

## 8 Advanced Counting techniques

### Key Terms and Results

#### TERMS

**回归关系** **recurrence relation:** a formula expressing terms of a sequence, except for some initial terms, as a function of one or more previous terms of the sequence

**initial conditions for a recurrence relation:** the values of the terms of a sequence satisfying the recurrence relation before this relation takes effect

**动态规划** **dynamic programming:** an algorithmic paradigm that finds the solution to an optimization problem by recursively breaking down the problem into overlapping subproblems and combining their solutions with the help of a recurrence relation

**线性齐次常系数回归关系** **linear homogeneous recurrence relation with constant coefficients:** a recurrence relation that expresses the terms of a sequence, except initial terms, as a linear combination of previous terms

**特征根** **characteristic roots of a linear homogeneous recurrence relation with constant coefficients:** the roots of the polynomial associated with a linear homogeneous recurrence relation with constant coefficients

#### 非齐次回归关系

**linear nonhomogeneous recurrence relation with constant coefficients:** a recurrence relation that expresses the terms of a sequence, except for initial terms, as a linear combination of previous terms plus a function that is not identically zero that depends only on the index

**divide-and-conquer algorithm:** an algorithm that solves a problem recursively by splitting it into a fixed number of smaller non-overlapping subproblems of the same type

**generating function of a sequence:** the formal series that has the  $n$ th term of the sequence as the coefficient of  $x^n$

**sieve of Eratosthenes:** a procedure for finding the primes less than a specified positive integer

**错排** **derangement:** a permutation of objects such that no object is in its original place

分治算法

序列的生成函数

埃拉托斯特尼筛法

#### RESULTS

**the formula for the number of elements in the union of two finite sets:**

$$|A \cup B| = |A| + |B| - |A \cap B|$$

**the formula for the number of elements in the union of three finite sets:**

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

**the principle of inclusion-exclusion:** 包容-排斥原理

$$\begin{aligned} |A_1 \cup A_2 \cup \dots \cup A_n| &= \sum_{1 \leq i \leq n} |A_i| - \sum_{1 \leq i < j \leq n} |A_i \cap A_j| \\ &+ \sum_{1 \leq i < j < k \leq n} |A_i \cap A_j \cap A_k| \\ &- \dots + (-1)^{n+1} |A_1 \cap A_2 \cap \dots \cap A_n| \end{aligned}$$

**the number of onto functions from a set with  $m$  elements to a set with  $n$  elements:**

$$\begin{aligned} n^m - C(n, 1)(n-1)^m + C(n, 2)(n-2)^m \\ - \dots + (-1)^{n-1} C(n, n-1) \cdot 1^m \end{aligned}$$

**the number of derangements of  $n$  objects:**

$$D_n = n! \left[ 1 - \frac{1}{1!} + \frac{1}{2!} - \dots + (-1)^n \frac{1}{n!} \right]$$