概率统计作业 22CST 蒋沅翔

Section 4.3

Ex.30

(a) $\Phi(a)=0.91$, from Table A.3 we know: $\Phi(1.34)=0.9099$, so $a\approx 1.34$

(b) $\Phi(b)=0.09$, because the normal distribution is symmetric, so $b\approx -1.34$

(c) $\Phi(c)=0.75$, from Table A.3 we know: $\Phi(0.67)=0.7486$, so $c\approx 0.67$

(d) $\Phi(d)=0.25$, the same reason as question "b", $d=-0.67$

(e) $\Phi(e)=0.06$, from Table A.3 we know: $\Phi(-1.56)=0.594$, so $e\approx -1.56$

Ex.44 Let X denotes the length of the bolt thread:

(a) $P(\mu-1.5\sigma\le Z\le\mu+1.5\sigma)=\Phi(-1.5\le Z\le 1.5)=\Phi(1.5)-\Phi(-1.5)=0.9332-0.0668=0.8664$

~~(b) $P(Z\le\mu+2.5\sigma)=\Phi(Z\le 2.5)=$~~

(b) $P(Z\le\mu-2.5\sigma \text{ or } Z\ge\mu+2.5\sigma)=\Phi 1-\Phi(-2.5\le Z\le 2.5)=1-(0.9938-0.0062)=0.0124$

(c) $P(\mu-2\sigma\le Z\le\mu-\sigma \text{ or } \mu+\sigma\le Z\le\mu+2\sigma)=\Phi(-2\le Z\le -1 \text{ or } 1\le Z\le 2)=2\Phi(1\le Z\le 2)=2\times 0.1359=0.2718$

Ex.48.

(a) $P(-1.72\le Z\le -0.55)=P(0.55\le Z\le 1.72)=\Phi(1.72)-\Phi(0.55)$ (symmetry)

(b) $P(-1.72\le Z\le 0.55)=\Phi(0.55)-\Phi(-1.72)=\Phi(0.55)-(1-\Phi(1.72))=\Phi(0.55)+\Phi(1.72)-1$

Table A.1

Ex.53.

	binomial distribution ($P(15\le X\le 20)$)	normal approximation (Table A.3)
(a) $p=0.5$	$B(20;25,0.5)-B(15;25,0.5)=0.2122$	$\Phi P(14.5\le Z\le 20.5)=\Phi\left(\frac{14.5-12.5}{2.5}\le Z\le\frac{20.5-12.5}{2.5}\right)=0.3112$
$p=0.6$	the same as ↑ : 0.577	the same as ↑ : 0.5668
$p=0.8$	the same as ↑ : 0.573	the same as ↑ : 0.5957

	binomial distribution ($P(X\le 15)$)	normal approximation:
(b) $p=0.5$	$B(15;25,0.5)=0.885$	$P(Z\le 15.5)=\Phi\left(\frac{15.5-12.5}{2.5}\right)=\Phi(1.2)=0.8849$
$p=0.6$	the same as ↑, 0.575	the same as ↑ : 0.5793
$p=0.8$	the same as ↑ : 0.017	the same as ↑ : 0.122

	binomial distribution ($P(20\le X)$)	normal approximation:
(c) $p=0.5$	$1-B(19;25,0.5)=0.002$	$P(Z\ge 19.5)=\Phi 1-\Phi\left(\frac{19.5-12.5}{2.5}\right)=0.0026$
$p=0.6$	the same as ↑ : 0.029	the same as ↑ : 0.0329
$p=0.8$	the same as ↑ : 0.617	the same as ↑ : 0.5987

we find in (c) problem: the difference of approximation is a little bigger than the approximation in (a) and (b) (because in (c) $np=25\times 0.8=20$, but $npq=25\times 0.8\times 0.2=4\le 10$)

Section 4.4

Ex-56.

※ Let p^{100} be the (100p)th percentile of the standard 正态 normal distribution, then relation 式 is that: $\mu + p^{100}$

proof: $P(X \le 即 \mu + p^{100}) = P(\frac{X-\mu}{\sigma} \le p^{100}) = P(Z \le p^{100}) = p$

(替换成标准正态: $\frac{X-\mu}{\sigma}$ 来解题)

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9th Homework Section 4.4 and 4.6

F Section 4.4.

Ex.59 exponential distribution

(a) $E(X)=\frac{1}{\lambda}=1$

(b) $V(X)=\frac{1}{\lambda^2}=1$, $\sigma_X=\sqrt{V(X)}=1$

(c) $P(X\le 4)=F(4)=1-e^{-1\times 4}=1-e^{-4}\approx 0.982$

(d) $P(2\le X\le 5)=F(5)-F(2)=e^{-2}-e^{-5}\approx 0.129$

Ex.67.

(a.) $\begin{cases}\alpha\beta=24\\ \alpha\beta^2=12^2=144\end{cases}\Rightarrow\begin{cases}\alpha=4\\ \beta=6\end{cases}$

$P(12\le X\le 24)=F(4;4)-F(2;4)=0.567-0.143=0.424$

(b) $P(X\le 24)=F(4;4)=0.567$.

Yes. The median is less than 24, because $P(X\le 24)=0.567$, which means the graph is mostly like a positive skewed form.

(c) 99th percentile: According to Table A.4 $F(10;4)=0.99$, so $\frac{x}{6}=10$, $x=60$.

(d.) According to Table A.4: $F(11;4)=0.995$. So: $\frac{x}{6}=11$, $x=66$

Ex.70.

Let $F(x)=p$, then: $p=1-e^{-\lambda x}$

$e^{-\lambda x}=1-p$

$-\lambda x=\ln(1-p)$

$x=-\frac{\ln(1-p)}{\lambda}$

median: Let $p=0.5$

then: $x=\frac{\ln 0.5}{\lambda}$

No.

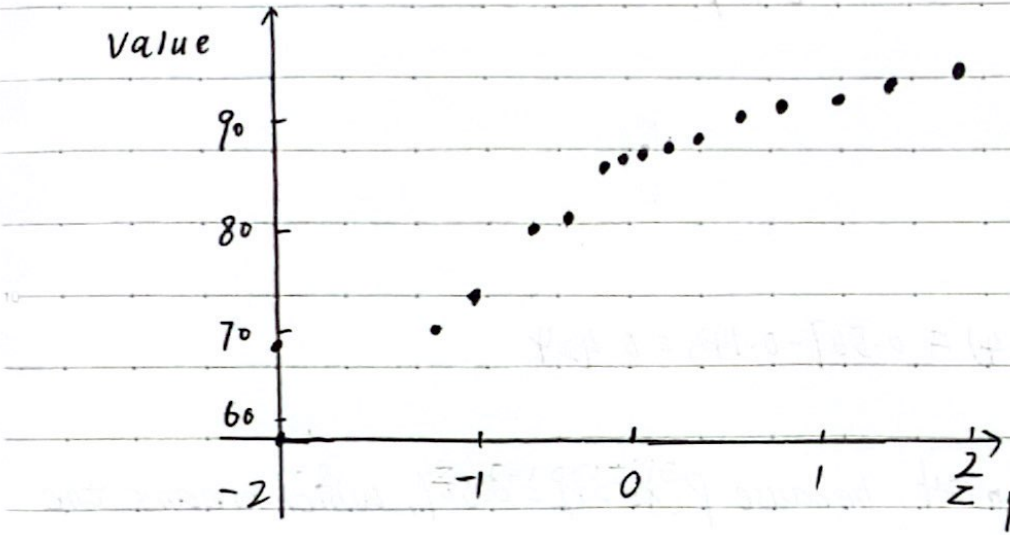
Date

Section 4.6

Ex. 87.

The graph seems linear, so it can be seen as a normal approximately. (tension distribution)

Ex. 88.



It is clear that the dot may not construct a line, so it's not plausible the distribution is normal.