No.

Date

概率统计　22CST 蒋云翔

homework 05 and homework 06

Section 3.3

Ex. 29

(a) $E(X)=\sum_{x\in D} x\,p(x) = 1\times0.05 + 2\times0.10 + 4\times0.35 + 8\times0.40 + 16\times0.10 = 6.45$ GB

(b) $V(X)=\sum_{x\in D}(x-\mu)^2 p(x) = (1-6.45)^2\times0.05 + (2-6.45)^2\times0.05 + \cdots + (16-6.45)^2\times0.10 = 15.6475$ GB

(c) $\sigma=\sqrt{V(X)} = 3.956$ GB

(d) $E(X^2)=\sum_{x\in D}x^2 p(x) = 57.25$; $E(X)^2 = 6.45^2$

$V(X) = E(X^2) - E(X)^2 = 57.25 - (6.45)^2 = 15.6475$

Ex. 33

(a) $E(X^2) = 0^2(1-p) + 1^2\cdot p = p$

(b) $V(X) = (0-p)^2(1-p) + (1-p)^2\cdot p = p(1-p)$

(c) $E(X^{79}) = p$, it is the same as question (a)

Ex (38.)

Solution: ~~E(h(x)~~ $E(h(X)) = E(\frac{1}{X}) = \sum_{x=1}^{6}\frac{1}{x}p(x) = 0.408$

as for: $1/3.5 = 0.286 < E(\frac{1}{X})$, so if u gamble, u will win more as expected

Ex (41.)

proof: $V(aX+b) = \sum(ax+b-E(aX+b))^2 p(x)$

$= \sum(ax+b-aE(X)-b)^2\cdot p(x)$

$= \sum(ax-aE(X))^2\cdot p(x)$

$= a^2\sum(x-E(X))^2\cdot p(x)$

$= a^2\cdot V(X)$

$= a^2\sigma_X^2$

No.

Date

Section 3.4

Ex.(46.)

(a). $b(3;8,0.35) = C_8^3 \cdot (0.35)^3 (1-0.35)^5 = 0.279$

(b) $b(5;8,0.6) = C_8^5 (0.6)^5 \cdot (1-0.6)^3 = 0.279$

~~(c) $P(3 \le X \le 5) = b(3;7,0.6) = C_7^3 (0.6)^3 (1-0.6)^4$~~

(c) $P(3 \le X \le 5) = b(3;7,0.6) + b(4;7,0.6) + b(5;7,0.6) = 0.745$

(d) $P(1 \le X) = 1 - P(X=0) = 1 - C_9^0 \cdot (0.1)^0 (1-0.1)^9 = 0.613$

Ex.47

(a) $B(4;15,0.3) = 0.515$

(b) $b(4;15,0.3) = B(4;15,0.3) - B(3;15,0.3) = 0.219$

(c) $b(6;15,0.7) = B(6;15,0.7) - \cancel{B(5;} B(5;15,0.7) = 0.012$

(d) $P(2 \le X \le 4) = B(4;15,0.3) - B(1;15,0.3) = 0.480$

(e) $P(2 \le X) = 1 - B(1;15,0.3) = 0.965$

(f) $P(X \le 1) = B(1;15,0.7) = 0.000$

(g) $P(2 < X < 6) = B(5;15,0.3) - B(2;15,0.3) = 0.595$

Ex.48

(a) $P(X \le 2) = B(2;25,0.05) = 0.873$

(b) $P(X \le 5) = 1 - P(X < 5) = 1 - B(4;25,0.05) = 1 - 0.993 = 0.007$

(c) $P(1 \le X \le 4) = B(4;25,0.05) - B(0;25,0.05) = 0.716$

(d) $P(X=0) = P(0;25,0.05) = 0.277$

(e) $E(X) = np = 25 \times 0.05 = 1.25$

$\sigma_X \cancel{V(X)} = \sqrt{np(1-p)} \approx 1.09$

No.

Date

Ex.54 Let X denote the number of people who buys an oversized racket.

(a) $P(X\geq 6) = 1 - P(X<6) = 1 - B(5;10,0.6) = 1-0.367 = 0.633$

(b) $E(X) = np = 10\times 0.6 = 6$, $\sigma = \sqrt{np(1-p)} = 1.55$ so the range is $(4.45, 7.55)$

$P(4.45 < X < 7.75) = B(7;10,0.6) - B(4;10,0.6) = 0.667$

(c) This situation can be expressed like: $P(3\leq X\leq 7) = B(7;10;0.6) - B(2;10,0.6) = 0.667$

Section 3.5

Ex.68.

(a) It is clear that it is a hypergeomatic distribution. $X \sim h(X; 6, 12, 20)$

(b) $P(X=2) = \frac{C_{12}^{2}\cdot C_{8}^{4}}{C_{20}^{6}} = \frac{66\times 70}{38760} = 0.119$

$P(X\leq 2) = P(X=0) + P(X=1) + P(X=2) = \frac{C_{12}^{0}\cdot C_{8}^{6}}{C_{20}^{6}} + \frac{C_{12}^{1}\cdot C_{8}^{5}}{C_{20}^{6}} + P(X=2) = 0.137$

$P(X\geq 2) = 1 - P(X\leq 1) = 0.982$

(c) $E(X) = n\cdot\frac{M}{N} = 6\times\frac{12}{20} = 3.6$; $V(X) = \frac{6\times 12(20-6)(20-12)}{20^2(20-1)} = 1.06$

$\sigma(X) = \sqrt{V(X)} = 1.03$

Ex 69.

(a) $P(X=5) = \frac{C_{7}^{5}\cdot C_{5}^{1}}{C_{12}^{6}} = 0.114$

(b) $P(X\leq 4) = 1 - P(X=5) - P(X=6) = 1 - 0.114 - \frac{C_{7}^{6}\cdot C_{5}^{0}}{C_{12}^{6}} = 0.879$

(c) $E(X) = 6\times\frac{7}{12} = 3.5$; $V(X) = \frac{\cancel{7}\,6\,(12-6)\cdot(12-7)}{12^2(12-1)} = 0.795$; $\sigma_X = \sqrt{V(X)} = 0.892$

$P(X > \mu+\sigma) = P(X > 4.392) = P(X=5) + P(X=6) = 0.121$

(d) View it as a binomial distribution

$P(X\leq 5) = B(5;15,0.1) = 0.998$

Ex.72.

(a). $\frac{C_{4}^{x}\cdot C_{7}^{6-x}}{C_{11}^{6}}$

(b) $E(X) = 6\times\frac{4}{11} = 2.18$, 2.18 as expected will be interviewed at 1st day.

Ex.75. It is a negative Binomial distribution

(a) $P(X=x)=C_{x+2-1}^{2-1}(0.5)^2\cdot(1-0.5)^x=(0.5)^{x+2}\cdot(x+1)$

(b) $P(X=2)=(2+1)\times(0.5)^{2+2}=0.188$

(c) $P(X\le 2)=0.5^2+2\cdot(0.5)^3+0.188=0.688$

(d) $E(X)=\frac{2(1-0.5)}{0.5}=2$, so the family is expected to have $2+2=4$ children.

E\ Section 3.6

Ex.79

(a) $P(X\le 8)=F(8,5)=0.932$

(b) $P(X=8)=F(8,5)-F(7,5)=0.065$

(c) $P(X\ge 9)=1-F(8,5)=0.068$

(d) $P(5\le X\le 8)=F(8,5)-F(4,5)=0.492$

(e) $P(5<X<8)=F(7,5)-F(5,5)=0.251$

Ex. 84.

(a) $\mu=np=10000\times 0.001=10$, $\sigma=\sqrt{V(x)}=\sqrt{10000\times 0.001\times 0.999}=3.16$

(b) $P(X>10)=1-P(X\le 10)=1-F(10;10)=1-0.583=0.417$ (泊松分布中 $\mu=\lambda$, $V(X)=\lambda$)

(c) $P(X=0)=F(0;10)=0.000045$

Ex.86

(a) $F(4;5)=0.175$

(b) $P(X\ge 4)=1-P(X\le 3)=1-F(3;5)=0.735$

(c) expect: $\mu\times 0.75=5\times 0.75=3.75$

Ex.87

(a) $F(10;8)-F(9;8)=0.99$

(b) $F(0,2)=0.135$

(c) $E(X)$ $\mu=2$