$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy$$
$$= \int_{20}^{30}\int_{20}^{30} k(x^2+y^2)\,dx\,dy$$

A

$$= 10k\int_{20}^{30} x^2\,dx + 10k\int_{20}^{30} y^2\,dy$$
$$= 20k\cdot\left(\frac{19000}{3}\right)$$
$$\Rightarrow k = \frac{3}{380000}$$

b. $P(X<26 \text{ and } Y<26)$
$$= \int_{20}^{26}\int_{20}^{26} k(x^2+y^2)\,dx\,dy$$
$$= 12k\int_{20}^{26} x^2\,dx \quad 4k\,x^3\Big|_{20}^{26}$$
$$= .3024$$

c. $P(|X-Y|\le 2) = \iint f(x,y)\,dx\,dy$
$$1 - \iint (x,y)\,dx\,dy - \iint f(x,y)\,dx\,dy$$
$$= .3593$$

d. $f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$
$$= \int_{20}^{30} k(x^2+y^2)\,dy$$
$$= 10kx^2 + .05$$

e. X and Y are not independent.

12. a. $P(X>3) = \int_3^\infty \int_0^\infty x e^{-x(1+y)}\,dy\,dx$

$= \int_3^\infty e^{-x}\,dx = .05$

b. $\int_0^\infty x e^{-x(1+y)}\,dy = e^{-x}$

of Y is $\int_3^\infty x e^{-x(1+y)}\,dx = \frac{1}{(1+y)^2}$

so the two are not independent

c. $P = 1 - P(X \le 3 \text{ and } Y \le 3)$

$= 1 - \int_0^3 \int_0^3 x e^{-x(1+y)}\,dy\,dx$

$= 1 - \int_0^3 e^{-x}(1 - e^{-3x})\,dx = e^{-3} + .25 - .25e^{-12} = .302$

18. a. $p_{y|x}(0|1) = \frac{.08}{.34} = .2353$

$p_{y|x}(1|1) = \frac{.20}{.34} = .5882$

$p_{y|x}(2|1) = \frac{.06}{.34} = .1765$

b.

y	0	1	2
$p_{y\|x}(y\|2)$	.12	.28	.60

c. $P(Y \le 1 | X = 2) = p_{y|x}(0|2) + p_{y|x}(1|2)$

$= .12 + .28 = .40$

d.

x	0	1	2
$p_{x\|y}(x\|2)$	.0526	.1579	.7895

$$f_{X|Y}(x|y) = \frac{f(x,y)}{f_X(x)} = \frac{k(x^2+y^2)}{10kx^2+.05}$$

$$f_{X|Y}(x|y) = \frac{k(x^2+y^2)}{10ky^2+.05}$$

$$k = \frac{3}{380000}$$

b. $P(Y \geq 25 \mid X = 22) = \int_{25}^{30} f_{Y|X}(y|22)\,dy$

$$= \int_{25}^{30} \frac{k((22)^2+y^2)}{10k(22)^2+.05}\,dy = .783$$

$$P(Y \geq 25) = \int_{25}^{30} f_Y(y)\,dy = \int_{25}^{30} (10ky^2+.05)\,dy = .75$$

c. $E(Y^2 \mid X=22) = \int_{20}^{30} y^2 \cdot \frac{k((22)^2+y^2)}{10k(22)^2+.05}\,dy = 652.028640$

$V(Y|X=22) = E(Y^2|X=22) - [E(Y|X=22)]^2 = 8.243976$

24.

$h(x,y)$	y = 1	2	3	4	5	6
x = 1	-	2	3	4	3	2
2	2	-	2	3	4	3
3	3	2	-	2	3	4
4	4	3	2	-	2	3
5	3	4	3	2	-	2
6	2	3	4	3	2	-

$$E[h(x,y)] = \sum_x \sum_y h(x,y) \cdot \frac{1}{30} = \frac{84}{30}$$

26. $E = E(3X+10Y)$

$$= \sum_{x=0}^{5} \sum_{y=0}^{2} (3x+10y) \cdot p(x,y) = 0 \cdot p(0,0) + \dots + 35 \cdot p(5,2)$$

$$= 15.4$$

3. $E(XY) = E(X)\cdot E(Y)$ $Cov(X,Y) = E(XY) - E(X)E(Y)$

$Corr(X,Y) = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} = 0$

35. a. $Cov(aX+b, cY+d)$

$= E[acXY + adX + bcY + bd] - (aE(X)+b)(cE(Y)+d)$

$= acCov(X,Y)$

b. $Corr(aX+b, cY+d)$

$= \frac{Cov(aX+b, cY+d)}{\sqrt{Var(aX+b)}\sqrt{Var(cY+d)}}$

c. $Corr(aX+b, cY+d) = -Corr(X,Y)$