A

38. There are two traffic lights on a commuter's route to and from work. Let X_1 be the number of lights at which the commuter must stop on his way to work, and X_2 be the number of lights at which he must stop when returning from work. Suppose these two variables are independent, each with pmf given in the accompanying table (so X_1, X_2 is a random sample of size $n = 2$).

x_1	0	1	2
$p(x_1)$	.2	.5	.3

$\mu = 1.1, \sigma^2 = .49$

a. Determine the pmf of $T_o = X_1 + X_2$.
b. Calculate μ_{T_o}. How does it relate to μ, the population mean?
c. Calculate $\sigma^2_{T_o}$. How does it relate to σ^2, the population variance?
d. Let X_3 and X_4 be the number of lights at which a stop is required when driving to and from work on a second day assumed independent of the first day. With T_o = the sum of all four X_i's, what now are the values of $E(T_o)$ and $V(T_o)$?
e. Referring back to (d), what are the values of $P(T_o = 8)$ and $P(T_o \geq 7)$ [*Hint*: Don't even think of listing all possible outcomes!]

e. $P(T_0=8) = 0.3(0.3)(0.3)(0.3)$
$= 0.0081$
$P(T_0=7) = 4[0.5(0.3)^3]$
$= 0.54$
$P(T_0 \geq 7) = 0.54 + 0.0081$
$= 0.0621$

a. X may be 0 or 1 or 2, so T_0 may be 0,1,2,3,4.
$P(T_0=0) = P(X_1=0 \text{ and } X_2=0) = 0.2(0.2) = 0.04$
$P(T_0=1) = P((X_1=0 \text{ and } X_2=1) \text{ OR } (X_1=1 \text{ and } X_2=0))$
$= 0.2(0.5) + 0.5(0.2)$
$= 0.20$
$P(T_0=2) = 0.2(0.3) + 0.3(0.2) + 0.5^2 = 0.37$
$P(T_0=3) = 0.5(0.3) + 0.3(0.5) = 0.3$
$P(T_0=4) = 0.3(0.3) = 0.09$

T_0	0	1	2	3	4
$p(T_0)$	0.04	0.2	0.37	0.3	0.9

b. $E(T_0) = 0(0.04) + 1(0.2) + 2(0.37) + 3(0.3) + 4(0.9) = 2.2$
$2\mu = 2(1.1) = 2.2 = E(T_0)$

c. $E(T_0^2) = 0^2(0.04) + 1^2(0.2) + 2^2(0.37) + 3^2(0.3) + 4^2(0.9) = 5.82.$
$V(T_0) = 5.82 - (2.2)^2 = 0.98$
$2\sigma^2 = 2(0.49) = 0.98 = V(T_0)$

d. $T_0 = X_1 + X_2 + X_3 + X_4$
$E(T_0) = 4\mu = 4(1.1) = 4.4$
$V(T_0) = 4\sigma^2 = 4(0.49) = 1.96$

41. Let X be the number of packages being mailed by a randomly selected customer at a certain shipping facility. Suppose the distribution of X is as follows:

x	1	2	3	4
$p(x)$	.4	.3	.2	.1

a. Consider a random sample of size $n = 2$ (two customers), and let $\overline{X}$ be the sample mean number of packages shipped. Obtain the probability distribution of $\overline{X}$.

b. Refer to part (a) and calculate $P(\overline{X} \le 2.5)$.

c. Again consider a random sample of size $n = 2$, but now focus on the statistic R = the sample range (difference between the largest and smallest values in the sample). Obtain the distribution of R. [*Hint*: Calculate the value of R for each outcome and use the probabilities from part (a).]

d. If a random sample of size $n = 4$ is selected, what is $P(\overline{X} \le 1.5)$? [*Hint*: You should not have to list all possible outcomes, only those for which $\bar{x} \le 1.5$.]

(x_1, x_2)	(1,1)	(1,2)	(1,3)	(1,4)	(2,1)	(2,2)	(2,3)	(2,4)
probability	0.16	0.12	0.08	0.04	0.12	0.09	0.06	0.03
$\bar{x}$	1	1.5	2	2.5	1.5	2	2.5	3
r	0	1	2	3	1	0	1	2
(x_1, x_2)	(3,1)	(3,2)	(3,3)	(3,4)	(4,1)	(4,2)	(4,3)	(4,4)
probability	0.08	0.06	0.04	0.02	0.04	0.03	0.02	0.01
$\bar{x}$	2	2.5	3	3.5	2.5	3	3.5	4
r	2	1	0	1	3	2	1	2

a.

$\bar{x}$	1	1.5	2	2.5	3	3.5	4
$p(\bar{x})$	0.16	0.24	0.25	0.2	0.1	0.04	0.01

b. $p(\bar{x} \le 2.5) = 0.16 + 0.24 + 0.25 + 0.2$
$= 0.85$

c.

r	0	1	2	3
p(r)	0.3	0.40	0.22	0.08

d. $p(\bar{x} \le 1.5) = (0.4)^4 + 4[(0.4)^3(0.3)] + 4[(0.4)^2(0.2)^2] + 6[(0.4)^2(0.3)^2]$
$= 0.24$

40. The inside diameter of a randomly selected piston ring is a random variable with mean value 12 cm and standard deviation .04 cm.

a. If $\overline{X}$ is the sample mean diameter for a random sample of $n = 16$ rings, where is the sampling distribution of $\overline{X}$ centered, and what is the standard deviation of the $\overline{X}$ distribution?

b. Answer the questions posed in part (a) for a sample size of $n = 64$ rings.

c. For which of the two random samples, the one of part (a) or the one of part (b), is $\overline{X}$ more likely to be within .01 cm of 12 cm? Explain your reasoning.

a. $E(\bar{x}) = \mu = 12$

$\sigma_{\bar{x}} = \frac{\sigma_{\bar{x}}}{\sqrt{n}} = \frac{0.04}{\sqrt{16}} = 0.01$ ✓

b. When $n = 64$,

$E(\bar{x}) = \mu = 12$

$\sigma_{\bar{x}} = \frac{0.04}{\sqrt{64}} = 0.005$ ✓

c. Because variability of $\bar{x}$ is decreased, so the sample size became larger. ✓

51. The time taken by a randomly selected applicant for a mortgage to fill out a certain form has a normal distribution with mean value 10 min and standard deviation 2 min. If five individuals fill out a form on one day and six on another, what is the probability that the sample average amount of time taken on each day is at most 11 min?

Given $X \sim N(10, 2)$,

For day 1 and $n = 5$,

$p(\bar{x} \leq 11) = P\left(Z \leq \frac{11-10}{\frac{2}{\sqrt{5}}}\right) = P(Z \leq 1.12) = 0.8686$

For day 2 and $n = 6$,

$p(\bar{x} \leq 11) = P\left(Z \leq \frac{11-10}{\frac{2}{\sqrt{6}}}\right) = P(Z \leq 1.22) = 0.8888$

$\therefore$ The probability $= 0.8686\,(0.8888)$
$= 0.7720$

55. The number of parking tickets issued in a certain city on any given weekday has a Poisson distribution with parameter $\mu = 50$. What is the approximate probability that

a. Between 35 and 70 tickets are given out on a particular day? [*Hint*: When μ is large, a Poisson rv has approximately a normal distribution.]

b. The total number of tickets given out during a 5-day week is between 225 and 275?

a. In Poisson distribution, its $\mu = \sigma^2 = \lambda$.

Since $\mu = 50$, $\sigma = \sqrt{50} = 7.071$, suppose X = the number of tickets.

$P(35 \le X \le 70) = P\left(\frac{35-50}{7.071} \le Z \le \frac{70-50}{7.071}\right)$

$= P(-2.12 \le Z \le 2.83)$

$= 0.9977 - 0.0170$

$= 0.9807$

b. when day is 5, $\mu = 250$, $\sigma = \sqrt{250} = 15.811$,

$P(225 \le X \le 275) = P\left(\frac{225-250}{15.811} \le Z \le \frac{275-250}{15.811}\right)$

$= P(-1.58 \le Z \le 1.58)$

$= 0.9429 - 0.0571$

$= 0.8858$

58. A shipping company handles containers in three different sizes: (1) 27 ft^3 ($3 \times 3 \times 3$), (2) 125 ft^3, and (3) 512 ft^3. Let X_i ($i = 1, 2, 3$) denote the number of type i containers shipped during a given week. With $\mu_i = E(X_i)$ and $\sigma_i^2 = V(X_i)$, suppose that the mean values and standard deviations are as follows:

$$\mu_1 = 200 \qquad \mu_2 = 250 \qquad \mu_3 = 100$$
$$\sigma_1 = 10 \qquad \sigma_2 = 12 \qquad \sigma_3 = 8$$

a. Assuming that X_1, X_2, X_3 are independent, calculate the expected value and variance of the total volume shipped. [*Hint*: Volume $= 27X_1 + 125X_2 + 512X_3$.]

b. Would your calculations necessarily be correct if the X_i's were not independent? Explain.

a. $E(27X_1 + 125X_2 + 512X_3)$
$= 27E(X_1) + 125E(X_2) + 512E(X_3)$
$= 27(200) + 125(250) + 512(100)$
$= 87850$
$V(27X_1 + 125X_2 + 512X_3)$
$= 27^2V(X_1) + 125^2V(X_2) + 512^2V(X_3)$
$= 27^2(10)^2 + 125^2(12)^2 + 512^2(8)^2$
$= 19100116$

b. Expected value would be still correct.
But the variance would not, because of the coveriences change to the varience.

Consider a random sample of size n from a continuous distribution having median 0 so that the probability of any one observation being positive is .5. Disregarding the signs of the observations, rank them from smallest to largest in absolute value, and let W = the sum of the ranks of the observations having positive signs. For example, if the observations are $-.3$, $+.7$, $+2.1$, and -2.5, then the ranks of positive observations are 2 and 3, so $W = 5$. In Chapter 15, W will be called *Wilcoxon's signed-rank statistic*. W can be represented as follows:

$$W = 1 \cdot Y_1 + 2 \cdot Y_2 + 3 \cdot Y_3 + \cdots + n \cdot Y_n$$
$$= \sum_{i=1}^{n} i \cdot Y_i$$

where the Y_i's are independent Bernoulli rv's, each with $p = .5$ ($Y_i = 1$ corresponds to the observation with rank i being positive).

a. Determine $E(Y_i)$ and then $E(W)$ using the equation for W. [*Hint*: The first n positive integers sum to $n(n + 1)/2$.]

b. Determine $V(Y_i)$ and then $V(W)$. [*Hint*: The sum of the squares of the first n positive integers can be expressed as $n(n + 1)(2n + 1)/6$.]

a. $E(Y_i) = \frac{1}{2}$

$E(W) = \sum_{i=1}^{n} i[E(Y_i)] = \frac{1}{2}\sum_{i=1}^{n} i = \frac{n(n+1)}{4}$

b. $V(Y_i) = \frac{1}{4}$

$V(W) = \sum_{i=1}^{n} i^2 [V(Y_i)] = \frac{1}{4}\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{24}$

73. Suppose the expected tensile strength of type-A steel is 105 ksi and the standard deviation of tensile strength is 8 ksi. For type-B steel, suppose the expected tensile strength and standard deviation of tensile strength are 100 ksi and 6 ksi, respectively. Let $\overline{X}$ = the sample average tensile strength of a random sample of 40 type-A specimens, and let $\overline{Y}$ = the sample average tensile strength of a random sample of 35 type-B specimens.

a. What is the approximate distribution of $\overline{X}$? Of $\overline{Y}$?

b. What is the approximate distribution of $\overline{X} - \overline{Y}$? Justify your answer.

c. Calculate (approximately) $P(-1 \leq \overline{X} - \overline{Y} \leq 1)$.

d. Calculate $P(\overline{X} - \overline{Y} \geq 10)$. If you actually observed $\overline{X} - \overline{Y} \geq 10$, would you doubt that $\mu_1 - \mu_2 = 5$?

a. Since Type-A and type-B steels are both $\sim N(\mu, \sigma)$, their sample distribution of sample average tensile strength also satisfy normal distribution.

b. $\because \bar{X}, \bar{Y} \sim N(\mu, \sigma)$, their linear combination $\bar{X}-\bar{Y}$ is also a normal distribution.

$\mu_{\bar{X}-\bar{Y}} = 105 - 100 = 5$

$\sigma_{\bar{X}-\bar{Y}}^2 = \frac{8^2}{40} - \frac{6^2}{35} = 2.6286$

$\sigma = \sqrt{2.6286} = 1.6213$

c. $P(-1 \leq \bar{X}-\bar{Y} \leq 1) = P\left(\frac{-1-5}{1.6213} \leq Z \leq \frac{1-5}{1.6213}\right)$

$= P(-3.70 \leq Z \leq -2.47)$

$= 0.0068 - 0$

$= 0.0068$

d. $P(\bar{x}-\bar{y} \geq 10) = P(Z \geq \frac{10-5}{1.6213})$

$= P(Z \geq 3.08)$

$= 1 - 0.9990$

$= 0.0010$

$\because$ The probability is so small

$\therefore$ It is not satisfy with $\mu_1 - \mu_2 = 5$