2021103523 黄毓乾

注：老师好，我的作业是左右两栏竖列排版。

A+

Homework 11 2021103523 黄毓乾

Section 5.3 38 41

Ex.38 a.)

The each X is 0, 1, or 2, and the possible values of T_0 are 1, 2, 3 and 4.

$P(T_0=0)=P(X_1=0 \text{ and } X_2=0)=0.2\times0.2=0.04$

Since X_1 and X_2 are independent.

$P(T_0=1)=P(X_1=1 \text{ and } X_2=0 \,||\, X_1=0 \text{ and } X_2=1)$

$=0.5\times0.2+0.2\times0.5=0.20$

Then we can also get ~~P(T₀)=~~ $P(T_0=2)=0.37$,

$P(T_0=3)=0.30$, $P(T_0=4)=0.09$

We draw a ~~pdf~~ pmf table to show it:

t_0	0	1	2	3	4
$p(t_0)$	0.04	0.20	0.37	0.30	0.09

b.)

We can get that:

$E(T_0)=0\times0.04+1\times0.20+2\times0.37+3\times0.30+4\times0.09=2.2$

There is an exact twice the population mean: $E(T_0)=2\mu$.

c.)

$E(T_0^2)=0^2\times0.04+1^2\times0.20+2^2\times0.37+3^2\times0.30+4^2\times0.09$

$=5.82$; And then $V(T_0)=5.82-(2.2)^2=0.98$

There is an exact twice the population variance:

~~$V(T_0)=2\sigma_0^2$~~ $V(T_0)=2\sigma^2$

d.)

The pattern persists all the time, when $T_0=X_1+X_2+X_3+X_4$ we have $E(T_0)=4\mu=4\times1.1=4.4$; At the same time $V(T_0)=4\sigma^2=1.96$

e.) The event $\{T_0=8\}$ occurs when we encounter 2 lights on all 4 trips.

ie, $X_i=2$ for each X_i. Hence, we assume the X_i are independent.

$P(T_0=8)=P(X_1=2\cap X_2=2\cap X_3=2\cap X_4=2)=P(X_1=2)\cdots$

$P(X_4=2)=0.3^4=0.0081$

We also can write $T_0=7$ iff exactly three of the X_i are 2 and the remaining X_i is 1.

The probability of the event is

$P(T_0=7)=0.3\times0.3\times0.3\times0.5+0.3\times0.3\times0.5\times0.3+\ldots$

$=4\times0.3^3\times0.5=0.054$

Hence, $P(T_0\geq7)=P(T_0=7)+P(T_0=8)=0.054+0.0081$

$=0.0621$

Ex.41

We build a table below to describe all 16 possible (x_1,x_2) pairs. And probabilities are calculated using the independence of X_1 and X_2.

(x_1,x_2)	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4
Probability	0.16	0.12	0.08	0.04	0.12	0.09	0.06	0.03
$\bar{x}$	1	1.5	2	2.5	1.5	2	2.5	3
r	0	1	2	3	1	0	1	2

(x_1,x_2)	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
probability	0.08	0.06	0.04	0.02	0.04	0.03	0.02	0.01
$\bar{x}$	2	2.5	3	3.5	2.5	3	3.5	4
r	2	1	0	1	3	2	1	2

a) We obtain the $\bar{x}$ values from the table above yields the pmf table below:

$\bar{x}$	1	1.5	2	2.5	3	3.5	4
$p(\bar{x})$	0.16	0.24	0.25	0.20	0.10	0.04	0.01

b) $p(\bar{x} \le 2.5) = 0.16 + 0.24 + 0.25 + 0.20 = 0.85$

c) We obtain the r values from the table above yields the pmf table below:

r	0	1	2	3
p(r)	0.30	0.40	0.22	0.08

d) When $n=6$, there are many ways to get a sample average ≤ 1.5.

Since $\bar{X} \le 1.5$ iff the sum of X_i is at most 6.

$$P(\bar{X} \le 1.5) = P(1,1,1,1) + P(2,1,1,1) + \dots + P(1,1,1,2)$$
$$+ P(1,1,2,2) + \dots + P(2,2,1,1) + P(3,1,1,1) + \dots P(1,1,1,3)$$
$$= 0.4^4 + 4(0.4)^3 + 6(0.4)^2(0.3)^2 + 4(0.4)^2(0.2)^2 = 0.2400$$

Section 5-4 46, 51, 55

Ex.46

a) The sampling distribution of $\bar{X}$ is centered at $E(\bar{X}) = \mu = 12$ cm. The standard deviation of the $\bar{X}$ distribution is $\sigma_{\bar{x}} = \frac{\sigma_{\bar{x}}}{\sqrt{n}} = \frac{0.04}{\sqrt{16}} = 0.01$ cm

b) When $n=64$, the sampling distribution of $\bar{X}$ is still centered at $E(\bar{X}) = \mu = 12$ cm, but the standard deviation of the $\bar{X}$ distribution is

$$\sigma_{\bar{X}} = \frac{\sigma_{\bar{x}}}{\sqrt{n}} = \frac{0.04}{\sqrt{64}} = 0.005 \text{ cm}$$

c) Because the decreased variability of $\bar{X}$ that comes with a larger sample size, then $\bar{X}$ is more likely to be within 0.01 cm of the mean with the second, larger, sample.

Ex.51

Individual times are given by $X \sim N(10, 2)$.

① For day 1, $n=5$:

$$P(\bar{X} \le 11) = P\left(Z \le \frac{11-10}{\frac{2}{\sqrt{5}}}\right) = P(Z \le 1.12) = 0.8686$$

② For day 2, $n=6$:

$$P(\bar{X} \le 11) = P\left(Z \le \frac{11-10}{\frac{2}{\sqrt{6}}}\right) = P(Z \le 1.22) = 0.8888$$

Therefore, we assume the results of the two days are independent, the probability the sample average is at most 11 min on both day =

$$0.8686 \times 0.8888 = 0.7720$$

Ex.55

a) 11 P.M. − 6:50 P.M. = 250 min.

$T_0 = X_1 + \dots + X_{40}$ = total grading time,

$\mu_{T_0} = n\mu = 40 \times 6 = 240$

$\sigma_{T_0} = \sigma \cdot \sqrt{n} = 37.95$

$$\Rightarrow P(T_0 \le 250) \approx P\left(Z \le \frac{250-240}{37.95}\right) = P(Z \le 0.26)$$
$$= 0.6026$$

b) We know that the sports report begins 260 min after he begins grading papers.

$$P(T_0 > 260) = P\left(Z > \frac{260-240}{37.95}\right) = P(Z > 0.53) = 0.2981$$

Section 5.5 58, 70, 73

Ex.58

a) $E(27X_1+125X_2+512X_3) = 27E(X_1)+125E(X_2)+512E(X_3)$

$=27\times 200+125\times 250+512\times 100=87850$

$V(27X_1+125X_2+512X_3)=27^2V(X_1)+125^2V(X_2)+512^2V(X_3)$

$=27^2\times 10^2+125^2\times 12^2+512^2\times 8^2$

$=19100116$

b) We can say that:

The expected value is correct, but the variance is not, because the covariances now also contribute to the variance.

Ex.70

a) $E(Y_i)=\frac{1}{2}$

$\Rightarrow E(W)=\sum_{i=1}^{n} i\cdot E(Y_i)=\frac{1}{2}\sum_{i=1}^{n} i=\frac{n(n+1)}{4}$

b) $V(Y_i)=\frac{1}{4}$

$\Rightarrow \cancel{E}V(W)=\sum_{i=1}^{n} i^2 V(Y_i)=\frac{1}{4}\sum_{i=1}^{n} i^2=\frac{n(n+1)(2n+1)}{24}$

Ex.73

a) Because of the Central Limit Theorem, both are approximately normal.

b) The linear combination of normal rvs has a normal distribution.

Therefore, $\bar{X}-\bar{Y}$ has approximately a normal distribution with $\mu_{\bar{X}-\bar{Y}}=5$ and

$\sigma_{\bar{X}-\bar{Y}}=\sqrt{\frac{8^2}{40}+\frac{6^2}{35}}=1.621$

c) $P(-1\le \bar{X}-\bar{Y}\le 1)\approx P(\frac{-1-5}{1.6213}\le Z\le \frac{1-5}{1.6213})$

$=P(-3.70\le Z\le -2.47)\approx 0.0068$

d) If $\mu_1-\mu_2=5$

$P(\bar{X}-\bar{Y}\ge 10)\approx P(Z\ge \frac{10-5}{1.6213})=P(Z\ge 3.08)=0.0010$

Because the probability is very small, which ~~该~~ occurs unlikely. (if $\mu_1-\mu_2=5$)

Hence, we will doubt the claim.