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HW5

A

3.3) 29)

a) $E(X) = \sum_{\text{all } x} xp(x) = 1\cdot 0.05 + 2\cdot 0.1 + 4\cdot 0.35 + 8\cdot 0.4 + 16\cdot 0.1$
$= 6.45$

b) $V(X) = \sum_{\text{all } x} (x-\mu)^2 p(x) = (1-6.45)^2(0.05) + (2-6.45)^2(0.10) \cdots + (16-6.45)^2(0.1)$
$= 15.6475$

c) $\sigma = \sqrt{V(X)} = \sqrt{15.6475} = 3.956$

d) $E(X^2) = \sum_{\text{all } x} x^2 p(x) = 1^2(0.05) + 2^2(0.1) + 4^2(0.35) + 8^2(0.4) + 16^2(0.1)$
$= 57.25$

33)

a) $E(X^2) = \sum_{x=0}^{1} x^2 \cdot p(x) = 0^2(1-p) + 1^2 p$
$= p$

b) $V(X) = E(X^2) - [E(X)]^2$
$= p - p^2$
$= p(1-p)$

c) $E(X^{79}) = 0^{79}(1-p) + 1^{79}(p)$
$= p$

38) $\left(\frac{1}{3.5}\right) = 0.286$
$\sum_{x=1}^{6}\left(\frac{1}{x}\right)\cdot\frac{1}{6} = \frac{1}{6}\sum_{x=1}^{6}\frac{1}{x} = 0.408$

winning more

41) $V(aX+b) = \sum [aX+b - E(aX+b)]^2 \cdot p(x)$
$= \sum [aX - aE(X)]^2 \cdot p(x)$
$= a^2 \cdot \sum [X - E(X)]^2 \cdot p(x)$
$= a^2 \cdot \sigma^2 x$

HW6

A+

3.4) 46) a) $b(3;8,0.35) = \binom{8}{3}(0.35)^3(0.65)^5 = 0.279$

b) $b(5;8,0.6) = \binom{8}{5}(0.6)^5(0.4)^3 = 0.279$

c) $P(3 \leq X \leq 5) = b(3;7,0.6) + b(4;7,0.6) + b(5;7,0.6)$
$= 0.745$

d) $P(1 \leq X) = 1 - P(X=0)$
$= 1 - \binom{9}{0} \cdot 1 \cdot (0.9)^9$
$= 1 - (0.9)^9$
$= 0.613$

47) a) $B(4;15,0.3) = 0.515$

b) $b(4;15,0.3)$
$= B(4;15,0.3) - B(3;15,0.3)$
$= 0.219$

c) $b(6;15,0.7)$
$= B(6;15,0.7) - B(5;15,0.7)$
$= 0.012$

d) $P(2 \leq X \leq 4)$
$= B(4;15,0.3) - B(1;15,0.3)$
$= 0.48$

e) $P(2 \leq X)$
$= 1 - P(X \leq 1)$
$= 1 - B(1;15,0.3)$
$= 0.965$

f) $P(X \leq 1)$
$= B(1;15,0.7)$
$= 0$

g) $P(2 < X < 6)$
$= P(2 < X \leq 5)$
$= B(5;15,0.3) - B(2;15,0.3)$
$= 0.595$

48) a) $P(X \le 2) = B(2; 25, 0.05) = 0.873$

b) $P(X \geq 5)$
$= 1 - P(X \le 4)$
$= 1 - B(4; 25, 0.05)$
$= 0.1 - 0.993$
$= 0.007$

c) $P(1 \le X \le 4)$
$= P(X \le 4) - P(X \le 0)$
$= 0.993 - 0.277$
$= 0.716$

d) $P(X = 0)$
$= P(X \le 0)$
$= 0.277$

e) $E(X) = np$
$= (25)(0.05)$
$= \sqrt{np(1-p)}$
$= \sqrt{25(0.05)(0.95)}$
$= 1.09$

54 a) $P(X \geq 6)$ $\quad X \sim B(10, 0.6)$
$= 1 - P(X \le 5)$
$= 1 - 0.367$
$= 0.633$

b) $np = 10(0.6) = 6$ $\quad \sigma = \sqrt{10(0.6)(0.4)} = 1.55$ $\quad \mu \pm \sigma = (4.45, 7.55)$
$P(4.45 < X < 7.55)$
$= P(5 \le X \le 7)$
$= P(X \le 7) - P(X \le 4)$
$= 0.833 - 0.166$
$= 0.667$

54) c) $P(3 \leq X \leq 7)$

$= P(X \leq 7) - P(X \leq 2)$

$= 0.833 - 0.012$

$= 0.821$

68) a) $N = 20$, $M = 12$, $n = 6$, X is geometric

b) $P(X=2) = \frac{\binom{12}{2}\binom{20-12}{6-2}}{\binom{20}{6}} = \frac{\binom{12}{2}\binom{8}{4}}{\binom{20}{6}} = \frac{(66)(70)}{(38760)} = 0.1192$

$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= \frac{\binom{12}{0}\binom{8}{6}}{\binom{20}{6}} + \frac{\binom{12}{1}\binom{8}{5}}{\binom{20}{6}} + 0.1192$

$= 0.1373$

$P(X \geq 2) = 1 - P(X \leq 1)$

$= 1 - [P(X=0) + P(X=1)]$

$= 1 - 0.0007 - 0.0174$

$= 0.9819$

c) $E(X) = n \cdot \frac{M}{N}$

$= 6 \cdot \frac{12}{20}$

$= 6 \cdot (0.6)$

$= 3.6$

$V(X) = \left(\frac{20-6}{20-1}\right) \cdot 6(0.6)(1-0.6)$

$= 1.061$

$\sigma = 1.03$

69) a) X is hypergeometric, $n=6$, $N=12$, $M=7$

$P(X=5)$

$= \dfrac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}}$

$= \dfrac{105}{924}$

$= 0.114$

b) $P(X\leq 4)$

$=1-P(X>4)$

$=1-[P(X=5)+P(X=6)]$

$=1-\dfrac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}}+\dfrac{\binom{7}{6}\binom{5}{0}}{\binom{12}{6}}$

$=1-0.114-0.007$

$=0.879$

c) $E(X)=n\cdot\dfrac{M}{N}$

$=6\cdot\dfrac{7}{12}$

$=3.5$

$V(X)=\left(\dfrac{12-6}{12-1}\right)\cdot 6\cdot\left(\dfrac{7}{12}\right)\left(1-\dfrac{7}{12}\right)$

$=0.795$

$\sigma=0.892$

$P(X>\mu+\sigma)=P(X>3.5+0.892)$

$=P(X>4.392)$

$=P(X=5 \mid X=6)$

$=0.121$

69) d) $b(x;15,0.1)$

$B(5;15,0.1) = 0.998$

72) a) $N=11$, $M=4$, $n=6$

$h(x;6,4,11)$

$$= \frac{\binom{4}{x}\binom{7}{6-x}}{\binom{11}{6}}$$

b) $E(X) = n \cdot \frac{M}{N} = 6 \cdot \frac{4}{11} = 2.18$

75) a) $S=$ female $F=$ male $X=$ number of male before second female

$P(X=x) = nb(x;2,0.5)$

$$= \binom{x+2-1}{2-1}(0.5)^2(1-0.5)^x$$

$$= (x+1)(0.5)^{x+2}$$

b) $P(\text{exactly 4 children}) = P(X=2)$

$= nb(2;2,0.5)$

$= (2+1)(0.5)^4$

$= 0.188$

c) $P(\text{at most 4 children}) = P(X \leq 2)$

$$= \sum_{x=0}^{2} nb(x;2,0.5)$$

$= 0.25 + 0.25 + 0.188$

$= 0.688$

d) $E(X) = \frac{r(1-p)}{p} = \frac{2(1-0.5)}{0.5} = 2$

$E(X+2) = 4$

3.6) 79) a) $F(x;5)$

$P(X \le 8) = F(8;5) = 0.932$

b) $P(X=8) = F(8;5) - F(7;5) = 0.065$

c) $P(X \ge 9) = 1 - P(X \le 8) = 0.068$

d) $P(5 \le X \le 8) = F(8;5) - F(4;5) = 0.492$

e) $P(5 < X < 8) = F(7;5) - F(5;5) = 0.867 - 0.616 = 0.251$

84) a) $\sigma = \sqrt{npq} = \sqrt{(1\times10^4)(0.001)(0.999)} = 3.16$

b) $\mu = 10$

$P(X > 10) = 1 - F(10;10) = 1 - 0.583 = 0.417$

c) $P(X=0) = \frac{e^{-10}10^0}{0!} = e^{-10} = 454\times10^{7}$

86) a) $P(X=4) = \frac{e^{-5}\cdot5^4}{4!} = 0.175$

b) $P(X \ge 4) = 1 - P(X \le 3)$

$= 1 - F(3;5)$

$= 1 - 0.265$

$= 0.735$

c) Arrive at rate of $5/h$, $\mu = (5)(0.75) = 3.75$

87) a) $\lambda t = 4 \cdot 2 = 8$

$P(X=10) = F(10;8) - F(9;8)$

$= 0.099$

b) $\lambda t = 4(0.5) = 2$

$P(X=0) = F(0,2) = 0.135$

c) $E(X) = \lambda t = 2$