

Chapter 5

Section 5.1

Ex. 9.

$$a) \int_{20}^{30} \int_{20}^{30} k(x^2 + y^2) dx dy = 1, \text{ then: } k \int_{20}^{30} \int_{20}^{30} x^2 dx dy + k \int_{20}^{30} \int_{20}^{30} y^2 dx dy = 1$$

$$\text{then: } 10k \int_{20}^{30} x^2 dx + 10k \int_{20}^{30} y^2 dy = 1$$

$$\text{then } 20k \times \frac{9000}{3} = 1$$

$$k = \frac{3}{380000}$$

$$b) P(X < 26 \text{ and } Y < 26) = \int_{20}^{26} \int_{20}^{26} k(x^2 + y^2) dx dy = k \int_{20}^{26} [x^2 y + \frac{y^3}{3}]_{20}^{26} dx$$

$$= k \int_{20}^{26} (6x^2 + 3192) dx$$

$$= k(38304) = 0.3024$$

$$c) P(|X - Y| \leq 2) = 1 - \int_{20}^{28} \int_{x+2}^{30} f(x, y) dy dx - \int_{22}^{30} \int_{20}^{x-2} f(x, y) dy dx$$

$$= 0.3593$$

$$d) f_X(x) = \int_{20}^{30} k(x^2 + y^2) dy = 10kx^2 + 0.05 \quad (20 \leq x \leq 30)$$

$$e) f(x, y) \neq f_X(x) \cdot f_Y(y), \text{ so } X, Y \text{ are not independent.}$$

Ex. 12.

$$a) P(X > 3) = \int_3^{\infty} \int_0^{\infty} x e^{-x(1+y)} dy dx = \int_3^{\infty} e^{-x} dx = 0.05$$

$$b) f_X(x) = \int_0^{\infty} x e^{-x(1+y)} dy = e^{-x} \quad (x > 0)$$

$$f_Y(y) = \int_0^{\infty} x e^{-x(1+y)} dx = \frac{1}{(1+y)^2} \quad (y > 0)$$

$$f(x, y) \neq f_X(x) \cdot f_Y(y), \text{ so } X \text{ and } Y \text{ are not independent.}$$

$$c) P(X > 3 \text{ or } Y > 3) = 1 - P(X \leq 3 \text{ and } Y \leq 3) = 1 - \int_0^3 \int_0^3 x e^{-x(1+y)} dy dx$$

$$= 1 - \int_0^3 \int_0^3 x e^{-x} e^{-xy} dy dx$$

$$= 1 - \int_0^3 e^{-x} (1 - e^{-3x}) dx$$

$$= 0.3$$

Ex. 18.

$$a) \text{ we know that: } P_{X|Y}(0|1) = \frac{0.08}{0.34} = 0.2353$$

$$P_{Y|X}(1|1) = \frac{0.20}{0.34} = 0.5882$$

$$P_{Y|X}(2|1) = \frac{0.06}{0.34} = 0.1765$$



b)	y	0	1	2
$P_{Y X}(y 2)$		0.12	0.28	0.60

$$c) P(Y \leq 1 | X=2) = P_{Y|X}(0|2) + P_{Y|X}(1|2) = 0.12 + 0.28 = 0.4$$

d)	X	0	1	2
$P_{X Y}(x 2)$		0.0526	0.1579	0.7895

Ex. 19.

$$a) f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} = \frac{k(x^2+y^2)}{10kx^2+0.05} \quad (20 \leq y \leq 30) \quad , k \text{ is a constant } \left(\frac{0.3}{380000}\right)$$

$$f_{X|Y}(x|y) = \frac{k(x^2+y^2)}{10ky^2+0.05} \quad , (x: 20 \leq x \leq 30)$$

$$b) P(Y \geq 25 | X=22) = \int_{25}^{30} f_{Y|X}(y|22) dy = \int_{25}^{30} \frac{k(22^2+y^2)}{10k22^2+0.05} dy = 0.556$$

$$P(Y \geq 25) = \int_{25}^{30} f_Y(y) dy = 0.549$$

$$c) E(Y | X=22) = \int_{20}^{30} y \cdot \frac{k(22^2+y^2)}{10k22^2+0.05} dy = 25.372912$$

$$E(Y^2 | X=22) = \int_{20}^{30} y^2 \cdot \frac{k(22^2+y^2)}{10k22^2+0.05} dy = 652.02864$$

$$V(Y | X=22) = E(Y^2 | X=22) - E(Y | X=22)^2 = 8.243976$$

$$\sigma = \sqrt{V(Y | X=22)} = 2.87$$



Section 5.2

Ex. 24:

we can draw a table:

	$h(x,y)$	y					
		1	2	3	4	5	6
x	1	-	2	3	4	3	2
	2	2	-	2	3	4	3
	3	3	2	-	2	3	4
	4	4	3	2	-	2	3
	5	3	4	3	2	-	2
	6	2	3	4	3	2	-

every $p(x,y) = \frac{1}{30}$

$$\text{so: } E(h(x,y)) = \sum_x \sum_y h(x,y) \cdot p(x,y) = \frac{84}{30} = 2.8$$

Ex. 26.

$$\begin{aligned} \text{revenue} &= 3X + 10Y, \text{ so } E(\text{revenue}) = E(3X + 10Y) \\ &= \sum_{x=1}^6 \sum_{y=1}^6 (3x + 10y) \cdot p(x,y) = 15.4 \$ \end{aligned}$$

Ex. 33

$$E(XY) = E(X) \cdot E(Y), \text{ Cov}(X,Y) = E(XY) - E(X) \cdot E(Y) = 0$$

$$\text{so: } \text{Corr}(X,Y) = \frac{\text{Cov}(X,Y)}{\sigma_X \sigma_Y} = 0$$

Ex. 35.

$$\begin{aligned} \text{a) } \text{Cov}(aX+b, cY+d) &= E[(aX+b)(cY+d)] - E(aX+b) \cdot E(cY+d) \\ &= E(acXY + adX + bcY + bd) - [aE(X)+b][cE(Y)+d] \\ &= acE(XY) - acE(X)E(Y) = ac(E(XY) - E(X)E(Y)) \\ &= ac \text{Cov}(X,Y) \end{aligned}$$

$$\text{b) } \text{Corr}(aX+b, cY+d) = \frac{\text{Cov}(aX+b, cY+d)}{SD(aX+b)SD(cY+d)} = \frac{ac}{|ac|} \text{Corr}(X,Y)$$

as, a and c have same sign, so: $\text{Corr}(aX+b, cY+d) = \text{Corr}(X,Y)$

$$\text{c) } ac = -|ac|, \text{ so: } \text{Corr}(aX+b, cY+d) = -\text{Corr}(X,Y)$$

