No.

概率统计作业　22CST 蒋云翔 2022102330　Date

Homework chapter 4 (7^{th})

Section 4.1

Ex.2

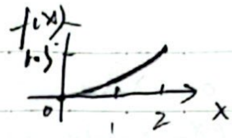
(a) $f(x;-5,5)=\begin{cases}\frac{1}{10} & (-5\le x\le 5)\\ 0, & \text{otherwise}\end{cases}$, $P(X<0)=\int_{-5}^{0}\frac{1}{10}dx=\frac{1}{2}$

(b) $P(-2.5<X<2.5)=\int_{-2.5}^{2.5}\frac{1}{10}dx=\frac{1}{2}$

(c) $P(-2\le X\le 3)=\int_{-2}^{3}\frac{1}{10}dx=\frac{1}{2}$

(d) $P(k<x<k+4)=\int_{k}^{k+4}\frac{1}{10}dx=\frac{2}{5}$

Ex.5



(a) $\int_0^2 kx^2dx=1$, then we get: $k=\frac{3}{8}$

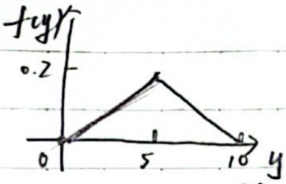
(b) $P(X<1)=\int_0^1\frac{3}{8}x^2dx=\frac{1}{8}$

(c) $P(1<X<1.5)=\int_1^{1.5}\frac{3}{8}x^2dx=\frac{19}{64}$

(d) $P(1.5<X<2)=\int_{1.5}^{2}\frac{3}{8}x^2dx=\frac{37}{64}$

Ex.8

(a)



(b) $\int_{-\infty}^{\infty}f(y)dy=\int_0^5\frac{1}{25}y\,dy+\int_5^{10}(\frac{2}{5}-\frac{1}{25}y)dy=0.5+0.5=1$

(c) $P(Y<3)=\int_0^3\frac{1}{25}y\,dy=\frac{9}{50}$

(d) $P(Y<8)=\int_0^5\frac{1}{25}y\,dy+\int_5^8(\frac{2}{5}-\frac{1}{25}y)dy=\frac{23}{25}$

(e) $P(3<Y<8)=\int_3^5\frac{1}{25}y\,dy+\int_5^8(\frac{2}{5}-\frac{1}{25}y)dy=\frac{37}{50}$

(f) $P(Y<2)+P(Y>6)=\int_0^2(\frac{1}{25})y\,dy+\int_6^{10}(\frac{2}{5}-\frac{1}{25}y)dy=\frac{9}{50}+\frac{11}{50}=\frac{2}{5}$

Section 4.2

Ex.12

(a) $P(X<0)=F(0)=\frac{1}{2}$

(b) $P(-1<X<1)=F(1)-F(-1)=\frac{3}{32}(4-\frac{1}{3})-\frac{3}{32}(-4+\frac{1}{3})=\frac{11}{16}$

(c) $P(0.5<X)=1-F(0.5)=1-(\frac{1}{2}+\frac{3}{32}(2-\frac{1}{24}))=1-0.684=0.316$

(d) $F'(x)=\frac{d}{dx}(\frac{1}{2}+\frac{3}{32}(4x-\frac{x^3}{3}))=\frac{3}{32}(4-x^2)=0.09375(4-x^2)=f(x)$

(e) try to solve: $F(\tilde{\mu})=0.5$, our $F(0)=\frac{1}{2}\Rightarrow\tilde{\mu}=0$

No.

Date

Ex.17

(a) cdf is: $\frac{X-A}{B-A}$ which is also represents "p", so: $\frac{X-A}{B-A}=p$ then $X = A + p(B-A)$

(b) $E(X) = \int_A^B x \cdot \frac{1}{B-A} dx = \frac{1}{B-A} \cdot (\frac{1}{2}x^2)\Big|_A^B = \frac{B^2-A^2}{2(B-A)} = \frac{B+A}{2}$

~~the same: if we wa~~

$V(X) = E(X^2) - E^2(X) = \int_A^B x^2 \frac{1}{B-A} dx - (\frac{B+A}{2})^2 = \frac{A^2+B^2+AB}{3} - (\frac{B+A}{2})^2 = \frac{(B-A)^2}{12}$

$\sigma_X = \frac{B-A}{\sqrt{12}}$

(c) $E(X^n) = \int_A^B x^n \frac{1}{B-A} dx = \frac{1}{B-A} \cdot \frac{1}{n+1} x^{n+1}\Big|_A^B = \frac{B^{n+1} - A^{n+1}}{(n+1)(B-A)}$

Ex.22

(a) cdf of X: $F(x) = \begin{cases} 0, & x < 1 \\ \int_1^x f(x)dx, & 1 \le x \le 2 \quad (\text{that is: } 2(x+\frac{1}{x}) - 4,\ 1 \le x \le 2) \\ 1, & x \ge 2 \end{cases}$

(b) $F(\tilde{\mu}) = 0.5$, $2(\tilde{\mu} + \frac{1}{\tilde{\mu}}) - 4 = 0.5 \Rightarrow \tilde{\mu}^2 - \frac{9}{4}\tilde{\mu} + 1 = 0 \Rightarrow \tilde{\mu} = 1.64 \text{ or } 0.61$

(c) $E(X)\ \int_1^2 x \cdot f(x)\,dx = \int_1^2 2(x - \frac{1}{x}) = 2 \cdot (\frac{1}{2}x^2 - \ln x)\Big|_1^2 = 1.614$

$V(X) = E(X^2) - E^2(X)$

$= 2\int_1^2 (x^2-1)\,dx - 1.614^2$

$= \frac{8}{3} - 1.614^2$

$= 0.063$

(d) Based on the problem: $h(x) = \max(1.5-x, 0)$

$E(h(x)) = \int_1^2 \max(1.5-x, 0) \cdot f(x)\,dx = \int_1^{1.5} (1.5-x) \cdot 2(1 - \frac{1}{x^2})\,dx = 0.061$

Ex.23

Let X denotes °C, then $°F = 1.8X + 32$

$E(°F) = 1.8E(X) + 32 = 1.8 \times 120 + 32 = 248$

$V(°F) = 1.8^2 \times 2^2 = 12.96$

$\sigma_{°F} = 3.6$