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注：老师好，我的作业是左右两栏竖列排版。

Homework05 2021103523 黄毓乾

§ Section 3.3 29, 33, 38, 41

A+

Ex.29

a. $E(X)=\sum_D x\cdot p(x) = 1\times0.05+2\times0.10+4\times0.35+8\times0.40+16\times0.10 = 6.45$ (GB)

b. $V(X)=\sum_D (x-\mu)^2\cdot p(x) = (1-6.45)^2\times0.05+(2-6.45)^2\times(0.10)+(4-6.45)^2\times0.35+(8-6.45)^2\times0.40+(16-6.45)^2\times0.10$
$= 15.6475$

c. $\sigma=\sqrt{\sigma^2}=\sqrt{V(X)}=\sqrt{15.6475}=3.956$ (GB)

d.
$V(X)=E(X^2)-\mu^2$

$\mu^2=(6.45)^2$

$E(X^2)=\sum_D x^2 p(x) = 1^2\times0.05+2^2\times0.10+4^2\times0.35+8^2\times0.40+16^2\times0.10 = 57.25$

$V(X)=E(X^2)-\mu^2 = 57.25-(6.45)^2 = 15.6475.$

Ex.33

a. $E(X^2)=\sum_{x=0}^{1} x^2 p(x) = 0^2(1-p)+1^2(p)=p$

b. $V(X)=E(X^2)-[E(X)]^2 = p-p^2 = p(1-p)$

c. $E(X^{79}) = \cancel{\sum_D x p(x)}$
$0^{79}(1-p)+1^{79}\cdot p = p$

We find that $E(X^n)=p$, $n\geq 0$

Ex.38

We can make a comparison.

① accept the guaranteed amount $=\frac{1}{3.5}=\$\ \cancel{2.86}$ $0.286\$$

② gamble.

$E[h(X)] = E\left(\frac{1}{X}\right) = \sum_{x=1}^{6}\left(\frac{1}{x}\right)\cdot p(x) = \sum_{x=1}^{6}\left(\frac{1}{x}\right)\cdot\frac{1}{6} = 0.408\ \$$

Hence, if you want more money, gamble is better.

Ex.41

We want to prove that $V(aX+b)=a^2\cdot\sigma_X^2$

Proof:

$$V(aX+b) = \sum[aX+b-E(aX+b)]^2\cdot p(X)$$
$$= \sum[aX+b-aE(X)-b]^2\cdot p(X)$$
$$= \sum[aX-aE(X)]^2\cdot p(X)$$
$$= a^2\cdot\sum[X-E(X)]^2 p(X)$$
$$= a^2\cdot\sum[X-\mu]^2\cdot p(x)$$

Since $\sigma_X^2 = V(X) = \sum[X-\mu]^2\cdot p(x)$

$V(aX+b)=a^2\cdot\sigma_X^2$

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Section 3.4 46, 47, 48, 54

A+

Ex. 46

a) $b(3;8,0.35) = \binom{8}{3}(0.35)^3(0.65)^5 = 0.279$

b) $b(5;8,0.6) = \binom{8}{5}(0.6)^5(0.4)^3 = 0.279$

c) $P(3 \le X \le 5) = b(3;7,0.6) + b(4;7,0.6) + b(5;7,0.6)$
$= 0.745$

d) $P(1 \le X) = 1 - P(X=0) = 1 - \binom{9}{0}(0.1)^0(0.9)^9$
$= 1 - (0.9)^9 = 0.613$

Ex. 47

a) $B(4;15,0.3) = 0.515$

b) $b(4;15,0.3) = B(4;15,0.3) - B(3;15,0.3) = 0.219$

c) $b(6;15,0.7) = B(6;15,0.7) - B(5;15,0.7) = 0.012$

d) $P(2 \le X \le 4) = B(4;15,0.3) - B(1;15,0.3) = 0.480$

e) $P(2 \le X) = 1 - P(X \le 1) = 1 - B(1;15,0.3) = 0.965$

f) $P(X \le 1) = B(1;15,0.7) = 0.3^{14} \times 15 \times 0.7$
$= 10.5 \times 0.3^{14}$

g) $P(2 < X < 6) = P(2 < X \le 5) = B(5;15,0.3) -$
$B(2;15,0.3) = 0.595$

Ex. 48

For $X \sim Bin(25, 0.05)$ we have:

a) $P(X \le 2) = B(2;25,0.05) = 0.873$

b) $P(X \ge 5) = 1 - P(X \le 4) = 1 - B(4;25,0.05)$
$= 1 - 0.993 = 0.007$

c) $P(1 \le X \le 4) = P(X \le 4) - P(X \le 0) =$
$0.993 - 0.277 = 0.716$

d) $P(X=0) = P(X \le 0) = 0.277$

e) $E(X) = np = (25)(0.05) = 1.25,$
$\Rightarrow SD(X) = \sqrt{np(1-p)} = \sqrt{25(0.05)(0.95)} = 1.09$

Ex. 54

Let X be the number of customers who choose an oversize racket. Then we can write that: $X \sim Bin(10, 0.6)$

a) $P(X \ge 6) = 1 - P(X \le 5) = 1 - B(5;20,0.6) =$
$1 - 0.367 = 0.633$

b) $\mu = np = 10 \times 0.6 = 6$
$\sigma = \sqrt{10 \times 0.6 \times 0.4} = 1.55$, Hence $\mu \pm \sigma =$ $(4.45, 7.55)$

$P(4.45 < X < 7.55) = P(5 \le X \le 7) = P(X \le 7) - P(X \le 4)$
$= 0.833 - 0.166 = 0.667$

c) we find that:
$\cancel{P(\le} P(3 \le X \le 7) = P(X \le 7) - P(X \le 2) = 0.833 - 0.012$
$= 0.821$

Section 3.5 68, 69, 72, 75

~~68~~

Ex 68 ~~total~~

a) Total items ~~in~~ numbers are 20, and 12 of which are "successes". In these 20 items, 6 of which have been randomly selected to be put under the shelf. Therefore, the random variable X is hypergeometric, with $N=20$, $M=12$, $n=6$.

b) $P(X=2)=\dfrac{\binom{12}{2}\binom{20-12}{6-2}}{\binom{20}{6}}=\dfrac{(66)\times 70}{38760}=0.1192$

$P(X\le 2)=P(X=0)+P(X=1)+P(X=2)=$

$\dfrac{\binom{12}{0}\binom{8}{6}}{\binom{20}{6}}+\dfrac{\binom{12}{1}\binom{8}{5}}{\binom{20}{6}}+0.1192=0.1373$

$P(X\ge 2)=1-P(X\le 1)=1-[P(X=0)+P(X=1)]$

$=1-(0.007+0.074)=0.9819$

c) $E(X)=n\cdot\frac{M}{N}=6\cdot\frac{12}{20}=6\times 0.6=3.6$

$V(X)=\left(\frac{14}{19}\right)\times 0.6\times 6\times 0.4=1.061$

$\sigma=\sqrt{V(X)}=1.030$

Ex.69 X is hypergeometric, with $n=6$, $N=12$, $M=7$.

a) $P(X=5)=\dfrac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}}=\dfrac{105}{924}=0.114$

b. $P(X\le 4)=1-P(X>4)=1-[P(X=5)+P(X=6)]=$

$1-\left[\dfrac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}}+\dfrac{\binom{7}{6}\binom{5}{0}}{\binom{12}{6}}\right]=1-(0.114+0.07)=0.879$

c)

$E(X)=n\cdot\frac{M}{N}=6\cdot\frac{7}{12}=3.5$

$V(X)=\left(\frac{12-6}{12-1}\right)6\left(\frac{7}{12}\right)\left(1-\frac{7}{12}\right)=0.795$

$\sigma=\sqrt{V(X)}=0.892$

$\Rightarrow P(X>\mu+\sigma)=P(X>3.5+0.892)$

$=P(X>4.392)=P(X=5\text{ or }6)=0.121$.

d) Because the population size and the number of successes are large, then we approximate the hypergeometric distribution with binomial.

$\begin{cases} n=15 \\ \frac{M}{N}=\frac{40}{400}=0.1 \end{cases}$

$h(x;15,40,400)\approx b(x;15,0.10)$

$P(X\le 5)=B(5;15,0.10)=0.998$

Ex.72

a) $\begin{cases} N=11 \\ M=4 \\ n=6 \end{cases}$

$h(x;6,4,11)=\dfrac{\binom{4}{x}\binom{7}{6-x}}{\binom{11}{6}}$

b) With X = the number of "top four" interview candidates on 1st day, we can get $E(X)$.

$E(X)=n\cdot\frac{M}{N}=6\cdot\frac{4}{11}=2.18$

Ex.75

a) $P(X=x)=nb(x;2,0.5)=\binom{x+2-1}{2-1}(0.5)^2(1-0.5)^x=(x+1)(0.5)^{x+2}$

b) P(exactly 4 children) = P(exactly 2 males) = $P(X=2)$

$=nb(2;2,0.5)=(2+1)(0.5)^4=0.188$

c) P(at most 4 children) $=P(X\le 2)=\sum_{x=0}^{2}nb(x;2,0.5)$

$=0.25+0.25+0.188=0.688$

d) $E(X)=\dfrac{r(1-p)}{p}=\dfrac{2(1-0.5)}{0.5}=2$, so the expected number of children is equal to

~~$E(X)=$~~ $E(X+2)=E(X)+2=4$

Section 3.6 79, 84, 86, 87

Ex.79

Follow the cumulative Poisson table,

$F(x;\mu) = F(x;5)$

a) $P(X \le 8) = F(8;5) = 0.932$

b) $P(X=8) = F(8;5) - F(7;5) = \cancel{0.65}\ 0.065$

c) $P(X \ge 9) = 1 - P(X \le 8) = 0.068$

d) $P(5 \le X \le 8) = F(8;5) - F(4;5) = 0.492$

e) $P(5 < X < 8) = F(7;5) - F(5;5) = 0.867 - 0.616$
$= 0.251$

Ex.84

a) This is binomial with $\begin{cases} n = 10000 \\ p = 0.001 \end{cases}$

$\Rightarrow \mu = np = 10, \quad \sigma = \sqrt{npq} = \sqrt{10000(0.001)(0.999)}$
$= 3.16$

b) The X approximate to a Poisson distribution, with $\mu = 10$, Hence $\cancel{P(X)}\ P(X > 10) \approx 1 - F(10;10)$
$= 1 - 0.583 = 0.417$

c) Also the Poisson approximation,

$P(X=0) \approx \frac{e^{-10}10^0}{0!} = e^{-10} = 0.0000454$

Ex.86

a) $P(X=4) = \frac{e^{-5}5^4}{4!} = 0.175$

b) $P(X \ge 4) = 1 - P(X \le 3) = 1 - F(3;5) = 1 - 0.265$
$= 0.735$

c) The ~~per~~ people arrivals occur at the rate of 5 per hour.

Therefore, a 45-minute period the mean
$= \mu = 5 \times 0.75 = 3.75$.

Ex.87

a) The parameter of distribution is

$\lambda t = 4 \times 2 = 8$

so $P(X=10) = F(10;8) - F(9;8) = 0.099$

b) For a 30 minute period

$\Rightarrow \lambda t = 4 \times 0.5 = 2$

Hence, $P(X=0) = F(0,2) = 0.135$

c) $E(X) = \lambda t = 2$

Hence, there are 2 calls expect during their break.