A

2.4 : 46, 50, 58, 63

46) Let A be individual ≥ ft tall. Let B be individual who is a ^prof^ basketball player

$P(A|B)$ = probability of individual ≥ 6ft, given that he is a ^prof^ basketball player.

$P(B|A)$ = " " " who is a basketball player, given he is ≥ 6 ft

$P(A|B)$ is larger, because most of the ^prof^ basketball player are very tall. On the other hand, not all 6ft+ individual are prof basketball player. And amount of pro basketbal players are very small in relation to amount of 6ft+

50)

a) $P(M \cap LS \cap PR) = 0.05$

b) $P(M \cap PR) = P(M \cap LS \cap PR) + P(M \cap SS \cap PR) = 0.05 + 0.07 = 0.12$

c) $P(SS)$ = Sum of all probabilities in Short Sleeved table = 0.04 + 0.024 $\cdots + 0.08 = 0.56$

$P(LS) = 1 - P(SS) = 1 - 0.56 = 0.44$

d) $P(M) = 0.08 + 0.07 + \cdots + 0.07 = 0.49$

$P(Pr) = 0.02 + 0.07 + \cdots 0.02 = 0.25$

e) $P(M | SS \cap PL) = \dfrac{P(M \cap SS \cap PL)}{P(SS \cap PL)} = \dfrac{0.08}{0.04 + 0.08 + 0.03} = 0.533$

f) $P(SS | M \cap PL) = \dfrac{P(SS \cap M \cap PL)}{P(M \cap PL)} = \dfrac{0.08}{0.08 + 0.1} = 0.444$

$P(LS | M \cap PL) = \dfrac{P(LS \cap M \cap PL)}{P(M \cap PL)} = \dfrac{0.1}{0.1 + 0.08} = 0.556$

or $= 1 - P(SS | M \cap PL) = 1 - 0.444 = 0.556$

58)

$$P(A \cup B | C) = \frac{P[(A \cup B) \cap C]}{P(C)} = \frac{P[(A \cap C) \cup (B \cap C)]}{P(C)} = \frac{P[(A \cap C) + P(B \cap C) - P(A \cap B \cap C)]}{P(C)}$$

$$= \frac{P(A \cap C)}{P(C)} + \frac{P(B \cap C)}{P(C)} - \frac{P[(A \cap B) \cap C]}{P(C)} = P(A|C) + P(B|C) + P(A \cap B | C)$$

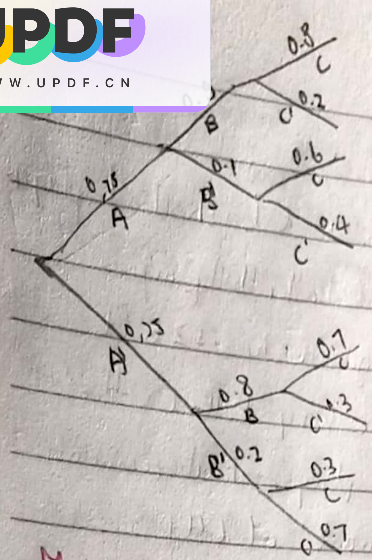
*63)

Tree diagram

Initial branches represent first layer of events, and adequate probabilitic are in diagram

2nd gen " " 2nd " " " , " " conditional " " " "

3rd " " " 3rd " " " , " " " " " " "



1st: $P(A) = 0.75$, $P(A') = 1 - P(A) = 0.25$

2nd: $P(B|A) = 0.9$, $P(B'|A) = 1 - P(B|A) = 0.1$

$P(B|A') = 0.8$

$P(B'|A') = 1 - P(B|A') = 0.2$

3rd: $P(C|A \cap B) = 0.8 \Rightarrow P(C'|A \cap B) = 0.2$

$P(C|A \cap B') = 0.6 \Rightarrow P(C'|A \cap B') = 0.4$

$P(C|A' \cap B) = 0.7 \Rightarrow P(C'|A' \cap B) = 0.3$

$P(C|A' \cap B') = 0.3 \Rightarrow P(C'|A' \cap B') = 0.7$

Multiplication Rule:

$P(A \cap B) = P(A|B) . P(B)$.

b) $P(A \cap B \cap C) = P(C|A \cap B) . P(A \cap B) = P(C|A \cap B) . P(B|A) . P(A)$

$= 0.75 . 0.9 . 08 = 0.54$

c) $P(B \cap C) = P(B \cap C \cap A) + P(B \cap C + A') = 0.54 + 0.25 . 0.8 . 0.7 = 0.68$

d) $P(C) = P(A \cap B \cap C) + P(A \cap B' \cap C) + P(A' \cap B C) + P(A' \cap B' \cap C)$

$= (0.75 . 0.9 . 0.8) + (0.75 . 0.1 . 0.6) + (0.25 . 0.8 . 0.7) + (0.25 . 0.2 . 0.3)$

$= 0.54 + 0.045 + 0.14 + 0.015 = 0.74$

e) use conditional probability.

$P(A|B \cap C) = \frac{P(A \cap B \cap C)}{P(B \cap C)} = \frac{0.54}{0.68} = 0.7941$

A

2.5 71, 72, 80, 84

3.1 4, 5, 8, 10

3.2 12, 23, 25

71)

a) Propositions: If A and B are independent, then

- A' and B are independent.
- A' and B' " "
- A " B' " "

Thus, $P(B'|A') = P(B') = 1 - 0.7 = 0.3$

b) Proposition: 2 events A and B are independent iff $P(A \cap B) = P(A).P(B)$

Proposition: For every 2 events A and B: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.7 - (0.4 \cdot 0.7) = 0.82$

c) Event that atleast 1 of 2 projects is succesful: $A \cup B$

" , where only the Asian project is " : $A \cap B'$

$$P(A \cap B' | A \cup B) = \frac{P[(A \cap B') \cap (A \cup B)]}{P(A \cup B)} = \frac{P(A \cap B')}{P(A \cup B)} = \frac{P(A) \cdot P(B')}{P(A \cup B)} = \frac{0.4 \cdot (1 - 0.7)}{0.82} = 0.146$$

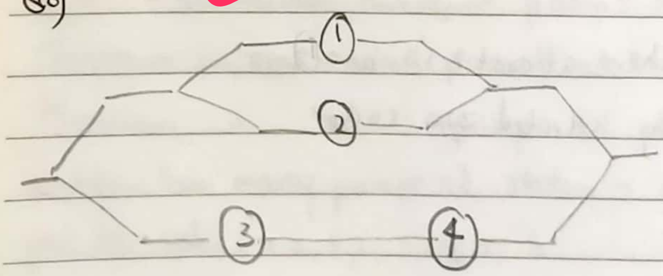
event $A \cap B'$ is a sub set of event $A \cup B$

72)

Multiplication property: 2 events A and B are independent iff $P(A \cap B) = P(A) \cdot P(B)$

From exercise 13	From exercise 72	dependency.
$P(A_1 \cap A_2) = 0.11$	$P(A_1) \cdot P(A_2) = 0.22 . 0.25 = 0.055$	$0.11 \neq 0.055$ => dependent
$P(A_1 \cap A_3) = 0.05$	$P(A_1) P(A_3) = 0.22 . 0.28 = 0.0616$	$0.05 \neq 0.066$ => "
$P(A_2 \cap A_3) = 0.07$	$P(A_2) . P(A_3) = 0.25 . 0.28 = 0.07$	$0.07 = 0.07$ => independent

80)



... [illegible]aining 1 and 2 as ~~sy~~ sub system A
" " 3 " 4 " " B.

$P(A \text{ works}) = P(1 \text{ works} \cup 2 \text{ works}) = P(1 \text{works}) + P(2 \text{works}) - P(1 \text{ works} \cap 2 \text{ works})$
$= 0.9 + 0.9 - (0.9 \cdot 0.9) = 0.99$

$P(B \text{ works}) = P(3 \text{ works} \cap 4 \text{ works}) = P(3 \text{works}) \cdot P(4 \text{works}) = 0.9 \cdot 0.9 = 0.81$

$P(\text{System works}) = P(A \text{ works} \cup B \text{ works}) = P(A \text{works}) + P(B \text{works}) - P(A \text{works}) \cdot (B \text{ works})$
$= P(A \text{works}) + P(B \text{ works}) - (P(A \text{works}) \cdot P(B \text{works}))$
$= 0.99 + 0.81 - 0.99 \cdot 0.81 = 0.9981$

84) $P(\text{pass}) = 0.7$

a) Assuming that succesiveive vehicles pass / fail independently, use multiplication rule:
$P(\text{All 3 pass}) = P(\text{pass}) \times P(\text{pass}) \times P(\text{pass}) = 0.7 \times 0.7 \times 0.7 = 34.3\%$

b) Atleast 1 fail is complement of all 3 pass:
$P(\text{atleast 1 fail}) = 1 - P(\text{all 3 pass}) = 1 - 34.3\% = 65.7\%$

c) Events are independent, so do complements:
$P([A_1 \cap A_2 \cap A_3'] \cup [A_1 \cap A_2' \cap A_3] \cup [A_1' \cap A_2 \cap A_3]) = (0.7 \cdot 0.3 \cdot 0.3) + (0.7 \cdot 0.3 \cdot 0.3) + (0.7 \cdot 0.3 \cdot 0.3) = 0.189 = 18,9\%$

d) $P(\text{None pass}) = P(\text{fail}) \cdot P(\text{fail}) \cdot P(\text{fail}) = 0.3 \cdot 0.3 \cdot 0.3 = 0.027$
Atmost 1 vehicle pass means none pass or exactly 1 pass. Since, it's not possible that both happen at the same time, the events are mutually exclusive, so use addition rule for mutually exclusive events:
$P(\text{atmost 1 pass}) = P(\text{None pass}) + P(\text{exactly 1 pass}) = 0.027 + 0.189 = 0,216 = 21,6\%$

e) $P(\text{Atleast 1 pass}) = 1 - P(\text{none pass}) = 1 - 0.027 = 0.973$
$P(\text{All 3 pass} \mid \text{atleast 1 pass}) = \frac{P(\text{all 3 pass and atleast 1 pass})}{P(\text{atleast 1 pass})} = \frac{P([A_1 \cap A_2 \cap A_3] \cap [A_1 \cup A_2 \cup A_3])}{P(A_1 \cup A_2 \cup A_3)}$
$= \frac{P(\text{all 3 pass})}{P(\text{atleast 1 pass})} = \frac{P([A_1 \cap A_2 \cap A_3])}{P(A_1 \cup A_2 \cup A_3)} = \frac{0.343}{0.973} = 35,25\%$

4) Zip codes usually have 5 possible digits, where atleast 1 is non zero.
X = number of non zero digits in randomly selected zip code.
possible values of $X = 1, 2, 3, 4, 5$

f.e zip code	X value
90210	3
10002	2
22313	5

7) ... In the experiment in which a coin is tossed repeatedly until a H results, let $Y = 1$ if the experiment terminates with at most 5 tosses and $Y = 0$ otherwise. The sample space is infinite, yet Y has only 2 possible values.

8) The least possible value of Y is 3. All possible values of Y are $3, 4, 5, \ldots$

$Y = 3$: SSS
$Y = 4$: FSSS
$Y = 5$; FFSSS, SFSSS
$Y = 6$; FFFSSS, SFFSSS, FSFSSS, SSFSSS
$Y = 7$; SFFSSS, SFSFSSS, SFFFSSS, FSSFSSS, FSFFSSS, FFSFSSS, FFFFSSS

terminate the string when 3 consecutive Success happened

10)

a) T = total number of pumps in use.
minimum of pumps use for a 6-pump station and 4-pump station = 0
maximum " " " " " " " " " " = 10
possible values = $0, 1, 2, \ldots, 10$

b) X = difference between the numbers in use at stations 1 and 2.
suppose station 1 is 6 pump station
" " 2 " 4 " "
Minimum values for difference between the numbers in use at station 1 and station 2 is if there are no pump in use in station 1 and ~~the~~ all 4 pumps are used in station 2: $0 - 4 = -4$.
Maximum " would be when all 6 pumps of station 1 are in use, while none on station 2: $6 - 0 = 6$
possible values = $-4, -3, -2, \ldots, 6$

c) U = the maximum number of pumps in use at either station.
Minimum value is when pumps at both stations are not in use, so $U = 0$.
Maximum " " " pumps at station 1 is fully used, because it doesn't matter how many pumps at station 2 are used there are only 4 pumps (5, 6)
possible values = $0, 1, 2, 3, 4, 5, 6$

d) Z = number of stations having exactly 2 pumps in use.

All cases	value
none of the stations have exactly 2 pumps in use.	0
Station 1 have exactly 2 pumps in use, but station 2 doesn't have exactly 2 in use	1
both station 1 and station 2 have exactly 2 pumps in use	2
Station 2 have exactly 2 pumps in use, but station 1 doesn't have exactly 2 in use	1

All possible value: 0, 1, 2

12)

Probability Mass Function (pmf) of a discrete random variable x is

a) $p(x) = P(X = x) = P(\omega \in S : X(\omega) = x)$ for every number x.

The probability that the flight will accomodate all ticketed passengers who show up, means take y values ≤ 50.

$P(Y \leq 50) = P(45) + P(46) + \cdots + P(50) = 0.05 + 0.1 + \cdots + 0.17 = 0.83$

b) If not all passengers can get accomodated, means more than 50 passengers have tickets $(Y > 50)$.

$P(Y > 50) = 1 - P(\leq 50) = 1 - 0.83 = 0.17$

c) Standby list appear when <50 have booked tickets. There are 50 available seats, so atmost 49 can show up, to have a standby list.

$P(y \leq 49) = P(45) + P(46) + \cdots + P(49) = 0.05 + 0.1 + \cdots + 0.25 = 0.66$

If there can appear a 3rd person in a standby list, so $y \leq 47$

$P(Y \leq 47) = P(45) + P(46) + P(47) = 0.05 + 0.1 + 0.12 = 0.27$

23)

Cumulative Distribution Function (cdf) $F(x)$ of a discrete random variable is $F(x) = P(X \leq x) = \sum_{u: u \leq x} p(u)$ for every number x, where $p(x)$ is the probability Mass Function of x.

Proposition: Let $a, b \in \mathbb{R}$, $a \leq b$, the following holds: $P(a \leq X \leq b) = F(b) - F(a-)$.

$a-$ stands for the largest possible value of X that is (strictly) less than a.

Assume that only possible values are integers, in this case:

$P(a \leq x \leq b) = P(X = a \text{ or } X = a+1 \text{ or } \cdots \text{ or } X = b) = F(b) - F(a-1)$.

a) $p(2) = P(X=2) = P(2 \leq X \leq 2) = F(2) - F(1) = 0.39 - 0.19 = 0.2$ $\quad b=2$, $a-1=1$

b) $P(x > 3) = P(3 < X \leq 6) = P(4 \leq X \leq 6) = F(6) - F(3) = 1 - 0.67 = 0.33$ $\quad a-1 = 4-1 = 3$

$b=5$
$a-1 = 2-1 = 1$

$\ldots X \le 5) = F(5) - F(1) = 0.97 - 0.19 = 0.78$

$b=4$
$a-1 = 3-1 = 2$

d) $P(2 < X < 5) = P(3 \le X \le 4) = F(4) - F(2) = 0.92 - 0.39 = 0.58$

25)

P(# girls born before experiment terminates) cannot be negative. It must be some non-negative integer, therefor $y \notin \mathbb{N}_0$

$p(y) = P(Y = x) = 0$

Let $y=0$, $p(0) = P(Y=0) = P[\{\text{first born is boy}\}] = p$

" $y=1$, $p(1) = P(Y=1) = P[(GB)] = P(G) \cdot P(B) = (1-p)p$ — the events are independent

" $y=2$, $p(2) = P(Y=2) = P((GGB)) = P(G) \cdot P(G) \cdot P(B) = (1-p)^2 \cdot p$

Multiplication Property:

For events $A_1, A_2, \cdots, A_n$, $n \in \mathbb{N}$, they are mutually independent if $P(A_{i_1} \cap A_{i_2} \ldots A_{i_k}) = P(A_{i_1}) \cdot P(A_{i_2}) \cdots P(A_{i_k})$ for every $k \in \{2, 3, \cdots, n\}$, and every subset of indices $i_1, i_2, \cdots, i_k$.

Let $y \in \mathbb{N}_0$, $p(y) = P(Y=y) = P(GG \ldots GB) = P(G) \cdot P(G) \ldots P(G) \cdot P(B) = (1-p)^y \cdot p$

$$p(x) = \begin{cases} (1-p)^y \cdot p, & x \in \mathbb{N}_0; \\ 0, & \text{otherwise.} \end{cases}$$