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46. a. $b(3;8,0.35)$

$= C_8^3 \, 0.35^3 \cdot 0.65^5$

$= 0.2786$

b. $b(5;8.0.6)$

$= C_8^5 \, 0.6^5 \cdot 0.4^3$

$= 0.2787$

c. $p(3 \le 4 \le 5)$

$= C_7^3 \, 0.6^3 \times 0.4^4 + C_7^4 \, 0.6^4 \times 0.4^3$

$+ C_7^5 \, 0.6^5 \times 0.4^2$

$= 0.193536 + 0.290304 + 0.2612736$

$= 0.7451136$

d. $P(1 \le x)$

$= 1 - p(x=0)$

$= 1 - C_9^0 \cdot 0.1^0 \times 0.9^9$

$= 0.61257951$

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$$= 0.6123\ 14311$$

47. a. $B(4;15,0.3) = 0.515$

b. $b(4;15,0.3)$

$$= B(4;15,0.3) - B(3;15,0.3)$$
$$= 0.515 - 0.297$$
$$= 0.218$$

c. $b(6;15,0.7)$

$$= B(6;15,0.7) - B(5;15,0.7)$$
$$= 0.015 - 0.004$$
$$= 0.009$$

d. $p(2 \le X \le 4)$ $\quad X \sim Bin(15, 0.3)$

$$= B(4;15,0.3) - B(1,15,0.3)$$
$$= 0.515 - 0.035$$
$$= 0.48$$

e. $p(2 \le X)$ $\quad X \sim Bin(15, 0.3)$

$$= 1 - B(2, 15, 0.3)$$
$$= 0.873$$

f. $p(X \le 1)$ $\quad X \sim Bin(15 \cdot 0.7)$

$$= B(1, 15, 0.7)$$
$$= 0.000$$

$= 0.000$

g. $P(2<X<6) \quad X \sim Bin(15, 0.3)$

$= B(5, 15, 0.3) - B(2, 15, 0.3)$

$= 0.722 - 0.127$

$= 0.595$

48. a. $P(X \le 2) \quad X \sim Bin(25, 0.05)$

$= B(2; 25, 0.05)$

$= 0.873$

b. $P(X \ge 5) \quad X \sim Bin(25, 0.05)$

$= 1 - B(5; 25, 0.05)$

$= 0.001$

c. $P(1 \le X \le 4) \quad X \sim Bin(25, 0.05)$

$= B(4; 25, 0.05) - B(0, 25, 0.05)$

$= 0.993 - 0.277$

$= 0.716$

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d. $P(X=0)$

$= B(0; 25, 0.05)$

$= 0.277$

$= 0.277$

e. For any single trial

$E(X) = np = 0.05 \times 25$

$= 0.125$

$\sigma_X = \sqrt{npq} = \sqrt{25 \times 0.05 \times 0.95}$

$= 1.08972$

54. a. $P(X \geq 6)$ $X \sim Bin(10, 0.6)$

$= 1 - B(5; 10, 0.6)$

$= 0.633$

b. $\mu = np = 10(0.6) = 6$ $\sigma = \sqrt{10 \times 0.6 \times 0.4} = 1.55$

$\mu \pm \sigma = (4.45, 7.55)$

$P(4.45 < X < 7.55) = P(5 \leq X \leq 7)$

$= P(X \leq 7) - P(X \leq 4)$

$= 0.667$

c. It occurs if between 3 and 7 customers want the oversize racket

$P(3 \leq X \leq 7) = P(X \leq 7) - P(X \leq 2)$

$= 0.833 - 0.012$

$= 0.821$

3.5

62.

$= 0.821$

3.5

68. a. Hypergeometric distribution

$h(x;6,12,20)$

b. $P(X=2)$

$$= \frac{C_{12}^{2} \times C_{8}^{4}}{C_{20}^{6}}$$

$$= \frac{\frac{12\times11}{2\times1} \times \frac{8\times7\times6\times5}{4\times3\times2\times1}}{\frac{20\times19\times18\times17\times16\times15}{6\times5\times4\times3\times2\times1}} \qquad 38760$$

$= 0.119195$

$P(X\le 2)$

$= P(X=0)+P(X=1)+P(X=2)$

$$= \frac{C_{12}^{0}\cdot C_{8}^{6}}{C_{20}^{6}} + \frac{C_{12}^{1}\cdot C_{8}^{5}}{C_{20}^{6}} + \frac{C_{12}^{2}\cdot C_{8}^{4}}{C_{20}^{6}}$$

$= 0.0007223 + 0.0173374 + 0.1191950$

$= 0.1372547$

$P(X\ge 2)$

$= 1-(P(X\le 2)-P(X<2))$

$= 0.9819403$

69. a. $P(X=5) = \frac{C_{7}^{5}\cdot C_{6}^{1}}{\quad}$

B+

69. a. $p(X=5) = \dfrac{C_7^5 \cdot C_6^1}{C_{12}^6}$ 5544

$= 0.02273$

b. $p(X \le 4) = 1 - p(X=5) - p(X=6)$

$= 1 - 0.2273 - \dfrac{C_7^6 \cdot C_6^0}{C_{12}^6}$

$= 1 - 0.2273 - 0.001263$

$= 0.771437$

c. $\mu = np = 6 \times \frac{7}{12} = 3.5$

$\sigma = \dfrac{12-6}{11} \times 6 \times \dfrac{7}{12} \times \dfrac{5}{12}$

$= 0.7955$

$\mu + \sigma = 4.2955$

$p(X > 4.2955) = p(X \ge 5)$

$= 1 - p(X \le 4)$

$= 0.228563$

d. Binomial Distributions

Let $p = \frac{40}{400} = 0.1$

$p(X \le 5) = C_{15}^0 \times 0.1^0 \times 0.9^{15} + C_{15}^1 \times 0.1^1 \times 0.9^{14}$

$+ C_{15}^2 \times 0.1^2 \times 0.9^{13} + C_{15}^3 \times 0.1^3 \times 0.9^{12}$

$+ C_{15}^4 \times 0.1^4 \times 0.9^{11} + C_{15}^5 \times 0.1^5 \times 0.9^{10}$

$= B(5; 15, 0.10)$

$= 0.998$

$= B(3, 15, 0.10)$

$= 0.998$

72. a. $p(X=x) = \dfrac{C_6^x \, C_5^{4-x}}{C_{11}^4}$

b. $E(X) = np = 4 \times \frac{6}{11} = \frac{24}{11} = 2.182$

75. a. $p(X=x) = nb(x; 2, 0.5)$

$= C_{x+2-1}^{2-1} (0.5)^2 (1-0.5)^x = (x+1)(0.5)^{x+2}$

b. $p(X=2) = C_3^2 \times \left(\frac{1}{2}\right)^4$

$= \frac{3}{16}$

c. At most four children.

$p = p(X=0) + p(X=1) + p(X=2)$

$= \left(\frac{1}{2}\right)^2 + C_2^1 \left(\frac{1}{2}\right)^3 + C_3^2 \cdot \left(\frac{1}{2}\right)^4$

$= \frac{1}{4} + \frac{1}{4} + \frac{3}{16}$

$= \frac{11}{16}$

d. $\mu = \dfrac{2(1-\frac{1}{2})}{\frac{1}{2}} = 2$

$V(X) = \dfrac{2(1-\frac{1}{2})}{(\frac{1}{2})^2} = 4$

36 79. a. $P(X \le 8) = 0.932$

$$\mu = \frac{2(1-\frac{1}{2})}{\frac{1}{2}} = 2$$

$$V(X) = \frac{2(1-\frac{1}{2})}{(\frac{1}{2})^2} = 4$$

36 79. a. $p(X \le 8) = 0.932$

b. $p(X = 8) = F(8;5) - F(7;5)$
$= 0.932 - 0.867$
$= 0.065$

c. $p(9 \le X) = 1 - F(8;5)$
$= 0.068$

d. $p(5 \le X \le 8)$
$= F(8;5) - F(4;5)$
$= 0.932 - 0.440$
$= 0.492$

e. $p(5 < X < 8)$
$= p(6 \le X \le 7)$
$= F(7;5) - F(5;5)$
$= 0.867 - 0.616$
$= 0.251$

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84. a. $\mu = np = 0.1\% \times 10000$

$= 0.251$

84. a. $\mu = np = 0.1\% \times 10000$

$= 10$

$\sigma = \sqrt{npq} = \sqrt{10000 \times 0.1\% \times 99.9\%}$

$= 3.16$

b. X has appromaxitely poisson probability Distribution

$P(X>10) = 1 - F(10;10)$

$= 1 - 0.583$

$= 0.417$

c. $P(X=0) = \dfrac{e^{-10} \cdot 10^0}{0!}$

$= 0.0000454$

86. a. $P(X=4) = F(4;5) - F(3;5)$

$= 0.440 - 0.265$

$= 0.175$

b. $P(X \geq 4) = 1 - F(3;5)$

$= 1 - 0.265$

$= 0.735$

c. $\mu = np = 45\text{min} \times \dfrac{5}{60\text{min}}$

$= 3.75$

$= 0.135$

c. $\mu = np = 45\text{min} \times \frac{5}{60\text{min}}$

$= 3.75$

87. a. For 2 hour

$\mu = 2 \times 4 = 8$

$p(x=10) = F(10;8) - F(9;8)$

$= 0.816 - 0.717$

$= 0.099$

b. $\mu = 0.5 \times d = 2$

$p(x=0) = F(0;2)$

$= 0.135$

c. $\mu = np = 30\text{min} \times \frac{4}{60\text{min}}$

$= 2$