A+

5.1 9.12.18.19

5.2 24.26.33.35

9. a. $1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dxdy$

$= \int_{20}^{30}\int_{20}^{30} k(x^2+y^2)\,dxdy$

$= 10k\int_{20}^{30} x^2\,dx + 10k\int_{20}^{30} y^2\,dy$

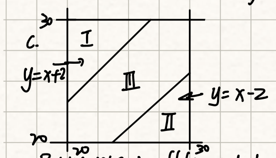
$= 20k\cdot\frac{19000}{3}$

$\therefore\ k = \frac{3}{380000}$

b. $P(X<26 \text{ and } Y<26) = \int_{20}^{26}\int_{20}^{26} k(x^2+y^2)\,dxdy = k\int_{20}^{26}\left[x^2y+\frac{y^3}{3}\right]_{30}^{26}dx$

$= k\int_{20}^{26}(6x^2+3192)\,dx = k(38304) = 0.3024$

c.



$P(|X-Y|\le 2) = \iint_{III} f(x,y)\,dxdy = 1-\iint_{I} f(x,y)\,dxdy - \iint_{II} f(x,y)\,dxdy$

$= 1-\int_{20}^{28}\int_{x+2}^{30} f(x,y)\,dydx - \int_{22}^{30}\int_{20}^{x-2} f(x,y)\,dydx$

$= 0.3593$

d. $f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_{20}^{30} k(x^2+y^2)\,dy = 10kx^2 + k\frac{y^3}{3}\Big|_{20}^{30} =$
$10kx^2+0.05$, for $20\le x\le 30$

e. $f(x,y)\ne f_X(x)\cdot f_Y(y)$, so X and Y are not independent.

12. a. $P(X>3) = \int_3^{\infty}\int_0^{\infty} xe^{-x(1+y)}\,dydx = \int_3^{\infty} e^{-x}\,dx = 0.050$

b. X: $f_X(x) = \int_0^{\infty} xe^{-x(1+y)}\,dy = e^{-x}$, $x\ge 0$
Y: $f_Y(y) = \int_3^{\infty} xe^{-x(1+y)}\,dx = \frac{1}{(1+y)^2}$, $y\ge 0$

$\because f(x,y)$ is not the product of the marginal pdfs.
so the two rvs are not independent.

c. P(at least on exceeds 3) $= P(X>3 \text{ or } Y>3) = 1-P(X\le 3 \text{ and } Y\le 3)$
$= 1-\int_0^3\int_0^3 xe^{-x(1+y)}\,dydx = 1-\int_0^3\int_0^3 xe^{-x}e^{-xy}\,dy$
$= 1-\int_0^3 e^{-x}(1-e^{-3x})\,dx = e^{-3}+0.25-0.25e^{-12} = 0.300$

18. a. $P_{Y|X}(y|1)$ results from dividing each entry in $X=1$ row of the joint probability table by $P_X(1)=0.34$:

$P_{Y|X}(0|1) = \frac{0.08}{0.34} = 0.2353$

$P_{Y|X}(1|1) = \frac{0.20}{0.34} = 0.5882$

$P_{Y|X}(2|1) = \frac{0.06}{0.34} = 0.1765$

b. $P_{Y|X}(x|2)$ is requested. to obtain this divide each entry in the $y=2$ row by $P_X(2)=.50$:

y	0	1	2
$P_{Y\mid X}(y\mid 2)$	0.12	0.28	0.60

c. $P(Y\le 1|X=2) = P_{Y|X}(0|2)+P_{Y|X}(1|2) = 0.12+0.28 = 0.40$

d. $P_{X|Y}(x|2)$ results from dividing each entry in the $y=2$ column by $P_Y(2)=0.38$:

x	0	1	2
$P_{X\mid Y}(x\mid 2)$	0.0526	0.1579	0.7895

19. a. $f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} = \frac{k(x^2+y^2)}{10kx^2+0.05}$, $20\le y\le 30$

$f_{X|Y}(x|y) = \frac{k(x^2+y^2)}{10ky^2+0.05}$ $20\le x\le 30$. $k=\frac{3}{380000}$

b. $P(Y \geq 25 \mid X=22) = \int_{25}^{30} f_{Y|X}(y|22)\,dy$

$= \int_{25}^{30} \frac{k((22)^2+y^2)}{10k(22)^2+0.05}\,dy = 0.556$

$P(Y \geq 25) = \int_{25}^{30} f_Y(y)\,dy = \int_{25}^{30} (10ky^2 + 0.05)\,dy = 0.549$

c. $E(Y|X=22) = \int_{-\infty}^{\infty} y \cdot f_{Y|X}(y|22)\,dy = \int_{20}^{30} y \cdot \frac{k(22^2+y^2)}{10k22^2+0.05}\,dy$

$= 25.372912$

$E(Y^2|X=22) = \int_{20}^{30} y^2 \cdot \frac{k(22^2+y^2)}{10k(22^2+0.05}\,dy = 652.028640$

$V(Y|X=22) = E(Y^2|X=22) - [E(Y|X=22)]^2 = 8.243976$

$\sigma = \sqrt{V(Y|X=22)} = 2.87$

24. Let $h(X,Y)$ = the number of individuals

		y					
	$h(x,y)$	1	2	3	4	5	6
	1	–	2	3	4	3	2
	2	2	–	2	3	4	3
x	3	3	2	–	2	3	4
	4	4	3	2	–	2	3
	5	3	4	3	2	–	2
	6	2	3	4	3	2	–

Since $p(x,y) = \frac{1}{30}$ for each possible (x,y).

$E[h(X,Y)] = \sum_x \sum_y h(x,y) \cdot p(x,y) = \sum_x \sum_y h(x,y) \frac{1}{30} = 2.80$

26. $E(\text{revenue}) = E(3X+10Y)$

$= \sum_{x=0}^{5} \sum_{y=0}^{2} (3x+10y) \cdot p(x,y) = 0 \cdot p(0,0) + \cdots + 35 \cdot p(5,2) = 15.4 = \$15.4$

33. Since $E(XY) = E(X) \cdot E(Y)$. $Cov(X,Y) = E(XY) - E(X) \cdot E(Y) = 0$

Since $corr(X,Y) = \frac{cov(X,Y)}{\sigma_X \sigma_Y}$. $corr(X,Y) = 0$.

35. a. $Cov(aX+b, cY+d) = E[(aX+b)(cY+d)] - E(aX+b)E(cY+d)$

$= E[acXY + adX + bcY + bd] - (aE(X)+b)(cE(Y)+d)$

$= acE(XY) - acE(X)E(Y)$

$= ac[E(XY) - E(X)E(Y)]$

$= ac\,Cov(X,Y)$

b. $Corr(aX+b, cY+d) = \frac{Cov(aX+b, cY+d)}{SD(aX+b)\,SD(cY+d)}$

$= \frac{ac\,Cov(X,Y)}{|a| \cdot |c| \cdot SD(X)SD(Y)} = \frac{ac}{|ac|} Corr(X,Y)$.

when a and c have the same signs, $ac = |ac|$. and we have $Corr(aX+b, cY+b) = Corr(X,Y)$

c. $|ac| = -ac$

$\therefore Corr(aX+b, cY+b) = -Corr(X,Y)$.