概率统计作业 $12^{th}$ 22CST 蒋祥[illegible]

## Chapter 6

### Section 6.1.

**Ex. 1**

(a) Use mean value: $\bar{X} = \frac{\sum x_i}{n} = \frac{219.8}{27} = 8.14$

(b) Use median value: $\tilde{X} = 7.7$

(c) Use: $\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{\sum (x_i^2 - 2x_i\bar{x} + \bar{x}^2)}{n-1} = \frac{1860.94 - 2\times 8.14 \times 219.8 + 27 \times 8.14^2}{27-1} = 2.75$

$\sigma = \sqrt{2.75} = 1.66$

(d) Use: $\frac{4}{27} = 0.148$

(e) Use: $\frac{\sigma}{\bar{X}} = \frac{1.66}{8.14} = 0.204$

**Ex. (8)**

(a) $1 - \frac{12}{80} = 0.85$

(b) It is clear that: $p = 0.85$

so: the answer is: $p^2 = 0.85^2 = 0.7225$

**Ex. (9)**

(a) $E(\bar{X}) = \mu_{\bar{X}} = \mu = E(X) = \frac{0\times 18 + 1\times 37 + 2\times 42 + \cdots + 7\times 1}{150} = 2.11$

(b) ~~As for poisson distribution~~

(b): $\sigma_X^2 = \mu$ so: $\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{\sqrt{2.11}}{\sqrt{150}} = 0.119$

**Ex. (13)**

first calculate $\mu = E(X) = E(\bar{X}) = \int_{-1}^{1} x f(x)\,dx = \int_{-1}^{1} x \cdot (0.5(1+\theta x))\,dx = \frac{\theta}{3}$

so: $\hat{\theta} = 3\bar{X}$, $E(\hat{\theta}) = E(3\bar{X}) = 3E(\bar{X}) = 3 \times \frac{\theta}{3} = \theta \Rightarrow$ unbiased estimator

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Section 6.2

Ex: 20

(a.) The example satisfy binomial distribution: so: $f(p) = p^3(1-p)^{17}$

Then: $\ln f(p) = 3\ln p + 17\ln(1-p)$

$(\ln f(p))' = \frac{3}{p} - \frac{17}{1-p}$

let: $\frac{3}{p} - \frac{17}{1-p} = 0$ then: $p = \frac{3}{20} = \frac{x}{n}$

(b) $E(\hat{p}) = E\left(\frac{X}{n}\right) = \frac{1}{n}E(X) = \frac{1}{n}\cdot np = p$

so: $\hat{p}$ is unbiased

(c) $(1-\hat{p})^5 = (0.85)^5 = 0.444$

Ex. (21)

(a) We use moment method can derive the ~~ans~~ answer.

(b) $E(X) = \beta\cdot\Gamma(1+1/\alpha)$

$E(X^2) = V(X) + E(X)^2 = \beta^2(\Gamma(1+2/\alpha) - [\Gamma(1+1/\alpha)]^2) + \beta\cdot\Gamma(1+1/\alpha)$

Then: $\frac{\bar{X}^2}{\frac{1}{n}\cdot\Sigma(X_i^2)} = \frac{28^2}{825} = 0.95 = \frac{\Gamma(1.2)^2}{\Gamma(1.4)} = \frac{\Gamma(1+1/\alpha)^2}{\Gamma(1+2/\alpha)} \Rightarrow \alpha = 5$

then: $\beta = \frac{28}{\Gamma(1.2)}$

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Ex. 29.

(a) According to the problem: $\hat{\theta} = \min(X_i)$

$f(x_1 x_2 \cdots x_n) = \lambda e^{-\lambda(x_1-\theta)} \cdot \lambda e^{-\lambda(x_2-\theta)} \cdots \lambda e^{-\lambda(x_n-\theta)}$

$= \lambda^n e^{-\lambda(\Sigma x_i - n\theta)}$

differentiate $f(x_1 x_2 \cdots x_n)$ w.r.t $\lambda$: $\ln f(x_1 x_2 \cdots x_n) = n \ln\lambda - \lambda(\Sigma x_i - n\theta)$

$(\ln f(x_1 x_2 \cdots x_n))' = \frac{n}{\lambda} - \Sigma x_i + n\theta = 0$

so: $\lambda = \frac{n}{\Sigma x_i - n\theta} = \frac{n}{\Sigma x_i - n \cdot \min(x_i)}$

(2) $\hat{\theta} = 0.64$

$\hat{\lambda} = \frac{10}{\Sigma x_i - 10 \times 0.64} = 0.202$

Ex. 32.

(a) $P(Y \le y) = P(X_1 \le y, X_2 \le y, \cdots X_n \le y)$

$= \frac{y}{\theta} \cdot \frac{y}{\theta} \cdots \frac{y}{\theta}$

$= (\frac{y}{\theta})^n$

so: ~~f(y)~~ $f_Y(y) = \frac{d(\frac{y}{\theta})^n}{dy} = \frac{n y^{n-1}}{\theta^n}$

(b) $E(\hat{\theta}) = \int_0^{\theta} y f_Y(y)\, dy = \frac{n}{n+1} \max(X_i)$ $(\hat{\theta} = \max(X_i))$

So: $E(\frac{n+1}{n}\hat{\theta}) = \frac{n+1}{n} E(\hat{\theta}) = \theta$ is unbiased.