Section 5.1

Ex. 56.

$A^{n+1} = A \cdot A^n = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \cdot \begin{bmatrix} a^n & 0 \\ 0 & b^n \end{bmatrix} = \begin{bmatrix} a^{n+1} & 0 \\ 0 & b^{n+1} \end{bmatrix}$

So we give the proof that $A^n = \begin{bmatrix} a^n & 0 \\ 0 & b^n \end{bmatrix}$

Section 5.2

Ex. 12.

Basis step: $1=2^0$, $2=2^1$, $3=2^0+2^1$, $4=2^2$, $5=2^0+2^2$, $6=2^1+2^2$ ......

Inductive step: The inductive ~~p~~hypothesis is k can be written as a sum of distinct powers of two.

When $k+1$ is even, ~~is~~ it is clear that $k+1$ can be written as a sum of distinct powers of two without $2^0=1$ because 1 is odd.

When $k+1$ is odd, we can know k is even, so we can add 2^0 to k and then add some 2^x to construct $k+1$.

Therefore, by strong ~~co~~induction, we have proved that every integer n can be written as a sum of ~~a subset~~ distinct powers of two.

Section 5.3

Ex. 26.

a) ~~(2,3) ∈ S, (3,2) ∈ S, (4,6)~~
(10,15), (11,14), (12,13), ~~(14~~, (13,12), (14,11), (15,10)

b) Basis step: when $a=b=0$, $5 \mid 0$ holds $\rightarrow P(0)$

Inductive step: The inductive hypothesis is ~~p(n) can~~ $5 \mid a+b$ at n's applications of the recursive definition $\rightarrow P(n)$

we can know $P(n+1)$ is $5 \mid a+3+b+2$ or $5 \mid a+2+b+3$, and they are equal to $5 \mid a+b+5$, since $P(n)$ holds, so $5 \mid a+b+5$ holds

Therefore, we prove it by strong induction.

c)

Basis step: $5 \mid 0+0$

recursive step: suppose that $P(n)$ holds

then we know $P(n+1) = 5 \mid a+3+b+2 = 5 \mid a+b+5$ ~~ab~~ is clearly holds

so we prove it by structural induction