Homework 01 22CS | 蒋云翔

Section 8.1

Ex.8

(a) There are 4 cases under such situation:

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so: the recurrence relation is clear: $a_n = 2^{n-3} + a_{n-1} + a_{n-2} + a_{n-3}$ $(n \geq 3)$

(b) initial condition is: $a_0 = 0$, $a_1 = 0$, $a_2 = 0$

(c) $a_7 = 2^{7-3} + a_6 + a_5 + a_4$

$= 2^4 + (2^3 + a_5 + a_4 + a_3) + (2^2 + a_4 + a_3 + a_2) + (2^1 + a_3 + a_2 + a_1)$

$= 2^4 + 2^3 + 2^2 + 2^1 + (2^2 + a_4 + a_3 + a_2) + (2^1 + a_3 + a_2 + a_1) + a_3$

$+ (2^1 + a_3 + a_2 + a_1) + 1 + 1$

$= 16 + 8 + 4 + 2 + 4 + 3 + 1 + 3 + 1 + 3 + 1 + 1$

$= 47$

Ex. Section 8.2

Ex.2

(a) $a_n = 3a_{n-2}$ Linear and homogeous with constant coefficients Degree: 2

(b) $a_n = 3$ Linear with constant coeffients

(c) $a_n = a_{n-1}^2$ Not linear

(d) $a_n = a_{n-1} + 2a_{n-3}$ Linear and homogeneous with constant coefficients Degree: 3

(e) $a_n = a_{n-1}/n$ Linear and homogeneous but no constant coeffieents

(f) $a_n = a_{n-1} + a_{n-2} + n + 3$ Linear with constant coefficients

(g) $a_n = 4a_{n-2} + 5a_{n-4} + 9a_{n-7}$ Linear and homogeneous with ~~constan~~ constant coeffients Degree: 7

Ex.4.

(c) $a_n = 6a_{n-1} - 8a_{n-2}$ for $n \geq 2$, $a_0 = 4$, $a_1 = 10$

Solution: the characteristic equation is: $r^2 - 6r + 8 = 0$

It's roots are $r=2$ and $r=4$. Therefore, $\{a_n\}$ is a solution to the recurrence relation if and only if $a_n = \alpha_1 2^n + \alpha_2 (4)^n$

then: $a_0 = \alpha_1 + \alpha_2$ $\Rightarrow$ $\begin{cases} \alpha_1 = 3 \\ \alpha_2 = 1 \end{cases}$

$a_1 = 2\alpha_1 + 4\alpha_2$

so: $a_n = 3 \cdot 2^n + 4^n$

(d) $a_n = 2a_{n-1} - a_{n-2}$, for $n \geq 2$, $a_0 = 4$, $a_1 = 1$

Solution: The characteristic equation is: $r^2 - 2r + 1 = 0$

It's roots are both $r=1$, therefore: $\{a_n\}$ is a solution to the recurrence relation if and only if $a_n = \alpha_1 (1)^n + \alpha_2 n (1)^n$

then: $\begin{cases} a_0 = \alpha_1 \\ a_1 = \alpha_1 + \alpha_2 \end{cases} \Rightarrow \begin{cases} \alpha_1 = 4 \\ \alpha_2 = -3 \end{cases}$ So: $a_n = 4 \cdot -3n$

Ex.22

$a_n = (a_{1,0} + a_{1,1}n + a_{1,2}n^2) \cdot (-1)^n + (a_{2,0} + a_{2,1}n) \cdot (2)^n$
$+ (a_{3,0} + a_{3,1}n)(5)^n + a_{4,0}(7)^n$

Ex.24.

(a) the right-hand equation: $2(n-1) \cdot 2^{n-1} + 2^n = (n-1)2^n + 2^n = n \cdot 2^n = a_n$

(b) the associated linear homogeneous equation is $a_n = 2a_{n-1}$

It's ~~associate~~ solutions are $a_n^{(h)} = \alpha \cdot 2^n$

$a_n = \alpha \cdot 2^n + n \cdot 2^n$

(c) $a_0 = \alpha \cdot 2^0 + 0 \cdot 2^0$

$\therefore 2$

So: $a_n = 2^{n+1} + n \cdot 2^n$

$= (n+2) \cdot 2^n$