Physics CST (2023-24) Homework 4

Please send the completed file to my mailbox yy.lam@qq.com by November 11th, with using the filename format:

student_number-name-cst-hw4

Please answer the questions by filling on these sheets. Or alternatively, do the homework as usual by using papers, then take the pictures and paste them onto these question sheets.

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1. How much mass of water vapour initially at 135°C is needed to warm 220 g of water in a 130-g glass container from 20.0°C to 65.0°C at thermal equilibrium? (Specific heats of water, steam and glass are 4.19×10^3, 2.01×10^3 and 837 J/kg · °C, respectively. The latent of vaporization of water is 2.26×10^6 J/kg.)

   Solution. *As the heat source solely from the steam transports to the glass of water to attain the final thermal equilibrium at $65°C$. Let m be the mass of steam from $135°C$ dropping to $100°C$ steam, the amount of thermal energy is*

   $$2.01 \times 10^3 \times (100 - 135)m = -7.035 \times 10^4 m \ J/kg.$$

   Converting $100°C$ of steam to $100°C$ of water gives the lalent heat

   $$-2.26 \times 10^6 m \ J/kg.$$

   Finally the water dropping from $100°C$ to $65°C$ gives

   $$4.19 \times 10^3 \times (65 - 100)m = -1.4665 \times 10^5 m \ J/kg.$$

   The total heat loss is

   $$-7.035 \times 10^4 m - 2.26 \times 10^6 m - 1.4665 \times 10^5 m = -2.4770 \times 10^6 m \ J/kg.$$

   The heat absorption of the whole glass of water from $20°C$ to $65°C$ reaching thermal equilibrium is

   $$(0.22 \times 4.19 \times 10^3 + 0.13 \times 837)(65 - 20) = 4.6377 \times 10^4 \ J.$$

   Equating the heat loss and heat gain gives

   $$m = \frac{4.6377 \times 10^4}{2.4770 \times 10^6} = 0.0187 \ kg,$$

   or 18.7 g of steam.

2. At a distance of 3.5 m from a source the sound level is 74 dB. How far away has the level dropped to 58 dB?

   Solution. *According to the inverse square law, $\frac{d^2}{3.5^2} = \frac{I_{3.5}}{I_d}$ where d is the distance to be determined, I_d, $I_{3.5}$ are the corresponding intensities measured at d and at 3.5 metre away* *from the source, respectively. As I_0 stands for the threshold of hearing, we get,*

$$\begin{aligned} 58 - 74 &= 10\left(\log\frac{I_d}{I_0} - \log\frac{I_{3.5}}{I_0}\right) \\ &= 10\log\frac{I_d}{I_{3.5}} \\ -1.6 &= \log\frac{3.5^2}{d^2} \\ 10^{-1.6} &= \frac{3.5^2}{d^2} \\ d &= 22.08\ m \end{aligned}$$

3. How does the rate of heat transfer by conduction change of an object when the volume is decreased by half?

   Solution. *Let L be the dimension of the original object, the area and linear dimensions are simply L^2 and L, respectively. Halfing the volume corresponds to the linear dimensions and area changing to*

   $$(L^3/2)^{1/3} = 2^{-1/3}L \quad \textit{and} \quad (L^3/2)^{2/3} = 2^{-2/3}L^2$$

   The new heat flow is hence

   $$\frac{k(2^{-2/3}L^2)\Delta T}{2^{-1/3}L} = \frac{1}{2}\cdot\frac{kL^2\Delta T}{L} = \frac{1}{2} \times \textit{the orginal heat flow}$$

   The heat flow happens dropping to the half of the original heat flow.

4. A box with volume V contains 16 particles with each of mass m having various speeds. Four have speed v; three have speed $2v$; two have speed $4v$; four have speed half v; the other three have speed $3v$. (a) Find the root-mean-square speed of the particles. (b) Use the ideal gas law to find the pressure inside the container.

   Solution. *(a) The mean-square speed is*

   $$\langle v^2\rangle = \frac{4v^2 + 3(2v)^2 + 2(4v)^2 + 4(v/2)^2 + 3(3v)^2}{16} = \frac{4+12+32+1+27}{16}v^2 = 4.75v^2$$

   It follows the root-mean-square speed $2.180v$ (b) Using the ideal gas law, the pressure is

   $$P = \frac{2}{3}\left(\frac{N}{V}\right)\left(\frac{1}{2}m\langle v^2\rangle\right) = \frac{2}{3}\frac{16}{V}\frac{1}{2}m(4.75v^2) = 25.33mv^2/V$$

5. A box with 8 cells contains 2 different balls. The balls are allowed to sit on the same cell. What is the change of entropy if it is replaced by a box with only 4 cells?

   Solution. *The change of space of configuration is from ${}^8P_2$ to ${}^4P_2$. The change of entropy is*

   $$k\ln{}^4P_2 - k\ln{}^8P_2 = 1.38\times10^{-23}\times\ln\left(\frac{4!}{2!}/\frac{8!}{6!}\right) = -2.126\times10^{-23}\ JK^{-1}$$

6. A box contains 10^{22} gas molecules, approximately, under normal room temperature and pressure. Approximate the change of entropy of the system if the volume of the box is double of the original. (For simplicity, assuming the molecules are distinguishable)

Solution. *As each molecule has 2 times more room to move around than the initial state, for* 10^{22} *molecules there are*

$$\sim 2^{10^{22}} \text{ times of increments of the freedom (the configuration space)}$$

The increase of the entropy of the system is

$$k \ln 2^{10^{22}} = 1.38 \times 10^{-23} \times 10^{22} \times ln2 = 0.09565 \ JK^{-1}$$

7. Ultrasound that has a frequency of 2.44 MHz is sent toward blood in an artery that is moving away the source at 20.0 cm/s. Use the speed of sound in human tissue as 1550 m/s. (a) What frequency does the blood receive? (b) What frequency returns to the source? Take your results to 6 decimal places.

   Solution. *(a) As the blood moving away from the source, it is the case of moving observer with stationary sound source. Thus the the blood receive the frequency with*

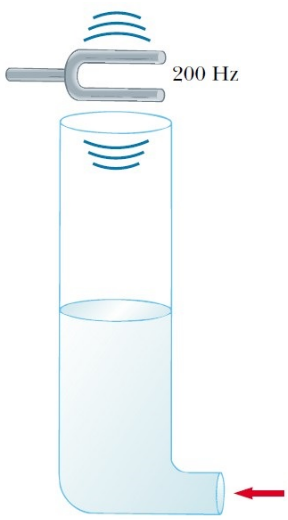
$$\nu \frac{v - v_o}{v} = 2.44 \times \frac{1550 - 0.2}{1550} = 2.439685 \ MHz.$$

   (b) The blood here becomes the moving (away) sound source due to the reflection of sound, the original sound source acts as an observer receiving

$$\nu \frac{v}{v + v_s} = 2.439685 \times \frac{1550}{1550 + 0.2} = 2.439370 \ MHz.$$

   Note that the 'new source' frequency $\nu = 2.439685$ *is used from the result of part (a).*

8. Water is pumped into a long vertical cylinder at a rate of 24 cm^3/s. The radius of the cylinder is 5 cm, and at the open top of the cylinder is tuning fork vibrating with a frequency of 200 Hz. As the water rises, how much time elapses between any successive resonances at 20 °C?



   Solution. *Resonance occurs when the length of the one-end-closed and one-end-opened pipe provides a length with an integral multiple of* $(2n-1)\lambda/4$*, where* λ *this time is a fixed wavelength according to the frequency of the tuning fork. The height of the water rising* *for two successive resonances is* $\lambda/2$. *Let* r *be the radius of the cylinder, the volume of rised water is*

$$\frac{r^2\pi\lambda}{2} = \frac{r^2\pi v}{2\nu} = 24t$$

where t is the time taken between successive resonances. Substituting the numerical values into the equation with speed of sound $v = 343 \times 10^2$ *cm/s, we obtain*

$$t = \frac{5^2\pi \times 343 \times 10^2}{2 \times 24 \times 200}$$
$$= 280.6 \ sec.$$