A+

5.1

$k(\frac{1}{3}x^3 + xy^2)$

9. a. $\iint k(x^2+y^2)\,dx\,dy$

$= \int k[(\frac{1}{3}\times 30^3 + 30y^2) - (\frac{1}{3}\times 20^3 + 20y^2)]\,dy$

$= k\int \frac{19000}{3} + 10y^2\,dy$

$= k \cdot \frac{380000}{3} = 1$

$k = \frac{3}{380000}$

b. $p = \int_{20}^{26}\int_{20}^{26} \frac{3}{380000}(x^2+y^2)\,dx\,dy$

$= \frac{3}{380000}\int_{20}^{26} 3192 + 10y^2\,dy$

$= \frac{3}{380000} \times (19152 + 31920)$

$= 0.3024$

c. $p = 2 \times \int_{20}^{30}\int_{20}^{y+2} \frac{3}{380000}\cdot(x^2+y^2)\,dx\,dy$

$= \frac{6}{380000}\int_{20}^{30}\int_{20}^{y+2}(x^2+y^2)\,dx\,dy$

$= 0.3593$

d. $f_X(x) = \int_{20}^{30} k(x^2+y^2)\,dy$

$= k(x^2y + \frac{1}{3}y^3)\Big|_{20}^{30}$

$= k(10x^2 + \frac{19000}{3})$

e. since $f(x,y) \neq f_X(x)\cdot f_Y(y)$

They are not independent

12. a. $P(X \geq 3) = \int_0^\infty \int_3^\infty x e^{-x(1+y)} \, dx \, dy$

$$= \int_3^\infty \int_0^\infty x e^{-x(1+y)} \, dy \, dx$$

$$= \int_3^\infty e^{-x} \, dx$$

$$\approx 0.05$$

b. $f_X(x) = \int_0^\infty x e^{-x(1+y)} \, dy$

$$= e^{-x}$$

$$f_Y(y) = \int_0^\infty x e^{-x(1+y)} \, dx$$

$$= \frac{1}{(1+y)^2}$$

$$f(x,y) \neq f_X(x) \cdot f_Y(y)$$

They are not independent.

c. $P(Y \geq 3)$

$$= \int_3^\infty \frac{1}{(1+y)^2} \, dy$$

$$= 0.0625$$

$$P(X \geq 3, Y \geq 3)$$

$$= \int_3^\infty \int_3^\infty x e^{-x(1+y)} \, dx \, dy$$

$$= 0.1875$$

$$P = P(X \geq 3) + P(Y \geq 3)$$

$$+ P(X \geq 3, Y \geq 3)$$

$$= 0.300$$

18. a. $P_{Y|X}(0|1) = \frac{0.08}{0.34} = 0.235$

$P_{Y|X}(1|1) = \frac{0.20}{0.34} = 0.588$

$P_{Y|X}(2|1) = \frac{0.06}{0.34} = 0.176$

b. $X = 2$

$P_{Y|X}(0|2) = \frac{0.06}{0.5} = 0.12$

$P_{Y|X}(1|2) = \frac{0.14}{0.5} = 0.28$

$P_{Y|X}(2|2) = \frac{0.30}{0.5} = 0.60$

The pmf $X = 2$

y	0	1	2
	0.12	0.28	0.60

c. $P(Y \le 1 \mid X = 2)$

$= 0.12 + 0.28$

$= 0.40$

d. $Y = 2$

$P_{X|Y}(0|2) = \frac{0.2}{0.38} = 0.05$

$P_{X|Y}(1|2) = \frac{0.06}{0.38} = 0.16$

$P_{X|Y}(2|2) = \frac{0.30}{0.38} = 0.79$

pmf $Y = 2$

X	0	1	2
	0.05	0.16	0.79

pmf Y = 2

X	0	1	2
	0.05	0.16	0.79

19. a. $f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} = \frac{k(x^2+y^2)}{10kx^2+0.5}$

$f_{X|Y}(x|y) = \frac{k(x^2+y^2)}{10ky^2+0.5}$ $\quad k = \frac{3}{380000}$

b. $P(Y \geq 25 \mid X=22) = \int_{25}^{30} f_{Y|X}(y|22)\,dy$

$= \int_{25}^{30} \frac{k((22)^2+y^2)}{10k(22)^2+0.05}\,dy$

$= 0.556$

$P(Y \geq 25) = \int_{25}^{30} f_Y(y)\,dy = 0.549$

c. $E(Y|X=22) = \int_{-\infty}^{\infty} y \cdot f_{Y|X}(y|22)\,dy$

$= 25.3729$

$E(Y^2|X=22) = \int_{20}^{30} y^2 \cdot f_{Y|X}(y|22)\,dy$

$= 652.0286$

$V(Y|X=22) = E(Y^2|X=22) - [E(Y|X=22)]^2$

$= 8.2440$

$\sigma = \sqrt{V(Y|X=22)} = 2.87$

5.2

$$6 = \sqrt{V(Y|X=22)} = 2.87$$

102%

5.2

24. Let $m(x,y)$ be the number of individuals of who handle the message

$p(x,y)$ be the single event $p(E)$'s probability.

$$p(x,y) = \frac{1}{6\times5} = \frac{1}{30}$$

$$E(m(x,y)) = \sum m(x,y)\cdot p(x,y)$$

$$= \sum_x \sum_y m(x,y)\cdot \frac{1}{30}$$

$$= 2.80$$

26. $$E_t = \sum_0^5 \sum_0^2 (3x+10y)\cdot p(x,y)$$

$$= 15.4$$

33. $E(XY) = E(X)\cdot E(Y)$

$$Cov(X,Y) = \sum_x \sum_y (x-E(x))(y-E(y))p(x,y)$$

0.4 0.6 0.8

26. $E_t = \sum_{0}^{5}\sum_{0}^{2} (3x+10y)\cdot p(x,y)$

$= 15.4$

33. $E(XY) = E(X)\cdot E(Y)$

$Cov(X,Y) = \sum_x\sum_y (x-E(x))(y-E(y))p(x,y)$

$= E(XY) - E(X)\cdot E(Y)$

$= 0$

$$Corr(X,Y) = \frac{Cov(X,Y)}{\sigma_x \cdot \sigma_y} = 0$$

So $Cov(X,Y) = Corr(X,Y) = 0$

35. a. $Cov(aX+b,\ cY+d)$

$= E[(aX+b)\cdot(cY+d)] - E(aX+b)\cdot E(cY+d)$

$= E(acXY) - E(aX)\cdot E(cY)$

$= ac[E(XY) - E(X)E(Y)]$

$= ac\,Cov(X,Y)$

b. $Corr(aX+b,\ cY+d)$

$$= \frac{Cov(aX+b,\ cY+d)}{SD(aX+b)\,SD(cY+d)}$$

$$= \frac{ac\ Cov(X,Y)}{|a|\cdot|c|\,SD(X)SD(Y)}$$

$$= \frac{ac}{|ac|}Corr(X,Y)$$

if $a \cdot c$ are same signs

$Corr(aX+b,\ cY+d) = Corr(X,Y)$

c. if not

$Corr(aX+b,\ cY+d) = -Corr(X,Y)$

0.4 0.6 0.8