No.

Date

概率统计作业　22CST 蒋云翔 2022102330

A+

Chapter 5

Section 5.1

Ex. 9.

a) $\int_{20}^{30}\int_{20}^{30} k(x^2+y^2)\,dx\,dy = 1$, then: $k\int_{20}^{30}\int_{20}^{30} x^2\,dx\,dy + k\int_{20}^{30}\int_{20}^{30} y^2\,dx\,dy = 1$

then: $10k\int_{20}^{30} x^2\,dx + 10k\int_{20}^{30} y^2\,dy = 1$

then $20k \times \frac{19000}{3} = 1$

$k = \frac{3}{380000}$

b) $P(X<26 \text{ and } Y<26) = \int_{20}^{26}\int_{20}^{26} k(x^2+y^2)\,dx\,dy = k\int_{20}^{26}\left[x^2y + \frac{y^3}{3}\right]_{20}^{26} dx$

$= k\int_{20}^{26}(6x^2+3192)\,dx$

$= k(38304) = 0.3024$

c) $P(|X-Y| \le 2) = 1 - \int_{20}^{28}\int_{x+2}^{30} f(x,y)\,dy\,dx - \int_{22}^{30}\int_{20}^{x-2} f(x,y)\,dy\,dx$

$= 0.3593$

d) $f_X(x) = \int_{20}^{30} k(x^2+y^2)\,dy = 10kx^2 + 0.05 \quad (20 \le x \le 30)$

e) $f(x,y) \ne f_X(x)\cdot f_Y(y)$, so X, Y are not independent.

Ex. 12.

a) $P(X>3) = \int_3^\infty\int_0^\infty x e^{-x(1+y)}\,dy\,dx = \int_3^\infty e^{-x}\,dx = 0.05$

b) $f_X(x) = \int_0^\infty x e^{-(1+y)x}\,dy = e^{-x} \quad (x \ge 0)$

$f_Y(y) = \int_3^\infty x e^{-(1+y)x}\,dx = \frac{1}{(1+y)^2} \quad (y \ge 0)$

$f(x,y) \ne f_X(x)\cdot f_Y(y)$, so X and Y are not independent.

c) $P(X>3 \text{ or } Y>3) = 1 - P(X \le 3 \text{ and } Y \le 3) = 1 - \int_0^3\int_0^3 x e^{-(1+y)x}\,dy\,dx$

$= 1 - \int_0^3\int_0^3 x e^{-x} e^{-xy}\,dy\,dx$

$= 1 - \int_0^3 e^{-x}(1-e^{-3x})\,dx$

$= 0.3$

Ex. 18.

a) we know that: $P_X(1) = 0.34$: then: $P_{Y|X}(0|1) = \frac{0.08}{0.34} = 0.2353$

$P_{Y|X}(1|1) = \frac{0.20}{0.34} = 0.5882$

$P_{Y|X}(2|1) = \frac{0.06}{0.34} = 0.1765$

b)

y	0	1	2
$P_{Y\|X}(y\|2)$	0.12	0.28	0.60

c) $P(Y \le 1 \mid X=2) = P_{Y|X}(0|2) + P_{Y|X}(1|2) = 0.12 + 0.28 = 0.4$

d)

x	0	1	2
$P_{X\|Y}(x\|2)$	0.0526	0.1579	0.7895

Ex.19.

a) $f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} = \frac{k(x^2+y^2)}{10kx^2+0.05}$ $(20 \le y \le 30)$, k is a constant $\frac{3}{380000}$)

$f_{X|Y}(x|y) = \frac{k(x^2+y^2)}{10ky^2+0.05}$, $(\cancel{x}\ 20 \le x \le 30)$

b) $P(Y \ge 25 \mid X=22) = \int_{25}^{30} f_{Y|X}(y|22)\,dy = \int_{25}^{30} \frac{k(22^2+y^2)}{10k\cdot 22^2+0.05}\,dy = 0.556$

$P(Y \ge 25) = \int_{25}^{30} f_Y(y)\,dy = 0.549$

c) $E(Y \mid X=22) = \int_{20}^{30} y \cdot \frac{k(22^2+y^2)}{10k\cdot 22^2+0.05}\,dy = 25.372912$

$E(Y^2 \mid X=22) = \int_{20}^{30} y^2 \cdot \frac{k(22^2+y^2)}{10k\cdot 22^2+0.05}\,dy = 652.02864$

$V(Y \mid X=22) = E(Y^2 \mid X=22) - E(Y \mid X=22)^2 = 8.243976$

$\sigma = \sqrt{V(Y \mid X=22)} = 2.87$

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Section 5.2

Ex.24:

we can draw a table:

every $p(x,y) = \frac{1}{30}$

$h(x,y)$	1	2	3	4	5	6
1	-	2	3	4	3	2
2	2	-	2	3	4	3
3	3	2	-	2	3	4
4	4	3	2	-	2	3
5	3	4	3	2	-	2
6	2	3	4	3	2	-

(rows: X, columns: y)

so: $E(h(X,Y)) = \sum_x \sum_y h(x,y) \cdot p(x,y) = \frac{84}{30} = 2.8$

Ex.26.

revenue $= 3X + 10Y$, so $E(\text{revenue}) = E(3X+10Y)$

$= \sum_{x=0}^{5} \sum_{y=0}^{2} (3x+10y) \cdot p(x,y) = 15.4\$$

Ex.33

$E(XY) = E(X) \cdot E(Y)$, $Cov(X,Y) = E(XY) - E(X) \cdot E(Y) = 0$

so: $Corr(X,Y) = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} = 0$

Ex.35.

a) $Cov(aX+b, cY+d) = E[(aX+b)(cY+d)] - E(aX+b) * E(cY+d)$

$= E(acXY + adX + bcY + bd) - [(aE(X)+b)(cE(Y)+d)]$

$= acE(XY) - acE(X)E(Y) = ac(E(XY) - E(X)E(Y))$

$= ac\, Cov(X,Y)$

b). $Corr(aX+b, cY+d) = \frac{Cov(aX+b, cY+d)}{SD(aX+b)SD(cY+d)} = \frac{ac}{|ac|} Corr(X,Y)$

as, a and c have same sign, so: $Corr(aX+b, cY+d) = Corr(X,Y)$

c) $ac = -|ac|$, so: $Corr(aX+b, cY+d) = -Corr(X,Y)$

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