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注：老师好，我的作业是左右两栏竖列排版。

Homework 03 2021103523 黄毓乾

Section 2.4: 46, 50, 58, 63

Ex.46

Let A be that the individual is more than 6 feet tall.
Let B be that the individual is a professional basketball player.

The $P(A|B)$ is larger.
Because most professional basketball players are tall, then the probability of an individual is more than 6 feet tall is very large in the reduced sample space.

Ex.50

a. $P(M \cap LS \cap PR) = 0.05$

b. $P(M \cap Pr) = P(M \cap LS \cap PR) + P(M \cap SS \cap PR)$
$= 0.05 + 0.07 = 0.12$

c. $P(SS) = 0.56$, $P(LS) = \cancel{1-0.56}\ 1 - 0.56 = 0.44$

d. $P(M) = 0.08 + 0.07 + 0.12 + 0.10 + 0.05 + 0.07 = 0.49$
$P(Pr) = 0.02 + 0.07 + 0.07 + 0.02 + 0.05 + 0.02 = 0.25$

e. $P(M|SS \cap Pl) = \frac{P(M \cap SS \cap Pl)}{P(SS \cap Pl)} = \frac{0.08}{0.04 + 0.08 + 0.03}$
$= 0.533$

f. $P(SS|M \cap Pl) = \frac{P(SS \cap M \cap Pl)}{P(M \cap Pl)} = \frac{0.08}{0.08 + 0.10}$
$= 0.444$

$P(LS|M \cap Pl) = 1 - P(SS|M \cap Pl) = 1 - 0.444 = 0.556$

A+

Ex.58

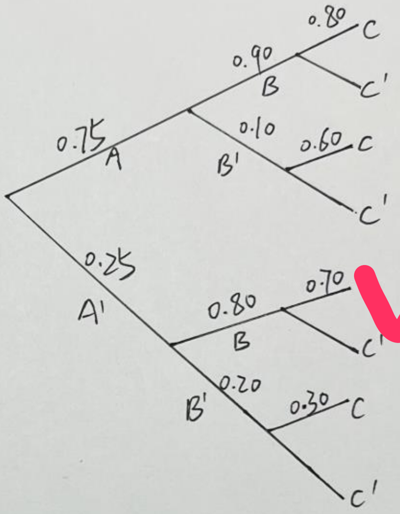
$P(A \cup B|C) = \frac{P[(A \cup B) \cap C]}{P(C)} = \frac{P[(A \cap C) \cup (B \cap C)]}{P(C)}$

$= \frac{P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)}{P(C)}$ $\cancel{= P(A)}$

$= P(A|C) + P(B|C) - P(A \cap B|C)$

Ex.63

a. First we draw a tree diagram.



b. $P(A \cap B \cap C) = 0.75 \times 0.9 \times 0.8 = 0.54$

c. $P(B \cap C) = P(A \cap B \cap C) + P(A' \cap B \cap C) = 0.54 + 0.25 \times 0.8 \times 0.7 = 0.68$

d. $P(C) = P(A \cap B \cap C) + P(A' \cap B \cap C) + P(A \cap B' \cap C) + P(A' \cap B' \cap C) = 0.54 + 0.045 + 0.14 + 0.015 = 0.74$

e. Rewrite the conditional probability

$\Rightarrow P(A|B \cap C) = \frac{P(A \cap B \cap C)}{P(B \cap C)} = \frac{0.54}{0.68} = 0.7941$

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Section 2.5 71, 72, 80, 84

Ex.71

event A: the Asian project is successful.
event B: the European project is successful.

a. Because both events are independent, we get A' and B' are independent.
$\Rightarrow P(B'|A') = P(B') = 1-0.7 = 0.3$

b. $P(A\cup B) = P(A)+P(B)-P(A\cap B)$
$= 0.4+0.7-0.4\times 0.7 = 0.82$
Because A and B both are independent.
Hence, the probability that at least one ~~$\Rightarrow P(A\cap B)$~~ of the two projects will be successful is 0.82.

c. This also involves conditional probability.

$$P(AB'|A\cup B) = \frac{P(AB'\cap(A\cup B))}{P(A\cup B)} = \frac{P(AB')}{P(A\cup B)} = \frac{P(A)P(B')}{P(A\cup B)} = \frac{0.4\times(1-0.7)}{0.82} = 0.1463$$

Ex.72

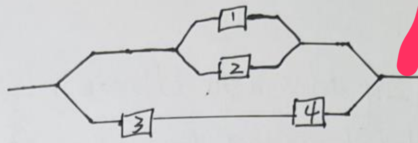
① For A_1 and A_2, they are not independent, since $P(A_1\cap A_2)=0.11$, but $P(A_1)P(A_2)=0.055$

② For A_1 and A_3, they are not independent, since $P(A_1\cap A_3)=0.05$, but $P(A_1)P(A_3)=0.0616$

③ For A_2 and A_3, they are independent, since $P(A_2\cap A_3)=0.07$, ~~but~~ and $P(A_2)P(A_3)=0.07$, they are equal.

Ex.80

A



let i be the component number.
Let A_i be the event that component i works.
(Note: $i = 1,2,3,4$)

The event for "System works" is $(A_1\cup A_2)\cup(A_3\cap A_4)$

$P(A_1\cup A_2) = P(A_1)+P(A_2)-P(A_1\cap A_2) = 0.99$
$P(A_3\cap A_4) = 0.9\times 0.9 = 0.81$
$\Rightarrow P((A_1\cup A_2)\cup(A_3\cap A_4)) = P(A_1\cup A_2)+P(A_3\cap A_4)-P(A_1\cup A_2)\cap(A_3\cap A_4)$
$= P(A_1\cup A_2)+P(A_3\cap A_4)-P(A_1\cup A_2)\times P(A_3\cap A_4)$
$= 0.99+0.81-0.99\times 0.81 = 0.9981$

Ex.84

let i be the vehicle number.
Let A_i be the event that vehicle i passes inspection ($i=1,2,3$)

a. $P(A_1\cap A_2\cap A_3) = P(A_1)\times P(A_2)\times P(A_3) = 0.7^3 = 0.343$

b. b is the complement of a.
$P(b) = 1-P(A_1\cap A_2\cap A_3) = 1-0.343 = 0.657$

c. $P(c) = P([A_1\cap A_2'\cap A_3']\cup[A_1'\cap A_2\cap A_3']\cup[A_1'\cap A_2'\cap A_3])$
$= 0.7\times 0.3\times 0.3+0.3\times 0.7\times 0.3+0.3\times 0.3\times 0.7 = 0.189$

d. P(at most one vehicle passes) = P(no one pass) + P(exactly one vehicle passes)
$P(\text{no one pass}) = P(A_1'\cap A_2'\cap A_3') = 0.3^3 = 0.027$
P(exactly one vehicle passes) = 0.189
$\Rightarrow$ P(d) = P(at most one vehicle passes) = 0.027+0.189 = 0.216

e. $P(A_1\cup A_2\cup A_3) = 1-0.027 = 0.973$

$$P(A_1\cap A_2\cap A_3|A_1\cup A_2\cup A_3) = \frac{P([A_1\cap A_2\cap A_3]\cap[A_1\cup A_2\cup A_3])}{P(A_1\cup A_2\cup A_3)} = 0.3525$$

Section 3.1 4, 5, 8, 10

Ex.4

We know that Zip codes in the US are 5 digits.
And we find that 00000 does not exist.
Hence, $X = 2, 3, 4, 5$
for $X = 3$, we have 80044
for $X = 4$, we have 44098
for $X = 5$, we have 35238

Ex.5

No. Consider a random experiment of rolling a fair six-sided die. The sample space S for this experiment is finite, consisting of the number $\{1,2,3,4,5,6\}$. Now let's define a random variable X as follows:
X represents the outcome of rolling the die minus 1.
In this case, X takes values from the set $\{0,1,2,3,4,5\}$, which is finite. Hence, it does not have an infinite set of possible values.

Ex.8

We know that the least possible value of Y is 3. And $Y \in \{3,4,5,6,7,8 \cdots\cdots\}$
$Y=3$, SSS; $Y=4$, FSSS; $Y=5$, FFSSS, SFSSS;
$Y=6$, SSFSSS, SFFSSS, FSFSSS, FFSSSS;
$Y=7$, SSFFSSS, SFSFSSS, SFFSSSS, FSSFSSS, FSFFSSS, FFSFSSS, FFFSSSS.

Ex.10

a. $V(T) = 0,1,2,3,4,5,6,7,8,9,10$
b. $V(X) = -4,-3,-2,-1,0,1,2,3,4,5$
c. $V(U) = 0,1,2,3,4,5,6$
d. $V(Z) = 0,1,2$

Section 3.2 12, 23, 25

Ex.12

a. $P(Y \le 50) = 0.05+0.10+0.12+0.14+0.25+0.17 = 0.83$
b. b is the complement of a.
$P(Y > 50) = 1 - P(Y \le 50) = 1 - 0.83 = 0.17$
c. ① For you're the first standby passenger:
$P(Y \le 49) = 0.05+0.10+0.12+0.14+0.25 = 0.66$
② For you're third standby passenger:
$P(Y \le 47) = 0.05+0.10+0.12 = 0.27$

Ex.23

a. $p(2) = P(X=2) = F(3) - F(2) = 0.39 - 0.19 = 0.20$
b. $P(X>3) = 1 - P(X \le 3) = 1 - F(3) = 1 - 0.67 = 0.33$
c. $P(2 \le X \le 5) = F(5) - F(2-1) = F(5) - F(1) = 0.78$
d. $P(2 < X < 5) = P(2 < X \le 4) = F(4) - F(2) = 0.53$

Ex.25

$P(B) = P$, $P(G) = 1-P$
$p(0) = P(Y=0) = P(B) = P$
$p(1) = P(Y=1) = P(GB) = (1-P)P$
$p(2) = P(Y=2) = P(GGB) = (1-P)^2 P$
$p(3) = P(Y=3) = P(GGGB) = (1-P)^3 P$
And so on, we get the general formula:
$p(Y) = P(Y\text{ Gs then a B}) = (1-P)^Y P$ for $Y = 0,1,2,3\ldots$