

section 6.1

1. (a) mean value  $\hat{x} = \frac{\sum x_i}{n} = \frac{219.8}{27} = 8.14$

(b) the median value 9

(c)  $\sigma = \sqrt{\sigma^2} = \sqrt{\frac{E(x^2)}{n} - \bar{x}^2} = \sqrt{\frac{1236}{27} - 8.14^2} = 1.66$

(d)  $\hat{p} = \frac{4}{27} = 0.148$

(e)  $\frac{\sigma}{\bar{x}} = \frac{1.66}{8.14} = 0.204$

8. (a)  $\hat{p} = \frac{80-12}{80} = 0.85$

(b)  $\hat{p} = (0.85)^2 = 0.7225$

9. (a)  $E(\bar{x}) = \mu = \frac{0 \times 8 + 1 \times 37 + \dots + 7 \times 1}{150} = 2.11$

(b)  $\sigma = \frac{s}{\sqrt{n}} = \sqrt{\frac{2.11}{n}} = 0.119$

9

13.  $f(x, \theta) = 0.5(1 + \theta x) \quad -1 \leq x \leq 1$

$E(x) = \int_{-1}^1 x f(x, \theta) dx$

$= \int_{-1}^1 \theta x^2 dx + \int_{-1}^1 0.5x dx$

$= \frac{\theta x^3}{3} \Big|_{-1}^1 + 0.5x \Big|_{-1}^1$

$=$

13.  $E(x) = \int_{-1}^1 x f(x, \theta) dx = \int_{-1}^1 0.5x(1 + \theta x) dx$   
 $= \int_{-1}^1 0.5x dx + \int_{-1}^1 0.5\theta x^2 dx$   
 $= \frac{x^2}{4} \Big|_{-1}^1 + \frac{\theta x^3}{6} \Big|_{-1}^1$   
 $= \frac{\theta}{3}$

$\bar{x} = \mu = E(x) = \frac{\theta}{3}$      $\theta = 3\bar{x}$



20. (a)  $p = \frac{x}{n} = \frac{3}{20}$

b, yes.

c,  $p' = (1-p)^5 = 0.44$

21. (a)  $E(\bar{x}) = \frac{1}{n} \sum x_i$   $E(x^2) = V(x) + [E(x)]^2$

$$= \beta \cdot \Gamma(1 + \frac{1}{d}) = \beta^2 \{ \Gamma(1 + \frac{2}{d}) - [\Gamma(1 + \frac{1}{d})]^2 \} + \beta^2 [\Gamma(1 + \frac{1}{d})]^2$$

$$\beta = \frac{\frac{1}{n} \sum x_i}{\Gamma(1 + \frac{1}{d})} = \beta^2 \Gamma(1 + \frac{2}{d}) = \frac{1}{n} \sum x_i^2$$

$$\frac{1}{n} \sum x_i^2 = \frac{(\frac{1}{n} \sum x_i)^2 \Gamma(1 + \frac{2}{d})}{[\Gamma(1 + \frac{1}{d})]^2}$$

Thus d can be solve if  $x_1, \dots, x_n$  is determined.

(b)  $\beta = \frac{28}{\Gamma(1 + \frac{1}{d})}$

$$\frac{\frac{1}{n} \sum x_i^2}{(\frac{1}{n} \sum x_i)^2} = \frac{\Gamma(1 + \frac{2}{d})}{[\Gamma(1 + \frac{1}{d})]^2} = 1.0522 \approx \frac{1}{0.95} \quad d=5 \quad \beta = \frac{28}{\Gamma(1.2)}$$

29. (a)  $E(x) = \frac{1}{n} \sum x_i = \bar{x}$   $V(x) = \frac{1}{n^2}$   $E(x^2) = V(x) + [E(x)]^2$   
 $= \frac{1}{n^2}$   $= \frac{2}{n^2} = \frac{1}{n} \sum x_i^2$

32. (a)  $F(x) = P(x_1 < x_2 < \dots < x_n) = \left[ \int_0^y \frac{1}{\theta} dy \right]^n = \left( \frac{y}{\theta} \right)^n$

$f_y(y) = \frac{ny^{n-1}}{\theta^n}$

(b)  $E(x) = \int_0^y y \cdot f_y(y) dy = \int_0^y \frac{ny^{n-1}}{\theta^n} dy = \frac{n}{n+1} \frac{y^{n+1}}{\theta^n} \Big|_0^y =$

