affected, E(T₀) do not change

but the V(T₀) will be changed, since when they're not independent, it may use covariances to get the V(T₀).

70. Consider a random sample of size n from a continuous distribution having median 0 so that the probability of any one observation being positive is .5. Disregarding the signs of the observations, rank them from smallest to largest in absolute value, and let W = the sum of the ranks of the observations having positive signs. For example, if the observations are −.3, +.7, +2.1, and −2.5, then the ranks of positive observations are 2 and 3, so $W = 5$. In Chapter 15, W will be called *Wilcoxon's signed-rank statistic*. W can be represented as follows:

→ 以绝对值从小到大.

→ 数值最小得秩1 次小得秩2.

$$W = 1 \cdot Y_1 + 2 \cdot Y_2 + 3 \cdot Y_3 + \cdots + n \cdot Y_n$$
$$= \sum_{i=1}^{n} i \cdot Y_i$$

where the Y_i's are independent Bernoulli rv's, each with $p = .5$ ($Y_i = 1$ corresponds to the observation with rank i being positive).

a. Determine $E(Y_i)$ and then $E(W)$ using the equation for W. [*Hint*: The first n positive integers sum to $n(n + 1)/2$.]

b. Determine $V(Y_i)$ and then $V(W)$. [*Hint*: The sum of the squares of the first n positive integers can be expressed as $n(n + 1)(2n + 1)/6$.]

$p = 0.5,\ q = 1-p = 0.5 \quad E(Y_i) = p,\ V(Y_i) = p(1-p)$

a). $E(Y_i) = \frac{1}{2}$ since it is an independent Bernoulli rv's

$$E(W) = \sum_{i=1}^{n} i E(Y_i) = \frac{1}{2}\sum_{i=1}^{n} i = \frac{1}{2}\cdot\frac{n(1+n)}{2} = \frac{n(1+n)}{4}$$

b). $V(Y_i) = \frac{1}{4}$ $\quad V(W) = \sum_{i=1}^{n} i^2 V(Y_i) = \frac{1}{4}\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{24}$

73. Suppose the expected tensile strength of type-A steel is 105 ksi and the standard deviation of tensile strength is 8 ksi. For type-B steel, suppose the expected tensile strength and standard deviation of tensile strength are 100 ksi and 6 ksi, respectively. Let $\overline{X}$ = the sample average tensile strength of a random sample of 40 type-A specimens, and let $\overline{Y}$ = the sample average tensile strength of a random sample of 35 type-B specimens.

a. What is the approximate distribution of $\overline{X}$? Of $\overline{Y}$?

b. What is the approximate distribution of $\overline{X} - \overline{Y}$? Justify your answer.

c. Calculate (approximately) $P(-1 \le \overline{X} - \overline{Y} \le 1)$.

d. Calculate $P(\overline{X} - \overline{Y} \ge 10)$. If you actually observed $\overline{X} - \overline{Y} \ge 10$, would you doubt that $\mu_1 - \mu_2 = 5$?

a)

A: $\mu_A = 105$ ksi, $\sigma_A = 8$ ksi

B: $\mu_B = 100$ ksi, $\sigma_B = 6$ ksi.

By the Central Limit Therom. that $\overline{X}$ and $\overline{Y}$ has approximately a normal distribution

b) $\overline{X} - \overline{Y}$ is a linear combinaction

the linear combination of normal rvs is also has a normal distribution.

Since $E(\overline{X}-\overline{Y}) = E(\overline{X}) - E(\overline{Y})$ (they are independent) $= 5$

$$V(\overline{X}-\overline{Y}) = V(\overline{X}) + V(\overline{Y}) = \frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B} = \frac{64}{40} + \frac{36}{35} = 2.6286.$$

$\sigma_{\overline{X}-\overline{Y}} \approx 1.621$

c) $P(-1 \le \overline{X} - \overline{Y} \le 1)$

since from the b) we know that $\overline{X} - \overline{Y}$ has a normal distribution.

and $E(\overline{X} - \overline{Y}) = 5$

$\sigma_{\overline{X}-\overline{Y}} = 1.621$

$$P(-1 \le \overline{X} - \overline{Y} \le 1) = P\left(\frac{-1-5}{1.621} \le Z \le \frac{1-5}{1.621}\right) = \Phi(-2.47) - \Phi(-3.70) \approx 0.0068$$

d) $P(\overline{X} - \overline{Y} \ge 10) \approx P\left(Z \ge \frac{10-5}{1.621}\right) = P(Z \ge 3.08) = 0.0010$

the probability is two small, I may doubt that $\mu_1 - \mu_2 = 5$