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6.1 1. a. I use the $\bar{x}$ to do this: $\hat{\mu} = \bar{x} = \frac{\sum x_i}{27} = 8.14$

b. We have $\hat{\mu}' = \frac{219.8 - 8.7 - 8.2 - 8.1 - 7.8 - 7.9 - 7.8}{27-6} = 8.16$, using sample mean

c. We use the sample variance: $\hat{\sigma}^2 = \frac{\sum x_i^2}{27} - 8.14^2 = 2.66 \Rightarrow \sigma = \sqrt{2.66} = 1.63$

d. Since there are 4 out of 27 that exceeds 10MPa, so we have $\hat{p} = \frac{4}{27}$.

e. We use the sample mean and the sample variance: $C = \frac{\sigma}{\mu} = \frac{1.63}{8.14} = 0.2$

8. a. We used the sample to estimate the population: $\hat{p} = \frac{80-12}{80} = 0.85$

b. $P(\text{system works}) = p^2 = 0.7225$

9. a. Since for possion distribution, $E(X) = \mu$, however,

$E(\bar{X}) = E(\frac{1}{n}\sum_{i=1}^{n} x_i) = \frac{1}{n}\sum_{i=1}^{n} E(x_i) = \frac{1}{n} \cdot n \cdot E(X) = \mu$

Therefore, $\bar{x}$ is an unbiased estimator of μ, and $E(\bar{x}) = \mu = 2.11$

b. $V(\bar{x}) = V(\frac{1}{n}\sum_{i=1}^{n} x_i) = \frac{1}{n^2}\sum_{i=1}^{n} V(x_i) = \frac{1}{n^2} \cdot n \cdot V(X) = \frac{\sigma^2}{n} = \frac{\mu}{n} = 0.0141 \Rightarrow \hat{\sigma} = 0.119$

13. Since $E(\hat{\theta}) = E(3\bar{x}) = 3E(\bar{x}) = 3E(\frac{1}{n}\sum_{i=1}^{n} x_i) = 3\frac{1}{n}\sum_{i=1}^{n} E(x_i) = \frac{3}{n} \cdot n \cdot \mu = 3\mu$

and $\mu = E(X) = \int_{-1}^{1} (0.5x + 0.5\theta x^2)dx = (\frac{1}{4}x^2 + \frac{1}{6}\theta x^3)\Big|_{-1}^{1} = \frac{1}{4} + \frac{1}{6}\theta - \frac{1}{4} + \frac{1}{6}\theta = \frac{1}{3}\theta$

So $E(\hat{\theta}) = 3\mu = \theta$, therefore, $\hat{\theta} = 3\bar{x}$ is an unbiased estimator of θ.

6.2 20. a. Since $f(p) = C_n^x p^x (1-p)^{n-x}$, so $\ln f(p) = \ln C_n^x + x\ln p + (n-x)\ln(1-p)$

$\frac{d\ln f(p)}{dp} = \frac{x}{p} - \frac{n-x}{1-p}$, let $\frac{x}{p} = \frac{n-x}{1-p} \Rightarrow p = \frac{x}{n}$, take $x = 3, n = 20$ we know $\hat{p} = 0.15$.

b. Since $E(\hat{p}) = E(\frac{X}{n}) = \frac{1}{n}\sum_{i=1}^{n} \frac{x_i}{n} = \frac{1}{n} \cdot n \cdot \frac{1}{n} \cdot x = \frac{x}{n} = p$, so $\hat{p}$ is unbiased.

c. That is $h(\hat{p}) = (1-p)^5 = 0.85^5 = 0.444$

21. a. $E(X^1) = \frac{1}{n}\sum_{i=1}^{n} x_i$, $E(X^2) = \frac{1}{n}\sum_{i=1}^{n} x_i^2$, and $V(X) = E(X^2) - [E(X)^2]$, so we have

$\frac{\Gamma^2(1+\frac{1}{\alpha})}{\Gamma(1+\frac{2}{\alpha})} = \frac{1}{n} \cdot \frac{(\sum x_i)^2}{\sum x_i^2}$, $\beta = \frac{\sum x_i}{n\Gamma(1+\frac{1}{\alpha})}$, in line with the problem descrition.

b. $\frac{\Gamma^2(1+\frac{1}{\alpha})}{\Gamma(1+\frac{2}{\alpha})} = 0.95 \Rightarrow \hat{\alpha} = 5$, then $\hat{\beta} = \frac{28}{\Gamma(1.2)}$.

a. Since $f(x;\lambda,\theta) = \lambda e^{-\lambda(x-\theta)}$, thus the joint pdf $f(x_1,x_2,\cdots,x_n;\lambda,\theta) = \lambda^n e^{-\lambda\Sigma(x_i-\theta)}$, then $\ln f(x_1,\cdots,x_n;\lambda,\theta) = n\ln\lambda + (-\lambda)\Sigma(x_i-\theta)$

We have $\frac{\delta \ln f(x_1,\cdots,x_n;\lambda,\theta)}{\delta\lambda} = \frac{n}{\lambda} - \Sigma(x_i-\theta) \Rightarrow \hat{\lambda} = \frac{n}{\Sigma(x_i-\hat{\theta})}$

and $\frac{\delta \ln f(x_1,\cdots,x_n;\lambda,\theta)}{\delta\theta} = -n\lambda \Rightarrow \hat{\theta} = -\frac{n^2}{\Sigma(x_i-\hat{\theta})}$

b. Then $\hat{\theta} = 7.007$, $\hat{\lambda} = -0.7007$

32. a. $F(y) = P(Y\le y) = P(\max(X_i)\le y) = P(x_1\le y)P(x_2\le y)\cdots P(x_n\le y)$

$= \int_0^{\theta}\frac{1}{\theta}dy \int_0^{\theta}\frac{1}{\theta}dy\cdots$

$= \left(\frac{y}{\theta}\right)^n$.

$f_Y(Y) = F'(y) = \frac{ny^{n-1}}{\theta^n}$, $0\le y\le\theta$, otherwise it's 0.

b. $E(Y) = \int_0^{\theta}\frac{ny^n}{\theta^n}dy = \frac{n}{n+1}\cdot\frac{y^{n+1}}{\theta^n}\Big|_0^{\theta} = \frac{n\theta}{n+1}$, so $E(\hat{\theta}) \ne E(Y)$,

but $E\left(\frac{n+1}{n}\right) = E(Y)$, so $\frac{n+1}{n}\max(X_i)$ is unbiased.