

chapter 2

A

2.4)

4b) A = individual is more than 6 feet tall

B = individual is a professional basketball player

 $P(A|B)$ = probability of the individual being more than 6 feet tall, knowing that the individual is prof. player $P(B|A)$ = probability of the individual being a professional player, knowing that they are taller than 6 feet $P(A|B) > P(B|A)$ most prof. player are tall but not all tall people are prof. player.5b) a) $P(M \cap LS \cap PR) = 0.05$ directly from the table of probabilityb) $P(M \cap Pr) = P(M \cap LS \cap PR) + P(M \cap SS \cap PR) = 0.05 + 0.07 = 0.12$ c) $P(SS)$ = sum of 9 probability in the ss table
= 0.56 $P(LS) = 1 - P(SS) = 0.44$ d) $P(M) = 0.08 + 0.07 + 0.12 + 0.1 + 0.05 + 0.07 = 0.49$ $P(Pr) = 0.02 + 0.07 + 0.07 + 0.02 + 0.05 + 0.02 = 0.25$ e) $P(M|SS \cap PI) = \frac{P(M \cap SS \cap PI)}{P(SS \cap PI)} = \frac{0.08}{0.04 + 0.08 + 0.03} = 0.533$ f) $P(SS|M \cap PI) = \frac{P(SS \cap M \cap PI)}{P(M \cap PI)} = \frac{0.08}{0.08 + 0.1} = 0.444$ $P(LS|M \cap PI) = 1 - P(SS|M \cap PI) = 1 - 0.444 = 0.556$

$$58) P(A \cup B | C) = \frac{P[(A \cup B) \cap C]}{P[C]}$$

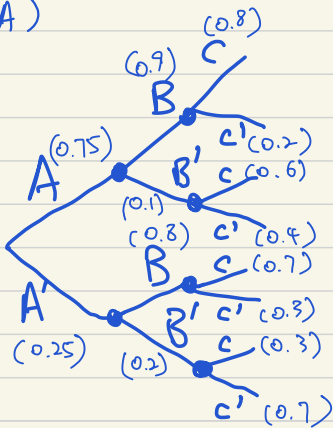
$$= \frac{P[(A \cap C) \cup (B \cap C)]}{P(C)}$$

$$= \frac{P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)}{P(C)}$$

$$= P(A)$$



A)



$$B) P(A \cap B \cap C) = 0.75 \cdot 0.9 \cdot 0.8 = 0.54$$

$$c) P(B \cap C) = P(A \cap B \cap C) + P(A' \cap B \cap C) = 0.54 + 0.25 \times 0.8 \times 0.7 = 0.68$$

$$d) P(C) = P(A \cap B \cap C) + P(A' \cap B \cap C) + P(A \cap B' \cap C) + P(A' \cap B' \cap C) = 0.54 + 0.045 + 0.14 + 0.015 = 0.74$$

$$e) P(A | B \cap C) = \frac{P(A \cap B \cap C)}{P(B \cap C)} = \frac{0.54}{0.68}$$

$$= 0.7941$$

2.5)

71) a) $P(B' | A') = 1 - 0.7 = 0.3$

b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 $= 0.4 + 0.7 - 0.4 \times 0.7$
 $= 0.82$

c) $P(AB' | A \cup B) = \frac{P(AB' \cap (A \cup B))}{P(A \cup B)}$

$= \frac{P(AB')}{P(A \cup B)}$

$= \frac{P(A)P(B')}{P(A \cup B)}$

$= \frac{(0.4)(1-0.7)}{0.82}$

$= \frac{0.12}{0.82}$

$= 0.146$

72) $P(A_1 \cap A_2) = 0.11$ when $P(A_1)P(A_2) \Rightarrow \therefore$ not independent
 $P(A_1 \cap A_3) = 0.05$ when $P(A_1)P(A_3) \Rightarrow \therefore$ not independent
 $P(A_2 \cap A_3) = 0.07$, $P(A_1) \& P(A_3) = 0.07 \Rightarrow$ they're equal, so is independent

80) A_i = component i works, $i = \{1, 2, 3, 4\}$

system work when $(A_1 \cup A_2) \cup (A_3 \cap A_4)$

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = 0.9 + 0.9 - 0.9 \times 0.9 = 0.99$$

$$P(A_3 \cap A_4) = 0.9 \times 0.9 = 0.81$$

$$\begin{aligned} P[(A_1 \cup A_2) \cup (A_3 \cap A_4)] &= P(A_1 \cup A_2) + P(A_3 \cap A_4) - P[(A_1 \cup A_2) \cap (A_3 \cap A_4)] \\ &= P(A_1 \cup A_2) + P(A_3 \cap A_4) - P(A_1 \cup A_2) \cdot P(A_3 \cap A_4) \\ &= 0.99 + 0.81 - 0.99 \times 0.81 \\ &= 0.99 \end{aligned}$$

84) A_i = vehicle i passes inspection. $i = \{1, 2, 3\}$, π = number of passes

a) $P(A_1 \cap A_2 \cap A_3) = 0.7 \times 0.7 \times 0.7 = 0.343$

b) $1 - 0.343 = 0.657$

c) $P[(A_1 \cap A_2' \cap A_3') \cup (A_1' \cap A_2 \cap A_3') \cup (A_1' \cap A_2' \cap A_3)]$
 $= 0.189$

d) $P(\pi \leq 1) = P(\pi=0) + P(\pi=1)$
 $= P(\pi=0) + 0.189$
 $= P(A_1' \cap A_2' \cap A_3') + 0.189$
 $= 0.027 + 0.189$
 $= 0.216$

e) $P(\pi \geq 1) = 1 - P(\pi=0)$
 $= 1 - 0.027$
 $= 0.973$

$$\begin{aligned} &P(A_1 \cap A_2 \cap A_3 \mid A_1 \cup A_2 \cup A_3) \\ &= \frac{P[(A_1 \cap A_2 \cap A_3) \cap (A_1 \cup A_2 \cup A_3)]}{P(A_1 \cup A_2 \cup A_3)} = \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1 \cup A_2 \cup A_3)} \end{aligned}$$

$$= \frac{0.343}{0.973} = 0.3525$$

3.1)

4)

- X can be 2, 3, 4, 5
- $X=5$, outcome = 15213
- $X=4$, outcome = 44074
- $X=3$, outcome = 90022

5)

No.

$I=1$ if the experiment stop at most 5 tosses.

$I=0$ for other situations. The sample space is finite, but I has only two possible values

8) The possible values are $Y = \{3, 4, 5, 6, 7, \dots\}$

$$y = \begin{cases} 3 : SSS \\ 4 : FSSS \\ 5 : FFSSS, SFSSS \\ 6 : SSFSSS, SFFSSS, FSFSSS, FFFSSS \\ 7 : SSFFSSS, SFSFSSS, SFFFSSS, FSSFSSS, FSFFSSS, FFSSFSSS, FFFFSSS \end{cases}$$

10) a) $T = i$, where $(0 \leq i \leq 10)$

b) $X = i$, where $(-4 \leq i \leq 6)$

c) $U = i$, where $(0 \leq i \leq 6)$

d) $Z = i$, where $(0 \leq i \leq 2)$

3.2)

12)

a) $P(Y \leq 50) = 0.05 + 0.1 + 0.12 + 0.14 + 0.25 + 0.17 = 0.83$

b) $1 - 0.83 = 0.17$

c) more than 49 people are not needed for 1st passenger
 $P(Y \leq 49) = 0.05 + 0.1 + 0.12 + 0.14 + 0.25 = 0.66$

more than 47 people are not needed for 3rd passenger
 $P(Y \leq 47) = 0.05 + 0.1 + 0.12 = 0.27$

23)

a) $P(2)$

$= P(X=2)$
 $= F(3) - F(2)$
 $= 0.39 - 0.19$
 $= 0.2$

b) $P(X > 3)$

$= 1 - P(X \leq 3)$
 $= 1 - F(3)$
 $= 1 - 0.67$
 $= 0.33$

c) $P(2 \leq X \leq 5)$

$= F(5) - F(1)$
 $= 0.92 - 0.19$
 $= 0.73$

d) $P(2 < X \leq 5)$

$= P(2 < X \leq 4) = F(4) - F(2) = 0.92 - 0.39 = 0.53$

$$25) P(0)$$

$$= P(Y=0)$$

$$= P(B \text{ first})$$

$$= p$$

$$P(1)$$

$$= P(Y=1)$$

$$= P(G, \text{ then } B)$$

$$= (1-p)p$$

$$P(2)$$

$$= P(Y=2)$$

$$= P(GG|B)$$

$$= (1-p)^2 p$$

$$P(y)$$

$$= P(y \text{ Gs, then } B)$$

$$= (1-p)^y p \text{ for } y = 0, 1, 2, \dots$$