A+

DATE /

3.3 29. a. $E(X) = 1\times0.05 + 2\times0.1 + 4\times0.35 + 8\times0.4 + 16\times0.1 = 0.05 + 0.2 + 1.4 + 3.2 + 1.6 = 6.45$

b. $V(X) = 5.45^2\times0.05 + 4.45^2\times0.1 + 2.45^2\times0.35 + 1.55^2\times0.4 + 9.55^2\times0.1 = 1.485125 + 1.98025 + 2.100875 + 0.961 + 9.12025 = 15.6475$

c. $\sigma = \sqrt{V(X)} = 3.956$

d. $V(X) = E(X^2) - [E(X)^2]$

$= 1^2\times0.05 + 2^2\times0.1 + 4^2\times0.35 + 8^2\times0.4 + 16^2\times0.1 - 6.45^2 = 57.25 - 41.6025$

$= 15.6475$

33. a. $E(X^2) = 0^2\cdot p(0) + 1^2\cdot p(1) = p$

b. Since $E(X) = p$, we know $V(X) = E(X^2) - [E(X)^2] = p - p^2 = p(1-p)$

c. $E(X^{79}) = 0^{79}\cdot p(0) + 1^{79}\cdot p(1) = p$

38. Since $E[h(X)] = \Sigma \frac{1}{x}\cdot p(x) = \frac{1}{1}\cdot\frac{1}{6} + \frac{1}{2}\cdot\frac{1}{6} + \frac{1}{3}\cdot\frac{1}{6} + \frac{1}{4}\cdot\frac{1}{6} + \frac{1}{5}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} = 0.408 >$ $0.286 = \frac{1}{3.5}$, so we should gamble.

41. $V(aX+b) = \Sigma[(ax+b) - E(aX+b)]^2\cdot p(x)$

$= \Sigma[ax+b - a\mu - b]^2\cdot p(x)$

$= \Sigma[ax - a\mu]^2\cdot p(x)$

$= a^2\Sigma(x-\mu)^2\cdot p(x)$

$= a^2\cdot\sigma_X^2$

B+

3.4 46. a. $b(3;8,0.35) = C_8^3\, 0.35^3\, 0.65^5 = 0.279$ b. $b(5;8,0.6) = C_8^5\, 0.6^5\, 0.4^3 = 0.279$

c. $P(3\le X\le5) = b(3;7,0.6) + b(4;7,0.6) + b(5;7,0.6) = 0.745$

d. $P(1\le X) = 1 - b(0;9,0.1) = 0.613$

47. a. $B(4;15,0.3) = 0.515$ b. $b(4;15,0.3) = B(4;15,0.3) - B(3;15,0.3) = 0.218$

c. $b(6;15,0.7) = B(6;15,0.7) - B(5;15,0.7) = 0.011$

d. $P(2\le X\le4) = B(4;15,0.3) - B(1;15,0.3) = 0.48$

e. $P(2\le X) = 1 - B(1;15,0.3) = 0.965$

f. $P(X\le1) = B(1;15,0.7) = 0$ g. $P(2<X<6) = B(5;15,0.3) - B(2;15,0.3)$

$= 0.595$

$P(X \le 2) = b(0;25,0.05) + b(1;25,0.05) + b(2;25,0.05)$

$= B(2;25,0.05) = 0.873$

b. $P(X \geq 5) = 1 - B(4;25,0.05) = 0.007$

c. $P(1 \le X \le 4) = B(4;25,0.05) - B(0;25,0.05) = 0.716$

d. $P(X=0) = 0.95^{25} = 0.277$

e. $E(X) = np = 25 \times 0.05 = 1.25$, $V(X) = np(1-p) = 1.25 \times 0.95 = 1.1875$

54. a. Let X denotes the number of people who wants the oversize version, then $P(X \geq 6) = 1 - B(5;10,0.6) = 0.633$

b. $P(5 \le X \le 7) = B(7;10,0.6) - B(4;10,0.6) = 0.667$

c. $P(3 \le X \le 7) = B(7;10,0.6) - B(2;10,0.6) = 0.821$

3.5 68. a. It's hypergeometric distribution, that is $X \sim h(X;6,12,20)$.

b. $P(X=2) = \dfrac{C_{12}^2 C_8^4}{C_{20}^6} = 0.1192$, $P(X \le 2) = \dfrac{C_8^6 + C_{12}^1 C_8^5 + C_{12}^2 C_8^4}{C_{20}^6} = 0.1373$

$P(X \geq 2) = 1 - P(X=0) - P(X=1) = 0.9819$

c. $E(X) = np = \dfrac{nM}{N} = \dfrac{12 \times 6}{20} = 3.6$

$V(X) = \left(\dfrac{14}{19}\right) 3.6 \left(1 - \dfrac{12}{20}\right) = \dfrac{36}{10} \times \dfrac{8}{20} \times \dfrac{14}{19} = 1.061$

so $\sigma = \sqrt{V(X)} = 1.03$.

69. a. $P(X=5) = \dfrac{C_7^5 C_5^1}{C_{12}^6} = 0.114$. b. $P(X \le 4) = 1 - P(X=5) - P(X=6) = 0.879$

c. $E(X) = 6 \times \dfrac{7}{12} = 3.5$, $V(X) = \dfrac{6}{11} \cdot 3.5 \cdot \dfrac{5}{12} = \dfrac{6}{11} \times \dfrac{35}{10} \times \dfrac{5}{12} = 0.795$, $\sigma = \sqrt{V(X)} = 0.892$

so $P = P(X=5) + P(X=6) = 0.121$

d. We can use the binomial distribution to do this, since the population is very large, we consider $X \sim (X;15,0.1)$, so $P(X \le 5) = B(5;15,0.1) = 0.998$.

72. a. $P = \frac{C_4^{?}C_7^{?}}{C_{11}^6}$. b. Since $X \sim h(x;6,4,11)$, so $E(X) = np = \frac{nM}{N} = \frac{6\times4}{11} \approx 2.18$.

75. a. Since $X \sim nb(x;2,0.5) = C_{x+1}^1\, 0.5^2\, 0.5^x = (x+1)\cdot(\frac{1}{2})^{x+2}$.

b. If they have 4 children, they have 2 male birth. $P(X=2) = 3\cdot(\frac{1}{2})^4 = \frac{3}{16}$.

c. That is $P(X=0)+P(X=1)+P(X=2) = \frac{1}{4}+\frac{1}{4}+\frac{3}{16} = \frac{11}{16}$.

d. $E(X) = \frac{2\times0.5}{0.5} = 2$, $E(X+2) = 4$, so the family is expected to have 4 children.

3.6 79. a. $P(X\le8) = 0.932$ b. $P(X=8) = 0.932-0.867 = 0.065$

c. $P(9\le X) = 1-P(X\le8) = 0.068$ d. $P(5\le X\le8) = 0.932-0.440 = 0.492$

e. $P(5<X<8) = 0.867-0.616 = 0.251$

84. a. $E(X) = 10000\times0.001 = 10$, $V(X) = 10\times0.999 = 9.999$, so $\sigma = \sqrt{V(X)} = 3.16$.

b. We use poisson to solve this (since $n = 10000$, $p = 0.001$, $np < 20$):

$\lambda = np = 10$, so $P(X>10) = 1-F(10;10) = 0.417$.

c. $P(X=0) = \frac{e^{-10}10^0}{0!} = e^{-10} \approx 0.00005$.

86. a. $P(X=4) = \frac{e^{-5}5^4}{4!} = 0.175$.

b. $P(X\ge4) = 1-F(3;5) = 0.735$.

c. $E(X) = \frac{45}{60}\times5 = 3.75$.

87. a. $P(X=\frac{10}{2}) = \frac{e^{-4}4^5}{5!} = 0.156$

b. $P(X=0) = F(0;2) = 0.175$

c. $E(X) = \lambda' = 2$