蒋云翔 2265T

Physics CST (2023-24) Homework 3

Please send the completed file to my mailbox yy.lam@qq.com by November 13th, with using the filename format:

student_number-name-cst-hw3

Please answer the questions by filling on these sheets. Or alternatively, do the homework as usual by using papers, then take the pictures and paste them onto these question sheets.

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1. A large diameter tank without lid placing on ground filled with water to a height h. The tank is punctured with a small hole at a height 0.5 m above the bottom. What is the water level h being able to eject a stream of water through the hole landing 3 m from the tank?

Sol: By Bernouli's equation: $P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$

It is clear that: $P_1 = \rho g(h - 0.5m)$

$V_1 = 0$

$h_1 = 0.5m$

$P_2 = 0$

$h_2 = 0$

then we know: $P_1 + \rho g h_1 = \rho g h = \frac{1}{2}\rho V_2^2$, $V_2 = 2\sqrt{gh}$ ... ①

$x = V_2 t = 3m$

$y = \frac{1}{2}gt^2 = 0.5m$ $\Rightarrow V_2 = x \cdot \sqrt{\frac{g}{2y}}$ ... ②

from ① ② we get: $h = \frac{x^2}{8y} = 2.25m$

2. A wind turbine is rotating counterclockwise at 0.5 rev/s and slows to a stop in 10 s. Its blades are 20 m in length. (a) What is the angular acceleration of the turbine? (b) What is the centripetal acceleration of the tip of the blades at $t = 0$ s? (c) What is the magnitude and direction of the total linear acceleration of the tip of the blades at $t = 0$ s?

Sol: (a) $\ddot{\theta} = \frac{\dot{\theta}}{t} = 0.05\ rev/s^2 = 0.1\pi\ rad/s^2$

Its direction is clockwise

(b) centripetal acceleration of the tip of the blades at t=0s is:

$a = \dot{\theta}^2 r = 20\pi^2\ m/s^2$

(c) centripetal: $a_c = 20\pi^2\ m/s^2$

slow down acceleration: $a_s = \ddot{\theta} r = 2\pi\ m/s^2$

total: $a = \sqrt{a_c^2 + a_s^2} = \sqrt{(20\pi^2)^2 + (2\pi)^2} = 197.49\ m/s^2$

Its direction is clockwise

3. To develop muscle tone, a man lifts a 8.0 kg weight held in his hand. He uses his biceps muscle to flex the lower arm through an angle of 60°. (a) What is the angular acceleration if the weight is 28 cm from the elbow joint, here forearm has a moment of inertia of 0.27 kgm², and the net force he exerts is 1230 N at an effective perpendicular lever arm of 2.5 cm? (b) What is the angular velocity at 60°? (c) How much work does she do?

Sol: (a) $I = I_{forearm} + I_w = 0.27 + 8\times0.28^2 = 0.8972\ kgm^2$

$\tau = rF = 2.5\times10^{-2}\times1230 = 30.75\ Nm$

$\ddot{\theta} = \frac{\tau}{I} = 12.251\ rev/s^2$

(b) By $V_t^2 - V_0^2 = 2as$, we know: $\dot{\theta}^2 - 0 = 2\cdot\ddot{\theta}\cdot\theta$ ··· ①

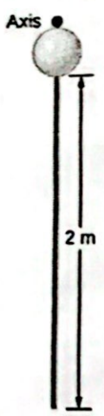
$\theta = 60°$ ②

By ①②, we get: $\dot{\theta} = 8.473\pi\ rad/s$

(c) $W = \tau\cdot\theta$

$= 10.25\pi J$

4. A pendulum consists of a rod of length 2 m and mass 2.4 kg with a solid sphere of mass 1.8 kg and radius 0.3 m attached at one end. (i) Find the moment of inertia of the pendulum about the axis. (ii) Find the centre of mass of it from the axis. (iii) What is the angular velocity of the pendulum at its lowest point if it is released from rest at an angle of 25°?



Sol: (i) As for the rod: $I_r = \int_0^L r^2 dm = \int_0^L r^2 \rho dr = [\rho \frac{r^3}{3}]_0^L = \frac{1}{3}mL^2 = 3.2\ kgm^2$

As for the sphere: $I_s = (r+L)^2\cdot m_s = 9.522\ kgm^2$

$I = I_r + I_s = 12.722\ kgm^2$

(ii) center of mass from axis: $\frac{2.4\times\frac{2}{2} + 1.8\times(2+0.3)}{2.4+1.8} = 1.557\ m$

(iii) By energy conservation law:

$mgx(1-\cos25°) = \frac{1}{2}mv^2$ ··· ①

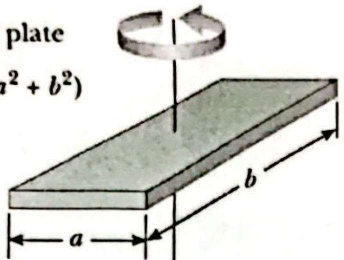
$\dot{\theta} = \frac{v}{x}$ ··· ②

by ①②, we get: $\dot{\theta} = 1.086\ rad/s$

5. (a) Drive the formula for I_{CM} as shown in the figure. (b) Find the new moment of inertia if the rotating axis is shifted to one of the corners.

Rectangular plate

$I_{CM} = \frac{1}{12}M(a^2+b^2)$



Sol: (a) $I_{cm} = \frac{1}{m}\sum_{i}^{n} m_i (x^2+y^2)$, $x \in [-\frac{a}{2}, \frac{a}{2}]$, $y \in [-\frac{b}{2}, \frac{b}{2}]$

$$\therefore I_{cm} = \int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}} (x^2+y^2)\,dm = \frac{M}{ab}\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}} (x^2+y^2)\,dx\,dy = \frac{M}{ab}\cdot\frac{1}{12}(a^3b+ab^3)$$

$$= \frac{M}{12}(a^2+b^2)$$

(b) By parallel-axis therom:

$$I' = I_{cm} + M\left(\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2\right)$$

$$= \frac{M}{3}(a^2+b^2)$$

6. Oil of density 730 kgm^{-3} is poured on top of a tank of water, and it floats on the water without mixing. A block of plastic of density 860 kgm^{-3} is placed in the tank, and it is about floating at the interface of the two liquids (completely immerses in the liquids). What fraction of the block's volume is immersed in water?

Sol: density of water = 1000 kgm^{-3}

$$\rho_o g V_o + \rho_b g V_o = \rho_w g V_w + \rho_b g V_w$$

then we get: $\frac{V_w}{V_o} = \frac{53}{62}$ so: $\frac{V_w}{V} = \frac{53}{115}$

7. (a) Using the law of conservation of energy, derive and calculate the escape velocity from the surface of earth. (b) What initial vertical speed is necessary to shoot a satellite to 300 km above the earth? (Given $G = 6.7 \times 10^{-11}\text{Nm}^2\text{kg}^2$, mass and radius of the earth 6.0×10^{24}kg and 6.4×10^6m, respectively.)

Sol: (a) By the energy conservation law: $\frac{1}{2}mv_1^2 = \frac{GMm}{r}$

then we get: $v_1 = \sqrt{\frac{2GM}{r}} = 11208.26$ m/s

(b) $\frac{1}{2}mv_2^2 = GMm\left(\frac{1}{r} - \frac{1}{r+3\times10^5}\right)$

then we get: $V_2 = 2371.71$ m/s

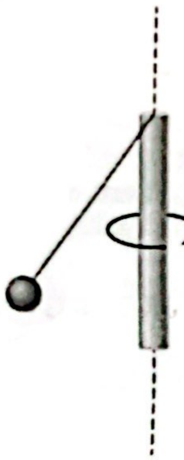
8. Assuming the density of Earth is constant, the gravitational acceleration $g(r)$ is a function of the radial distance. (a) Derive the equation of average value of the acceleration $\langle g(r)\rangle$ along the radial direction in terms of the G, M, R and r ($r < R$), being the gravitational constant, mass of Earth, radius of Earth and the radial distance from the centre. (b) Estimate the pressure at the centre of the earth.

Sol: (a) average: $\int_0^r g(r)\,dr$

$g(r) = \frac{GMr}{R^3}$

so: the average value is: $\frac{GMr}{2R^3}$

9. **(Angular momentum).** A small ball of mass 0.50 kg is attached by a massless string to a vertical rod that is spinning as shown. When the rod has an angular velocity of 6.0 rad s^{-1}, the string makes an angle of 30° with respect to the vertical. (a) If the angular velocity is increased to 10.0 rad s^{-1}, what is the new angle of the string? (b) Calculate the initial and final angular momenta of the ball. (c) Can the rod spin fast enough so that the ball is horizontal?



Sol: y direction: $T \cdot \cos 30^\circ = mg$ ... ①

x direction: $T \cdot \sin 30^\circ = m\omega^2 r$ ... ②

By ①, ②: we get: $r = 0.1572\,m$

length: $l = \frac{r}{\sin 30^\circ} = 0.3144\,m$

when $\omega' = 10\,rad/s$: $T' \cdot \sin\theta = m\omega'^2 l \sin\theta$

$T' \cdot \cos\theta = mg$

then we get: $\cos\theta = 0.3117 \Rightarrow \theta = 72.04^\circ$

(b) (initial) $L = mvr$

$= m\omega^2 r$

$= 0.074\ kg\,m^2 s^{-2}$

final: $L' = m\omega'^2 (l \cdot \sin\theta)^2$

$= 0.446\ kg\,m^2 s^{-2}$

(c) No, it can't.

Because the force in the horizontal direction can not be balanced.