6. Suppose an individual is randomly selected from the population of all adult males living in the United States. Let A be the event that the selected individual is over 6 ft in height, and let B be the event that the selected individual is a professional basketball player. Which do you think is larger, $P(A \mid B)$ or $P(B \mid A)$? Why?

$P(A|B) > P(B|A)$

of the following probabilities (a Venn diagram might help).

a. $P(B \mid A)$ b. $P(B' \mid A)$
c. $P(A \mid B)$ d. $P(A' \mid B)$
e. Given that the selected individual has at least one card, what is the probability that he or she has a Visa card?

50. A department store sells sport shirts in three sizes (small, medium, and large), three patterns (plaid, print, and stripe), and two sleeve lengths (long and short). The accompanying tables give the proportions of shirts sold in the various category combinations.

Short-sleeved

	Pattern		
Size	Pl	Pr	St
S	.04	.02	.05
M	.08	.07	.12
L	.03	.07	.08

Long-sleeved

	Pattern		
Size	Pl	Pr	St
S	.03	.02	.03
M	.10	.05	.07
L	.04	.02	.08

a. What is the probability that the next shirt sold is a medium, long-sleeved, print shirt?
b. What is the probability that the next shirt sold is a medium print shirt?
c. What is the probability that the next shirt sold is a short-sleeved shirt? A long-sleeved shirt?
d. What is the probability that the size of the next shirt sold is medium? That the pattern of the next shirt sold is a print?
e. Given that the shirt just sold was a short-sleeved plaid, what is the probability that its size was medium?
f. Given that the shirt just sold was a medium plaid, what is the probability that it was short-sleeved? Long-sleeved?

53. A certain shop repairs both audio and video components. Let A denote the event that the next component brought in for repair is an audio component, and let B be the event that the next component is a compact disc player (so the event B is contained in A). Suppose that $P(A) = .6$ and $P(B) = .05$. What is $P(B \mid A)$?

54. In Exercise 13, A_i = {awarded project i}, for $i = 1, 2, 3$. Use the probabilities given there to compute the following probabilities, and explain in words the meaning of each one.
a. $P(A_2 \mid A_1)$ b. $P(A_2 \cap A_3 \mid A_1)$
c. $P(A_2 \cup A_3 \mid A_1)$ d. $P(A_1 \cap A_2 \cap A_3 \mid A_1 \cup A_2 \cup A_3)$

55. Deer ticks can be carriers of either Lyme disease or human granulocytic ehrlichiosis (HGE). Based on a recent study, suppose that 16% of all ticks in a certain location carry Lyme disease, 10% carry HGE, and 10% of the ticks that carry at least one of these diseases in fact carry both of them. If a randomly selected tick is found to have carried HGE, what is the probability that the selected tick is also a carrier of Lyme disease?

56. For any events A and B with $P(B) > 0$, show that $P(A \mid B) + P(A' \mid B) = 1$.

57. If $P(B \mid A) > P(B)$, show that $P(B' \mid A) < P(B')$. [*Hint:* Add $P(B' \mid A)$ to both sides of the given inequality and then use the result of Exercise 56.]

58. Show that for any three events A, B, and C with $P(C) > 0$, $P(A \cup B \mid C) = P(A \mid C) + P(B \mid C) - P(A \cap B \mid C)$.

59. At a certain gas station, 40% of the customers use regular gas (A_1), 35% use plus gas (A_2), and 25% use premium (A_3). Of those customers using regular gas, only 30% fill their tanks (event B). Of those customers using plus, 60% fill their tanks, whereas of those using premium, 50% fill their tanks.
a. What is the probability that the next customer will request plus gas and fill the tank ($A_2 \cap B$)?

50

a $p = 0.05$

b $p = 0.05 + 0.07 = 0.12$

c $P(\text{short}) = 0.04 + \cdots + 0.08 = 0.56$

$P(\text{long}) = 1 - P(\text{short}) = 0.44$

d $P(M) = 0.22 + 0.27 = 0.49$

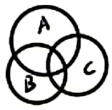
$P(Pr) = 0.09 + 0.16 = 0.25$

e $P(A|B) = \dfrac{P(AB)}{P(B)} = \dfrac{0.27}{0.56} = \dfrac{27}{56}$

f $P(L|M) = \dfrac{P(L \cap M)}{P(M)} = \dfrac{0.22}{0.49} = \dfrac{22}{49}$

$P(S|M) = \dfrac{P(S \cap M)}{P(M)} = \dfrac{0.27}{1.49} = \dfrac{27}{49}$

58



$$P(A \cup B \mid C) = \frac{P(C \cap (A \cup B))}{P(C)}$$

$$= \frac{P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)}{P(C)}$$

$$P(A|C) + P(B|C) - P(A \cap B|C)$$

$$= \frac{P(A \cap C)}{P(C)} + \frac{P(B \cap C)}{P(C)} - \frac{P(A \cap B \cap C)}{P(C)}$$

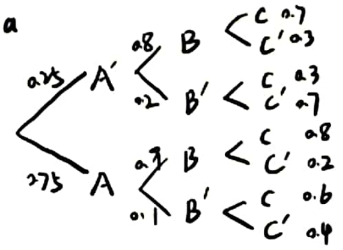
$$\therefore P(A \cup B|C) = P(A|C) + P(B|C) - P(A \cap B|C)$$

63. For customers purchasing a refrigerator at a certain appliance store, let A be the event that the refrigerator was manufactured in the U.S., B be the event that the refrigerator had an icemaker, and C be the event that the customer purchased an extended warranty. Relevant probabilities are

$P(A) = .75$ $P(B \mid A) = .9$ $P(B \mid A') = .8$
$P(C \mid A \cap B) = .8$ $P(C \mid A \cap B') = .6$
$P(C \mid A' \cap B) = .7$ $P(C \mid A' \cap B') = .3$

a. Construct a tree diagram consisting of first-, second-, and third-generation branches, and place an event label and appropriate probability next to each branch.
b. Compute $P(A \cap B \cap C)$.
c. Compute $P(B \cap C)$.
d. Compute $P(C)$.
e. Compute $P(A \mid B \cap C)$, the probability of a U.S. purchase given that an icemaker and extended warranty are also purchased.

a



b $P = \frac{3}{4} \times \frac{9}{10} \times \frac{4}{5} = \frac{27}{50} = 0.54$

c $P(B \cap C) = \frac{1}{4} \times \frac{4}{5} \times \frac{7}{10} + \frac{3}{4} \times \frac{9}{10} \times \frac{4}{5} = \frac{34}{50}$

d $P(C) = \frac{1}{4} \times \left(\frac{4}{5} \times \frac{7}{10} + \frac{1}{5} \times \frac{3}{10}\right) + \frac{3}{4} \times \left(\frac{9}{10} \times \frac{4}{5} + \frac{1}{10} \times \frac{3}{5}\right)$

$= \frac{31}{200} + \frac{117}{200} = \frac{37}{50}$

e $P(A|B \cap C) = \dfrac{P(A \cap B \cap C)}{P(B \cap C)}$

$= \dfrac{7}{34}$

B+

70. Reconsider the credit card scenario of Exercise 47 (Section 2.4), and show that A and B are dependent first by using the definition of independence and then by verifying that the multiplication property does not hold.

71. An oil exploration company currently has two active projects, one in Asia and the other in Europe. Let A be the event that the Asian project is successful and B be the event that the European project is successful. Suppose that A and B are independent events with $P(A) = .4$ and $P(B) = .7$.
   a. If the Asian project is not successful, what is the probability that the European project is also not successful? Explain your reasoning.
   b. What is the probability that at least one of the two projects will be successful?
   c. Given that at least one of the two projects is successful, what is the probability that only the Asian project is successful?

72. In Exercise 13, is any A_i independent of any other A_j? Answer using the multiplication property for independent events.

73. If A and B are independent events, show that A' and B are also independent. [*Hint*: First establish a relationship between $P(A' \cap B)$, $P(B)$, and $P(A \cap B)$.]

74. The proportions of blood phenotypes in the U.S. population are as follows:

A	B	AB	O
.40	.11	.04	.45

Assuming that the phenotypes of two randomly selected individuals are independent of one another, what is the probability that both phenotypes are O? What is the probability that the phenotypes of two randomly selected individuals match?

71.

a the probility that European project is not successful is 0.3

because event A and B are independent

b $P = P(A)\cdot P(B') + P(A')\cdot P(B) + P(A)\cdot P(B)$

$= 0.4\times0.3 + 0.6\times0.7 + 0.4\times0.7$

$= 0.82$

c Only Asian project is successful

$P' = \dfrac{P(A)\cdot P(B')}{P} = \dfrac{0.12}{0.82} = \dfrac{6}{41}$

72.

if A_i and A_j are independ,

$P(A_i \cap A_j) = P(A_i)\cdot P(A_j)$

$P(A_1 \cap A_2) = 0.11 \neq P(A_1)\cdot P(A_2) = 0.0505$

$P(A_1 \cap A_3) = 0.5 \neq P(A_3)\cdot P(A_1)$

$P(A_2 \cap A_3) = 0.7 \neq P(A_2)\cdot P(A_3)$

So they are independent

section 3.1

1. A concrete beam may fail either by shear (S) or flexure (F). Suppose that three failed beams are randomly selected and the type of failure is determined for each one. Let X = the number of beams among the three selected that failed by shear. List each outcome in the sample space along with the associated value of X.

2. Give three examples of Bernoulli rv's (other than those in the text).

3. Using the experiment in Example 3.3, define two more random variables and list the possible values of each.

4. Let X = the number of nonzero digits in a randomly selected zip code. What are the possible values of X? Give three possible outcomes and their associated X values.

5. If the sample space $\mathcal{S}$ is an infinite set, does this necessarily imply that any rv X defined from $\mathcal{S}$ will have an infinite set of possible values? If yes, say why. If no, give an example.

6. Starting at a fixed time, each car entering an intersection is observed to see whether it turns left (L), right (R), or goes straight ahead (A). The experiment terminates as soon as a car is observed to turn left. Let X = the number of cars observed. What are possible X values? List five outcomes and their associated X values.

7. For each random variable defined here, describe the set of possible values for the variable, and state whether the variable is discrete.

4. 001000 $x=1$

101000 $x=2$

101001 $x=3$

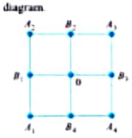
5. Yes, Because rv X defined from $\mathcal{S}$ and $\mathcal{S}$ is an infinite set. X could be any one of the set $\mathcal{S}$.

So X will have an infinite set of possible value.

e. Z = the amount of royalties earned from the sale of a first edition of 10,000 textbooks
f. Y = the pH of a randomly chosen soil sample
g. X = the tension (psi) at which a randomly selected tennis racket has been strung
h. X = the total number of coin tosses required for three individuals to obtain a match (HHH or TTT)

8. Each time a component is tested, the trial is a success (S) or failure (F). Suppose the component is tested repeatedly until a success occurs on three consecutive trials. Let Y denote the number of trials necessary to achieve this. List all outcomes corresponding to the five smallest possible values of Y, and state which Y value is associated with each one.

9. An individual named Claudius is located at the point 0 in the accompanying diagram.



the (new) adjacent points is determined by tossing an appropriate die or coin.
   a. Let X = the number of moves that Claudius makes before first returning to 0. What are possible values of X? Is X discrete or continuous?
   b. If moves are allowed also along the diagonal paths connecting 0 to A_1, A_2, A_3, and A_4, respectively, answer the questions in part (a).

10. The number of pumps in use at both a six-pump station and a four-pump station will be determined. Give the possible values for each of the following random variables:
   a. T = the total number of pumps in use
   b. X = the difference between the numbers in use at stations 1 and 2
   c. U = the maximum number of pumps in use at either station
   d. Z = the number of stations having exactly two pumps in use

8. $Y=3$ {SSS}

$Y=4$ {FSSS}

$Y=5$ {FFSSS, SFSSS}

$Y=6$ {FFFSSS, SFFSSS, SSFSSS, FSFSSS}

$Y=7$ {FFFFSSS, SSFFSSS, FSSFSSS, FFSFSSS, SFSFSSS}

10

a. $T = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

b. $X = \{0, 1, 2, 3, 4, 5, 6\}$

c. $U = \{6, 4\}$

d. $Z = \{0, 1, 2\}$

11. An automobile service facility specializing in engine tune-ups knows that 45% of all tune-ups are done on four-cylinder automobiles, 40% on six-cylinder automobiles, and 15% on eight-cylinder automobiles. Let X = the number of cylinders on the next car to be tuned.

a. What is the pmf of X?

b. Draw both a line graph and a probability histogram for the pmf of part (a).

c. What is the probability that the next car tuned has at least six cylinders? More than six cylinders?

12. Airlines sometimes overbook flights. Suppose that for a plane with 50 seats, 55 passengers have tickets. Define the random variable Y as the number of ticketed passengers who actually show up for the flight. The probability mass function of Y appears in the accompanying table.

y	45	46	47	48	49	50	51	52	53	54	55
$p(y)$	.05	.10	.12	.14	.25	.17	.06	.05	.03	.02	.01

a. What is the probability that the flight will accommodate all ticketed passengers who show up?

b. What is the probability that not all ticketed passengers who show up can be accommodated?

c. If you are the first person on the standby list (which means you will be the first one to get on the plane if there are any seats available after all ticketed passengers have been accommodated), what is the probability that you will be able to take the flight? What is this probability if you are the third person on the standby list?

13. A mail-order computer business has six telephone lines. Let X denote the number of lines in use at a specified time. Suppose the pmf of X is as given in the accompanying table.

x	0	1	2	3	4	5	6
$p(x)$	.10	.15	.20	.25	.20	.06	.04

Calculate the probability of each of the following events.

a. {at most three lines are in use}

b. {fewer than three lines are in use}

c. {at least three lines are in use}

d. {between two and five lines, inclusive, are in use}

e. {between two and four lines, inclusive, are not in use}

f. {at least four lines are not in use}

12.

a. $P(45 \leq y \leq 50) = 0.05 + 0.1 + 0.12 + 0.14 + 0.25 + 0.17$
$= 0.83$

b. $P(51 \leq y \leq 55) = 0.06 + 0.05 + 0.03 + 0.02 + 0.01 = 0.17$

c. if I am the first person on the standby list

$P = P(45 \leq y \leq 49) = 0.83 - 0.25 = 0.58$

if I am the third person on the standby list

$P = P(45 \leq y \leq 47) = 0.27$

23. A consumer organization that evaluates new automobiles customarily reports the number of major defects in each car examined. Let X denote the number of major defects in a randomly selected car of a certain type. The cdf of X is as follows:

$$F(x) = \begin{cases} 0 & x < 0 \\ .06 & 0 \leq x < 1 \\ .19 & 1 \leq x < 2 \\ .39 & 2 \leq x < 3 \\ .67 & 3 \leq x < 4 \\ .92 & 4 \leq x < 5 \\ .97 & 5 \leq x < 6 \\ 1 & 6 \leq x \end{cases}$$

Calculate the following probabilities directly from the cdf:

a. $p(2)$, that is, $P(X = 2)$ **b.** $P(X > 3)$

c. $P(2 \leq X \leq 5)$ **d.** $P(2 < X < 5)$

23.

a. $P(X=2) =$

b. $P(X>3) = 1 - P(0 < X \leq 3)$

c. $P(2 \leq X \leq 5) =$

d. $P(2 < X < 5) =$

25. In Example 3.12, let Y = the number of girls born before the experiment terminates. With $p = P(B)$ and $1 - p = P(G)$, what is the pmf of Y? [*Hint:* First list the possible values of Y, starting with the smallest, and proceed until you see a general formula.]

$Y = 0$ P

$Y = 1$ $(1-P)P$

$Y = 2$ $(1-P)^2 P$

let $Y = n$ $P = (1-P)^n \cdot P$