$$\ldots + y^2)\,dy = \left[x^2 y + \frac{y^3}{3}\right]_{20}^{30} = 10x^2 + \frac{30^3 - 20^3}{3}$$

$$= 10x^2 + \frac{27000 - 8000}{3}$$

$$= 10x^2 + \frac{19000}{3}$$

A

$$\int_{20}^{30} k\left(10x^2 + \frac{19000}{3}\right) dx = 1$$

$$k\left(\int_{20}^{30} 10x^2\,dx + \int_{20}^{30} \frac{19000}{3}\,dx\right) = 1$$

$$\int_{20}^{30} 10x^2\,dx = 10\int_{20}^{30} x^2\,dx = 10\left(\frac{x^3}{3}\right)_{20}^{30} = \frac{190000}{3}$$

$$\int_{20}^{30} \frac{19000}{3}\,dx = \frac{19000}{3}(30-20) = \frac{190000}{3}$$

$$k\left(\frac{190000}{3} + \frac{190000}{3}\right) = 1$$

$$k = \frac{3}{380000}$$

b, $P(X<26,\ Y<26) = \int_{20}^{26}\int_{20}^{26} \frac{3}{380000}(x^2+y^2)\,dy\,dx$

$$\int_{20}^{26} (x^2+y^2)\,dy = 6x^2 + \frac{26^3 - 20^3}{3} = 6x^2 + 3192$$

$$\int_{20}^{26} \frac{3}{380000}(6x^2 + 3192)\,dx = \frac{3}{380000}\left(6\cdot\frac{x^3}{3} + 3192x\right)_{20}^{26}$$

$$= \frac{3}{380000}(19152 + 19152)$$

$$= \frac{114912}{380000}$$

$$= 0.3024$$

c. $P(|X-Y| \le 2) = \left\{\int_{28}^{30}\int_{x-2}^{30} k(x^2+y^2)\,dy\,dx\right\} + \left\{\int_{20}^{22}\int_{20}^{x+2} k(x^2+y^2)\,dy\,dx\right\}$

$$+ \left\{\int_{22}^{28}\int_{x-2}^{x+2} k(x^2+y^2)\,dy\,dx\right\}$$

$$= k\left[\left\{\frac{14804}{15} + \frac{8204}{15} + \frac{45264}{15}\right\}\right]$$

$$= \frac{3}{38000}\cdot\frac{68272}{15}$$

$$= 0.3593$$

d. $f_X(x) = k\int (x^2+y^2)\,dy$

$$= k\left(x^2 y + \frac{y^3}{3}\right)_{20}^{30}$$

$$= \frac{3}{38000}\left(10x^2 + \frac{19000}{3}\right)$$

$$= 10kx^2 + \frac{1}{20}$$

e. $f_Y(y) = 10ky^2 + \frac{1}{20}$

$$f_X(x) f_Y(y) = \left(10kx^2 + \frac{1}{20}\right)\left(10ky^2 + \frac{1}{20}\right) \neq f(x,y)$$

$\therefore$ They are not independent.

$f(x) = \int_0^\infty xe^{-x(1+y)}dy$

$= xe^{-x}\int_0^\infty e^{-xy}dy$

$= xe^{-x}\left(\frac{e^{-xy}}{-x}\right)_0^\infty$

$= xe^{-x}(0+\frac{1}{x})$

$= e^{-x}$

$P(X>3) = \int_3^\infty e^{-x}dx = (-e^{-x})_3^\infty = 0.05$

b. $f(y) = \int_0^\infty xe^{-x(1+y)}dx$

$= \int_0^\infty \frac{w}{1+y}e^{-w}\frac{dw}{1+y}$

$= \frac{1}{(1+y)^2}\int_0^\infty we^{-w}dw$

$= \frac{1}{(1+y)^2}$

$f(x)\cdot f(y) = \frac{e^{-x}}{(1+y)^2} \neq f(x,y)$

∴ They are not independant.

c. $P(X>3, Y>3) = 1 - P(X\le 3, Y\le 3)$

$= 1 - \int_0^3\int_0^3 xe^{-x(1+y)}dy\,dx$

$= 1 - \int_0^3 e^{-x}\left(\int_0^3 xe^{-xy}dy\right)dx$

$= 1 - \left(\int_0^3 xe^{-x}dx - \int_0^3 xe^{-4x}dx\right)$

$= e^{-3} + \frac{1}{4} - \frac{e^{-12}}{4}$

$= 0.3$

18. a. $P_{Y|X}(0|1) = \frac{0.08}{0.34} = 0.2353$

$P_{Y|X}(1|1) = \frac{0.2}{0.34} = 0.5882$

$P_{Y|X}(2|1) = \frac{0.06}{0.34} = 0.1765$

b. $P_X(2) = 0.5$

y	0	1	2
$P_{Y\|X}(\frac{y}{2})$	0.12	0.28	0.6

c. $P(Y\le 1 \mid X=2) = P_{Y|X}(0|2) + P_{Y|X}(1|2) = 0.12+0.28 = 0.40$

d. $P_Y(2) = 0.38$

x	0	1	2
$P_{X\|Y}(\frac{x}{2})$	0.0526	0.1579	0.7895

$f_{Y|X}(\frac{y}{x}) = \frac{f(x,y)}{f_X(x)} = \frac{k(x^2+y^2)}{10kx^2+0.05}$ for $20 \le y \le 30$

$f_{X|Y}(\frac{x}{y}) = \frac{f(x,y)}{f_Y(y)} = \frac{k(x^2+y^2)}{10ky^2+0.05}$ for $20 \le y \le 30$

b. $P(Y \ge 25 \mid X = 22) = \int_{25}^{30} f_{Y|X}(\frac{y}{22})dy = \int_{25}^{30} \frac{k((22)^2+y^2)}{10k(22)^2+0.05} dy = 0.5559$

$P(Y \ge 25) = \int_{25}^{30} f_Y(y)dy = \int_{25}^{30} (10ky^2+0.05)\, dy = 0.75$

$0.5559 < 0.75$

c. $E(Y \mid X = 22) = \int_{-\infty}^{\infty} y \cdot f_{Y|X}(\frac{y}{22})dy = \int_{20}^{30} y \cdot \frac{k((22)^2+y^2)}{10k(22)^2+0.05} dy = 25.373$

$E(Y^2 \mid X=22) = \int_{20}^{30} y^2 \cdot \frac{k((22)^2+y^2)}{10k(22)^2+0.05} dy = 652.03$

$V(Y \mid X=22) = E(Y^2 \mid X=22) - [E(Y \mid X = 22)]^2$

$= 652.03 - (25.373)^2$

$= 8.24$

$SD(Y \mid X=22) = \sqrt{8.24} = 2.87$

5.2.24. $E[h(X,Y)] = \sum_x \sum_y h(x,y) \cdot p(x,y)$

$= 84 \cdot \frac{1}{30}$

$= 2.8$

26. $R = 3X + 10Y \qquad E(R) = E(3X+10Y)$

$\sum_{x=0}^{5} \sum_{y=0}^{2} (3x+10y) \cdot p(x,y) = 0 \cdot p(0,0) + 3 \cdot p(1,0) + \dots + 35 \cdot p(5,2)$

$= 15.4$

33. $E(XY) = E(X) \cdot E(Y)$

$Cov(X,Y) = E(XY) - E(X) \cdot E(Y)$

$= E(X) \cdot E(Y) - E(X) \cdot E(Y)$

$= 0$

$Corr(X,Y) = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} = 0$

35a. $Cov(aX+b, cY+d) = E[(aX+b)(cY+d)] - E(aX+b) \cdot E(cY+d)$

$= E[acXY + adX + bcY + bd] - (aE(X)+b)(cE(Y)+d)$

$= acE(XY) + adE(X) + bcE(Y) + bd - acE(X)E(Y) -$
$adE(X) - bcE(Y) - bd$

$= ac[E(XY) - E(X)E(Y)]$

$= ac\, Cov(X,Y)$

b. $Corr(aX+b, cY+d) = \frac{Cov(aX+b, cY+d)}{SD(aX+b)SD(cY+d)} = \frac{ac\,Cov(X,Y)}{|a| \cdot |c|\, SD(X) SD(Y)}$

$= \frac{ac}{|ac|} Corr(X,Y)$

when a and c have the same signs, $ac = |ac|$,

$Corr(aX+b, cY+d) = Corr(X,Y)$

c. when they have different signs, $|ac| = -ac$,

$Corr(aX+b, cY+d) = -Corr(X,Y)$