A+

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注：老师好，我的作业是左右两栏竖列排版。

Homework 10 2021103523 黄毓乾

Section 5.1 9, 12, 18, 19

Ex.9

a) The value of K what we want to find:

$$1=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy$$

$$=\int_{20}^{30}\int_{20}^{30} K(x^2+y^2)\,dx\,dy$$

$$=K\int_{20}^{30}\int_{20}^{30} x^2 dy dx+K\int_{20}^{30}\int_{20}^{30} y^2 dx dy$$

$$=10K\int_{20}^{30}x^2dx+10K\int_{20}^{30}y^2dy$$

$$=20K\times\frac{19000}{3}$$

$$\Rightarrow K=\frac{3}{380000}$$

b) $P(X<26 \text{ and } Y<26)=\int_{20}^{26}\int_{20}^{26}K(x^2+y^2)\,dx\,dy$

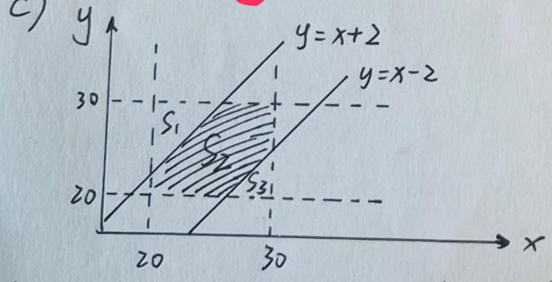
$$=K\int_{20}^{26}\left[x^2y+\frac{y^3}{3}\right]_{20}^{26}dx$$

$$=K\int_{20}^{26}(6x^2+3192)\,dx$$

$$=K\times 38304$$

$$=0.3024$$

c)



The integration region as the shadow shown, denote S_2.

$$P(|X-Y|\le 2)=\iint_{S_2} f(x,y)\,dx\,dy=1-\iint_{S_1} f(x,y)\,dx\,dy-\iint_{S_3} f(x,y)\,dx\,dy$$

$$=1-\int_{20}^{28}\int_{x+2}^{30}f(x,y)\,dy\,dx-\int_{22}^{30}\int_{20}^{x-2}f(x,y)\,dy\,dx$$

$$=0.3593$$

d) $f_X(x)=\int_{-\infty}^{\infty}f(x,y)\,dy=\int_{20}^{30}K(x^2+y^2)\,dy$

$$=10Kx^2+K\frac{dy}{dx}\cdot\frac{1}{3}\Big|_{20}$$

$$=10kx^2+0.05 \quad (x\in[20,30])$$

e) We can obtain the $f_Y(y)$ by substituting y for x in the question (d).

Hence, $f(x,y)\ne f_X(x)\cdot f_Y(y)$.

$\Rightarrow$ X and Y are not independent.

Ex.12

a) $P(X>3)=\int_3^{\infty}\int_0^{\infty}xe^{-x(1+y)}\,dy\,dx$

$$=\int_3^{\infty}e^{-x}dx$$

$$=0.050$$

b)

The marginal pdf of X:

$$f_X(x)=\int_0^{\infty}xe^{-x(1+y)}dy$$

$$=e^{-x} \quad (\text{For } x\ge 0)$$

The marginal pdf of Y:

$$f_Y(y)=\int_3^{\infty}xe^{-x(1+y)}dx=\frac{1}{(1+y)^2} \quad (\text{For } y\ge 0)$$

It is very obvious that $f(x,y)$ is not the product of the marginal pdfs;

Hence, we can determine the two rvs are not independent.

c) $P(\text{at least one}>3)=P(X>3 \text{ or } Y>3)$

$$=1-P(X\le 3 \text{ and } Y\le 3)$$

$$=1-\int_0^3\int_0^3 xe^{-x(1+y)}\,dy\,dx=1-\int_0^3\int_0^3 xe^{-x}e^{-xy}dy$$

$$=1-\int_0^3 e^{-x}(1-e^{-3x})\,dx=e^{-3}+0.25-0.25e^{-12}$$

$$=0.300$$

Ex.18

a)
$$\Rightarrow \begin{cases} P_{Y|X}(0|1) = \frac{0.08}{0.34} = 0.2353 \\ P_{Y|X}(1|1) = \frac{0.20}{0.34} = 0.5882 \\ P_{Y|X}(2|1) = \frac{0.06}{0.34} = 0.1765 \end{cases}$$

b)

$P_{Y|X}(x|2)$ is computed by this divide each entry in the $y=2$ row by $P_X(2)=0.50$:

y	0	1	2
$P_{Y|X}(y|2)$	0.12	0.28	0.60

c) $P(Y \le 1 | x=2)$

$= P_{Y|X}(0|2) + P_{Y|X}(1|2)$

$= 0.12 + 0.28$

$= 0.40$

d) $P_{X|Y}(X|2)$ results got from that it divide each entry in the $y=2$ column by $P_Y(2)=0.38$

X	0	1	2
$P_{X|Y}(X|2)$	0.0526	0.1579	0.7895

Ex.19

a) $f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} = \frac{k(x^2+y^2)}{10kx^2+0.05}$ $\quad y \in [20,30]$

$f_{X|Y}(x|y) = \frac{k(x^2+y^2)}{10ky^2+0.05}$ $\quad x \in [20,30]$

by the way: $k = \frac{3}{380000}$

~~b) P(Y≥25|X=~~

b) $P(Y \ge 25 | X=22) = \int_{25}^{30} f_{Y|X}(y|22)\,dy$

$= \int_{25}^{30} \frac{k((22)^2+y^2)}{10k\,22^2+0.05}\,dy$

$= 0.556$

$P(Y \ge 25) = \int_{25}^{30} f_Y(y)\,dy$

$= \int_{25}^{30} (10ky^2+0.05)\,dy$

$= 0.549$

c) $E(Y|X=22) = \int_{-\infty}^{\infty} y \cdot f_{Y|X}(y|22)\,dy$

$= \int_{20}^{30} y \cdot \frac{k(22^2+y^2)}{10k\,22^2+0.05}\,dy$

$= 25.372912$

$E(Y^2|X=22) = \int_{20}^{30} y^2 \frac{k(22^2+y^2)}{10k\,22^2+0.05}\,dy$

$= 652.028640$

$V(Y|X=22) = E(Y^2|X=22) - [E(Y|X=22)]^2$

$= 8.243976$

$\sigma = \sqrt{V(Y|X=22)} = 2.87$

Homework 10 (Cont.)

Section 5.2 24, 26, 33, 35

Ex.24

We set $h(X,Y)$ = the number of individuals who handle the ~~ma~~ message.

There are a table of the possible values of (X,Y) and $h(X,Y)$.

$h(X,Y)$	y=1	2	3	4	5	6
x=1	–	2	3	4	3	2
2	2	–	2	3	4	3
3	3	2	–	2	3	4
4	4	3	2	–	2	3
5	3	4	3	2	–	2
6	2	3	4	3	2	–

Because $p(x,y) = \frac{1}{30}$ for each one (x,y).

$$\Rightarrow E[h(X,Y)] = \sum_x\sum_y h(x,y)\cdot p(x,y)$$
$$= \sum_x\sum_y h(x,y)\cdot\frac{1}{30}$$
$$= 2.80$$

Ex.26

$$\text{Revenue} = 3X+10Y$$
$$E(\text{revenue}) = E(3X+10Y)$$
$$= \sum_{x=0}^{5}\sum_{y=0}^{2}(3x+10y)\cdot p(x,y)$$
$$= 0\cdot p(0,0)+\cdots+35\cdot p(5,2)$$
$$= 15.4 \Leftrightarrow 15.40(\$)$$

Ex.33

$$E(XY)=E(X)\cdot E(Y)$$
$$Cov(X,Y)=E(XY)-E(X)\cdot E(Y)$$
$$= E(X)\cdot E(Y)-E(X)\cdot E(Y)$$
$$= 0$$

Since $Corr(X,Y)=\frac{Cov(X,Y)}{\sigma_X\sigma_Y}$

Hence, $Corr(X,Y)=0$.

Ex.35

a) $Cov(aX+b, cY+d) =$

$$= E[(aX+b)(cY+d)] - E(aX+b)\cdot E(cY+d)$$
$$= E[acXY+adX+bcY+bd] - (aE(X)+b)(cE(Y)+d)$$
$$= acE(XY)+adE(X)+bcE(Y)+bd - [acE(X)E(Y)+adE(X)+bcE(Y)+bd]$$
$$= acE(XY)-acE(X)E(Y)$$
$$= ac[E(XY)-E(X)E(Y)]$$
$$= ac\,Cov(X,Y)$$

b) $Corr(aX+b, cY+d) = \frac{Cov(aX+b, cY+d)}{SD(aX+b)SD(cY+d)}$

$$= \frac{ac\,Cov(X,Y)}{|a|\cdot|c|SD(X)SD(Y)} = \frac{ac}{|ac|}Corr(X,Y)$$

If a and c have the same signs, $ac=|ac|$

we have $Corr(aX+b, cY+d) = Corr(X,Y)$

c) If a and c are different in sign,

$\Rightarrow |ac| = -ac$

we get that:

$$Corr(aX+b, cY+d) = -Corr(X,Y)$$