Physics CST (2022-23) Homework 3

Please send the completed file to my mailbox yy.lam@qq.com by October 26th, by using the filename format:

student_number-name-cst-hw3

Please answer the questions by filling on these sheets.

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1. Two projectiles of mass m_1 and m_2 are fired at the same speed but in opposite directions from two launch sites separated by a distance D. They both reach the same spot in their highest point and strike there. As a result of the impact they stick together and move as a single body afterwards. Show that the landing place will be $\frac{m_1}{m_1+m_2}D$ measured from the launching position of m_1.

   Solution. *Assuming the mass m_1 is launched at 0, m_2 launched at D in a regular Cartesian coordinate system. Let $v_{1x}, v_{2x} > 0$ be the horizontal components of the launching velocities, $v_{1y}, v_{2y} > 0$ the corresponding vertical components, respectively. Since the horizontal motions are in inertial motion we have*

$$\frac{d}{v_{1x}} = -\frac{D-d}{-v_{2x}} = t = -\frac{v_{1y}}{g} = -\frac{v_{2y}}{g}$$

   where t is the instant when they hit at the d horizontal distance from 0 as well as reaching the highest position. The conservation of momentum gives

$$m_1 v_{1x} - m_2 v_{2x} = (m_1+m_2)v_x$$

   where v_x is the combined velocity in x-direction. As the masses have reached the maximum height, the time taken of the combined mass falling to the ground is the same as the previous t. The horizontal range traveled by the combined mass from d is

$$v_x t = \frac{m_1 v_{1x} - m_2 v_{2x}}{m_1+m_2} t$$

   Inserting the v_{1x} and v_{2x} of the first equation into this expression we get

$$v_x t = \frac{m_1}{m_1+m_2} d - \frac{m_2}{m_1+m_2}(D-d) = d - \frac{m_2}{m_1+m_2} D$$

   Since The distance $v_x t$ is measured from the distance d, the range measured from 0 is

$$2d - \frac{m_2}{m_1+m_2} D.$$

   When they have the same speed, in order both to reach the highest position to hit together the launching angle must be the same. That is, $D = 2d$ and $v_{1x} = v_{2x}$. Therefore, the range measured from 0 is

$$\frac{m_1}{m_1+m_2} D$$

2. To develop muscle tone, a woman lifts a 3.0 kg weight held in her hand. She uses her biceps muscle to flex the lower arm through an angle of 60°. (a) What is the angular acceleration if the weight is 26 cm from the elbow joint, here forearm has a moment of inertia of 0.25 kgm^2, and the net force she exerts is 760 N at an effective perpendicular lever arm of 2 cm? (b) What is the angular velocity at 60°? (c) How much work does she do?

Solution. *(a) Since*

$$\textit{net torque} = \textit{distance of the perpendicular lever arm} \times \textit{net force}$$

and also equals to $I\ddot{\theta}$ *where* I *is the* ***total*** *moment of inertia of the system, thus*

$$(0.25 + 3 \times 0.26^2)\ddot{\theta} = 0.02 \times 760 \quad \Rightarrow \quad \ddot{\theta} = 33.6 \ rad \ s^{-2}$$

(b) Assuming the angular acceleration $\ddot{\theta}$ *being constant, the rotational kinematic equation* $\dot{\theta}^2 = 2\ddot{\theta}\theta$ *with the initial angular velocity zero gives*

$$\dot{\theta} = \sqrt{2 \times 33.6 \times (60 \times 2\pi/360)} = 8.4 \ rad \ s^{-1}$$

(c) As rotational energy can be given by $\tau\theta$*, we get*

$$0.02 \times 760 \times \frac{60 \times 2\pi}{360} = 15.9 \ J$$

Note. *Using* $K.E. = \frac{1}{2}I\dot{\theta}^2$ *will give the same answer.*

3. A wind turbine is rotating counterclockwise at 0.5 rev/s and slows to a stop in 10 s. Its blades are 20 m in length. (a) What is the angular acceleration of the turbine? (b) What is the centripetal acceleration of the tip of the blades at $t = 0$ s? (c) What is the magnitude and direction of the total linear acceleration of the tip of the blades at $t = 0$ s?

   Solution. *(a) The angular acceleration is*

$$\ddot{\theta} = \frac{0 - 0.5(2\pi)}{10} = -0.1\pi \ rad \ s^{-2}.$$

   (b) The centripetal acceleration is

$$a_c = r\dot{\theta}^2 = 20 \times (0.5(2\pi))^2 = 20\pi^2 \ ms^{-2}.$$

   (c) The tangential acceleration is

$$a_t = r\ddot{\theta} = 20 \times (-0.1\pi) = -2\pi \ ms^{-2}.$$

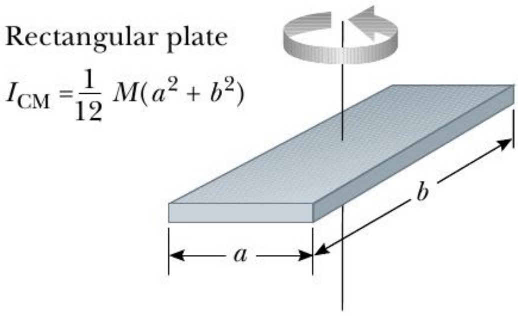
   The magnitude of the acceleration is

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{(20\pi^2)^2 + (2\pi)^2} = 197.5 \ ms^{-2}.$$

   The angle of direction is

$$\theta = \tan^{-1}\frac{-2\pi}{20\pi^2} = -1.82^\circ$$

4. Drive the formula for I_{CM} as shown in the figure.



Solution. *The formula of moment of inertia is given by* $\int r^2 \mathrm{d}m$. *Written in terms of Cartesian coordinate system that* $r^2 = x^2 + y^2$ *and* $\mathrm{d}m = \rho(\mathrm{d}x\mathrm{d}y)$ *where* ρ *is the mass per unit area. Thus,*

$$
\begin{aligned}
I &= \int_{-b/2}^{b/2} \int_{-a/2}^{a/2} \rho(x^2 + y^2)\mathrm{d}x\mathrm{d}y \\
&= \int_{-b/2}^{b/2} \int_{-a/2}^{a/2} \rho x^2 \mathrm{d}x\mathrm{d}y + \int_{-b/2}^{b/2} \int_{-a/2}^{a/2} \rho y^2 \mathrm{d}x\mathrm{d}y \\
&= \int_{-b/2}^{b/2} \rho \frac{x^3}{3}\Big|_{-a/2}^{a/2} \mathrm{d}y + \int_{-b/2}^{b/2} \rho x y^2\Big|_{-a/2}^{a/2} \mathrm{d}y \\
&= \frac{\rho}{12} a^3 y\Big|_{-b/2}^{b/2} + \rho a \frac{y^3}{3}\Big|_{-b/2}^{b/2} \\
&= \frac{\rho}{12} a^3 b + \frac{\rho}{12} a b^3
\end{aligned}
$$

Since $M = \rho ab$, *the moment of inertia is* $I_{CM} = \dfrac{1}{12} M(a^2 + b^2)$. □

5. A 200 kg rocket in deep space moves with a velocity of $(121\hat{\mathbf{i}} + 38\hat{\mathbf{j}})$ m/s. Suddenly, it explodes into three pieces, with the first (78 kg) moving at $(-321\hat{\mathbf{i}} + 228\hat{\mathbf{j}})$ m/s and the second (56 kg) moving at $(16\hat{\mathbf{i}} - 88\hat{\mathbf{j}})$ m/s. Find the velocity of the third piece.

   Solution. *The total linear momentum of the system before and after the explosion must conserve, thus*

   $$78(-321\hat{\mathbf{i}} + 228\hat{\mathbf{j}}) + 56(16\hat{\mathbf{i}} - 88\hat{\mathbf{j}}) + (200 - 78 - 56)(v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}) - 200(121\hat{\mathbf{i}} + 38\hat{\mathbf{j}}) = 0$$

   where v_x, v_y *are the components. It implies that the velocity of the third piece is* $732\hat{\mathbf{i}} - 79.6\hat{\mathbf{j}}$.

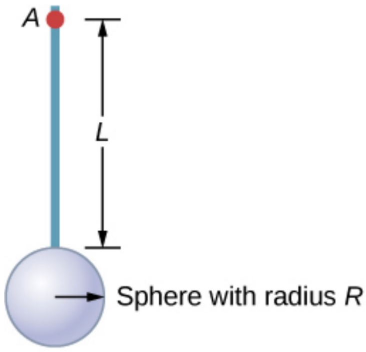
6. Find the moment of inertia of the rod and solid sphere combination about the (red colour) axis as shown. The rod has length 0.5 m and mass 2.0 kg. The radius of the sphere is 20.0 cm and has mass 1.0 kg.

   Solution. *The moments of inertia of a thin rod about one end and a solid sphere rotating about its centre are given by*

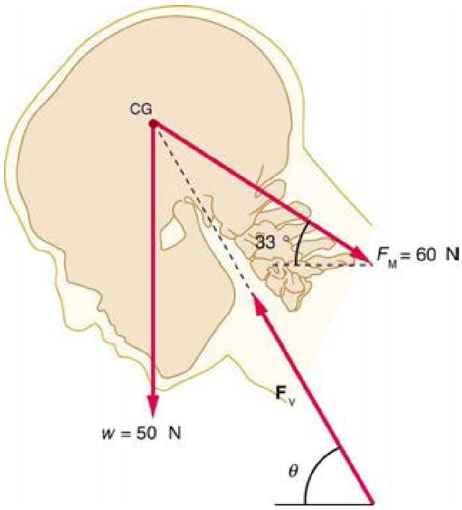
   $$I_r = \frac{1}{3} m_r l^2, \qquad I_{s(c.m.)} = \frac{2}{5} m_s R^2$$

   Using the parallel-axis theorem for the sphere, we obtain the total moment of inertia of the system

   $$I = \frac{1}{3} \times 2 \times 0.5^2 + \left(\frac{2}{5} \times 1 \times 0.2^2 + 1 \times (0.5 + 0.2)^2 \right) = 0.673 \ kgm^2.$$



7. A person holds her head as shown requiring muscle action to support the head. Calculate the direction and magnitude of the force supplied by the upper vertebrae $\mathbf{F}_v$ to hold the head stationary, assuming that this force acts along a line through the center of gravity (CG) as do the weight and muscle force.



Solution. *If we strictly use the vector product for the total torque, taking moment about the pivot point O somewhere at the base of the vertebrae, we have the vanishing of the total torque for rotational equilibrium:*

$$\boldsymbol{\ell} \times \mathbf{W} + \boldsymbol{\ell} \times \mathbf{F}_V + \boldsymbol{\ell} \times \mathbf{F}_M = 0$$

where $\boldsymbol{\ell}$ is the displacement from the pivot point to the CG. Carefully finding all the angles starting from $\boldsymbol{\ell}$ to the forces anti-clockwisely one should get

$$W \sin(\theta + 90) + 0 + F_M \sin(\theta + 147) = 0$$

The middle term vanishes as $\boldsymbol{\ell} \parallel \mathbf{F}_V$ and $\boldsymbol{\ell}$ is canceled in the expression. Inserting the values of the forces into the expression and expanding the trigonometric functions we get

$$50 \cos\theta + 60(\sin\theta \cos 147 + \sin 147 \cos\theta) = 0$$

Thus,

$$\tan\theta = -\frac{60 \sin 147 + 50}{60 \cos 147} \quad \Rightarrow \quad \theta = 58.7^\circ$$

The vanishing resultant force for linear equilibrium is

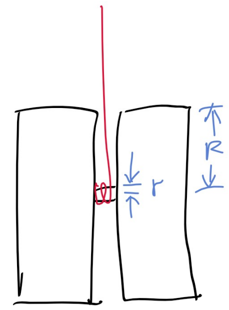
$$\mathbf{W} + \mathbf{F}_V + \mathbf{F}_M = 0$$

which gives

$$50 \sin\theta + 60\cos(\theta - 33) = F_V$$

Putting $\theta = 58.7°$ *into it we get* $F_V = 96.8\ N$.

8. A yo-yo of total mass m consists of two solid cylinders of radius R, connected by a small spindle of negligible mass and radius r. The top of the string is held motionless while the string unrolls from the spindle freely under gravity. Given the angular momentum of a cylinder $L = \pi m R^2/T$, find the linear acceleration of the yo-yo.



Solution. *The energy conservation of the system gives*

$$KE_R + KE = PE$$

where KE_R, KE *and* PE *are the rotational, the linear kinetic energies and the potential energy of the yo-yo. Let* v *be the falling velocity of the yo-yo, using the formula* $v^2 = 2as$ *we have*

$$KE_R + \frac{1}{2}mv^2 = mg\frac{v^2}{2a} \tag{1}$$

where a *is the falling acceleration of the yo-yo. For* I *is the moment of inertia of the yo-yo,* $L = I\dot{\theta}$ *where* $\dot{\theta}$ *is the angular velocity. Thus,*

$$KE_R = \frac{1}{2}I\dot{\theta}^2 = \frac{1}{2}L\dot{\theta}.$$

The tangential velocity of the spindle is equal to the linear falling velocity of the yo-yo $r\dot{\theta} = 2\pi r/T = v$ *where* T *being the period is not a constant in the system. Using the given angular momentum* $L = \pi m R^2/T$,

$$KE_R = \frac{1}{2}L\dot{\theta} = \frac{1}{2}\frac{\pi m R^2}{T}\dot{\theta} = \frac{mR^2}{4r^2}v^2.$$

Therefore, the conservation equation (1) *becomes*

$$\frac{mR^2}{4r^2}v^2 + \frac{1}{2}mv^2 = mg\frac{v^2}{2a} \quad \Rightarrow \quad \frac{R^2}{2r^2} + 1 = \frac{g}{a} \quad \Rightarrow \quad a = \frac{g}{1 + \frac{R^2}{2r^2}}$$