Exercises

Part one:　Choice questions

1. The order of the partial differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3+3ye^x=2x$ is (　　)

A.　1　　　　B.　2　　　　C. 3　　　　D. 0

2. Given $f(x,y)=\begin{cases}\frac{xy}{\sqrt{x^2+y^2}}, & x^2+y^2\neq 0\\ 0, & x^2+y^2=0\end{cases}$, then at $(0,0)$, $f(x,y)$ is (　　)

A. continuous and differentiable.　　　B. continuous and non-differentiable.
C. discontinuous and differentiable.　　D. discontinuous and non-differentiable.

3. Given $I_i=\iint\limits_{D_i} e^{-(x^2+y^2)}dxdy,\ i=1,\ 2,\ 3,$ where $D_1=\{(x,y)\,|\,x^2+y^2\le R^2\}$,

$D_2=\{(x,y)\,|\,x^2+y^2\le 2R^2\}$, $D_3=\{(x,y)\,|\,|x|+|y|\le 2R\}$, then（　　）

A.　$I_3>I_2>I_1$　　B. $I_1>I_2>I_3$　　C.　$I_1>I_3>I_2$　　D. $I_3>I_1>I_2$

4. If $z=f(ax^2+by^2)$, where f is differentiable, a, b are constants, then z satisfies the following equation（　　）

A.　$ax\frac{\partial z}{\partial x}+by\frac{\partial z}{\partial y}=0$　　　　B.　$ax\frac{\partial z}{\partial x}-by\frac{\partial z}{\partial y}=0$

C.　$by\frac{\partial z}{\partial x}-ax\frac{\partial z}{\partial y}=0$　　　　D.　$by\frac{\partial z}{\partial x}+ax\frac{\partial z}{\partial y}=0$

5. If $u(x,y)$ has continuous second order partial derivatives on the closed bounded region D, and satisfies $\frac{\partial^2 u}{\partial x\partial y}\neq 0$ and $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, then (　　)

A. both the absolute maximum and minimum occur in the interior of D.

B. both the absolute maximum and minimum occur on the boundary of D.

C. the absolute maximum occurs in the interior of D, and the absolute minimum occurs on the boundary of D

D. the absolute minimum occurs in the interior of D, and the absolute maximum occurs on the boundary of D

6. If $I_1=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\ln(\sin x)dx,\ \ I_2=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\ln(\cos x)dx,$ then ( ).

A. $I_2<I_1<0$; B. $I_1<I_2<0$; C. $0<I_1<I_2$; D. $0<I_2<I_1$

7. At $(0,0)$, $f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$ is ( ).

A. continuous and the partial derivatives exist;

B. continuous and the partial derivatives don't exist;

C. discontinuous and the partial derivatives exist;

D. discontinuous and the partial derivatives don't exist.

8. If $y_1(x),y_2(x),y_3(x)$ are linearly independent and they are the solutions of 2nd order ODE $y''+P(x)y'+Q(x)y=f(x)$, C_1,C_2 are arbitrary constants, then the general solution of this differential equation is ( ).

A. $C_1y_1+C_2y_2+y_3$ B. $C_1y_1+C_2y_2-(C_1+C_2)y_3$

C. $C_1y_1+C_2y_2-(1-C_1-C_2)y_3$ D. $C_1y_1+C_2y_2+(1-C_1-C_2)y_3$

9. If $I_1=\iint\limits_D\cos\sqrt{x^2+y^2}d\sigma, I_2=\iint\limits_D\cos(x^2+y^2)d\sigma, I_3=\iint\limits_D\cos(x^2+y^2)^2d\sigma$, where $D=\{(x,y)\,|\,x^2+y^2\le 1\}$, then ( ).

A. $I_3>I_2>I_1$ B. $I_1>I_2>I_3$ C. $I_2>I_1>I_3$ D. $I_3>I_1>I_2$

10. Considering the following properties of $f(x,y)$:

① $f(x,y)$ is continuous at (x_0,y_0),

②The partial derivatives of $f(x,y)$ is continuous at (x_0,y_0),

③ $f(x,y)$ is differentialble at (x_0,y_0),

④ The partial derivatives of $f(x,y)$ exist at (x_0,y_0),

The notation "$P\Rightarrow Q$" represents that property Q can be obtained by property P, then ( ).

A. ②⇒③⇒① B. ③⇒②⇒① C. ③⇒④⇒① D. ③⇒①⇒④

11. If $f(x,y)$ is continuous on a neighborhood of (0,0),and

$$\lim_{(x,y)\to(0,0)} \frac{f(x,y)-xy}{(x^2+y^2)^2}=1$$

then the correct one of the following four statements is（ ）

A. $f(0,0)$ is neither a local maximum nor a local minimum;

B. $f(0,0)$ is a local maximum;

C. $f(0,0)$ is a local minimum;

D. Unable to determine whether $f(0,0)$ is a local maximum or a local minimum from the given information;

12. If $y=\frac{1}{2}e^{2x}+(x-\frac{1}{3})e^{x}$ is a particular solution of 2nd order constant coefficients linear differential equation $y''+ay'+by=ce^{x}$, then ( ).

A. $a=-3,\ b=2,\ c=-1$ B. $a=3,\ b=2,\ c=-1$

C. $a=-3,\ b=2,\ c=1$ D. $a=3,\ b=2,\ c=1$

13. If the partial derivatives of $f(x,y)$ at (x_0,y_0) exist，then ( ).

A. $f(x,y)$ is continuous at (x_0,y_0); B. $f(x,y)$ is differentiable at (x_0,y_0);

C. Both $\lim\limits_{x\to x_0} f(x,y_0),\ \lim\limits_{y\to y_0} f(x_0,y)$ exist D. $\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y)$ exists

14. If the curve $y=f(x)$ passes through the origin，and the normal line at the origin is perpendicular to the line $y-3x=5$, the function $y=f(x)$ satisfies $y''+y'-2y=0$, then $f(x)=$（ ）

A. $e^{-2x}-e^{x}$, B. $e^{x}-e^{-2x}$ C. $\frac{1}{9}e^{-2x}-\frac{1}{9}e^{x}$ D. $\frac{1}{9}e^{x}-\frac{1}{9}e^{-2x}$

15. If $I_1=\iint\limits_D yx^3d\sigma, I_2=\iint\limits_D y^2x^3d\sigma, I_3=\iint\limits_D \sqrt{y}x^3d\sigma$, where D is a closed region in the second quadrant，and $0<y<1$，then ( ).

A. $I_3>I_2>I_1$ B. $I_1>I_2>I_3$ C. $I_2>I_1>I_3$ D. $I_3>I_1>I_2$

16. The particular solution of $y''-2y'+10y=e^{x}\cos 3x$ is in the form of ( )

A. $Ae^x\cos 3x$, B. $e^x(A\cos 3x+B\sin 3x)$, C. $Axe^x\cos 3x$, D. $e^x(Ax\cos 3x+Bx\sin 3x)$

17. For $f(x,y)=x|x|+|y|$, then ( )

A. $f_x(0,\ 0)$ exists, but $f_y(0,\ 0)$ doesn't exist;

B. $f_x(0,\ 0)$ doesn't exist, but $f_y(0,\ 0)$ exists;

C. Both $f_x(0,\ 0)$ and $f_y(0,\ 0)$ exist;

D. Neither $f_x(0,\ 0)$ nor $f_y(0,\ 0)$ exists.

18. For the following statements, ( ) is right.

A. If the function $f(x,y)$ is continuous at $P(x_0,\ y_0)$, then its partial derivatives exist at $P(x_0,\ y_0)$;

B. If the function $f(x,y)$ is differentiable at $P(x_0,\ y_0)$, then it is continuous at $P(x_0,\ y_0)$;

C. If the mixed second partial derivatives $f_{xy}(x,\ y),\ f_{yx}(x,\ y)$ exist at $P(x_0,\ y_0)$, then $f_{xy}(x_0,\ y_0)=f_{yx}(x_0,\ y_0)$;

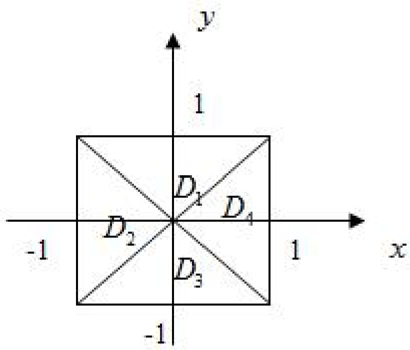
D. If the partial derivatives $f_x(x,\ y),\ f_y(x,\ y)$ exist at $P(x_0,\ y_0)$, then $f(x,y)$ is differentiable at $P(x_0,\ y_0)$.

19. If $D=\left\{(x,y)\,|\,|x|+|y|\le\frac{\pi}{2}\right\}$,and $I_1=\iint_D\sqrt{x^2+y^2}\,dxdy$, $I_2=\iint_D\sin\sqrt{x^2+y^2}\,dxdy$ $I_3=\iint_D\left(1-\cos\sqrt{x^2+y^2}\right)dxdy$, then ( )

A. $I_3<I_2<I_1$; B. $I_1<I_2<I_3$; C. $I_2<I_1<I_3$; D. $I_2<I_3<I_1$

20. Like as the right hand, The square $\{(x,y)\,|\,|x|\le 1,|y|\le 1\}$ is partitioned into $D_k\,(k=1,2,3,4)$ by the diagonals , $I_k=\iint\limits_{D_k}y\cos x\,dxdy$, then $\max\limits_{1\le k\le 4}\{I_k\}=$ ( )

A. I_1 B. I_2 C. I_3 D. I_4



21. If $z=z(x,y)$ is defined by $F(\frac{y}{x},\frac{z}{x})=0$, where F is a differentiable function, and $F_2'\neq 0$, then $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=($ $)$

A. x B. z C. $-x$ D. $-z$

22. $\lim\limits_{x\to\infty}\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}\frac{n}{(n+i)(n^2+j^2)}=($ $)$

A. $\int_0^1 dx\int_0^x \frac{1}{(1+x)(1+y^2)}dy$ B. $\int_0^1 dx\int_0^x \frac{1}{(1+x)(1+y)}dy$

C. $\int_0^1 dx\int_0^1 \frac{1}{(1+x)(1+y)}dy$ D. $\int_0^1 dx\int_0^1 \frac{1}{(1+x)(1+y^2)}dy$

23. If $f(x,y)$ is continuous at $(0,0)$, then the correct one of the following statements is （ ）

A. If $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{f(x,y)}{|x|+|y|}$ exists，then $f(x,y)$ is differentiable at $(0,0)$;

B. If $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{f(x,y)}{x^2+y^2}$ exists，then $f(x,y)$ is differentiable at $(0,0)$;

C. If $f(x,y)$ is differentiable at $(0,0)$, then $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{f(x,y)}{|x|+|y|}$ exists;

D. If $f(x,y)$ is differentiable at $(0,0)$, then $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{f(x,y)}{x^2+y^2}$ exists;

24. For the differential equation $y''-4y'+8y=e^{2x}(1+\cos 2x)$, the form of particular solution y^* is ( )

A. $Ae^{2x}+e^{2x}(B\cos 2x+C\sin 2x)$ B. $Axe^{2x}+e^{2x}(B\cos 2x+C\sin 2x)$

C. $Ae^{2x}+xe^{2x}(B\cos 2x+C\sin 2x)$ D. $Axe^{2x}+xe^{2x}(B\cos 2x+C\sin 2x)$

25. If $f(x,y)$ has partial derivatives， $\frac{\partial f(x,y)}{\partial x}>0, \frac{\partial f(x,y)}{\partial y}<0$ at any point (x, y)，

then ( )

A. $f(0,0)>f(1,1)$; B. $f(0,0)<f(1,1)$; C. $f(0,1)>f(1,0)$; D. $f(0,1)<f(1,0)$

26. If $I_k=\int_0^{k\pi} e^{x^2}\sin x\mathrm{d}x, (k=1,2,3)$, then ( )

A. $I_1<I_2<I_3$ B. $I_3<I_2<I_1$ C. $I_2<I_3<I_1$ D. $I_2<I_1<I_3$

27. If $f(x,y)$ is continuous, then $\int_{\frac{\pi}{2}}^{\pi}dx\int_{\sin x}^{1}f(x,y)dy$ =( ).

A. $\int_0^1 dy\int_{\pi+\arcsin y}^{\pi}f(x,y)dy$; B. $\int_0^1 dy\int_{\pi-\arcsin y}^{\pi}f(x,y)dy$

C. $\int_0^1 dy\int_{\frac{\pi}{2}}^{\pi+\arcsin y}f(x,y)dy$; D. $\int_0^1 dy\int_{\frac{\pi}{2}}^{\pi-\arcsin y}f(x,y)dy$

28. If $f(x)$ is a continuous function, $F(t)=\int_1^t dy\int_y^t f(x)dx$, then $F'(2)=$ ( )

A. $2f(2)$ B. $f(2)$ C. $-f(2)$ D. 0

29. If $D=\{(x,y)|x^2+y^2\le 4, x\ge 0, y\ge 0\}$, $f(x)$ is a positive continuous function on D，a, b are constants, then $\iint_D \frac{a\sqrt{f(x)}+b\sqrt{f(y)}}{\sqrt{f(x)}+\sqrt{f(y)}}d\sigma=$ ( )

$ab\pi$. B. $\frac{ab}{2}\pi$. C. $(a+b)\pi$. D. $\frac{a+b}{2}\pi$.

30. If $I_1=\int_0^{\frac{\pi}{4}}\frac{\tan x}{x}dx, I_2=\int_0^{\frac{\pi}{4}}\frac{x}{\tan x}dx$, then ( )

A. $I_1>I_2>1$. B. $1>I_1>I_2$. C. $I_2>I_1>1$. D. $1>I_2>I_1$.

31. If $f(x,y)$ is a continuous function, $\int_0^1 dy\int_{-\sqrt{1-y^2}}^{1-y}f(x,y)dx=$ ( )

A. $\int_0^1 dx\int_1^{x-1}f(x,y)dy+\int_{-1}^0 dx\int_0^{\sqrt{1-x^2}}f(x,y)dy$

B. $\int_0^1 dx\int_1^{1-x}f(x,y)dy+\int_{-1}^0 dx\int_{-\sqrt{1-x^2}}^0 f(x,y)dy$

C. $\int_0^{\frac{\pi}{2}} d\theta \int_0^{\frac{1}{\cos\theta+\sin\theta}} f(r\cos\theta, r\sin\theta)dr + \int_{\frac{\pi}{2}}^{\pi} dx \int_0^1 f(r\cos\theta, r\sin\theta)dr$

D. $\int_0^{\frac{\pi}{2}} d\theta \int_0^{\frac{1}{\cos\theta+\sin\theta}} f(r\cos\theta, r\sin\theta)rdr + \int_{\frac{\pi}{2}}^{\pi} dx \int_0^1 f(r\cos\theta, r\sin\theta)rdr$

32. If the curve $y = f(x)$ passes through the origin，and the normal line at the origin is perpendicular to the line $y - 3x = 5$, the function $y = f(x)$ satisfies $y'' + y' - 2y = 0$, then $f(x)=$ （ ）

A. $e^{-2x} - e^x$, B. $e^x - e^{-2x}$ C. $\frac{1}{9}e^{-2x} - \frac{1}{9}e^x$ D. $\frac{1}{9}e^x - \frac{1}{9}e^{-2x}$

Part Two: Filling blanks

1. The domain of $z = \arcsin\frac{y}{x} + \sqrt{xy}$ is ________________
2. The general solution of $y'' + 4y' + 4y = 0$ is ________________.
3. $\lim\limits_{(x,y)\to(0,0)} \frac{\sqrt{x^2+y^2} - \sin\sqrt{x^2+y^2}}{(\sqrt{x^2+y^2})^3} =$ ___________.
4. Reverse the order of iterated integrals $\int_0^2 dx \int_{x^2}^{2x} f(x,y)dy =$ _____________.
5. Let $D = \{(x,y), x^2 + y^2 \le 1\}$, then $\iint\limits_D e^{(x^2+y^2)} dxdy =$ _______________.
6. If $f(u)$ is differentiable, and $f'(0) = \frac{1}{2}$, then the total differential of $z = f(4x^2 - y^2)$ at $(1,2)$ is $dz\big|_{(1,2)} =$ ______________.
7. If $\overline{Y} + y_1^*$ is the general solution of $y'' + P(x)y' + Q(x)y = f_1(x)$, y_2^* is a particular solution of $y'' + P(x)y' + Q(x)y = f_2(x)$, then the general solution of $y'' + P(x)y' + Q(x)y = f_1(x) + f_2(x)$ is _______________.
9. If $f(u)$ is differentiable, and $f(0) = 0, f'(0) = 1$, then

$\lim\limits_{t\to0^+} \frac{1}{\pi t^3} \iint\limits_{x^2+y^2\le t^2} f(\sqrt{x^2+y^2})d\sigma =$ ____________________.

8. $\frac{\mathrm{d}}{\mathrm{d}x}\int_0^x \sin^{100}(x-t)\,\mathrm{d}t=$________

9. $\int_{-1}^{1}(x^2\sin^3 x + x\tan^2 x)dx=$________

10. If $f(x)$ is continuous, and $f(x)=3x^2-\int_0^2 f(x)dx-2$, then $f(x)=$________

11. $\iint\limits_{x^2+y^2\le a^2}(4-5\sin x+3y)d\sigma=$________

12. If the single variable function $f(u)$ has continuous derivative, and $z=f(e^x+e^y)$

satisfies $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=2$, and $f(1)=0$, then $f(u)=$_________

13. $\lim\limits_{(x,y)\to(0,0)}\frac{xy}{\sqrt{2-e^{xy}}-1}=$________.

14. If $z=\frac{y}{x}$,then when $x=2, y=1, \Delta x=0.1, \Delta y=-0.2$,the total differential is

$dz\big|_{(2,1)}=$________.

15. If $f(x)$ is a continuous function on $[0, 1]$, and $\int_0^1 f(x)\mathrm{d}x=A$, then

$\int_0^1 \mathrm{d}x\int_x^1 f(x)f(y)\mathrm{d}y=$________.

16. The area of region enclosed by the curve $y=\frac{4}{x}$ and the lines $y=x$, $y=4x$ in the first quadrant is _______

17. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} t^{20}\sin^5 t\,dt=$________.

18. $\lim\limits_{x\to 0}\frac{\int_0^x \cos t^2 dt}{x}=$_______

19. The total differential of the function $z=y\sin(2x-y)$ at the point (1, 2) is

20. $\iint\limits_{x^2+y^2\le 2}(4+5\sin x-3y)d\sigma=$___________

21. The domain of $z=\ln(xy)+\sqrt{4-x^2-y^2}$ is___________

22. $\lim\limits_{n\to\infty}\frac{1}{n}(\sin\frac{1}{n}+\sin\frac{2}{n}+\cdots+\sin\frac{n}{n})=$__________

23. The general solution of the differential equation $xy''+3y'=0$ is ________

24. If $y_1=e^{-x}$, $y=e^{3x}$ are two particular solutions of the homogeneous linear equation $y''+py'+qy=0$ (p, q are two constants), then the general solution of $y''+py'+qy=x$ is ___________

25. If the function $f(x,y)$ has continuous partial derivatives,

$$f(1,\ 1)=1,\quad f_x'(1,\ 1)=a \text{ and } f_y'(1,\ 1)=b,$$

Then the differential of $u(x)=f(x,f(x,x))$ at $x=1$ is ____________

26. Interchange the iterated integral order of $\int_2^3 dy\int_y^3 f(x,y)dx=$________

27. If $D=\{(x,y)\,|\,|x|+|y|\le 1\}$, then $\iint_D(\sqrt{5}+1)dxdy=$_________.

28. Transform the iterated integral in rectangular system into iterated integral in polar system: $I=\int_0^1 dx\int_{1-x}^{\sqrt{1-x^2}}\frac{1}{\sqrt{(x^2+y^2)^3}}dy=$________ and $I=$________

29. If $f(u,v)$ has continuous partial derivative, and $f_u(u,v)+f_v(u,v)=uv$, then $y(x)=e^{-2x}f(x,x)$ is the solution of the differential equation __________

and the general solution of this differential equation is __________.

30. If $f(u)$ is differentiable, $z=f(\sin y-\sin x)+xy$, then $\frac{1}{\cos x}\frac{\partial z}{\partial x}+\frac{1}{\cos y}\frac{\partial z}{\partial y}=$

____

31. If $f(u)$ is differentiable, $z=yf\left(\frac{y^2}{x}\right)$, then $2x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=$________

32. Reverse the order of iterated integrals $\int_0^2\int_{x^3}^8 f(x,y)dydx=$__________.

33. $\frac{d}{dx}\int_1^{\sin x}3t^2dt=$_________

34. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(1+t^{20}\sin^5 t)dt=$___________.

Part 3: Calculation:

1. If the function $y = y(x)$ satisfies the integral equation $y + \int_0^x t\, y(t) dt = e^{x^2}$, write the corresponding differential equation, and find its solution.

2. Find $I = \iint\limits_D \sqrt{1 - \sin^2(x+y)}\, dxdy$, where $D: 0 \le x \le \frac{\pi}{2}, 0 \le y \le \frac{\pi}{2}$.

3. The region enclosed by $2x = y^2$ and $x = \frac{1}{2}$ is revolved about the line $y = 1$ to generate a solid, find its volume.

4. $\int_0^{\frac{\pi}{2}} \sqrt{1-\sin 2x}\,\mathrm{d}x$

5. $\int_0^1 \frac{\ln(1+x)}{(2-x)^2} dx$

6. Let $z = f(x^2 - y^2, \mathrm{e}^{xy})$，where f has continuous second order partial derivatives，find $\frac{\partial z}{\partial x}, \frac{\partial^2 z}{\partial x \partial y}$.

7. If $y(x)$ has continuous derivative on $[0,+\infty)$, and satisfied

$$y(x)=-1+x+2\int_0^x (x-t)y(t)y'(t)dt$$

find $y(x)$.

8. The region enclosed by $2x=y^2$ and $x=2$ is revolved about the line $y=2$ to generate a solid, find the volume of the solid.

9. Find $\iint_D |\sin(y-x)|\,d\sigma$, where D is the region enclosed by the lines $x=0$, $y=2\pi$, $y=x$.

10. The region enclosed by $y = x^2$ and $x = y^2$ is revolved about y axis to generate a solid, find the volume of the solid.

11. The function $\varphi(x)$ is continuous, and satisfies

$$\varphi(x) = e^x + \int_0^x t\varphi(t)dt - x\int_0^x \varphi(t)dt,$$

find $\varphi(x)$.

12. Find $\iint_D \ln(1+x^2+y^2)d\sigma$, where D is the region enclosed by the circle $x^2+y^2=1$ and axes in the first quadrant.

13. Find the volume of the solid generated by revolving the circle $x^2+(y-5)^2=16$ about x axis.

14. Find the solution of initial value problem $\begin{cases} y'' + y = e^x \\ y|_{x=0} = 1, \;\; y'|_{x=0} = 1 \end{cases}$.

15. If $z = y^x$, find $\frac{\partial z}{\partial x}$, $\frac{\partial^2 z}{\partial x^2}$, $\frac{\partial^2 z}{\partial x \partial y}$.

16. Find $\iint\limits_D \arctan\frac{y}{x}d\sigma$, where D is the region enclosed by circles $x^2+y^2=4$, $x^2+y^2=1$ and lines $y=0,\ y=x$ in the first quadrant.

17. $\int_1^e \frac{1}{x\sqrt{2+\ln x}}\mathrm{d}x$

18. $\int_1^e x\ln x dx$

19. Find $\int_0^1 \frac{x+1}{x^2-2x+5}dx$.

20. Find $\lim\limits_{n\to\infty}\sum_{k=1}^{n}\frac{k}{n^2}\ln\left(1+\frac{k}{n}\right)$

21. Find the limit $\lim\limits_{x\to 0^+}\frac{\int_0^x \sqrt{x-t}\,e^t dt}{\sqrt{x^3}}$

22. If $f(x)$ is continuous and increasing on $[a,\ b]$, show that

$$\int_a^b xf(x)dx \ge \frac{a+b}{2}\int_a^b f(x)dx$$

23. Find the general solution of the following differential equation:

$$y''+2y'+y=x^2$$

24. If $z=\arctan\frac{x}{y}$, $x=u\cos v, y=u\sin v$, find $\frac{\partial z}{\partial u}$, $\frac{\partial z}{\partial v}$

25. Find the double integral: $\int_1^2 dy\int_y^2 \frac{x}{y\ln x}dx$.

26. Find double integral: $\iint_D |y^2-x^2|\,d\sigma$, where

$$D=\{(x,\ y)\,|\,x\in[-1,\ 1], y\in[0,\ 2]\}$$

27. If $D=\{(x,y)|\ x^2+y^2\le R^2\}$, find the volume of the cylinder with *D* as its base and with the top surface $z=e^{-x^2-y^2}$.

28. Find all the local extrema of the function $f(x,y)=x^2y+y^3-y$.

29. If $f(u,v)$ has continuous partial derivative, and $g(x,y)=xy-f(x+y,x-y)$, find $\frac{\partial^2 g}{\partial x^2}+\frac{\partial^2 g}{\partial x\partial y}+\frac{\partial^2 g}{\partial y^2}$.

30. The function $\boldsymbol{f(u)}$ has continuous second derivatives, and $\boldsymbol{z=f(e^x\cos y)}$ satisfies $\boldsymbol{\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=(4z+e^x\cos y)e^{2x}}$. If $\boldsymbol{f(0)=0, f'(0)=0}$, find $\boldsymbol{f(u)}$. .

31. Find $\int_0^1 \frac{f(x)}{\sqrt{x}}dx$, where $f(x)=\int_1^x \frac{\ln(t+1)}{t}dt$.

32. If $f(x,y)$ has continuous second partial derivatives， and $f(1,y)=0$ ， $f(x,1)=0$ ， $\iint\limits_D f(x,y)dxdy=a$ ， where $D=\{(x,y)\mid 0\le x\le 1, 0\le y\le 1\}$ ， find $I=\iint\limits_D xyf''_{xy}(x,y)dxdy$.

33. Find $\iint_R (x^2+y^2)dxdy$ where R is the triangular region with vertices (0, 0), (1, 0) and (0, 1).

34. From the origin to draw the tangent line of the curve $y=\ln x$, the region enclosed by the tangent line, the curve $y=\ln x$ and the *x*-axis is *D*.

(1) Find the area of *D*.

(2) Find the volume of the solid generated by revolving *D* about the line $x=e$.

35. Find the points on the curve $x^2+xy+y^2=1$ in the *xy*-plane that are nearest to and farthest from the origin.

Part Four: Application Problem

1. Suppose that a company sales the same product in two different markets, the demands are

$$P_1=18-2Q_1, \qquad P_2=12-Q_2$$

respectively, where P_1，P_2 are the prices (in \$thousand/ton)，Q_1，Q_2 represents the sales (in ton) in two markets，and the cost of the company is $C=2Q+5$，where Q represents the total sales in two markets，that is, $Q=Q_1+Q_2$.

(1) If the company performs the different prices strategy, try to determine the sales and prices in two markets that will maximize the largest profit.

(2) If the company performs the same price strategy, try to determine the sales and the same price in two markets that will maximize the largest profit.

(3) Determine which strategy is better.

2. If we want to design an open top rectangular container with volume V, what is the dimension of the container that will cost least material.

3. When a space probe with the shape of ellipsoid $4x^2 + y^2 + 4z^2 \le 16$ enters the earth atmosphere, its surface is heated, 1 hour later, the temperature of the probe at (x, y, z) is $T = 8x^2 + 4yz - 16z + 600$, find the hottest point on the surface of the probe.

4. Find the shortest distance from the point $(1, 0, -2)$ to the plane $x + 2y + z = 4$.

5. The highway department is planning to build a picnic area for motorists along a major highway. It is to be rectangular with an area of 800 square yards and is to be fenced off on the three sides not adjacent to the highway. What is the least amount of fencing that will be needed to complete the job?

Part Five: Proof Questions

1. If $z = f[e^{xy}, \cos(xy)]$ and f is differentiable，show the following equality：

$$x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 0$$

2. If $z=z(x,y)$ is defined by the equation $F(z+\frac{1}{x},z-\frac{1}{y})=0$ implicitly,where F has continuous second order partial derivatives， and $F_u(u,v)=F_v(u,v)\neq 0$,show that

$$x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y}=0 \quad \text{and} \quad x^3\frac{\partial^3 z}{\partial x^2}+xy(x+y)\frac{\partial^2 z}{\partial x\partial y}+y^3\frac{\partial^2 z}{\partial y^2}=0$$

3. If $u(x,y,z)=\dfrac{2}{\sqrt{x^2+y^2+z^2}}$, $(x,y,z)\neq(0,0,0)$, show that $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial^2 u}{\partial z^2}=0$