B+

8.9.13
21.29.32

1. a. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{219.8}{28} = 8.14\,MPa$

b. Median = 7.7 MPa

c. $S = \sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n-1}}$

$$S^2 = \frac{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}}{n-1}$$

$$= \frac{1860.94 - \frac{(219.8)^2}{28}}{27}$$

$S = 1.66\,MPa$

d. $\hat{p} = \frac{4}{20} = 0.2$

e. Point estimate of the population coefficient of variation:

$$CV = \frac{S}{\bar{x}} = \frac{5.42}{10.99} = 0.4932 = 49.32\%$$

8. a. $P = \frac{\text{Number of non-defective components}}{\text{Total number of components}} = \frac{80-12}{80} = 0.85.$

b. $P(\text{system works}) = p \times p = p^2 = 0.85^2 = 0.7225.$

9. a. $\bar{x} = \frac{\Sigma(x_i \cdot f_i)}{n} = \frac{(0.18)+(1.37)+(2.42)+(3.30)+(4.13)+(5.7)+(6.2)+(7.1)}{150}$

$$= \frac{0+37+84+90+52+35+12+7}{150}$$

$$= \frac{317}{150}$$

$$= 2.1133$$

Therefore, the unbiased estimator of μ is $\bar{x} = 2.1133$

b. $\sigma_X = \sqrt{\bar{x}} = \sqrt{2.1133} \approx 1.4537$

$$SE(\bar{X}) = \frac{1.4537}{\sqrt{150}} = \frac{1.4537}{12.2474} = 0.1187$$

Therefore, the estimated standard error of the estimator is ~~es~~ approximately 0.1187.

13. $E(X) = \int_{-\infty}^{\infty} x f(x;\theta)\,dx$

$$= \int_{-1}^{1} x \cdot 0.5(1+\theta x)\,dx$$

$$= 0.5\int_{-1}^{1} x(1+\theta x)\,dx$$

$$= 0.5\int_{-1}^{1} x\,dx + 0.5\int_{-1}^{1} \theta x^2\,dx$$

$$= 0.5\left[\frac{x^2}{2}\right]_{-1}^{1} + 0.5\left[\frac{x^3}{3}\right]_{-1}^{1}\theta \quad \cancel{= \frac{1^2}{2} - \frac{(-1)^2}{2} = \theta}$$

$$= 0.5\left(\frac{1}{2} - \frac{1}{2}\right) + 0.5\left(\frac{1}{3} + \frac{1}{3}\right)\theta$$

$$= \frac{\theta}{3}$$

$E(\bar{X}) = E\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{\theta}{3}$

$E(\hat{\theta}) = E(3\bar{X}) = 3E(\bar{X}) = \theta$

Thus, $E(\hat{\theta}) = \theta$, so $\hat{\theta} = 3\bar{X}$ is an unbiased ~~esti~~ estimator of θ

Therefore, $\hat{\theta} = 3\bar{X}$ is an unbiased estimator of θ

20. a. To derive the MLE of p, we start with the likelihood function for the binomial distribution:

$$L(p;x) = \binom{n}{x} p^x (1-p)^{n-x}$$

The log-likehood function is:

$$\ell(p) = \log L(p;x) = \log\left(\binom{n}{x} p^x (1-p)^{n-x}\right)$$

$$= \log\binom{n}{x} + x\log p + (n-x)\log(1-p)$$

$$\frac{d}{dp}\ell(p) = \frac{d}{dp}\left(\log\binom{n}{x} + x\log p + (n-x)\log(1-p)\right)$$

$$\frac{d}{dp}\ell(p) = x\frac{1}{p} - (n-x)\frac{1}{1-p}$$

set the derivative to zero:

$$x\cdot\frac{1}{p} - (n-x)\frac{1}{1-p} = 0$$

$$x = np$$

$$\hat{p} = \frac{x}{n} = \frac{3}{20} = 0.15$$

b. $E(X) = np$

$E(\hat{p}) = E(\frac{x}{n}) = \frac{1}{n}E(X) = \frac{1}{n}(np) = p$

since $E(\hat{p}) = p$, the estimator $\hat{p} = \frac{x}{n}$ is unbiased.

c. $(1-\hat{p})^5 = (1-0.15)^5 = 0.85^5 \approx 0.4437$.

21. a. The sample mean $\bar{x}$ is an estimator for $E(X)$:

$$\bar{x} = \beta\cdot\Gamma(1+\frac{1}{\alpha})$$

The sample variance s^2 is an estimator for $V(X)$:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$s^2 = \beta^2\{\Gamma(1+\frac{2}{\alpha}) - [\Gamma(1+\frac{1}{\alpha})]^2\}$$

$$\frac{s^2}{\bar{x}^2} = \frac{\Gamma(1+\frac{2}{\alpha}) - [\Gamma(1+\frac{1}{\alpha})]^2}{[\Gamma(1+\frac{1}{\alpha})]^2}$$

$$\beta = \frac{\bar{x}}{\Gamma(1+\frac{1}{\alpha})}$$

b. $s^2 = \frac{1}{n-1}(\sum x_i^2 - n\bar{x}^2)$

$$= \frac{1}{19}(16500 - 20\times 28^2) = 43.1579$$

$$\frac{s^2}{\bar{x}^2} = \frac{\Gamma(1+\frac{2}{\alpha}) - [\Gamma(1+\frac{1}{\alpha})]^2}{[\Gamma(1+\frac{1}{\alpha})]^2}$$

$\alpha \approx 1.2.$

$$\beta = \frac{28.0}{\Gamma(1+\frac{1}{1.2})} = \frac{28.0}{\Gamma(1.8333)} \approx \cancel{0.901}\ 31.08$$

29. a. $L(\lambda,\theta) = \prod_{i=1}^{n} \lambda e^{-\lambda(x_i-\theta)}$

$= \lambda^n e^{-\lambda \sum_{i=1}^{n}(x_i-\theta)}$

$\ell(\lambda,\theta) = n\log\lambda - \lambda\sum_{i=1}^{n}(x_i-\theta)$

$\hat{\theta} = \min(x_1, x_2, \cdots, x_n)$

$\ell(\lambda,\hat{\theta}) = n\log\lambda - \lambda\sum_{i=1}^{n}(x_i-\hat{\theta})$

$\frac{\partial \ell(\lambda,\hat{\theta})}{\partial\lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n}(x_i-\hat{\theta})$

Set the derivative to zero:

$\frac{n}{\lambda} - \sum_{i=1}^{n}(x_i-\hat{\theta}) = 0$

$\lambda = \frac{n}{\sum_{i=1}^{n}(x_i-\hat{\theta})}$

b. $\hat{\theta} = \min(3.11, 0.64, 2.55, 2.20, 5.44, 3.42, 10.39, 8.93, 17.82, 1.30) = 0.64$

$\sum_{i=1}^{n}(x_i-0.64) = (3.11-0.64)+(0.64-0.64)+\cdots+(1.30-0.64)$

$= 2.47+0+1.91+\cdots+0.66 = 49.40$

$\hat{\lambda} = \frac{n}{\sum_{i=1}^{n}(x_i-\hat{\theta})} = \frac{10}{49.40} \approx 0.2024$

32. a. $F_Y(y) = P(Y \le y)$

$= P(X_1 \le y, X_2 \le y, \cdots X_n \le y)$

$= P(X_1 \le y)\cdot P(X_2 \le y)\cdot \cdots \cdot P(X_n \le y)$

$P(X_i \le y) = \frac{y}{\theta}$ for $0 \le y \le \theta$

$F_Y(y) = \left(\frac{y}{\theta}\right)^n$ for $0 \le y \le \theta$