B

Section 6.1

Ex 1

a. we use $\bar{x}$ to estimate μ

$\hat{\mu} = \bar{x} = \frac{\sum x_i}{n} = \frac{219.8}{27} = 8.14$

b. we use sample median $\tilde{x}$

c. we use the sample standard ~~d~~ deviation, $s = \sqrt{s^2} = \sqrt{\frac{1860.94 - \frac{(219.8)^2}{27}}{26}} = 1.66$

d. "success" greater than 10, $x = \# \text{ of success} = 4$, $\hat{p} = \frac{x}{n} = \frac{4}{27} = 0.1481$

e. we use the sample (std dev)/(mean) or $\frac{s}{\bar{x}} = \frac{1.66}{8.1407} = 0.2039$

Ex 8

a. use q denote the true proportion of dective components

$\hat{q} = \frac{\#\text{ dective in sample}}{\text{sample size}} = \frac{12}{80} = 0.15$

b. $P(\text{system works}) = p^2$, so an estimate of this probability $= \hat{p}^2 = \left(\frac{68}{80}\right)^2 = .723$

Ex 9.

a. $E(\bar{X}) = \mu = E(X) = \lambda$; so $\bar{X}$ is an ~~unbia~~ unbiased estimator for the Possion λ; $\sum x_i = 0\times18 + 1\times37 + \cdots + 7\times1 = 317$, since $n = 150$, $\hat{\lambda} = \bar{x} = \frac{317}{150} = 2.11$

b. $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{\lambda}}{\sqrt{n}}$, so the estimate standard error is $\sqrt{\frac{\hat{\lambda}}{n}} = \frac{\sqrt{2.11}}{\sqrt{150}} = 0.119$

Ex 13

$E(X) = \int_{-1}^{1} x \cdot \frac{1}{2}(1+\theta x)\,dx = \frac{x^2}{4} + \frac{\theta x^3}{6}\Big|_{-1}^{1} = \frac{1}{3}\theta$

$E(\bar{X}) = \frac{1}{3}\theta$, $\hat{\theta} = 3\bar{X} \Rightarrow E(\hat{\theta}) = E(3\bar{X}) = 3E(\bar{X}) = 3(\frac{1}{3})\theta = \theta$

Section 6.2

Ex 20

a. $\frac{d}{dp}\left[\ln\binom{n}{x} + x\ln(p) + (n-x)\ln(1-p)\right] = \frac{x}{p} - \frac{n-x}{1-p} \Leftrightarrow \hat{p} = \frac{x}{n}$

we get: $n = 20$, $x = 3$, $\hat{p} = \frac{3}{20} = 0.15$

b. $E(\hat{p}) = E\left(\frac{X}{n}\right) = \frac{1}{n}E(X) = \frac{1}{n}(np) = p$, thus $\hat{p}$ is an unbiased estimator.

c. $(1-0.15)^5 = 0.4437$

Ex 2.1

a. $E(X) = \beta\Gamma(1+\frac{1}{\alpha})$ and $E(X^2) = Var(X) + E(X)^2 = \beta^2\Gamma(1+\frac{2}{\alpha})$

so $\bar{X} = \hat{\beta}\cdot\Gamma(1+\frac{1}{\hat{\alpha}})$, $\frac{1}{n}\sum X_i^2 = \hat{\beta}^2\Gamma(1+\frac{2}{\hat{\alpha}})$

b.

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Ex 2.9.

a. $f(x_1 \ldots x_n; \lambda; \theta) = \begin{cases} \lambda^n e^{-\lambda\sum(x_i-\theta)} & x_1 \geq \theta, \ldots x_n \geq \theta \\ 0 & \text{, otherwise} \end{cases}$

and that $x_1 \geq \theta, \ldots x_n \geq \theta$, iff $\min(x_i) \geq \theta$, and that $\lambda\sum(x_i-\theta) = \lambda\sum x_i - n\lambda\theta$

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So: $= \begin{cases} \lambda^n \exp(-\lambda\sum x_i)\exp(n\lambda\theta) & , \min(x_i) \geq \theta \\ 0 & , \min(x_i) < \theta \end{cases}$

b. $\hat{\theta} = \min(x_i) = 0.64$, $\sum x_i = 55.8$, so $\hat{\lambda} = \frac{10}{55.8 - 6.4} = 0.202$

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