Physics CST (2023-24) Homework 2

Please send the completed file to my mailbox yy.lam@qq.com by October 23rd, with using the filename format:

student_number-name-cst-hw2

Please answer the questions by filling on these sheets. Or alternatively, do the homework as usual by using papers, then take the pictures and paste them onto these question sheets.

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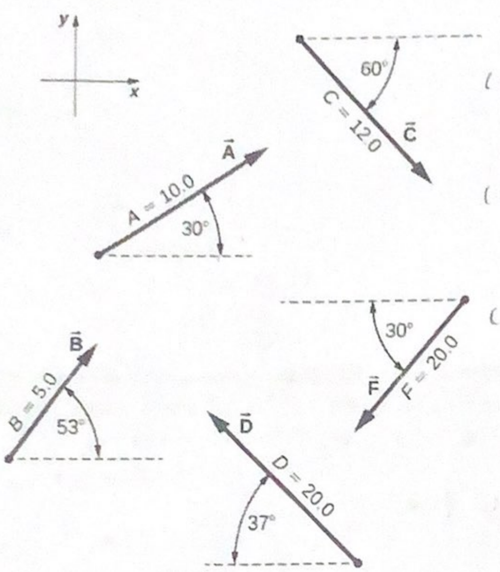
1. Find the following vector products: (a) $\mathbf{A} \times \mathbf{C}$, (b) $\mathbf{A} \times \mathbf{F}$, (c) $\mathbf{D} \times \mathbf{C}$, (d) $\mathbf{A} \times (\mathbf{F} + 2\mathbf{C})$, (e) $\hat{\mathbf{i}} \times \mathbf{B}$, (f) $\hat{\mathbf{j}} \times \mathbf{B}$, (g) $(3\hat{\mathbf{i}} - \hat{\mathbf{j}}) \times \mathbf{B}$, and (h) $\hat{\mathbf{B}} \times \mathbf{B}$.



Solution:

(a) $A \times C = |A| \cdot |C| \cdot \sin(-90°)$

$= -120$

(b) $A \times F = |A| \cdot |F| \cdot \sin 180°$

$= 0$

(c) $D \times C = |D||C| \cdot \sin(37° + 90°)$

$= 20 \times 12 \times \cos 37°$

$= 20 \times 12 \times \frac{4}{5}$

$= 192$

(d) $A \times (F + 2C)$

$= A \times F + 2 \cdot A \times C$

$= 0 + (-120) \times 2$

$= -240$

(e) $\hat{i} \times B = |B| \cdot \sin 53°$

$= 5 \times \frac{4}{5}$

$= 4$

(f) $\hat{j} \times B = |B| \cdot \sin(-37°)$

$= 5 \times (-\frac{3}{5})$

$= -3$

(g) $(3\hat{i} - \hat{j}) \times B$

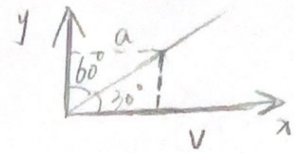
$= 3\hat{i} \times B - \hat{j} \times B$

$= 3 \times 4 - (-3)$

$= 15$

(h) $\hat{B} \times B = |B| \times \sin 0°$

$= 0$



$5t^2 \times \frac{2}{\sqrt{3}}$

2. The velocity of a particle moving along the x axis varies in time according to the expression $v_x = (40 - 5t^2)\ ms^{-1}$, where t is in seconds. (a) Find the average acceleration in the time interval $t = 0$ to $t = 2$ s along the direction $30°$ from the x-axis. (b) Determine the acceleration at $t = 2$ s. (c) What is the acceleration along the y-axis?

Solution:

(a) Differentiate the V_x wrt t

$$\frac{dv_x}{dt} = a_x = -10t\ (m/s^2)$$

By decomposing acceleration:

we know that: $a \cdot \cos 30° = a_x$

so $a = -\frac{20t}{\sqrt{3}}\ (m/s^2)$

(b) when $t = 2s$

$a = -\frac{40}{\sqrt{3}}\ m/s^2$

(c) $a_y = a \cdot \sin 30°$

$= -\frac{10t}{\sqrt{3}}\ m/s^2$

3. Assuming earth is a rigid body with a constant density. Find the gravitational acceleration at a point p inside earth. Express your results in terms of the radius of earth R, the distance r of the point p from the centre, and M the mass of earth.

Solution: By Newton's gravitational force law:

$$F = \frac{Gm_1m_2}{R_1^2}$$

we see the earth is consist of infinite ball layer:

$$dF = \frac{G \cdot m_1 \cdot m_2\, dr}{R_1^2}$$

$$dF = \frac{Gm_1\, dm_2}{R_1^2}$$

$$a = \int dF = \int \left(\frac{G \cdot m_1 \cdot dm_2}{R_1^2}\right)$$

$$m_1 = \frac{4}{3}\pi R_1^3 \cdot \rho$$

$$V = \frac{4}{3}\pi R_1^3$$

$$a = \int dF = \frac{\frac{4}{3}\pi R_1^3 G R_1^3 \rho \cdot \frac{4}{3}\pi R_1^2 dr}{R_1^2} = \frac{GM_1 r}{R_1^3}$$

so we get:

$$a = \frac{GM \cdot r}{R^3}$$

静止的

4. A person driving a car traveling 30 m/s passes a stationary motorcycle police officer. 2.5 s after the car passes, the police starts to move and accelerates in pursuit of the speeding car. The motorcycle has constant acceleration of 3.7 m/s^2. (a) How fast will the police officer be traveling when he overtakes the car? (b) Draw curves of x versus t for both the motorcycle and the car, taking $t = 0$ at the moment the car passes the stationary police officer.

Solution: (a) Assume that the motorcycle overtook the car at t s from it's starting

so: $S_{car} = V_c \cdot (t+2.5)$
$= 30t + 75$ (m)

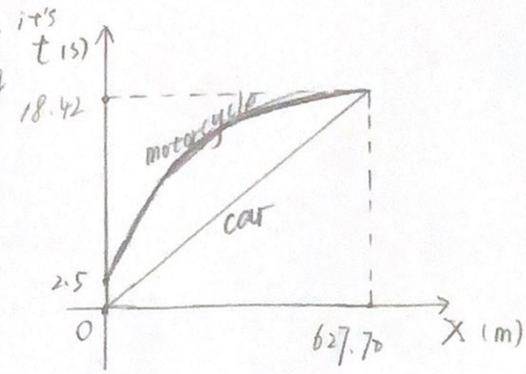
$S_{motor} = \frac{1}{2}at^2$
$= 1.85t^2$ (m)

we have: $S_{car} = S_{motor}$

that is: $30t + 75 = 1.85t^2$
$t = 18.42$ s

so: $V_{motor} = at = 68.154$ m/s

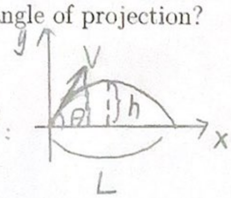
(b)



水平 =3最高

5. A projectile is fired in such a way that its horizontal range is equal to three times its maximum height. What is the angle of projection?

Solution: Assume the angle is θ: the initial velocity is V:



Let upward and foward direction be positive.

$V_x = V \cdot \cos\theta$

$V_y = V \cdot \sin\theta$

$a_y = -g = -9.8\ m/s^2$

$a_x = 0\ m/s^2$

in y direction: $2a_y h = 0 - V_y^2$,

that is: $19.6h = V^2 \sin^2\theta$ ... ①

$0 = V_y + a_y t$

that is: $t = \frac{V \cdot \sin\theta}{9.8}$ s

in x direction: $L = 3h$

$L = V_x t$

$3h = V \cdot \cos\theta \cdot \frac{V \sin\theta}{9.8}$

then we get: $3h = \frac{V^2 \sin\theta \cdot \cos\theta}{9.8}$ ... ②

From ①, ②, we get: $\tan\theta = \frac{2}{3}$

so $\theta = 33.69°$

6. Given the distance of the centre to centre distance of Earth and the Moon 3.84×10^5 km, the time interval for a month 27.3 days, (a) find the acceleration due to Earth's gravity at the distance of the moon. (b) Given the radius of Earth 6370 km, calculate the period of an artificial satellite orbiting at an average altitude of 1500 km above Earth's surface.

Solution: let M be the mass of earth, m be Moon's, r be the radius of earth

(a) Let $R = 3.84 \times 10^5$ km

$F = m(\frac{2\pi}{T})^2 \cdot R$

$a = \frac{F}{m}$

$= (\frac{2\pi}{T})^2 \cdot R$

$= 35.314$ Km/h

(b) $H = r + h = 7870$ (km)

By kepler's third law:

$\frac{T_m^2}{T_s^2} = \frac{R_m^3}{H^3}$

then we get $T_s = 0.08$ (day)

$= 1.92$ (h)

$= 115.2$ (min)

7. A hockey puck of mass 0.18 kg is shot across a rough floor with the roughness different at different places, which can be described by a position-dependent coefficient of kinetic friction. For a puck moving along the x-axis the coefficient of kinetic friction is the following function of x, where x is in m: $\mu(x) = 0.1 + 0.05x$. Find the work done by the kinetic friction force on the hockey p when it has moved (a) from $x = 0$ to $x = 1$ m, and (b) from $x = 1$ m to $x = 3$ m.

Solution: Let the direction of the puck's movement be positive

(a) from the function of kinetic friction:

$\mu(x) = 0.1 + 0.05x$

as it is linear, so the kinetic friction is also ~~to~~ linear.

when $x = 0$ (m)

$F_1 = \mu(0) mg$

$= 0.1764$ N

when $x = 1$ (m)

$F_2 = \mu(1) mg$

$= 0.2646$N

The average friction is: $\bar{F} = \frac{F_1 + F_2}{2}$

$= 0.2205$ N

so the work done by the kinetic friction is: $W = \bar{F} \cdot \Delta x = 0.2205$ J

(b) The same as the (a) quesion.

when $x = 3$ (m), $F_3 = \mu(3) mg$

$= 0.441$ N

$\bar{F}' = \frac{F_2 + F_3}{2}$

$= 0.3528$N

$W' = \bar{F}' \Delta x'$

$= 0.3528 \times (3 - 1)$

$= 0.7056$ (J)

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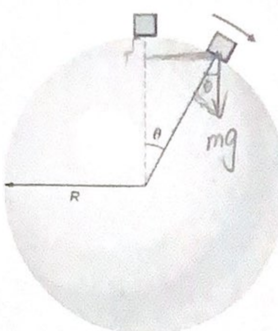
8. A body of mass m and negligible size starts from rest and slides down the surface of a frictionless solid sphere of radius R as shown. Find the angle θ while the body leaves the sphere.



Solution:

$$mg\cdot\cos\theta = m\frac{V^2}{R} \quad \cdots ①$$

$$mg(R - R\cdot\cos\theta) = \frac{1}{2}mV^2 \quad \cdots ②$$

By ①, ②, we know: $\cos\theta = \frac{2}{3}$

So $\theta \approx 48°$

9. A 75 kg crate rests on the bed of a truck. The coefficients of kinetic and static friction between the surfaces are $\mu_k = 0.3$ and $\mu_s = 0.4$, respectively. Find the frictional force on the crate and describe the state of motion of it when the truck is accelerating forward relative to the ground at (a) 2.0 ms^{-2}, and (b) 5.0 ms^{-2}.

Solution: (a)

The static friction can most provide the crate with a acceleration at most is:

$$a_{max} = \frac{\mu_s mg}{m} = 3.92\ m/s^2$$

So when truck's acceleration is only $2\ m/s^2$ the crate remains staic. and its acceleration is the same as truck's.

Then we know: $a_1 = \frac{f}{m}$

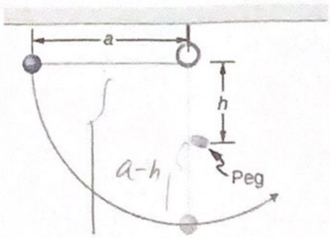
$$f = 150\ N$$

(b) As $5\ m/s^2 > 3.92\ m/s^2$

So: the crate begins to move, the friction transfer to the kinetic one.

$$f' = \mu_k mg = 220.5\ N$$

10. A small ball of mass m attached to a string of length a. A small peg is located a distance h below the point where the string is supported. If the ball is released when the string is horizontal, show that h must be greater than $3a/5$ if the ball is to swing completely around the peg. (*Hint:* Set up equations for the centripetal force and conservation law of energy)



Solution: The critical condition is: the ball's gravity acts as its centripetal force. and its radius is a-h

$$mg = m\frac{v^2}{(a-h)} \quad \cdots ①$$

$$mg[a-2(a-h)] = \frac{1}{2}mv^2 \quad \cdots ②$$

by ①, ②, we get: $h = \frac{3}{5}a$

so: only when $h > \frac{3a}{5}$ can the small ball satisfy the condition.