A

5.1 | 9.

a. Since $\int_{20}^{30}\int_{20}^{30} K(x^2+y^2)\,dx\,dy = \int_{20}^{30} K(\frac{1}{3}x^3+xy^2)\cdot x\Big|_{20}^{30} dy$

$= \int_{20}^{30} K(\frac{1}{3}(30^3-20^3)+(30-20)y^2)\,dy$

$= \int_{20}^{30} K(\frac{19000}{3}+10y^2)\,dy$

$= K(\frac{19000}{3}y+\frac{10}{3}y^3)\cdot y\Big|_{20}^{30}$

$= K(\frac{190000}{3}+\frac{190000}{3})$

$= \frac{380000}{3}K = 1$

$K = \frac{3}{380000}.$

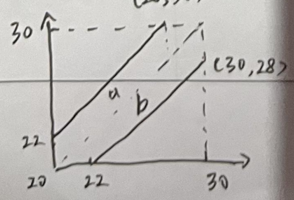
b. $P(X<26 \text{ and } Y<26) = \int_{20}^{26}\int_{20}^{26} \frac{3}{380000}(x^2+y^2)\,dx\,dy$

$= \int_{20}^{26} \frac{3}{380000}(3192+6y^2)\,dy$

$= \frac{3}{380000}(19152+28728)$

$= 0.378$

c.




we can find that the area of a and b is the case.

P(different between two tires is at most 2 psi)

$= \frac{\text{Area}(a+b)}{\text{Area}(S)} = \frac{36}{100} = 0.360$

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d. $f_X(y) = \int_{20}^{30} \frac{3}{380000}(x^2+y^2)\,dy = \frac{3}{380000}\left(\frac{190000}{3} + 10x^2\right) = \frac{1}{2} + \frac{3}{38000}x^2 \quad (20 \le x \le 30)$

e. Since $f_Y(x) = \frac{1}{2} + \frac{3}{38000}y^2 \quad (20 \le y \le 30)$

$f(x,y) \ne f_X(y) \cdot f_Y(x)$

X and Y are not independent.

12. a. $P(X \ge 3) = \int_3^{\infty}\int_0^{\infty} xe^{-x(1+y)}\,dy\,dx = \int_3^{\infty} xe^{-x}\,dx = 0.5$

b. $f_X(y) = \int_0^{\infty} xe^{-x(1+y)}\,dy = e^{-x} \quad (x \ge 0)$

$f_Y(x) = \int_0^{\infty} xe^{-x(1+y)}\,dx = \frac{1}{(1+y)^2} \quad (y \ge 0)$

$f(x,y) \ne f_X(y) \cdot f_Y(x)$, X, Y are not independent.

c. $P(X \ge 3 \text{ and } Y \ge 3) = 1 - P(X \le 3 \text{ and } Y < 3)$

$= 1 - \int_0^3\int_0^3 xe^{-x(1+y)}\,dx\,dy$

$= 1 - \int_0^3 e^{x}(1-e^{-3})\,dx$

$= 0.30$

13. a. Since $P_X(1) = 0.34$.

$P_{Y|X}(0|1) = \frac{P(1,0)}{P_X(1)} = 0.2353$

$P_{Y|X}(1|1) = \frac{P(1,1)}{P_X(1)} = 0.5882$

$P_{Y|X}(2|1) = \frac{P(1,2)}{P_X(1)} = 0.1765$

b.

| y | 0 | 1 | 2 |
|---|---|---|---|
| $P_{Y\|X}(y\|2)$ | 0.12 | 0.28 | 0.60 |

c. $P(Y \le 1 | X = 2) = P_{Y|X}(0|2) + P_{Y|X}(1|2) = 0.40$

d.

| x | 0 | 1 | 2 |
|---|---|---|---|
| $P_{X\|Y}(x\|2)$ | 0.0526 | 0.1579 | 0.7895 |

**教师评语**
**学生反思**

19. a. $f_{Y|X}(y|x) = \dfrac{f(x,y)}{f_X(x)} = \dfrac{k(x^2+y^2)}{10kx^2+0.05}$ $(20 \le y \le 30$ and $k = \dfrac{3}{380\,000})$

$f_{X|Y}(x|y) = \dfrac{f(x,y)}{f_Y(y)} = \dfrac{k(x^2+y^2)}{10ky^2+0.05}$ $(20 \le x \le 30$ and $k = \dfrac{3}{380\,000})$

b. $P(Y \geq 25 | X=22) = \int_{25}^{30} f_{Y|X}(y|22)\,dy = \int_{25}^{30} \dfrac{k(22^2+y^2)}{10k \cdot 22^2+0.05}\,dy = 0.556$

$P(Y \geq 25) = \int_{25}^{30} f_Y(y)\,dy = \int_{25}^{30} (10ky^2+0.05)\,dy = 0.549$

c. $E(Y|X=22) = \int_{20}^{30} y \cdot f_{Y|X}(y|22)\,dy = \int_{20}^{30} y \cdot \dfrac{k(22^2+y^2)}{10k \cdot 22^2+0.05}\,dy = 25.37$

$E(Y^2|X=22) = \int_{20}^{30} y^2 \cdot f_{Y|X}(y|22)\,dy = \int_{20}^{30} y^2 \cdot \dfrac{k(22^2+y^2)}{10k \cdot 22^2+0.05}\,dy = 652.03$

$V(Y|X=22) = E(Y^2|X=22) - E^2(Y|X=22) = 8.39$

$\sigma = \sqrt{V(Y|X=22)} = 2.90$

5.2

24. $h(x,y)$

| $x \backslash y$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | – | 2 | 3 | 4 | 5 | 6 |
| 2 | 2 | – | 2 | 3 | 4 | 5 |
| 3 | 3 | 2 | – | 2 | 3 | 4 |
| 4 | 4 | 3 | 2 | – | 2 | 3 |
| 5 | 5 | 4 | 3 | 2 | – | 2 |
| 6 | 6 | 5 | 4 | 3 | 2 | – |

$f(x,y)$ is the number of individuals handle the message when A in x and B in y

there for. $E[h(x,y)] = \sum_x \sum_y h(x,y) \cdot p(x,y) = 2.8$

26. $E(3x+10y) = \sum_{x=0}^{5} \sum_{y=0}^{2} (3x+10y) \cdot p(x,y) = 15.4$

33. Since $E(XY) = E(X) \cdot E(Y)$

$Cov(X,Y) = E(XY) - \mu_X \mu_Y = E(X) \cdot E(y) - E(x) E(y) = 0$

$Corr(X,Y) = \dfrac{Cov(X,Y)}{\sigma_X \sigma_Y} = 0$

therfor, $Cov(X,Y) = Corr(X,Y) = 0$

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35. a. $Cov(aX+b, cY+d) = E(aX+b, cY+d) - E(aX+b) \cdot E(cY+d)$

$= E(acXY + adX + bcY + bd) - (aE(X) + b)(cE(Y) + d)$

$= acE(XY) + adE(X) + bcE(Y) + bd - [acE(X)E(Y) + adE(X) + bcE(Y) + bd]$

$= acE(XY) - acE(X)E(Y)$

$= ac \cdot Cov(X, Y)$

b. $Corr(aX+b, cY+d) = \dfrac{Cov(aX+b, cY+d)}{\sigma(aX+b)\,\sigma(cY+d)}$

$= \dfrac{ac\,Cov(X,Y)}{|a||c|\,\sigma(X)\,\sigma(Y)}$

$= Corr(X, Y)$

c. Since a, b have different in sign, $|a| \cdot |c| = -ac$

therefor, $Corr(aX+b, cY+d) = -Corr(X, Y)$