B+

Section 5.1.

9. a. $\int_{20}^{30}\int_{20}^{30} k(x^2+y^2)\,dy\,dx = 1$

$\cancel{\Rightarrow k\int_{20}^{30} x^2 y + \frac{y^3}{3}\Big|_{20}^{30} = 1}$

$\Rightarrow k\int_{20}^{30}\int_{20}^{30} x^2\,dx\,dy + k\int_{20}^{30}\int_{20}^{30} y^2\,dx\,dy = 1$

$\Rightarrow k\int_{20}^{30} \frac{x^3}{3}\Big|_{20}^{30} dy + k\int_{20}^{30} 10y^2 = 1$

$\Rightarrow k = \frac{3}{380000}$

b. $P(x<26,\ y<26) = \int_{20}^{26}\int_{20}^{26} k(x^2+y^2)\,dx\,dy$

$= k\int_{20}^{26}\int_{20}^{26} x^2\,dy\,dx + \int_{20}^{26}\int_{20}^{26} y^2\,dx\,dy$

$= k\int_{20}^{26} 6x^2\,dx + k\int_{20}^{26} 6y^2\,dy$

$= k\left(2x^3\Big|_{20}^{26} + 2y^3\Big|_{20}^{26}\right)$

$= 38304k$

$= 0.3024$

c. $P(|x-y|\le 2) = 1-\int_{20}^{28}\int_{x+2}^{30} k(x^2+y^2)\,dy\,dx - \int_{22}^{30}\int_{20}^{x-2} k(x^2+y^2)\,dy\,dx$

$= 1-k\int_{20}^{28}\int_{x+2}^{30} x^2\,dy\,dx - k\int_{20}^{28}\int_{x+2}^{30} y^2\,dy\,dx - k\int_{28}^{30}\int_{20}^{x-2} x^2\,dy\,dx - k\int_{28}^{30}\int_{20}^{x-2} y^2\,dy\,dx$

$= 1-k\int_{20}^{28} (30-x-2)x^2 - k\int_{20}^{28} \frac{y^3}{3}\Big|_{x+2}^{30} dx - k\int_{28}^{30} (x-2-20)x^2\,dx - k\int_{28}^{30} \frac{y^3}{3}\Big|_{20}^{x-2} dx$

$= 0.35$.

d. $f_X(x) = k\int_{20}^{30} (x^2+y^2)\,dy = 10kx^2 + 0.05$

e. No. $f_Y(y) = 10ky^2 + 0.05$ $\quad f_Y(y)\cdot f_X(x) \ne f(x,y)$.

12. a. $P(X\ge 3) = \int_3^\infty\int_0^\infty x e^{-x(1+y)}\,dy\,dx$

$= \int_3^\infty -e^{-x(1+y)}\Big|_0^\infty$

$= \int_3^\infty e^{-x} = 0.05$

b. $F_X(x) = \int_0^\infty x e^{-x(1-y)}\,dy = e^{-x}$

$f_Y(y) = \int_0^\infty x e^{-x(1-y)}\,dx = \left[\frac{-3}{(y+1)} - \frac{1}{(1+y)^2}\right] e^{-3-3y}$

they are not indepent as $f_Y(y)\cdot f_X(x) \ne f(x,y)$.

c. $P(x>3 \text{ or } y>3) = 1 - P(x\le 3 \text{ and } y<3)$

$$1-P(x<3 \text{ and } y<3) = 1-\int_0^3\int_0^3 xe^{-x(1+y)}\,dy\,dx$$

$$= 1-\int_0^3\int_0^3 xe^{-x}\cdot e^{-xy}\,dy\,dx$$

$$= 1-\int_0^3 -e^{-x}\cdot e^{-xy}\Big|_0^3\,dx$$

$$= 1-\int_0^3 -e^{-4x}+e^{-x}\,dx$$

$$= 1-\left(\tfrac{1}{4}e^{-4x}-e^{-x}\Big|_0^3\right)$$

$$= 1-\left(\tfrac{1}{4}e^{-12}-e^{-3}-\tfrac{1}{4}+1\right)$$

$$= 0.300$$

18. a. $p_{Y|X}(0|1) = \frac{f(1,0)}{f_X(1)} = 0.2353$

$p_{Y|X}(1|1) = \frac{f(1,1)}{f_X(1)} = 0.5882$

$p_{Y|X}(2|1) = \frac{f(1,2)}{f_X(1)} = 0.1765$

b. $P_{(Y|X)}(Y|2)$

$P_{(Y|X)}(0|2) = \frac{f(2,0)}{f_X(2)} = 0.12$

$P_{Y|X}(1|2) = \frac{f(2,1)}{f_X(2)} = 0.28$

$P_{Y|X}(2|2) = \frac{f(2,2)}{f_X(2)} = 0.6$.

c. $P(Y\le 1|X=2) = (f(2,0)+f(2,1))/f_X(2) = 0.4$.

d. $P_{X|Y}(X|2)$

$P_{X|Y}(0|2) = \frac{f(0,2)}{p_Y(2)} = 0.0526$.

$P_{X|Y}(1|2) = \frac{f(1,2)}{p_Y(2)} = 0.1579$

$P_{X|Y}(2|2) = \frac{f(2,2)}{p_Y(2)} = 0.7895$.

19. a. $f_{Y|X}(Y|X) = \frac{f(x,y)}{f_X(x)} = \frac{k(x^2+y^2)}{10kx^2+0.05}$

$f_{X|Y}(x|Y) = \frac{f(x,y)}{f_Y(y)} = \frac{k(x^2+y^2)}{10ky^2+0.05}$.

b. $P(Y\ge 25|X=22) = \int_{25}^{30} f_{Y|X}(y|22)\,dy$

$$= \int_{25}^{30} \frac{k(22^2+y^2)}{10ky^2+0.05}\,dy$$

$$= 0.783.$$

c. $E = \int_{20}^{30} x \, P_{X|Y}(x|22)$

$= \int_{20}^{30} x \cdot \frac{k(x^2+22^2)}{10k22^2+0.05}$

$= 25.37$

$E(x^2|22) = \int_{20}^{30} x^2 P_{X|Y}(x|22)\,dx$

$= 652.02$

$V(x|22) = E(x^2|22) - [E]^2$

$= 8.24$

Section 5.2.

24. $|X-Y|$ $P = \frac{1}{6} \times \frac{1}{5} = \frac{1}{30}$

Y \ X	1	2	3	4	5	6
1	x	1	2	3	4	5
2	1	x	1	2	3	4
3	2	1	x	1	2	3
4	3	2	1	x	1	2
5	4	3	2	1	x	1
6	5	4	3	2	1	x

$E = \sum_{x=1}^{6} \sum_{y=1}^{6} P|X-Y|$

$= 2.33$

26. $E = \sum_{x=0}^{5} \sum_{y=0}^{2} (3x+10y) \cdot P = 15.4$

33. $Cov(X,Y) = E[(X-\mu_X)(Y-\mu_Y)]$ $Corr(X,Y) = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$

$= E(X,Y) - \mu_X \mu_Y$.

when X, Y are independent

$f(x,y) = f_X(x) \cdot f_Y(y)$

$E(X,Y) = E(X) \cdot E(Y)$

$Cov(X,Y) = 0$

$Corr(X,Y) = 0$

$$\cancel{35.\ Corr(aX+b, cY+d) = \frac{Cov(aX+b, cY+b)}{\sigma(aX+b)\,\sigma(cY+b)} = \frac{E(aX+b)(cY+b) - \mu_{aX+b}\,\mu_{cY+b}}{a^2\sigma X \cdot c^2\sigma Y}}$$

$\Rightarrow$

35. $Cov(aX+b, cY+d) = E[(aX+b)(cY+b)] - \mu_{aX+b} \cdot \mu_{cY+b}$

$= E[caXY + adX + bcY + bd] - (a\mu_x + b)(c\mu_y + b)$

$= ac\,Cov(X, Y).$

b. $Corr(aX+b, cY+d) = \dfrac{Cov(aX+b, cY+b)}{\sigma_{aX+b} \cdot \sigma_{cY+b}} = \dfrac{ac\,Cov(X,Y)}{\sqrt{a^2}\sigma_X \cdot \sqrt{c^2}\sigma_Y} = \dfrac{Cov(X,Y)}{\sigma_X \cdot \sigma_Y}$

$= Corr(X, Y)$

c. If ac are in different sign $\quad ac = -\sqrt{a^2}\sqrt{c^2}$

$Corr(aX+b, cY+d) = -Corr(X, Y).$