Physics CST (2022-23) Homework 4

Please send the completed file to my mailbox yy.lam@qq.com by November 15th, by using the filename format:

student_number-name-cst-hw4

Please answer the questions by filling on these sheets.

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1. Estimate the pressure at the centre of the earth in terms of the gravitational constant G, mass and radius of the earth M and R, respectively, assuming it is of constant density throughout.

   Solution. *Gravitational force at the centre of the the earth is zero, however, the pressure is the maximum due to the non-directional property of pressure. We may use the familiar formula $P = \rho\bar{g}R$ to estimate the pressure at the centre where R is the height which is just the radius of the earth. Since the gravitational acceleration g is not a constant which varies along the radial direction. We have put $\bar{g}$ as the mean value of the gravitational accelerations over the radial interval. The gravitational acceleration function $g(r)$ can be found as*

   $$g(r) = \frac{GMr}{R^3}$$

   which is measured at r from the centre of the earth. For the average gravitational field $\bar{g}$, integrate the field over the interval and divide by the interval:

   $$\bar{g} = \frac{1}{R}\int_0^R \frac{GMr}{R^3}\mathrm{d}r = \frac{1}{R}\frac{GMr^2}{2R^3}\bigg|_0^R = \frac{GM}{2R^2}$$

   Thus the pressure at the centre of the earth is

   $$P = \rho\bar{g}R = \left(\frac{M}{\frac{4}{3}R^3\pi}\right)\left(\frac{GM}{2R^2}\right)R = \frac{3GM^2}{8\pi R^3}.$$

2. Oil of density 720 kgm^{-3} is poured on top of a tank of water, and it floats on the water without mixing. A block of plastic of density 850 kgm^{-3} is placed in the tank, and it is about floating at the interface of the two liquids (completely immerses in the liquids). What fraction of the block's volume is immersed in water?

   Solution. *Let x be the fraction of the volume of the wooden block in water. Thus the volume xV is in water, and volume $(1-x)V$ is in oil, where V is the volume of the block. We have,*

   $$\rho V g = \rho_w x V g + \rho_o(1-x)Vg$$

   where ρ, ρ_w, ρ_o are the densities of the block, water and oil. Therefore,

   $$x = \frac{\rho - \rho_o}{\rho_w - \rho_o} = \frac{850 - 720}{1000 - 720} = 0.464$$

3. Firemen use a hose of inside diameter 5.0 cm to deliver 800 litres of water per minute. A nozzle is attached to the hose, and they want to squirt the water up to a window 25 m above the nozzle. (a) With what speed must the water leave the nozzle? (b) What is the inside diameter of the nozzle? (c) What pressure in the hose is required?

Solution. *(a) Assume water leaving from the nozzle upward without air resistance. The Bernoulli's equation gives*

$$\frac{1}{2}\rho u^2 + 0 = \frac{1}{2}\rho(0)^2 + \rho g h$$

where we have ignored the difference of pressures between the altitudes $P_a \approx P_a + P_{a-h}$. *The equation equivalent to the suvat-equation gives*

$$v = \sqrt{2 \times 9.8 \times 25} = 22.14 \ ms^{-1}.$$

(b) Given the flow $0.8/60 \ m^3 s^{-1}$, *the radius inside of the nozzle is*

$$\frac{0.8}{60} = r^2 \pi \times 22.14 \quad \Rightarrow \quad r = 0.0138 \ m.$$

Thus, the diameter is 2.76 cm. (c) The pressure P ***inside*** *the hose is given by*

$$P + \frac{1}{2}\rho \left(\frac{flow}{area}\right)^2 = 1 \ atm + \frac{1}{2}\rho \times 22.14^2$$

Density of water being $1000 \ kgm^{-3}$, *1 atmospheric pressure being* $10^5 \ Pa$ *for the calculation we get*

$$P = 10^5 + \frac{1}{2} \times 1000 \times 22.14^2 - \frac{1}{2} \times 1000 \times \left(\frac{0.8/60}{0.025^2 \pi}\right)^2 \approx 322,000 \ Pa.$$

Therefore, the pressure is 3.22 atmospheric pressure inside the hose.

4. A physicist makes a cup of instant coffee and notices that, as the coffee cools, its level drops 2.60 mm in the glass cup. (a) Show that this decrease cannot be due to thermal contraction by calculating the decrease in level if the 300 cm^3 of coffee is in a 7.20-cm-diameter cup and decreases in temperature from 98.0 °C to 45 °C. (You may find the coefficients on page 480 of the College Physics e-book.) (b) Then why does the coffee (water) level drop?

   Solution. *(a) The coefficient of volume expansion of glass is much smaller than water,* $27 \times 10^{-6} < 210 \times 10^{-6} \ K^{-1}$, *we may only calculate the linear expansion of water in vertical direction. The change of height of water is*

$$\Delta L = \frac{300}{(7.2/2)^2 \pi} \cdot \frac{210 \times 10^{-6}}{3}(45 - 98) = -0.0273 \ cm = -0.273 \ mm$$

   which is greatly smaller than the $2.60 \ mm$ *drop even though we did not count the expansion of the glass cup and the fact that the change of coffee (water) level will even be smaller if the expansion of the cup is included. Notice that the coefficient of linear expansion of water has been used by dividing the coefficient for volume by 3. (b) The most obvious reason of the decrease of the coffee level is part of the water becoming water vapour evaporating to air.*

5. What mass of steam initially at 130°C is needed to warm 200 g of water in a 100-g glass container from 20.0°C to 50.0°C at thermal equilibrium? (Specific heats of water, steam and glass are 4.19×10^3, 2.01×10^3 and 837 J/kg · °C, respectively. The latent of vaporization of water is 2.26×10^6 J/kg.)

Solution. *As the heat source solely from the steam transports to the glass of water to attain the thermal equilibrium at* $50°C$*. Consider first* m *quantity of steam from* $130°C$ *to* $100°C$ *steam, the amount of thermal energy is*

$$2.01 \times 10^3 \times (100 - 130)m = -6.03 \times 10^4 m \ J/kg.$$

Converting $100°C$ *of steam to* $100°C$ *of water gives the lalent heat*

$$-2.26 \times 10^6 m \ J/kg.$$

Finally the water dropping from $100°C$ *to* $50°C$ *gives*

$$4.19 \times 10^3 \times (50 - 100)m = -2.095 \times 10^5 m \ J/kg.$$

The total heat loss is

$$-6.03 \times 10^4 m - 2.26 \times 10^6 m - 2.095 \times 10^5 m = -2.5298 \times 10^6 m \ J/kg.$$

The heat absorption of the glass of water from $20°C$ *to* $50°C$ *reaching thermal equilibrium is*

$$(0.2 \times 4.19 \times 10^3 + 0.1 \times 837)(50 - 20) = 2.7651 \times 10^4 \ J.$$

Equating the heat loss and heat gain gives

$$m = \frac{2.7651 \times 10^4}{2.5298 \times 10^6} = 0.0109 \ kg,$$

or 10.9 g *of steam.*

6. Given the coefficient of conductivity of human tissue 18 Cal cm/m^2-hr-°C, calculate the heat transfer in 1.5 hours by conduction in the unit of kilo-Joule. Assume that the thickness of tissue is 2.5 cm, the average area of the cross section is 1.65 m^2 and the temperature difference is 2.2 °C between inner body and skin.

   Solution. *The heat flow is*

   $$H = \frac{K_c A \Delta T}{L} = \frac{18 \times 1.65 \times 2.2}{2.5} = 26.14 \ Cal/hr.$$

   In 1.5 hours, the heat energy is $26.14 \times 1.5 = 39.20$ Cal*. In term of kilo-Joule it is*

   $$70.36 \times 4.186 = 164.11 \ kJ$$

7. How does the rate of heat transfer by conduction change when the volume an object is doubled by keeping the same shape? Use your result to discuss the heat gaining of an adult and a child from a heater (suppose the volume difference is just double).

   Solution. *Let* L *be the dimension of the original object, the area and linear dimensions are simply* L^2 *and* L*, respectively. Doubling the volume corresponds to the area and linear dimensions becoming*

   $$(2L^3)^{1/3} = 2^{1/3}L \quad and \quad (2L^3)^{2/3} = 2^{2/3}L^2$$

   The new heat flow is hence

   $$\frac{k(2^{2/3}L^2)\Delta T}{2^{1/3}L} = 2^{1/3} \cdot \frac{kL^2 \Delta T}{L} = 2^{1/3} \times \text{the orginal heat flow}$$

   The heat flow increases by a factor of 1.260*. Suppose the volume of an adult is a double of a child, the adult heat absorbing rate is* 1.26 *times faster than the child. However, the reverse is not always the same, as the amount of heat generated by an adult is greater than a child, the difference of heat loss between an adult and a child are probably similar.*

8. Twelve particles, each of mass m and confined to a container with volume V, have various speeds; three have speed $3v$; four have speed v; one has speed $4v$; the other four have speed $2v$. (a) Find the mean-square speed of the particles. (b) Use the ideal gas law to find the pressure that the particles exerting on the walls of the container.

Solution. *The mean-square speed is*

$$\langle v^2 \rangle = \frac{3(3v)^2 + 4v^2 + (4v)^2 + 4(2v)^2}{12} = \frac{27 + 4 + 16 + 16}{12} v^2 = 5.25v^2$$

Using the ideal gas law, the pressure is

$$P = \frac{2}{3}\left(\frac{N}{V}\right)\left(\frac{1}{2}m\langle v^2 \rangle\right) = \frac{2}{3}\frac{12}{V}\frac{1}{2}m(5.25v^2) = \frac{21mv^2}{V}$$