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$\int_{20}^{30}\int_{20}^{30} K(x^2+y^2)\,dx\,dy = 1 \Rightarrow K = \frac{3}{380000}$.

A

b. P(both tires are underfilled) $= \int_{20}^{26}\int_{20}^{26} \frac{3}{380000}(x^2+y^2)\,dx\,dy$

$= \int_{20}^{26}\int_{20}^{26} \frac{3}{380000} x^2\,dx\,dy + \int_{20}^{26}\int_{20}^{26} \frac{3}{380000} y^2\,dy\,dx$

$= \frac{2\times 3\times 114912}{380000} = 0.3024$

c. That is to find the area between line $l_1: y = x+2$, $l_2: y = x-2$, we have P(difference at most 2 psi) $= 1 - 2\cdot\int_{20}^{x-2}\int_{22}^{30} f(x,y)\,dx\,dy = 0.3593$

d. $f_X(x) = \int_{20}^{30} \frac{3}{380000}(x^2+y^2)\,dy = \frac{3}{38000}x^2 + 0.05 \quad (20 \le x \le 30)$

e. Since x can be replaced by y in $f_X(x)$, so it's not possible that $f(x,y) = f(x)\cdot f(y)$, so they're not independent.

12. a. P(the first component lifetime exceeds 3) $= \int_3^\infty\int_0^\infty x e^{-x(1+y)}\,dy\,dx = e^{-3} = 0.05$

b. $f_X(x) = \int_0^\infty x e^{-x(1+y)}\,dy = e^{-x}$, $f_Y(y) = \int_0^\infty x e^{-x(1+y)}\,dx = -\frac{1}{1+y}$, so $f(x,y) \ne f_X(x)\cdot f_Y(y)$, so they're not independent.

c. P(at least one lifetime exceeds 3) $= 1 - \int_0^3\int_0^3 x e^{-x(1+y)}\,dy\,dx = 0.3$

13. a. $P_{Y|X}(0|1) = 0.235$

$P_{Y|X}(1|1) = 0.588$

$P_{Y|X}(2|1) = 0.177$

b. $P_{Y|X}(0|2) = 0.12$

$P_{Y|X}(1|2) = 0.28$

$P_{Y|X}(2|2) = 0.6$

c. $P(Y \le 1 | X = 2) = P_{Y|X}(0|2) + P_{Y|X}(1|2) = 0.4$

d. $P_{X|Y}(0|2) = 0.053$

$P_{X|Y}(1|2) = 0.158$

$P_{X|Y}(2|2) = 0.789$

the pmf is as follows:

y	0	1	2
$P_{Y\|X}(y\|2)$	0.12	0.28	0.6

the pmf is as follows:

x	0	1	2
$P_{X\|Y}(x\|2)$	0.053	0.158	0.789

19. a. $f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} = \frac{K(x^2+y^2)}{10Kx^2+0.05}$

$f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)} = \frac{K(x^2+y^2)}{10Ky^2+0.05}$

b. $P(X=22 \text{ and } Y \ge 25) = \int_{25}^{30} \frac{f(22,y)}{f_X(22)} = \frac{K(22^2+y^2)}{10K\cdot 22^2+0.05}\,dy = 0.558$

$P(Y \ge 25) = \int_{25}^{30} f_Y(y)\,dy = 0.550$ They are almost the same.

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$\cdots = 22) = \int_{20}^{30} y \cdot \dfrac{K(22^2+y^2)}{10K \cdot 22^2 + 0.05}\, dy = 25.37$

$E(Y^2 | X=22) = \int_{20}^{30} y^2 \cdot \dfrac{K(22^2+y^2)}{10K(22^2)+0.05}\, dy = 652.03$

$\Rightarrow V(Y|X=22) = E(Y^2|X=22) - E^2(Y|X=22) = 8.24$

$\sigma = \sqrt{V} = 2.87$.

5.2 24. Let $h(x)$ denotes the number of people handling message, then

$h(x,y)$	y=1	2	3	4	5	6
x=1	1	2	3	4	3	2
2	2	1	2	3	4	3
3	3	2	1	2	3	4
4	4	3	2	1	2	3
5	3	4	3	2	1	2
6	2	3	4	3	2	1

and $p(x,y) = \dfrac{1}{A_6^2} = \dfrac{1}{30}$

So $E[h(x,y)] = \sum\sum h(x,y) \cdot p(x,y) = (2+2+\cdots+2) \cdot \frac{1}{30} = 2.8$

26. Let $h(x,y)$ denotes the fee when there're X cars and y buses, then

$h(x,y) = 3x + 10y$.

So $E[h(x,y)] = \sum\sum h(x,y) \cdot p(x,y) = 0.025 \times 0 + 0.015 \times 10 + \cdots + 0.02 \times 35 = 15.4$

33. Since X, Y are independent, we have $E(XY) = E(X) \cdot E(Y)$.

And $Cov(X,Y) = E(XY) - \mu_X \mu_Y = E(X) \cdot E(Y) - E(X) \cdot E(Y) = 0$

Then, $corr(X,Y) = \dfrac{cov(X,Y)}{\sigma_X \cdot \sigma_Y} = 0$, too.

35. a. Since $cov(aX+b, cY+d) = E[(aX+b)(cY+d)] - E(aX+b)E(cY+d)$

$= E(acXY + adX + bcY + bd) - (aE(X)+b)(cE(Y)+d)$

$= ac E(XY) + ad E(X) + bc E(Y) + bd - ac E(X)E(Y) - ad E(X) - bc E(Y) - bd$

$= ac[E(XY) - E(X)E(Y)] = ac\, Cov(X,Y)$.

b. $Corr(aX+b, cX+d) = \dfrac{ac\, cov(X,Y)}{\sigma(aX+b)\sigma(cY+d)} = \dfrac{ac\, cov(X,Y)}{|ac|\sigma(X)\sigma(Y)} = \dfrac{cov(X,Y)}{\sigma(X)\sigma(Y)} = corr(X,Y)$

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c. $corr(aX+b, cX+d) = -corr(X,Y)$.