

Sec2Ex

46.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

I think  $P(A)$  the percentage of male American over 6ft selected randomly is higher than  $P(B)$  that the ratio of basketball player. Thus I think  $P(A|B) > P(B|A)$ .

50.

a. .05

b. Ls:Long-sleeved Ss::Short-sleeved

$$P(Ls) + P(Ss) = 1$$

$$P(Ls \cap Ss) = \phi$$

$$\Rightarrow P(M \cap Pr)$$

$$= P(M \cap Pr | Ls)P(Ls) + P(M \cap Pr | Ss)P(Ss)$$

$$= P(M \cap Pr \cap Ls) + P(M \cap Pr \cap Ss)$$

$$= .05 + .07 = .12$$

c.

(S,M,L) and (Pl,Pr,St) the two groups on there own both are mutually independent and exhaustive

$$P(Ss) = \sum P(Ss \cap Size \cap Pattern)$$

$$= .04 + .02 + .05 + .08 + .07 + .12 + .03 + .07 + .08$$

$$= .56$$

$P(Ls) = 1 - P(Ss) = .44$  for Ls and Ss are independent and exhaustive

d.

$$P(M) = .08 + .07 + .12 + .10 + .05 + .07 = .49$$

$$P(Pr) = .02 + .07 + .07 + .02 + .05 + .02 = .25$$

e.

$$P(M | Ss \cap Pl) = \frac{P(Ss \cap Pl \cap M)}{P(Ss \cap Pl)}$$

$$= \frac{.08}{.04 + .08 + .03} = \frac{8}{15}$$

f.

$$P(Ss | M \cap Pl) = \frac{P(Ss \cap M \cap Pl)}{P(M \cap Pl)} = \frac{.08}{.08 + .10} = \frac{4}{9}$$

$$P(Ls | M \cap Pl) = 1 - \frac{4}{9} = \frac{5}{9}$$

58.

$$\begin{aligned} & P(A \cup B | C) \\ &= \frac{P((A \cap C) \cup (B \cap C))}{P(C)} \\ &= \frac{P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)}{P(C)} \\ &= P(A | C) + P(B | C) - P(A \cap B | C) \end{aligned}$$

Q.E.D.

63.

a.

b.

$$\begin{aligned} & P(A \cap B \cap C) \\ &= P(C | A \cap B)P(A \cap B) \\ &= P(C | A \cap B)P(B | A)P(A) \\ &= .8 \times .9 \times .75 = .54 \end{aligned}$$

c.

$$\begin{aligned} & P(B \cap C) \\ &= P(B \cap C | A)P(A) + P(B \cap C | A')P(A') \\ &= P(A \cap B \cap C) + P(A' \cap B \cap C) \\ &= .8 \times .9 \times .75 + .7 \times .8 \times .25 = .68 \end{aligned}$$

d.

$$\begin{aligned} & P(C) \\ &= P(C | A \cap B)P(A \cap B) + P(C | A' \cap B)P(A' \cap B) \\ &\quad + P(C | A \cap B')P(A \cap B') + P(C | A' \cap B')P(A' \cap B') \\ &= .8 \times .9 \times .75 + .7 \times .8 \times .25 + .6 \times .1 \times .75 + .3 \times .2 \times .25 \\ &= .74 \end{aligned}$$

e.

$$P(A | B \cap C) = \frac{P(A \cap B \cap C)}{P(B \cap C)} = \frac{.54}{.68} = \frac{27}{34}$$

71.

a.

$$\begin{aligned}
 &P(B'|A') \\
 &= \frac{P(A' \cap B')}{P(A')} \\
 &= \frac{P(A') + P(B') - P(A' \cup B')}{P(A')} \\
 &= \frac{1 - P(A) + 1 - P(B) - (1 - P(A \cap B))}{1 - P(A)} \\
 &= \frac{1 - P(A) + 1 - P(B) - (1 - P(A|B)P(B))}{1 - P(A)} \\
 &= \frac{1 - P(A) + 1 - P(B) - (1 - P(A)P(B))}{1 - P(A)} \\
 &= \frac{.6 + .3 - (1 - .4 \times .7)}{.6} = .3
 \end{aligned}$$

$B^+$

b.

$$\begin{aligned}
 P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\
 &= P(A) + P(B) - P(A|B)P(B) \\
 &= P(A) + P(B) - P(A)P(B) \\
 &= .4 + .7 - .4 \times .7 = 0.82
 \end{aligned}$$

c.

$$\begin{aligned}
 P(A|A \cup B) &= \frac{P(A \cap (A \cup B))}{P(A \cup B)} \\
 &= \frac{P(A)}{P(A \cup B)} = \frac{.4}{.82} = \frac{20}{41}
 \end{aligned}$$

72.

$$P(A_2 | A_3) = \frac{P(A_2 \cap A_3)}{P(A_2)} = \frac{.07}{.28} = .25 = P(A_2)$$

which indicates  $A_2$  and  $A_3$  are independent.

Other pairs are dependant.

80.

denote 1,2,3,4 each of the components working well as A,B,C,D

$$\begin{aligned}
 &P(A \cup B \cup (C \cap D)) \\
 &= P(A \cup B) + P(C \cap D) - P((A \cup B) \cap C \cap D) \\
 &= P(A \cup B) + P(C \cap D) - P(A \cap C \cap D) - P(B \cap C \cap D) + P(A \cap B \cap C \cap D) \\
 &= .9 + .9 - .9^2 + .9^2 - .9^3 \times 2 + .9^4 = 0.9981
 \end{aligned}$$

84.

denote  $i$ th car passes as  $A_i$

a.  $P(\bigcap_{i=1}^3 A_i) = \prod_{i=1}^3 P(A_i) = .345$

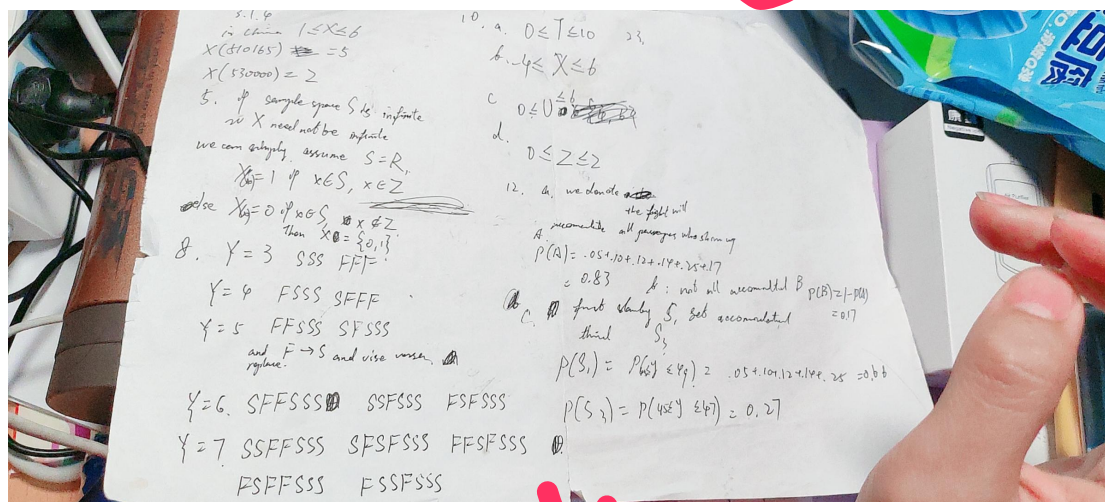
b.  $P((\bigcap_{i=1}^3 A_i)') = 1 - P(\bigcap_{i=1}^3 A_i) = 1 - 0.345 = 0.655$

c.  $P((A_1 \cap A_2 \cap A_3) \cup (A_1 \cap A_2 \cap A_3') \cup (A_1 \cap A_2 \cap A_3'))$   
 $= .3 \times .7 \times .7 \times 3 = 0.441$

d.  $P((A_1 \cap A_2 \cap A_3') \cup (A_1 \cap A_2 \cap A_3) \cup (A_1 \cap A_2 \cap A_3') \cup (A_1 \cap A_2 \cap A_3'))$   
 $= .3^3 + .3^2 \times .7 \times 3 = .216$

denote this accident as D

e.  $P(\bigcap_{i=1}^3 A_i | D') = P(\bigcap_{i=1}^3 A_i) / P(D') = 345 / 559$



$23. a. P(X=2) = 0.39 - 0.19 = 0.2$   
 $b. P(X \geq 3) = 1 - P(X \leq 3) = 1 - 0.67 = 0.33$   
 $c. P(2 \leq X \leq 5) = 0.97 - 0.19 = 0.78$   
 $d. P(2 < X < 5) = 0.92 - 0.39 = 0.53$

25.  ~~$Y \geq 0$~~

$P(Y=0) = p$

$P(Y=1) = (1-p)p$

$\begin{cases} P(Y=x) = (1-p)^x p, & x \in \mathbb{N}^* \\ 0 & \text{otherwise.} \end{cases}$   
 which is the pmf of  $Y$ .