

The Examination Paper of Jinan University

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| For Instructor Only | Academic Year : 20 <u>22</u> –20 <u>23</u> Semester: 1 st [<input checked="" type="checkbox"/>] 2 nd [<input type="checkbox"/>] | Course Type Compulsory [<input checked="" type="checkbox"/>] Elective [<input type="checkbox"/>] |
| | Course Title: <u>Advanced Mathematics I</u> | Form of the Examination Open-book [<input type="checkbox"/>] Closed-book [<input checked="" type="checkbox"/>] |
| | Date of the Examination <u>26/12/2022</u> | Paper A [<input checked="" type="checkbox"/>] Paper B [<input type="checkbox"/>] Total Pages <u>7</u> |
| | Instructor's Name <u>Lianghui Xia</u> | |
| For Student Only | School/College _____ Major _____ Name _____ Student No. _____ Mainland Student [<input type="checkbox"/>] Non-mainland Student [<input type="checkbox"/>] | |

| Section No. | I | II | III | IV | V | VI | VII | VIII | Total Score |
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| Score | | | | | | | | | |
| Evaluator | | | | | | | | | |

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| Score | Section I: Choice questions (There are 10 questions, each question has four choices, but only one is true, choose the one which is true, each question is 2 marks, and the total score of this section is 20 marks) |
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1. For the following limits, () exists

A. $\lim_{x \rightarrow 0} \frac{1}{e^x - 1}$; B. $\lim_{x \rightarrow \infty} \frac{x^2}{1 - x^2}$; C. $\lim_{x \rightarrow \infty} \sin x$; D. $\lim_{x \rightarrow 0} e^{\frac{1}{x}}$.

2. If $f(x) = \begin{cases} x \sin \frac{1}{x}, & x > 0 \\ \ln(a + x^2), & x \leq 0 \end{cases}$ is continuous on $(-\infty, +\infty)$, then $a = (\quad)$

A. 0; B. 1; C. 2; D. 3

3. If $f(x)$ satisfies $\lim_{x \rightarrow 1} \frac{f(x)}{\ln x} = 1$, then ()
- A. $f(1) = 0$; B. $\lim_{x \rightarrow 1} f(x) = 0$; C. $f'(1) = 1$; D. $\lim_{x \rightarrow 1} f'(x) = 1$

4. If $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then at $x = 0$, $f(x)$ is ()

- A. discontinuous; B. continuous but non-differentiable;
C. continuous and differentiable; D. discontinuous but differentiable.

5. $\lim_{n \rightarrow \infty} \left(\frac{n-2}{n+1} \right)^n = ()$

- A. e ; B. e^{-1} ; C. e^{-2} ; D. e^{-3}

6. If $f(x)$ is differentiable at $x = a$, then $f'(a) = ()$

- A. $\lim_{h \rightarrow 0} \frac{f(a) - f(a+h)}{h}$; B. $\lim_{h \rightarrow 0} \frac{f(a-h) - f(a)}{h}$;
C. $\lim_{h \rightarrow 0} \frac{f(a+2h) - f(a)}{h}$; D. $\lim_{h \rightarrow 0} \frac{f(a+2h) - f(a+h)}{h}$.

7. If $y = x^x$, then $y' = ()$

- A. x^{x-1} ; B. $x^x \ln x$; C. $x^x (\ln x + \frac{1}{x})$; D. $x^x (1 + \ln x)$

8. If the derivative of $y = f(x)$ is continuous, and $\lim_{x \rightarrow a} \frac{f'(x)}{x-a} = 2$, then ()

- A. $f(x)$ has a local maximum at $x = a$;
B. $f(x)$ has a local minimum at $x = a$;
C. $(a, f(a))$ is an inflection point;
D. there is no local extremum at $x = a$.

9. If $\int x f(x) dx = e^{-x^2} + C$, then $f(x) = ()$

- A. $x e^{-x^2}$; B. $-x e^{-x^2}$; C. $2 e^{-x^2}$; D. $-2 e^{-x^2}$.

10. $\int \frac{1}{1-\sqrt{x}} dx = ()$

- A. $\sqrt{x} - \ln(1 - \sqrt{x}) + C$; B. $2(\sqrt{x} - \ln(1 - \sqrt{x})) + C$;
 C. $\sqrt{x} + \ln(1 - \sqrt{x}) + C$; D. $-2(\sqrt{x} + \ln(1 - \sqrt{x})) + C$.

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| Score | Section II: Filling blanks (There are 8 questions, each question is 3 marks, the total score of this section is 24 marks) |
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- The domain of the function $f(x) = \sqrt{4-x^2} + \ln(x^2-1)$ is _____.
- $\lim_{x \rightarrow 0} \frac{x + x \cos x}{\sin x \cos x} =$ _____.
- $\lim_{x \rightarrow \infty} \left(\frac{x}{1+x} \right)^x =$ _____.
- The discontinuous points of the function $f(x) = \frac{x-1}{x^2-3x+2}$ are _____.
- If $f'(x_0) = 5$, then $\lim_{\varepsilon \rightarrow 0} \frac{f(x_0-3\varepsilon) - f(x_0)}{5\varepsilon} =$ _____.
- The vertical asymptote of the curve $f(x) = \frac{-2x-4}{x^3+2x^2}$ is _____.
- The horizontal asymptote of the curve $y = \frac{x^2+2x-3}{2x^2-x+1}$ is _____.
- $\int \left(\frac{1}{x} - \cos x + x - \frac{1}{\sqrt{1-x^2}} + 2 \right) dx =$ _____.

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| Score | Section III: Calculation (There are 7 questions, each question is 6 marks, the total score of this section is 42 marks) |
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- Find $\lim_{x \rightarrow 9} \frac{\sqrt{x}-3}{x-9}$

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2. Find $\lim_{n \rightarrow \infty} \left(\frac{1}{n^2 + \pi} + \frac{2}{n^2 + 2\pi} + \cdots + \frac{n}{n^2 + n\pi} \right)$

3. Find the derivative of $h(x) = \left(\frac{1}{x^2} - 5 \right)^{-2}$.

4. Given $y - xy^2 + x^2 + 1 = 0$. Find y' .

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5. Find $\lim_{x \rightarrow 1} \left(\frac{x}{x-1} - \frac{1}{\ln x} \right)$.

6. Find $\int \frac{1}{1+e^x} dx$.

7. Find $\int x \arctan x dx$

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| Score | Section IV: Application problems (There are two questions, each one is 7 marks, the total score of this section is 14 marks.) |
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1. (7 marks) For $f(x) = x^3(x + 2)$

- (1) Find all critical points.
- (2) Find the open intervals on which the function is increasing and decreasing.
Identify the function's local extreme values, if any, saying where they occur.
- (3) Find where the graph of f is concave up and where it is concave down.
- (4) Find all inflection points.

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2. (7 marks) Find the volume of the largest right circular cone that can be inscribed in a sphere of radius 3.

