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5.3 30. a. The pmf of T_0 is as follows:

T_0	0	1	2	3	4
$P(T_0)$	0.04	0.2	0.37	0.3	0.09

b. $\mu_{T_0} = 0.2 + 0.74 + 0.9 + 0.36 = 2.2 = \frac{\mu}{2}$

c. $\sigma^2_{T_0} = 1.2^2 \times 0.2 + 2.2^2 \times 0.04 + 0.2^2 \times 0.37 + 0.8^2 \times 0.3 + 1.8^2 \times 0.09 = 0.98 = \frac{\sigma^2}{2}$

d. Now $E(T_0) = 4\mu = 4.4$, $V(T_0) = 4\sigma^2 = 1.96$

e. $P(T_0 = 8) = P(4) \cdot P(4) = 0.09 \cdot 0.09 = 0.0081$

$P(T_0 \geq 7) = P(T_0 = 7) + P(T_0 = 8) = 0.3 \cdot 0.09 \cdot 2 + 0.0081 = 0.0621$

41. a. The probability distribution is as follows:

$\bar{X}$	1	1.5	2	2.5	3	3.5	4
$P(\bar{X})$	0.16	0.24	0.25	0.2	0.1	0.04	0.01

b. $P(\bar{X} \leq 2.5) = 0.85$

c. The distribution of R is shown below:

R	0	1	2	3
P(R)	0.3	0.4	0.22	0.08

d. $P(\bar{X} \leq 1.5) = P(1)P(1)P(1)P(1) + P(2)P(1)P(1)P(1) + \cdots + P(3)P(1)P(1)P(1)$

$= 0.4^4 + 4 \times 0.4^3 \times 0.3 + 6 \times 0.4^2 \times 0.3^2 + 4 \times 0.2 \times 0.4^3 = 0.24$

5.4 46. a. It centers at $\mu = 12cm$. $\sigma(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{0.04}{4} = 0.01cm$.

b. Also centers at $\mu = 12cm$. $\sigma(\bar{X}') = \frac{\sigma}{\sqrt{n}} = \frac{0.04}{8} = 0.005cm$.

c. (b). The bigger n, the smaller variance.

51. For the first day, we have $\mu_{\bar{X}} = \mu = 10$, $\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = 0.8945$

So $P(t \leq 11) = \Phi\left(\frac{11-10}{0.8945}\right) = \Phi(1.118) = 0.8686$

Similarly, the later day we have $\mu_{\bar{X}}' = \mu = 10$, $\sigma_{\bar{X}}' = \frac{\sigma_X}{\sqrt{n'}} = 0.82$

So $P'(t \leq 11) = \Phi\left(\frac{1}{0.82}\right) = \Phi(1.22) = 0.8888$

Then, $P = P(t \leq 11) P'(t \leq 11) = 0.772$

0.5 $2\times 0.5^3 x$

0.5

55 a. Since $\mu = \sigma^2 = 50$, then $P(35 \le X \le 70) = \Phi\left(\frac{20}{\sqrt{50}}\right) - \Phi\left(\frac{-15}{\sqrt{50}}\right) = \Phi(2.83) - \Phi(-2.12)$

$= 0.9977 - 0.017$

$= 0.9807$

b. Since $T_0 = X_1 + \dots + X_5$, then $\mu_{T_0} = 5\mu = 250$, $\sigma_{T_0} = \sqrt{250}$

So $P(225 \le X' \le 275) = \Phi\left(\frac{25}{\sqrt{250}}\right) - \Phi\left(\frac{-25}{\sqrt{250}}\right) = \Phi(1.58) - \Phi(-1.58) = 0.9429 - 0.0571$

$= 0.8858$

58\. a. $E(Y) = 200 \times 27 + 250 \times 125 + 100 \times 512 = 87850$

$V(Y) = 27^2 10^2 + 125^2 12^2 + 8^2 512^2 = 19100116$

b. The $E(Y)$ is correct, but $V(Y)$ is not. If we want to get correct $V(Y)$, we also need the covariance.

70\. a. $E(Y_i) = 0.5$, $E(W) = (1+2+3+\dots+n)0.5 = \frac{n(n+1)}{4}$.

b. $V(Y_i) = 0.25$, $V(W) = (1^2+2^2+\dots+n^2)0.25 = \frac{n(n+1)(2n+1)}{24}$

73\. a. approximate normal distribution.

b. Also approximate normal distribution, because $\bar{X} - \bar{Y}$ can be seen as another linear combination.

c. Since $E(\bar{X} - \bar{Y}) = E(\bar{X}) - E(\bar{Y}) = 5$, $V(\bar{X} - \bar{Y}) = V(\bar{X}) + V(\bar{Y}) = \frac{\sigma_1^2}{40} + \frac{\sigma_2^2}{35} = 1.621^2$

So $P(-1 \le \bar{X} - \bar{Y} \le 1) = \Phi\left(\frac{-4}{1.621}\right) - \Phi\left(\frac{-6}{1.621}\right) = \Phi(-2.47) - \Phi(-3.7) = 0.0068$.

d. $P(\bar{X} - \bar{Y} \geq 10) = 1 - \Phi\left(\frac{5}{1.621}\right) = 1 - \Phi(3.08) = 0.001$, I would doubt that.