B+

3. There are two traffic lights on a commuter's route to and from work. Let X_1 be the number of lights at which the commuter must stop on his way to work, and X_2 be the number of lights at which he must stop when returning from work. Suppose these two variables are independent, each with pmf given in the accompanying table (so X_1, X_2 is a random sample of size $n = 2$).

x_1	0	1	2
$p(x_1)$	.2	.5	.3

$\mu = 1.1, \sigma^2 = .49$

a. Determine the pmf of $T_o = X_1 + X_2$.
b. Calculate μ_{T_o}. How does it relate to μ, the population mean?
c. Calculate $\sigma^2_{T_o}$. How does it relate to σ^2, the population variance?
d. Let X_3 and X_4 be the number of lights at which a stop is required when driving to and from work on a second day assumed independent of the first day. With T_o = the sum of all four X_i's, what now are the values of $E(T_o)$ and $V(T_o)$?
e. Referring back to (d), what are the values of $P(T_o = 8)$ and $P(T_o \geq 7)$ [*Hint*: Don't even think of listing all possible outcomes!]

a. $T_0 = X_1 + X_2$ T_0 can be $0 \sim 4$

X_1 and X_2 are independent

so we can get

T_0	0	1	2	3	4
$P(T_0)$	0.04	0.2	0.37	0.3	0.09

b $\mu_{T_0} = n\mu = 2 \times 1.1 = 2.2$

c $\sigma^2_{T_0} = n\sigma^2 = 2 \times 0.49 = 0.98$

d $T_0 = X_1 + X_2 + X_3 + X_4$

$E(T_0) = n \cdot \mu = 4.4$ $V(T_0) = n\sigma^2 = 1.96$

e

$P(T_0 = 8) = (0.3)^4 = 0.0081$

$P(T_0 \geq 7) = (0.3)^4 + C_4^1 (0.5)\times(0.3)^3$
$= 0.0621$

41. Let X be the number of packages being mailed by a randomly selected customer at a certain shipping facility. Suppose the distribution of X is as follows:

x	1	2	3	4
$p(x)$	.4	.3	.2	.1

a. Consider a random sample of size $n = 2$ (two customers), and let $\bar{X}$ be the sample mean number of packages shipped. Obtain the probability distribution of $\bar{X}$.
b. Refer to part (a) and calculate $P(\bar{X} \leq 2.5)$.
c. Again consider a random sample of size $n = 2$, but now focus on the statistic R = the sample range (difference between the largest and smallest values in the sample). Obtain the distribution of R. [*Hint*: Calculate the value of R for each outcome and use the probabilities from part (a).]
d. If a random sample of size $n = 4$ is selected, what is $P(\bar{X} \leq 1.5)$? [*Hint*: You should not have to list all possible outcomes, only those for which $\bar{x} \leq 1.5$.]

$x_1 x_2$	(1,1)	(1,2)	(1,3)	(1,4)
P	0.16	0.12	0.08	0.04
$\bar{X}$	1	1.5	2	2.5

$x_1 x_2$	(2,1)	(2,2)	(2,3)	(2,4)
P	0.12	0.09	0.06	0.03
$\bar{X}$	1.5	2	2.5	3

$x_1 x_2$	(3,1)	(3,2)	(3,3)	(3,4)
P	0.08	0.06	0.04	0.02
$\bar{X}$	2	2.5	3	3.5

$x_1 x_2$	(4,1)	(4,2)	(4,3)	(4,4)
P	0.04	0.03	0.02	0.01
$\bar{X}$	2.5	3	3.5	4

a

X	1	1.5	2	2.5	3	3.5	4
$p(\bar{X})$	0.16	0.24	0.25	0.2	0.1	0.04	0.01

b $P(\bar{X} \leq 2.5) = 0.85$

c

R	0	1	2	3
$P(R)$	0.3	0.4	0.22	0.08

d $P(\bar{X} \leq 1.5) = 0.24$

6. The inside diameter of a randomly selected piston ring is a random variable with mean value 12 cm and standard deviation .04 cm.

a. If $\bar{X}$ is the sample mean diameter for a random sample of $n = 16$ rings, where is the sampling distribution of $\bar{X}$ centered, and what is the standard deviation of the $\bar{X}$ distribution?

b. Answer the questions posed in part (a) for a sample size of $n = 64$ rings.

c. For which of the two random samples, the one of part (a) or the one of part (b), is $\bar{X}$ more likely to be within .01 cm of 12 cm? Explain your reasoning.

a $\mu = 12 \quad \sigma = 0.04$

$\mu_{16} = 12$

$\sigma_{16} = \sqrt{V_{16}^2} = \sqrt{\frac{\sigma^2}{16}} = 0.01$

b $\mu_{64} = 12$

$\sigma_{64} = 0.005$

c (b) 64 is more likely within 0.01

$\sigma_{64} < \sigma_{16}$

51. The time taken by a randomly selected applicant for a mortgage to fill out a certain form has a normal distribution with mean value 10 min and standard deviation 2 min. If five individuals fill out a form on one day and six on another, what is the probability that the sample average amount of time taken on each day is at most 11 min?

day 1 $\quad n = 5$

$P_1(\bar{X} \leq 11) = P(t \leq \frac{11-10}{2/\sqrt{5}})$

day 2 $\quad n = 6$

$P_2(\bar{X} \leq 11) = P(t \leq \frac{11-10}{2/\sqrt{6}})$

$P(\bar{X}' \leq 11) = P_1 \cdot P_2 \approx 0.772$

55. The number of parking tickets issued in a certain city on any given weekday has a Poisson distribution with parameter $\mu = 50$. What is the approximate probability that

773

a. Between 35 and 70 tickets are given out on a particular day? [*Hint*: When μ is large, a Poisson rv has approximately a normal distribution.]

b. The total number of tickets given out during a 5-day week is between 225 and 275?

a this poisson distribution has approximately a normal distribution

$\mu = 50 \quad \sigma = \sqrt{50} \approx 7.071$

$P(35 \leq X \leq 70) = P(\frac{34.5-50}{7.071} \leq t \leq \frac{70.5-5}{7.071}$

≈ 0.9838

b $\mu = n\mu = 250 \quad \sigma = \sqrt{250} \approx 15.81$

$P(225 \leq X \leq 275) = P(\frac{225.5-250}{\sigma} \leq t \leq \frac{275.5-250}{\sigma}$

≈ 0.8926

58. A shipping company handles containers in three different sizes: (1) 27 ft³ (3 × 3 × 3), (2) 125 ft³, and (3) 512 ft³. Let X_i ($i = 1, 2, 3$) denote the number of type i containers shipped during a given week. With $\mu_i = E(X_i)$ and $\sigma_i^2 = V(X_i)$, suppose that the mean values and standard deviations are as follows:

$$\mu_1 = 200 \qquad \mu_2 = 250 \qquad \mu_3 = 100$$
$$\sigma_1 = 10 \qquad \sigma_2 = 12 \qquad \sigma_3 = 8$$

a. Assuming that X_1, X_2, X_3 are independent, calculate the expected value and variance of the total volume shipped. [*Hint*: Volume $= 27X_1 + 125X_2 + 512X_3$.]

b. Would your calculations necessarily be correct if the X_i's were not independent? Explain.

a $E(27X_1 + 125X_2 + 512X_3)$

$= \sum_{i=1}^{n} a_i E(X_i)$

$= 27 \times 200 + 250 \times 125 + 512 \times 100$

$= 87850$

X_1, X_2, X_3 are independent

$V = a_1^2\sigma_1^2 + \cdots + a_3^2\sigma_3^2$

$= (27)^2 \times 10^2 + (250)^2 \times 12^2 + (512)^2 \times 8^2$

$= 19100116$

b the expected value is correct but the variance is not correct

70. Consider a random sample of size n from a continuous distribution having median 0 so that the probability of any one observation being positive is .5. Disregarding the signs of the observations, rank them from smallest to largest in absolute value, and let W = the sum of the ranks of the observations having positive signs. For example, if the observations are $-.3$, $+.7$, $+2.1$, and -2.5, then the ranks of positive observations are 2 and 3, so $W = 5$. In Chapter 15, W will be called *Wilcoxon's signed-rank statistic*. W can be represented as follows:

$$W = 1 \cdot Y_1 + 2 \cdot Y_2 + 3 \cdot Y_3 + \cdots + n \cdot Y_n$$
$$= \sum_{i=1}^{n} i \cdot Y_i$$

where the Y_i's are independent Bernoulli rv's, each with $p = .5$ ($Y_i = 1$ corresponds to the observation with rank i being positive).

a. Determine $E(Y_i)$ and then $E(W)$ using the equation for W. [*Hint*: The first n positive integers sum to $n(n + 1)/2$.]

b. Determine $V(Y_i)$ and then $V(W)$. [*Hint*: The sum of the squares of the first n positive integers can be expressed as $n(n + 1)(2n + 1)/6$.]

a $E(Y_i) = 0.5$

$E(W) = \sum_{i=1}^{n} i \cdot E(Y_i) = \frac{1}{2} \times \sum_{i=1}^{n} i$

$= \frac{n(n+1)}{4}$

b $V(Y_i) = 0.25$

$V(W) = \sum_{i=1}^{n} i^2 V(Y_i) = \frac{n(n+1)(2n+1)}{24}$

Suppose the expected tensile strength of type-A steel is 105 ksi and the standard deviation of tensile strength is 8 ksi. For type-B steel, suppose the expected tensile strength and standard deviation of tensile strength are 100 ksi and 6 ksi, respectively. Let $\overline{X}$ = the sample average tensile strength of a random sample of 40 type-A specimens, and let $\overline{Y}$ = the sample average tensile strength of a random sample of 35 type-B specimens.

a. What is the approximate distribution of $\overline{X}$? Of $\overline{Y}$?

b. What is the approximate distribution of $\overline{X} - \overline{Y}$? Justify your answer.

c. Calculate (approximately) $P(-1 \le \overline{X} - \overline{Y} \le 1)$.

d. Calculate $P(\overline{X} - \overline{Y} \ge 10)$. If you actually observed $\overline{X} - \overline{Y} \ge 10$, would you doubt that $\mu_1 - \mu_2 = 5$?

$\mu_A = 105 \quad \sigma_A = 8$

$\mu_B = 100 \quad \sigma_B = 6$

a. Both $\bar{x}$, $\bar{Y}$ approximate normally by CLT

b normal distribution

$\bar{X} - \bar{Y}$. is a linear combination of normal distributions.

c

$\mu_{\bar{X}-\bar{Y}} = 5$

$\sigma^2_{\bar{X}-\bar{Y}} = \frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y} = 2.629$

$\sigma_{\bar{X}-\bar{Y}} = 1.621$

$P(-1 \le \bar{X}-\bar{Y} \le 1) = P(\frac{-6}{1.621} \le t \le \frac{-4}{1.621})$

≈ 0.0068

d

$P(\bar{X} - \bar{Y} \geqslant 10)$

$= P(t \geqslant \frac{5}{1.621}) \approx 0.001$

It's too small, doubt $\mu_1 - \mu_2 =$