HW 5-3, 5-4, 5-5 A+

38) a) $X \in \{0, 1, 2\}$

$T_0 \in \{0, 1, 2, 3, 4\}$

$P(T_0 = 0) = P(X_1 = 0 \text{ and } X_2 = 0) = 0.2 \times 0.2 = 0.4$

X_1 and X_2 are independent

$P(T_0 = 1) = P(X_1 = 1 \text{ and } X_2 = 0, \text{ or } X_1 = 0 \text{ and } X_2 = 1) = 2(0.5 \times 0.2)$

$= 0.2$

$P(T_0 = 2) = 0.37$

$P(T_0 = 3) = 0.3$

$P(T_0 = 4) = 0.09$

b) $E(T_0) = 0(0.04) + 1(0.2) + 2(0.37) + 3(0.3) + 4(0.09)$

$= 2.2$

$E(T_0) = 2\mu$

c) $E(T_0^2) = 0^2(0.04) + 1^2(0.2) + 2^2(0.37) + 3^2(0.3) + 4^2(0.09)$

$= 5.82$

$V(T_0) = 5.82 - (2.2)^2$

$= 0.98$

$V(T_0) = 2\sigma^2$

d) $T_0 = X_1 + X_2 + X_3 + X_4$, $E(T_0) = 4\mu = 4(1.1) = 4.4$

$V(T_0) = 4\sigma^2 = 4(0.49) = 1.96$

e) $P(T_0 = 8)$

$= P(X_1 = 2 \cap X_2 = 2 \cap X_3 = 2 \cap X_4 = 2)$

$= P(X_1 = 2) \cdots P(X_4 = 2)$

$= (0.3)^4$

$= 0.0081$

$P(T_0 = 7)$

$= (0.3 \times 0.3 \times 0.3 \times 0.5) + (0.3 \times 0.3 \times 0.5 \times 0.3) \cdots$

$= 4(0.3)^3(0.5)$

$P(T_0 \geq 7) = P(T_0 = 7) + P(T_0 = 8)$

$= 0.054 + 0.0081$

$= 0.0621$

41) a)

$\bar{x}$	1	1.5	2	2.5	3	3.5	4
$P(\bar{x})$	0.16	0.24	0.25	0.20	0.10	0.04	0.01

b) $P(\bar{x} \leq 2.5) = 0.16 + 0.24 + 0.25 + 0.20 = 0.85$

c)

r	0	1	2	3
P(r)	0.30	0.40	0.22	0.08

d) $P(\bar{x} \leq 1.5) = P(1,1,1,1) + P(2,1,1,1) + \dots + P(1,1,1,2) + P(1,1,2,2) + \dots + P(2,2,1,1) + P(3,1,1,1)$

$+ \dots + P(1,1,1,3)$

$= (0.4)^4 + 4(0.4)^3(0.3) + 6(0.4)^2(0.3)^2 + 4(0.4)^2(0.2)^2$

$= 0.24$

46) a) sample distribution $\bar{x}$ centered at $E(\bar{x}) = \mu = 12\text{cm}$, $\sigma_{\bar{x}} = \frac{\sigma_{\bar{x}}}{\sqrt{n}} = \frac{0.04}{\sqrt{16}} = 0.01$

b) $n = 64$, $\bar{x}$ is still centered at $E(\bar{x}) = \mu = 12\text{cm}$, but $\sigma_{\bar{x}} = \frac{0.04}{\sqrt{64}} = 0.005$ cm

c) decreased variability of $\bar{x}$ have larger sample size, so $\bar{x}$ is more likely to be within 0.01 cm of the mean of the mean

51) $x \sim N(10, 2)$

day 1 : $n=5$

$P(\bar{x} \leq 11) = P(z \leq \frac{11-10}{2/\sqrt{5}})$

$= P(z \leq 1.12)$

$= 0.8686$

day 2 : $n=6$

$P(\bar{x} \leq 11) = P(\bar{x} \leq 11)$

$= P(z \leq \frac{11-10}{2/\sqrt{6}})$

$= P(z \leq 1.22)$

$= 0.8888$

Assuming they are independent, the probability of the sample average is at most 11 min on both days is $0.8686 \times 0.8888 = 0.772$

55) a) $11\text{ PM} - 6{:}50\text{ PM} = 250$ min, $T_0 = X_1 + \cdots + X_{40}$ = total grading time

$\mu_T = n\mu = 40 \times 6 = 240$

$\sigma_{T_0} = \sigma \cdot \sqrt{n} = 37.95$

$P(T_0 \le 250) \approx P\left(z \le \frac{250-240}{37.95}\right) = P(Z \le 0.26) = 0.6026$

b) sport report begin 260 min after he begin grading paper

$P(T_0 > 260) = P\left(z > \frac{260-240}{37.95}\right) = P(Z > 0.53) = 0.2981$

58) a) $E(27X_1 + 125X_2 + 512X_3)$

$= 27E(X_1) + 125E(X_2) + 512E(X_3)$

$= 27 \times 200 + 125 \times 250 + 512 \times 100$

$= 87850$

$V(27X_1 + 125X_2 + 512X_3)$

$= 27^2 V(X_1) + 125^2 V(X_2) + 512^2 V(X_3)$

$= 27^2(10)^2 + 125^2(12)^2 + 512^2(8)^2$

$= 19100116$

b) expected value is correct

variance is not correct becaus covariances now contribute to the variance

70) a) $E(Y_i) = \frac{1}{2}$, $E(W) = \sum_{i=1}^{n} i \cdot E(Y_i) = \frac{1}{2}\sum_{i=1}^{n} i = \frac{n(n+1)}{4}$

b) $V(Y_i) = \frac{1}{4}$, $V(W) = \sum_{i=1}^{n} i^2 \cdot V(Y_i) = \frac{1}{4}\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{24}$

73) a) both are approximately normal by the central limit theorem

b) difference between two rvs is just an example of linear combination, and linear combination of normal rvs has a normal distribution, so $\bar{X} - \bar{Y}$ has approximately a normal distribution with $\mu_{\bar{X}-\bar{Y}} = 5$ and $\sigma_{\bar{X}-\bar{Y}} = \sqrt{\frac{8^2}{40} + \frac{6^2}{35}} = 1.621$

c) $P(-1 \le \bar{X} - \bar{Y} \le 1) \approx P\left(\frac{-1-5}{1.6213} \le Z \le \frac{1-5}{1.6213}\right) = P(-3.70 \le Z \le -2.47) \approx 0.0068$

d) $P(\bar{X} - \bar{Y} \ge 10) \approx P\left(Z \ge \frac{10-5}{1.6213}\right) = P(Z \ge 3.08) \cdot 0.001$

occurance is not likely to happen if $\mu_1 - \mu_2 = 5$, so we reject this claim