A+

$\frac{219.8}{27} = 8.1407$

$\bar{x} = 7.7$

c. $s = \sqrt{s^2} = \sqrt{\frac{1860.94 - \frac{(219.8)^2}{27}}{26}} = 1.660$

d. $\hat{p} = \frac{x}{n} = \frac{4}{27} = 0.1481$

e. $\frac{s}{\bar{x}} = \frac{1.660}{8.1407} = 0.2039$

8. a. $\hat{p} = \frac{80-12}{80} = 0.85$

b. $\hat{p}^2 = (0.85)^2 = 0.723$

9. a. $n = 150$

$$\hat{\mu} = \bar{x} = \frac{\sum x_i}{n} = \frac{(0)(18)+(1)(37)+(2)(42)+(3)(30)+(4)(13)+(5)(7)+(6)(2)+(7)(1)}{150}$$

$= 2.11$

b. $s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{\sqrt{\mu}}{\sqrt{n}} = \frac{\sqrt{2.11}}{\sqrt{150}} = 0.119$

13. $\mu = E(X) = \int_{-1}^{1} \frac{x}{2}(1+\theta x)\,dx = \left[\frac{x^2}{4} + \frac{\theta x^3}{6}\right]_{-1}^{1} = \frac{1}{3}\theta \Rightarrow \theta = 3\mu$

$\Rightarrow \hat{\theta} = 3\bar{x} \Rightarrow E(\hat{\theta}) = E(3\bar{x}) = 3E(\bar{x}) = 3\mu = 3\cdot\frac{1}{3}\theta = \theta$

6.2. 20. a. $l = \ln\left[\binom{n}{x} p^x (1-p)^{n-x}\right] = \ln\binom{n}{x} + x\ln p + (n-x)\ln(1-p)$

$l' = \frac{d}{dp}\left[\ln\binom{n}{x} + x\ln(p) + (n-x)\ln(1-p)\right] = \frac{x}{p} - \frac{n-x}{1-p} = 0$

$\Rightarrow x(1-p) = p(n-x)$

$\Rightarrow p = \frac{x}{n}$

$= \frac{3}{20}$

$= 0.15$

b. It is an unbiased estimator of p.

c. $(1-p)^5 = (1-\hat{p})^5 = (1-0.15)^5 = 0.4437$

21. $E(X) = \beta\Gamma(1+\frac{1}{\alpha})$ $\qquad E(X^2) = V(X) + [E(X)]^2 = \beta^2\Gamma(1+\frac{2}{\alpha})$

$\bar{x} = \hat{\beta}\Gamma(1+\frac{1}{\hat{\alpha}})$ $\qquad \frac{1}{n}\sum x_i^2 = \hat{\beta}^2\Gamma(1+\frac{2}{\hat{\alpha}})$

$\hat{\beta} = \frac{\bar{x}}{\Gamma(1+\frac{1}{\hat{\alpha}})}$ $\qquad \bar{x}^2 = \hat{\beta}^2\cdot\Gamma^2(1+\frac{1}{\hat{\alpha}})$

$\frac{1}{n}\sum\frac{x_i^2}{\bar{x}^2} = \frac{\Gamma(1+\frac{2}{\hat{\alpha}})}{\Gamma^2(1+\frac{1}{\hat{\alpha}})}$

$\frac{1}{20}\left(\frac{16500}{28^2}\right) = 1.05 = \frac{\Gamma(1+\frac{2}{\hat{\alpha}})}{\Gamma^2(1+\frac{1}{\hat{\alpha}})}$ $\qquad \frac{\Gamma^2(1+\frac{1}{\hat{\alpha}})}{\Gamma(1+\frac{2}{\hat{\alpha}})} = 0.95$

$\frac{1}{\hat{\alpha}} = 0.2 \Rightarrow \hat{\alpha} = 5$ $\qquad \hat{\beta} = \frac{\bar{x}}{\Gamma(1.2)} = \frac{28}{\Gamma(1.2)}$

The joint pdf (likehood function):

$$f(x_1, \dots, x_n; \lambda, \theta) = \begin{cases} \lambda^n e^{-\lambda \Sigma (x_i - \theta)} & x_1 \geq \theta, \dots, x_n \geq \theta \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Likehood} = \begin{cases} \lambda^n \exp(-\lambda \Sigma x_i) \exp(n \lambda \theta) & \min(x_i) \geq \theta \\ 0 & \min(x_i) < \theta \end{cases}$$

$$\hat{\theta} = \min(x_i)$$

The log likehood $= n \ln(\lambda) - \lambda \Sigma(x_i - \hat{\theta})$

$$\hat{\lambda} = \frac{n}{\Sigma(x_i - \hat{\theta})} = \frac{n}{\Sigma x_i - n\hat{\theta}}$$

b. $\hat{\theta} = 0.64$ $\quad \Sigma x_i = 55.8$ $\quad \hat{\lambda} = \frac{10}{55.8 - 6.4} = 0.202$

32. a. $F_Y(y) = P(Y \leq y) = P(X_1 \leq y, \dots, X_n \leq y) = P(X_1 \leq y) \dots P(X_n \leq y)$

$$= \left(\frac{y}{\theta}\right)^n \quad \text{for } 0 \leq y \leq \theta$$

$$f_Y(y) = \frac{ny^{n-1}}{\theta^n}$$

b. $E(Y) = \int_0^\theta y \cdot \frac{ny^{n-1}}{\theta^n} dy = \frac{n}{n+1}\theta$

$\hat{\theta} = Y$ is not unbiased, $\frac{n+1}{n} Y$ is since

$$E\left(\frac{n+1}{n} Y\right) = \frac{n+1}{n} E(Y) = \frac{n+1}{n} \cdot \frac{n}{n+1} \theta = \theta$$