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概率统计 Test 2

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1. $F(x) = \int_{-\infty}^{x} f(y)\,dy = \int_{0}^{x} (\frac{1}{8} + \frac{3}{8}y)\,dy = \frac{x}{8} + \frac{3}{16}x^2$

So:

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{8} + \frac{3}{16}x^2, & 0 \le x \le 2 \\ 1, & x > 2 \end{cases}$$

$$\begin{aligned} p(1 \le x \le 1.5) &= F(1.5) - F(1) \\ &= (\frac{1.5}{8} + \frac{3}{16} \times 1.5^2) - (\frac{1}{8} + \frac{3}{16}) \\ &= 0.297 \end{aligned}$$

$$\begin{aligned} p(X > 1) = 1 - F(1) &= 1 - (\frac{1}{8} + \frac{3}{16}) \\ &= 0.688 \end{aligned}$$

2.

(A) It is clear that $f(x) \geq 0$ (Always)

Then we have to prove: $\int_{-\infty}^{+\infty} f(x)\,dx = 1$

$$\begin{aligned} \int_{-\infty}^{+\infty} f(x)\,dx = \int_{0.5}^{\infty} 0.15 e^{-0.15(X-0.5)} &= 0.15 e^{0.075} \int_{0.5}^{\infty} e^{-0.15X}\,dx \\ &= 0.15 e^{0.075} \cdot \frac{1}{0.15} e^{-(0.15)(0.5)} \\ &= 1 \end{aligned}$$

So, formula (1) satisfy the pdf condition

(B)
$$\begin{aligned} p(X \le 5) = \int_{-\infty}^{5} f(x)\,dx = \int_{0.5}^{5} 0.15 e^{-0.15(X-0.5)}\,dX &= 0.15 e^{0.075} \int_{0.5}^{5} e^{-0.15X}\,dx \\ &= 0.15 e^{0.075} \times \left(-\frac{1}{0.15} e^{-0.15X}\right)\Big|_{0.5}^{5} \\ &= 0.491 \end{aligned}$$

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3. Poisson distribution

(A) $P(X=5) = \dfrac{e^{-4.5}(4.5)^5}{5!} = 0.1708$

(B) $P(X \le 5) = \sum_{x=0}^{5} \dfrac{e^{-4.5}(4.5)^x}{x!}$

$= \left(1 + 4.5 + \dfrac{4.5^2}{2!} + \dfrac{4.5^3}{3!} + \dfrac{4.5^4}{4!} + \dfrac{4.5^5}{5!}\right)e^{-4.5}$

$= 0.7029$

4. According to the problem:

$P(\mu-\sigma \le X \le \mu+\sigma) = P\left(\dfrac{\mu-\sigma-\mu}{\sigma} \le Z \le \dfrac{\mu+\sigma-\mu}{\sigma}\right)$

$= P(-1 \le Z \le 1)$

$= \Phi(1) - \Phi(-1) = 0.6826$ (Table A.3)

5.

(A) $P(0 \le X \le \frac{1}{4}, 0 \le Y \le \frac{1}{4}) = \dfrac{6}{5}\int_0^{\frac{1}{4}}\int_0^{\frac{1}{4}}(x+y^2)\,dx\,dy$

$= \dfrac{6}{5}\int_0^{\frac{1}{4}}\left(\frac{1}{2}x^2 + y^2x\right)\Big|_0^{\frac{1}{4}}\,dy$

$= \dfrac{6}{5}\int_0^{\frac{1}{4}}\left(\frac{1}{32} + \frac{1}{4}y^2\right)dy$

$= \dfrac{6}{5}\left(\frac{1}{32}y + \frac{1}{12}y^3\right)\Big|_0^{\frac{1}{4}}$

$= \dfrac{6}{5}\left(\frac{1}{128} + \frac{1}{12} \times \frac{1}{64}\right)$

$= \dfrac{7}{640}$

6.

(A) To find A: we use $F(x,y)=1$

that is: $\int_0^1\int_0^1 Axy\,dx\,dy=1$

$$\int_0^1 \frac{A}{2}y\,dy=1$$

$$\frac{A}{4}=1$$

$$A=4$$

(B) Marginal pdf of X and Y:

$$f_X(x)=\int_0^1 f(x,y)\,dy=\int_0^1 4xy\,dy=4x\cdot\left(\tfrac{1}{2}y^2\right)\Big|_0^1=2x$$

$f_Y(y)$ is: $2y$, as the structure is the same.

so: $f_X(x)=\begin{cases}2x, & 0\le x\le 1\\ 0, & \text{otherwise}\end{cases}$, $f_Y(y)=\begin{cases}2y, & 0\le y\le 1\\ 0, & \text{otherwise.}\end{cases}$

(C) $f(x,y)=4xy=f_X(x)\cdot f_Y(y)$

So: ~~A~~ X and Y are independent.

7. Because $X_1, X_2 \cdots X_n$ are independent of one another:

(A) so Joint pdf is:

$$f(x_1, x_2, \cdots x_n) = (\lambda e^{-\lambda x_1}) \cdot (\lambda e^{-\lambda x_2}) \cdots (\lambda e^{-\lambda x_n})$$

$$\text{so: } f(x_1, x_2 \cdots x_n) = \begin{cases} \lambda^n \cdot e^{-\lambda \Sigma x_i}, & x_i \geq 0 \text{ for } i = 1, 2, 3 \cdots n \\ 0, & \text{otherwise.} \end{cases}$$

(2) According to this describtion:

$$\begin{aligned} P(X_1 > t, X_2 > t, \cdots X_n > t) &= \int_t^\infty \cdots \int_t^\infty f(x_1, x_2 \cdots x_n)\, dx_1 \cdots dx_n \\ &= \int_t^\infty \lambda e^{-\lambda x_1} dx_1 \cdots \cdots \int_t^\infty \lambda e^{-\lambda x_n} dx_n \\ &= e^{-n\lambda t} \end{aligned}$$

8. According to this problem's describtion:

we know: $n=50$, which is larger than 30, so we can apply CLT Theorem to this problem:

As for $\bar{x}$: $\mu_{\bar{x}} = 4.0$, $\sigma_{\bar{x}} = \frac{1.5}{\sqrt{50}} = 0.2121$

So: $P(3.5 \le \bar{x} \le 3.8) \approx P\left(\frac{3.5-4.0}{0.2121} \le Z \le \frac{3.8-4.0}{0.2121}\right)$

$$= P(-2.36 \le Z \le -0.94)$$

$$= \Phi(-0.94) - \Phi(-2.36)$$

$$= 0.1736 - 0.0091$$

$$= 0.1645 \quad \text{(Table A.3)}$$