2.1 2 4 9

2.2 12 18 27

2.3 30 38 40

A

2) R: vehicle turns right

L: " " left

S: " goes straight

fe, event {RLS} means first vehicle turned right, second vehicle turns left and third vehicle ~~turns left~~ goes straight.

a) All outcomes of all 3 vehicles go in the same direction:

A = {RRR, LLL, SSS}.

b) All outcomes of all 3 vehicles go in different direction:

B = {RLS, RSL, LSR, LRS, SLR, SRL} (the order of the letter important)

c) All outcomes that exactly 2 of the 3 vehicles turn right

C = {RRL, RRS, RLR, RSR, LRR, SRR}

d) All outcomes that 2 vehicles goes in same direction

D = {RRL, RRS, RLR, RSR, LRR, SRR, LLR, LLS, LRL, LSL, RLL, SLL, SSR, SSL, SRS, SLS, RSS, LSS}

e) D' is the complement of event D. D' means all vehicle goes in the same direction or all go in different direction (to avoid that exactly goes in the same direction)

D' = {RRR, LLL, SSS, RLS, RSL, LSR, LRS, SLR, SRL}.

C∪D means event C is a subset of event D (exactly 2 vehicles turn right is "subset" of 2 vehicles going in the same direction).

C∪D = D (see item D)

C∩D means both event have ~~to be~~ contain same element.

C∩D = C (see item C)

4) There are 4 home mortgages classified as fixed (F) or variable (V) rate. Each row represent a sample. A column represent 1 of the 4 home mortgages.

d)

Outcome / Horse no.	1	2	3	4
1	F	F	F	F
2	F	F	F	V
3	F	F	V	F
4	F	F	V	V
5	F	V	F	F
6	F	V	F	V
7	F	V	V	F
8	F	V	V	V
9	V	F	F	F
10	V	F	F	V
11	V	F	V	F
12	V	F	V	V
13	V	V	F	F
14	V	V	F	V
15	V	V	V	F
16	V	V	V	V

b) Outcomes that exactly 3 are at fixed rate: FFFV, FFVF, FVFF, VFFF

c) outcomes that all are of the same type:
FFFF and VVVV

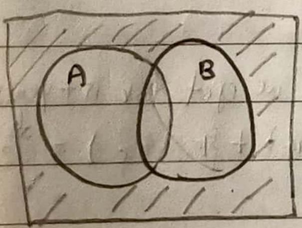
d) Event that atmost 1 of the 4 is a V: FFFF, FFFV, FFVF, FVFF, VFFF

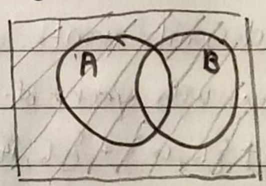
e) $C \cup D$ = FFFF, VVVV, FFFV, FFVF, FVFF, VFFF
$C \cap D$ = FFFF

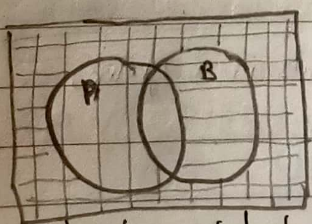
f) $B \cap C = \emptyset$
$B \cup C$ = FFFV, FFVF, FVFF, VFFF, FFFF, VVVV

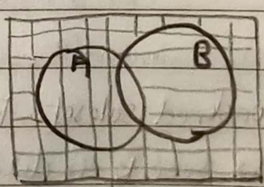
Note: for union, use OR when describing events.
" intersection, " AND " "

g) Verify:
a) $(A \cup B)' = A' \cap B'$ b) $(A \cap B)' = A' \cup B'$









A' = horizontal strips
B' : vertical "
} Union contains both horizontal & vertical strips

A: individual has a VISA card.

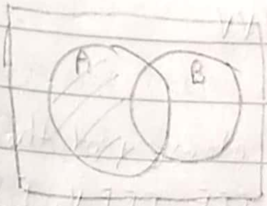
B : analogous events for Mastercard.

$P(A) = 0{,}5$, $P(B) = 0{,}4$ and $P(A \cap B) = 0{,}25$

a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0{,}5 + 0{,}4 - 0{,}25 = 0{,}65$

b) $P(A' \cap B') \overset{\text{De morgan}}{\equiv} P[(A \cup B)'] = 1 - P(A \cup B) = 1 - 0{,}65 = 0{,}35$

c) $P(A \cap B') = P(A) - P(A \cap B) = 0.5 - 0{,}25 = 0.25$



18) Atleast 2 bulb must be selected to obtain one 75 W means do not select 75 W at first try. 75 W can be selected at 2^{nd}, 3^{rd}, -- try, even though we dont care about after the 2^{nd} time.

Let A denote getting 75 W

$P(A) = \frac{4}{15}$

$P(A')$ means not getting 75 W $\Rightarrow 1 - \frac{4}{15} = \frac{11}{15}$

27) Selecting at random means we will have 10 equally likely outcomes: {A, B}

a), {A, Co}, {A, Cr}, {A, F}, {B, Co}, {B, Cr}, {B, F}, {Co, Cr}, {Co, F}, {Cr, F}.

For outcomes equally likely to happen for any simple event out of 10.

$P(\{A, B\}) = \frac{1}{10}$

b) P(atleast one C) = P[{A, Co} + {A, Cr} + {B, Co} + {B, Cr} + {Co, Cr}, {Co, F} + {Cr, F}] $= \frac{7}{10}$

"+" stands for union of disjoint event.

c) If you replace each letter with years of experience, than we get for 15+ years:

$P(15+ \text{years}) = P(\{3,14\} + \{6,10\} + \{6,14\} + \{7,10\} + \{7,14\} + \{10,14\}) = \frac{6}{10}$

30) A Permutation is an ordered subset. For n individuals in a group, the number of permutations size k is denoted as $P_{k,n}$.

$P_{k,n} = \frac{n!}{(n-k)!}$

a) Since order is important, use permutations. There are total of $n=8$ bottles zindafel and we want to serve $k=3$ bottles: $P_{k,n} = P_{3,8} = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 8 \cdot 7 \cdot 6 = 336$

b) A combination is an unordered subset. For n individuals in a group of n elements, we choose k, combinations of size k is denoted as $C_{k,n}$ or $\binom{n}{k}$

$C_{k,n} = \binom{n}{k} = \frac{P_{k,n}}{k!} = \frac{n!}{k!(n-k)!}$

For $n=30$ and $k=6$, we have $C_{k,n} = C_{6,30} = \binom{30}{6} = \left(\frac{30!}{6!(30-6)!}\right)$

$= \frac{30!}{6!\,24!} = 593775$

c) If you select first element of an order pair in n_1 ways and for each selected element, we can " " " n_2 ", then we have the number of pairs to be $n_1 n_2$.

$n_1 = C_{2,8} = \binom{8}{2} = \frac{8!}{2!(8-2)!} = \frac{8!}{2!\,6!} = 28$ (there are 28 ways to choose 2 zindafel from 8 available)

$n_2 = C_{2,10} = \binom{10}{2} = \frac{10!}{2!(10-2)!} = \frac{10!}{2!\,8!} = 45$

$n_3 = C_{2,12} = \binom{12}{2} = \frac{12!}{2!(12-2)!} = \frac{12!}{2!\,10!} = 66$

Because ~~the order~~ we don't care which 2 bottle we take (unordered), use combinations. $n_1 \cdot n_2 \cdot n_3 = 28 \cdot 45 \cdot 66 = 83160$

d) Probability of 2 bottles of each variety being chosen:

$P(D) = \frac{\text{\# favorable outcomes}}{\text{\# outcomes in the sample space}} = \frac{83160}{593775} = 0.14$

e) $P(A) = \frac{\text{\# favorable outcomes}}{\text{\# outcomes in the sample space}}$

$P(\{\text{all vine bottles are the same}\}) = P(\{\text{only zindafel}\}) + P(\{\text{only merlott}\}) + P(\{\text{only cabernet}\}) = \binom{8}{6} + \binom{10}{6} + \binom{12}{6} = 28 + 210 + 924 = 1162$

$P(E) = \frac{1162}{593775} = 0{,}002$

38)

a) Pick 2 75W out of 6, the other 1 can be from either 40W or 60W.

$C_{2,6} \times C_{1,9} = \binom{6}{2} \cdot \binom{9}{1} = 135$

Total number of outcomes in sample space is $C_{3,15} = \binom{15}{3} = 455$

$P(A) = \frac{\text{\# favorable outcomes}}{\text{\# outcomes in the sample space}} = \frac{135}{455} = 0{,}297$

b) B = all 3 randomly selected are from the same group

B_1, B_2 and B_3 are disjoint.

$P(B) = P(B_1 \cup B_2 \cup B_3) = P(B_1) + P(B_2) + P(B_3)$

$= \frac{C_{3,4}}{C_{3,15}} + \frac{C_{3,5}}{C_{3,15}} + \frac{C_{3,6}}{C_{3,15}} = \frac{4}{455} + \frac{10}{455} + \frac{20}{455} = \frac{34}{455} = 0.075$

c) $P(\text{3 different type}) = \dfrac{C_{1,4} \cdot C_{1,5} \cdot C_{1,6}}{C_{3,15}} = \dfrac{4 \cdot 5 \cdot 6}{455} = 0{,}264$

d) D = first 5 bulb are non 75W. only the 6th is.

$P(D) = \binom{9}{5} : \binom{15}{5} = \dfrac{126}{3003} = 0.042$

4a) If each of the A's, B's, C's and D's were distinguishable from each other, then there would be $3 \times 4 = 12$ places to put order of a chain molecule, where the order is important, so use permutations.

$n = 12$ and because a chain molecule size is 12, so $k = 12$

$P_{12,12} = \dfrac{12!}{(12-12)!} = 12! = 479001600$

Some examples include:

$A_1 A_2 A_3 B_2 C_3 C_1 D_3 C_2 D_1 D_2 B_3 B_1$

$A_1 A_3 A_2 B_2 C_3 C_1 D_3 C_2 D_1 D_2 B_3 B_1$

$A_3 A_2 A_1 B_2 C_3 C_1 D_3 C_2 D_1 D_2 B_3 B_1$

If you remove subscript from A_i, then every 6 samples of 479001600 becomes 1 sample.

$A_1 A_2 A_3$
$A_2 A_1 A_3$
$A_1 A_3 A_2$ $\quad P_{3,3} = \dfrac{3!}{(3-3)!} = 6$
$A_3 A_2 A_1$
$A_2 A_3 A_1$
$A_3 A_1 A_2$

If the A ~~are~~ don't have subscripts, we get $\dfrac{12!}{3!}$ ~~permutations~~ combinations $= 79833600$

If you remove the subsripst from B, C and D, we get $\dfrac{12!}{3!3!3!3!} = 369600$ combinations.

b) Find the number of ways to put the 4 letters in some order of 4 elements.

Since order is important, there are $P_{4,4} = \dfrac{4!}{(4-4)!} = 4! = 24$ ways

$P(A) = \dfrac{\#\text{ favorable outcomes}}{\#\text{ outcomes in sample space}} = \dfrac{24}{369600} = 6{,}49 \times 10^{-5}$

Conditional probability of A given the event B has occurred, for which

$P(B) > 0$, $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

? (independent) Proposition: for every 2 events A and B: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(disjoint/mutually exclusive) Addition rule: $P(A \cup B) = P(A \text{ or } B) = P(A) + P(B)$