B+

6.1 1.

a) estimate $= \frac{\sum x_i}{n} = 8.14$

b) Since the point is a median

sort the data:

5.9	6.3	6.3	6.5	6.8	6.8	7.0
7.0	7.2	7.3	7.4	7.6	7.7	7.7
7.8	7.8	7.9	8.1	8.2	8.7	9.0
9.7	9.7	10.7	11.3	11.6	11.8	

therefor the median is 7.8

c) $\sigma^2 = E(X^2) - E^2(X) = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = 2.66$

d) Since the number of that exceeds 10 MPa is 4, noted. S.

$p = \frac{S}{n} = 0.148$

e) $\sigma/\mu = \sqrt{\sigma^2} / \left(\frac{\sum x_i}{n}\right) = 0.20$

8.

a) $p = \frac{\text{not defective}}{n} = \frac{80-12}{80} = 0.85$

b) Since it is a large propulation

$P(\text{system works}) = p \cdot p = 0.7225$.

9.

a) $\hat{m} = \bar{x} = \frac{\sum x_i}{n} = \frac{18\times0+37\times1+42\times2+30\times3+13\times4+7\times5+2\times6+1\times7}{150} = 2.25$

b) $\sigma = \sqrt{\sigma_{\bar{x}}^2} = \sqrt{\hat{m}} = 1.5$

13.

$E(X) = \int_{-1}^{1} x \cdot f(x,\theta)\,dx = \int_{-1}^{1} \left(\frac{1}{2}x + \frac{1}{2}\theta x^2\right)dx = \left(\frac{1}{4}x^2 + \frac{1}{6}\theta x^3\right)\Big|_{-1}^{1} = \frac{1}{3}\theta$

$E(3\bar{X}) = 3E(\bar{X}) = 3E(X) = \theta$

therefor, $\hat{\theta} = 3\bar{x}$ is an unbiased estimator of θ

6.2 20.

a) $P(X=3) = b(3;20,p) = C_{20}^{3} \cdot p^{3}(1-p)^{20-3} = 1140\,p^{3}(1-p)^{17}$

$\frac{d\ln P(X=3)}{dp} = \frac{d}{dp}[3\ln p + 17\ln(1-p)] = \frac{3}{p} + \frac{17}{1-p}\times(-1) = \frac{3}{p} - \frac{17}{1-p} = 0$

therefor, $\hat{p} = \frac{3}{20} = 0.15$

b) since $\hat{p} = \frac{X}{n}$, $E(X) = np$

$E(\hat{p}) = E(\frac{X}{n}) = \frac{1}{n}E(X) = p$

It is unbiased

c) $(1-p)^5 = (1-0.15)^5 = 0.4437$

21.

a) Since $V(X) = E(X^2) - E^2(X)$

$E(X^2) = V(X) - E^2(X) = \beta^2[\Gamma(1+2/\alpha) - \Gamma^2(1+1/\alpha)] - \beta^2\Gamma^2(1+1/\alpha) = \beta^2\{\Gamma(1+2/\alpha) - 2\Gamma^2(1+1/\alpha)\}$

Since $\beta = \frac{E(X)}{\Gamma(1+1/\alpha)}$, $E(X^2) = \{\frac{E(X)}{\Gamma(1+1/\alpha)}\}^2\{\Gamma(1+2/\alpha) - 2\Gamma^2(1+1/\alpha)\} = \frac{E^2(X)\cdot\Gamma(1+2/\alpha)}{\Gamma^2(1+1/\alpha)} - 2E^2(X)$

From that, we can find estimate of β with α.

b) Since $E(X) = \bar{x} = 28.0$, $E(X^2) = \Sigma x_i^2/n = 825$.

$V(X) = E(X^2) - E^2(X) = 41$

So $\frac{\Gamma(1+2/\alpha)}{\Gamma^2(1+1/\alpha)} = 1.052 \Rightarrow \frac{\Gamma^2(1+1/\alpha)}{\Gamma(1+2/\alpha)} = 0.95 \Rightarrow \alpha = 5$

$\beta = \frac{E(X)}{\Gamma(1+1/\alpha)} = \frac{28}{\Gamma(1.2)} = 30.49$

29.

a) Since $f(x_1, x_2 \dots x_n; \lambda, \theta) = (\lambda e^{-\lambda(x_1-\theta)})(\lambda e^{-\lambda(x_2-\theta)})\dots(\lambda e^{-\lambda(x_n-\theta)}) = \lambda^n e^{-\lambda\sum^{n}(x_i-\theta)}$

$\ln f(x_1, x_2 \dots x_n; \lambda, \theta) = n\ln\lambda - \lambda\sum^{n}(x_i-\theta) = n\ln\lambda - \lambda\Sigma x_i + n\lambda\theta$

$\frac{\partial}{\partial\lambda}\ln f(x_1, x_2 \dots x_n; \lambda, \theta) = \frac{n}{\lambda} - \sum^{n}(x_i-\theta) = 0$

$\frac{\partial}{\partial\theta}\ln f(x_1, x_2 \dots x_n; \lambda, \theta) = n\lambda = 0$

$\lambda = \frac{n}{\sum^{n}(x_i-\theta)}$ and $\theta = 0$

b) Since $E(X) = 5.672$, $V(X) = 11.93$

therefor, $\lambda = 0.0838$, $\theta = 0$

32.

a)