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A

1. a. $\hat{m} = \bar{x} = \frac{\sum x_i}{n} = \frac{219.8}{27} = 8.1407$

b. $\tilde{x} = 7.7$

c. $s = \sqrt{\frac{1860.94 - \frac{(219.8)^2}{27}}{26}} = 1.660$

d. $\hat{p} = \frac{x}{n} = \frac{4}{27} = 0.1481$

e. $\frac{s}{\bar{x}} = \frac{1.660}{8.1407} = 0.2039$

8. a. $\hat{q} = \frac{12}{80} = .150$

b. $\hat{p}^2 = \left(\frac{68}{80}\right)^2 = .723$

9. a. $I: \sum x_i = (0)(18) + (1)(37) + \cdots + (7)(1) = 317$

$\hat{I} = \bar{x} = \frac{317}{150} = 2.11$

b. $\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{\sqrt{I}}{\sqrt{n}}$ $\quad \frac{\sqrt{\hat{I}}}{\sqrt{n}} = \frac{\sqrt{2.11}}{\sqrt{150}} = .119$

13. $E(x) = \int_{-1}^{1} x \cdot \frac{1}{2}(1+qx)\,dx = \frac{x^2}{4} + \frac{qx^3}{6}\Big|_{-1}^{1}$

$= \frac{1}{3}q$

$E(\bar{x}) = \frac{1}{3}q \quad \hat{q} = 3\bar{x} \Rightarrow E(\hat{q}) = E(3\bar{x}) = 3E(\bar{x}) = q$

20. a. $\frac{d}{dp}\left[\ln\binom{n}{x} + x\ln(p) + (n-x)\ln(1-p)\right] = \frac{x}{p} - \frac{n-x}{1-p}$

$\hat{p} = \frac{x}{n} \quad n = 20 \Rightarrow x = 3 \quad \hat{p} = .15$

b. $E(\hat{p}) = E\left(\frac{X}{n}\right) = \frac{1}{n}E(x) = \frac{1}{n}(np) = p$

c. $(1-.15)^5 = .4437$

21. a. $E(x) = b \cdot \Gamma\left(1+\frac{1}{a}\right)$

$E(x^2) = Var(x) + [E(x)]^2 = b^2\Gamma\left(1+\frac{2}{a}\right)$

$\frac{1}{n}\sum\frac{x_i^2}{\bar{x}^2} = \frac{\Gamma\left(1+\frac{2}{\hat{a}}\right)}{\Gamma^2\left(1+\frac{1}{\hat{a}}\right)}$

b. $\frac{1}{20}\left(\frac{16500}{28^2}\right) = 1.05 = \frac{\Gamma\left(1+\frac{2}{\hat{a}}\right)}{\Gamma^2\left(1+\frac{1}{\hat{a}}\right)}$

$\frac{1}{\hat{a}} = .2 \quad \Rightarrow \hat{a} = 5$

$\hat{b} = \frac{\bar{x}}{\Gamma(1.2)} = \frac{28.0}{\Gamma(1.2)}$

29. a. $f(x_1, \dots x_n; l, q) = \begin{cases} l^n e^{-l\Sigma(x_i - q)} \\ 0 \end{cases}$

$\begin{cases} l^n \exp(-l\Sigma x_i) \exp(nlq) \\ 0 \end{cases}$

$\hat{l} = \frac{n}{\Sigma(x_i - \hat{q})} = \frac{n}{\Sigma x_i - n\hat{q}}$

b. $\hat{q} = \min(x_i) = .64$ $\quad \Sigma x_i = 55.8$ so $\hat{l} = .202$

32. a. $F_Y(y) = P(Y \le y) = P(X_1 \le y, \dots, X_n \le y)$

$= \left(\frac{y}{q}\right)^n \Rightarrow f_Y(y) = \frac{ny^{n-1}}{q^n}$

b. $E(Y) = \int_0^q y \cdot \frac{ny^{n-1}}{n} dy = \frac{n}{n+1} q$

$E\left[\frac{n+1}{n} Y\right] = \frac{n+1}{n} E(Y) = q$

$k = \frac{n+1}{n}$ does u trick.