6.1 1, 8, 9, 13

6.2 20, 21, 29, 32

A

1. a. Estimator = $\bar{x}$

estimate: $\bar{x} = \Sigma x_i / n = \frac{219.8}{27} = 8.14$

b. Estimator = $\tilde{x}$

estimate: $\tilde{x} = 0.77$

c. Estimator = S

estimate = $s = \sqrt{\frac{\Sigma x_i^2 - (\Sigma x_i)^2/n}{n-1}} = 1.66$

d. $p = \frac{4}{27} = 0.148$

e. Estimator = $\frac{s}{\bar{x}}$

estimate = $\frac{s}{\bar{x}} = \frac{1.66}{8.14} = 0.204$

8.

a. $p = \frac{80-12}{80} = 0.85$

b. ~~$P = p \cdot p = 0.7225$~~ $p = 0.85 \times \frac{67}{79}$

$= 0.72$

9. a、Estimator $\cancel{E(X)}$: $\bar{x}$

$$\cancel{E}(\bar{X}) = \frac{37+2\times42+3\times30+\cdots+7}{150} = 2.11$$

b、Estimated standard Error:

$$\sqrt{\frac{2.11}{150}} = 0.119$$

13、

$$E(X) = \sum x_i \cdot f(x;\theta)$$

$$E(\bar{X}) = \sum x_i \cdot f(x;\theta) = E(X)$$

$$\hat{\theta} = 3\bar{X}$$

$$E(\hat{\theta}) = 3E(\bar{X}) = E(3X) = \theta$$

So θ is an unbias estimator

20.

a. When X is binomial rv,

$$\hat{p} = \frac{X}{n} = \frac{3}{20} = 0.15$$

b. $E(\hat{p}) = E(\frac{X}{n}) = \frac{1}{n}E(X) = p$

so the estimator of part (a) is unbiased.

c. $p = (1 - \frac{3}{20})^5 = 0.44$

21. a.

$$E(X^2) = V(X) + E(X)^2$$

$$= \beta^2 \{ \Gamma(1 + 2/\alpha))$$

$$\bar{X}^2 = \beta^2 (\Gamma(1 + 2/\alpha))$$

$$\beta = \frac{\bar{X}}{\Gamma(1 + 1/\alpha)}$$

$\cancel{S^2 = (\frac{\bar{X}}{\Gamma(1+\frac{1}{\alpha})})^2}$ $\Gamma(1 + 1/\alpha)^2 = \Gamma(1 + 2/\alpha)$

$$\Gamma(1 + 1/\alpha)^2 / \Gamma(1 + 2/\alpha) = 1$$

b $[\Gamma(1.2)]^2 / \Gamma(1.4) = 0.95$ so $\alpha = 5$

$$b = \frac{28}{\Gamma(1.2)} = 28.0 / \Gamma(1.2)$$

31.

a. $E(X) = \mu + \frac{1}{\lambda}$

$V(X) = \frac{1}{\lambda^2}$

$\bar{X} = \mu + \frac{1}{\lambda}$

$S^2 = \frac{1}{\lambda^2}$

$\lambda = \frac{1}{S}$

so $\hat{\theta} = \min(X_i)$. $\hat{\lambda} = n / \Sigma(X_i - \min(X_i))$

b. $\hat{\theta} = \min(X_i) = 0.64$

$\hat{\lambda} = \frac{10}{\Sigma(X_i - \min(X_i))} = 0.202$

32 a. $F_Y(y) = P(Y \le y)$

$= P(X_1 \le y, X_2 \le y, \cdots, X_n \le y)$

$= (P(X \le y))^n$

For $(X_i \le y) = \frac{y}{\theta}$

$F_Y(y) = \begin{cases} (\frac{y}{\theta})^n & 0 < y < \theta \\ 0 & \text{otherwise} \end{cases}$

b. $E(\hat{\theta}) = E(Y)$

$$= \int_0^{\theta} y\, f_Y(y)\, dy$$

$$= \frac{n}{n+1}\theta$$

$$E\left(\frac{n+1}{n} Y\right) = \frac{n+1}{n} E(Y)$$

$$= \theta$$

Thus, it is an unbiased estimator for θ

So the mle is biased but that $(n+1)\max(X_i)/n$ is unbiased.