

**1. Sort the following functions in increasing order of asymptotic (big- $O$ ) complexity and explain why:**

$$f_1(n) = n^{\sqrt{n}}$$

$$f_2(n) = \log(n^{100000})$$

$$f_3(n) = 2^{2^{100000}}$$

$$f_4(n) = 2^n$$

$$f_5(n) = n^{10} \cdot 2^{n/2}$$

$$f_6(n) = n\sqrt{n}$$

$$f_7(n) = \sum_{i=1}^n i + 1$$

$$f_8(n) = 1000000n$$

**2. Solve the following recurrence relations and give a  $\theta$  bound for each of them.**

$$(1) T(n) = 9T(n/3) + n$$

$$(2) \begin{cases} T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + cn \\ T(1) = 1 \end{cases}$$

$$(3) T(n) = T(n-1) + \log n$$

**3. Divide a positive integer  $N$  into the sum of several positive integers. How many non-repetitive splitting schemes there are? Give an algorithm with the time complexity  $O(N^2)$  to do this.**

**For example,  $N = 5$  has seven splitting schemes:**

$$1) \quad 5=5$$

$$2) \quad 5=4+1$$

$$3) \quad 5=3+2$$

$$4) \quad 5=3+1+1$$

$$5) \quad 5=2+2+1$$

$$6) \quad 5=2+1+1+1$$

$$7) \quad 5=1+1+1+1+1$$