Date 概率统计作业 22CST [name]

Section 5.3 5.4 5.5

Ex. 38

a) pmf:

| $T_0$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $p(T_0)$ | 0.04 | 0.2 | 0.37 | 0.3 | 0.09 |

b) $E(T_0) = \mu_{T_0} = 0\times0.04 + 1\times0.2 + \cdots + 4\times0.09 = 2.2 = 2\mu$

c) $\sigma_{T_0}^2 = E(T_0^2) - \mu_{T_0}^2$ , $E(T_0^2) = 0^2\times0.04 + 1^2\times\cancel{0.02}^{0.2} + \cdots + 4^2\times0.09 = 5.82$

$\therefore \sigma_{T_0}^2 = 5.82 - 2.2^2 = 0.98$

d) Now: $E(T_0) = 4\times\mu = 4.4$ , $V(T_0) = 4\sigma^2 = 4\times0.49 = 1.96$

e) $p(T_0=8) = (0.3)^4 = 0.0081$

$p(T_0\geq7) = p(T_0=7) + p(T_0=8) = (0.3)^3\times0.5\times4 + 0.0081 = 0.0621$

Ex. 41

a)

| $\bar{x}$ | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 |
|---|---|---|---|---|---|---|---|
| $p(\bar{x})$ | 0.16 | 0.24 | 0.25 | 0.2 | 0.1 | 0.04 | 0.01 |

b) $p(\bar{X}\leq2.5) = 0.16 + 0.24 + 0.25 + 0.2 = 0.85$

c)

| R | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| P(R) | 0.3 | 0.4 | 0.22 | 0.08 |

d) $p(\bar{x}\leq1.5) = (0.4)^4 + C_4^2\cdot(0.4)^2(0.3)^2 + C_4^1\cdot(0.4)^3(0.2)$

$= 0.0256 + 6\times0.16\times0.09 + 4\times0.064\times0.2$

$= 0.1632$

Soution 5.4

Ex. 46

a) Center at $\mu=12$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{16}} = 0.01$ cm

b) Still center at $\mu=12$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{64}} = 0.005$ cm

c) part(b)'s $\bar{x}$ is more likely to be within 0.01cm of 12cm.

The reason is that: The larger sample size is, the ~~more normal~~ will be normaler (concentrated)

Ex.51.

The first day: $P(\bar{X} \le 11) = P(\bar{X} \le \frac{11-10}{2/\sqrt{5}}) = P(\bar{X} \le 1.12) = 0.8686$

The second day: $P(\bar{X} \le 11) = P(\bar{X} \le \frac{11-10}{2/\sqrt{6}}) = P(\bar{X} \le 1.22) = 0.8888$

So: $P = 0.8686 \times 0.8888 = 0.772$

Ex.55

a) As it is Poisson distribution: so: $\sigma^2 = \mu = 50$

~~It~~ $\mu = 50$ is large enough, so we can use normal ~ to do it.

$P(35 \le X \le 70) = P(\frac{35-50}{\sqrt{50}} \le X \le \frac{70-50}{\sqrt{50}}) = P(-2.12 \le X \le 2.83)$

$= P(2.83) - P(-2.12)$

$= 0.9977 - 0.017$

$= 0.9807$

b) $E(T_0) = 50 \times 5 = 250$, $V(T_0) = n\sigma^2 = 250$ $\sigma_{T_0} = \sqrt{250} = 15.81$

~~P~~ $P(225 \le T_0 \le 275) = P(\frac{225-250}{15.81} \le T_0 \le \frac{275-250}{15.81})$

$= P(1.58) - P(-1.58)$

$=$ ~~P(1.58)~~ $0.9429 - 0.0571$

$= 0.8858$

Section 5.5

Ex.58.

a) $E(\text{Volume}) = 27 \times 200 + 125 \times 250 + 512 \times 100 = 87850$

$V(\text{Volume}) = 27^2 \times (10)^2 + 125^2 \times (12)^2 + 512^2 (8)^2 = 19100116$

b) The expected value is still correct, but variance not, because the correlation will influence the ~~finall~~ final result.

Ex.70.

a) $E(Y_i)=0.5$, $E(W)=\sum_{i=1}^{n} i\,\cancel{Y_i}\,E(Y_i)=\dfrac{n(n+1)}{4}$

b) $V(Y_i)=0.5(1-0.5)=0.25$, $V(W)=\sum_{i=1}^{n} i^2 V(Y_i)=\dfrac{n(n+1)(2n+1)}{24}$

Ex.73

a) Normal distribution (CLT Theorem)

b) still normal distribution, linear combination will not break it if it is normal before.

c) $\mu=105-100=5$, $\sigma=\sqrt{\frac{64}{40}+\frac{36}{35}}=1.62$

$P(-1\le \bar{x}-\bar{y}\le 1)=P\left(\frac{-1-5}{1.62}\le \bar{x}-\bar{y}\le \frac{1-5}{1.62}\right)=P(-3.7\le \bar{x}-\bar{y}\le -2.47)=0.0068$

d) $P(\bar{x}-\bar{y}\geq 10)=P\left(\bar{x}-\bar{y}\geq \frac{10-5}{1.62}\right)=0.001$, too small!

we will doubt whether $\mu=5$