B+

Section 6.2

6.1

1\. a. estimator: $\hat{p} = \frac{\sum X_i}{N}$ $\quad \bar{X}$

estimate $= \frac{\sum x_i}{n} = \frac{219.8}{27} \approx 8.14$

b. estimator: $\hat{p} = \sum_{i=1}^{\frac{n-1}{2}} (X_{\frac{n+1}{2}+i} - X_i)$ $\quad \tilde{X}$

estimate =
$7.8-5.9+7.8-6.3+7.9-6.3+8.1-6.5$
$+8.2-6.8+8.7-6.8+9.0-7.0+9.7-7.0$
$+9.7-7.2+10.7-7.3+11.3-7.4+11.6-7.6$
$+11.8-7.7$
$= 32.5$

c. estimator S

$$\text{estimate} = \sqrt{\frac{\sum(x_i^2)}{n} - E^2(x)}$$
$$= \sqrt{\frac{1860.4}{27} - \left(\frac{219.8}{27}\right)^2}$$
$$= \sqrt{2.63} = 1.66$$

d. estimate $= \frac{4}{27} = 0.148$

e. estimator: $\frac{S}{\bar{X}}$

estimate $= \frac{1.66}{8.14} = 0.20$

estimate $= \frac{1.66}{[illegible]14} = 0.1$[illegible]

8.

a. estimator: $\hat{p} = 1 - \frac{X}{N}$

estimate $= 1 - \frac{12}{80} = 0.85$

b. If p denotes the probability that a component works properly,

$p = p \cdot \left(\frac{N-X-1}{N-1} \right)$)

So estimator $= p$

$p = 1 - \frac{12}{80} = 0.85$

estimate $= 0.85 \cdot \left(\frac{67}{79} \right.$

$= 0.72$

9.

a. Estimator: $\bar{X}$ [illegible]

$$estimate = 0.85 \cdot \left(\frac{\cdot}{79}\right)$$

$= 0.72$

85%

9.

a. Estimator: $\bar{X}$

$$\text{Estimate} = \frac{1\times37 + 2\times42 + 3\times30 + 4\times13 + 5\times7 + 6\times2 + 7\times1}{150}$$

$= 2.11$

b. $\sigma_x^2 = \mu$

$$\sigma_x = \sqrt{\frac{2.11}{150}} = 0.119$$

13. So: $E(x) = \Sigma x_i \cdot f(x;\theta)$

$E(\bar{X}) = \Sigma p \cdot x_i = \Sigma x_i \cdot f(x;\theta)$

$= E(x)$

$\hat{\theta} = 3\bar{X}$

$E(\hat{\theta}) = E(3\bar{X})$

$= 3E(\bar{X})$

$= 3E(x)$

$= E(3x)$

$= \theta$

So θ is an unbias estimator.

0.4 0.6 0.8

So θ is an unbias estimator.

6.2

20.

a. $\hat{p} = \frac{X}{N}$

estimate $= \frac{3}{20} = 0.15$

b. $\hat{p} = \frac{X}{N}$

$E(\hat{p}) = E(\frac{X}{n})$

$= \frac{1}{n} E(X)$

for binomial random variable

$E(X) = np$

$E(\hat{p}) = p$

Therefore $\hat{p} = \frac{X}{n}$ is an unbiased estimator

c. Denote A as that none of next five tests done on desease-free individuals are positive.

$\hat{p} = \frac{3}{20} = 0.15$

$P(A) = (1-0.15)^5 = 0.4437$

21.

a. $E(X) = b\Gamma(1+\frac{1}{a})$

$$V(X) = b^2\left\{\Gamma(1+\frac{2}{a}) - [\Gamma(1+\frac{1}{a})]^2\right\}$$

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

$$S^2 = \frac{1}{n}\sum_{i=1}^{n} X_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)^2$$

$$\bar{X} = b\Gamma(1+\frac{1}{a})$$

$$S^2 = b^2\left\{\Gamma(1+\frac{2}{a}) - [\Gamma(1+\frac{1}{a})]^2\right\}$$

$$b = \frac{\bar{X}}{\Gamma(1+\frac{1}{a})}$$

$$S^2 = \left(\frac{\bar{X}}{\Gamma(1+\frac{1}{a})}\right)^2\left\{\Gamma(1+\frac{2}{a}) - [\Gamma(1+\frac{1}{a})]^2\right\}$$

$$\Gamma(1+\frac{2}{a}) - [\Gamma(1+\frac{1}{a})]^2 = \frac{S^2}{\left(\frac{\bar{X}}{\Gamma(1+\frac{1}{a})}\right)^2}$$

b. $n = 20$

$\bar{X} = 28.0$

$\sum X_i^2 = 16500$

$$S^2 = \frac{1}{n}\sum_{i=1}^{n} X_i^2 - \bar{X}^2$$

$$= 825 - 784$$

$$= 41$$

$$\Gamma(1+\frac{2}{a}) - [\Gamma(1+\frac{1}{a})]^2 = \frac{41}{\left(\frac{28}{\Gamma(1+\frac{1}{a})}\right)^2}$$

$$y = \frac{1}{a}$$

$$\Gamma(1+2y) - [\Gamma(1+y)]^2 = \frac{41\Gamma(1+y)^2}{784}$$

$$\frac{[\Gamma(1+y)]^2}{\Gamma(1.4)} = 0.95$$

for $y = 0.2$ $a = 5$

$$b = \frac{\bar{X}}{\Gamma(1+\frac{1}{a})} = \frac{28}{\Gamma(1.2)} \approx 30.50$$

So $\hat{a} \approx 5$

$\hat{b} \approx 30.50$

The estimates are $\hat{a} \approx 5$ $\hat{b} \approx 30.50$

The estimates are $\hat{a} \approx 5$ $\hat{b} \approx 30.50$

29.

a. $E(X) = \mu + \frac{1}{\lambda}$

$V(X) = \frac{1}{\lambda^2}$

$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$

$\bar{X} = \mu + \frac{1}{\lambda}$

$S^2 = \frac{1}{\lambda^2}$

$\lambda = \frac{1}{S}$

$\bar{X} = \mu + S$

so $\mu = \bar{X} - S$

Thus $\hat{\lambda} = \frac{1}{S}$

$\hat{\mu} = \bar{X} - S$

(b.) $n = 20$

$\bar{X} = 28.0$

$\sum_{i=1}^{n} x_i^2 = 16500$

$\bar{X} = 28.0$

$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$

$S^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} X_i^2 - \frac{(\sum_{i=1}^{n} X_i)^2}{n}\right)$

$\sum_{i=1}^{n} X_i = n\bar{X} = 20 \times 28 = 560$

$S^2 = \frac{1}{19}\left(16500 - \frac{560^2}{20}\right)$

≈ 43.16

Now $\hat{\lambda} = \frac{1}{S} \approx 0.152$

$\hat{\mu} = \bar{X} - S \approx 21.43$

32.

a. CDF of $Y = \max(X_i)$

$$F_Y(y) = P(Y \le y)$$

$$= P(\max(X_1, X_2, \cdots, X_n) \le y)$$

$$= P(X_1 \le y, X_2 \le y, \cdots, X_n \le y)$$

$$= (P(X \le y))^n$$

For $X_i \sim \text{Uniform}(0, \theta)$

$P(X_i \le y) = \frac{y}{\theta}$ for $0 < y < \theta$

$$F_Y(y) = \begin{cases} \left(\frac{y}{\theta}\right)^n & 0 < y < \theta \\ 0 & \text{otherwise} \end{cases}$$

b. $E(\hat{\theta}) = E(Y)$

$$= \int_0^\theta y f_Y(y)\, dy$$

$$= \frac{n}{\theta^n} \int_0^\theta y^n\, dy$$

$$= \frac{n}{n+1} \theta$$

$\theta_{\text{unbiased}} = \frac{n+1}{n} Y$

Verification:

$$E\left(\frac{n+1}{n} Y\right) = \frac{n+1}{n} E(Y)$$

$$= \frac{n+1}{n} \cdot \frac{n}{n+1} \theta$$

$$= \theta$$

Thus, it's an unbiased estimator for θ.

Thus MLE $\hat{\theta} = \max(X_i)$ is biased. estimator $\frac{n+1}{n} \max(X_i)$ is unbiased.