B+

46. a. $b(3; 8, 0.35) = \binom{8}{3}(0.35)^3(1-0.35)^{8-3} = 0.2786$

b. $b(5; 8, 0.6) = \binom{8}{5}(0.6)^5(1-0.6)^{8-5} = 0.2787$

c. $P(3 \le x \le 5)$ when $n=7$ and $p=0.6$

$= b(3;7,0.6) + b(4;7,0.6) + b(5;7,0.6)$

$= 0.1935 + 0.2903 + 0.2613$

$= 0.7451$

d. $P(1 \le x)$ when $n=9$ and $p=0.1$

$= 1 - P(X=0)$

$= 1 - \binom{9}{0}(0.1)^3(1-0.1)^{9-0}$

$= 1 - 0.3874$

$= 0.6126$

47. a. $B(4; 15, 0.3) = 0.515$

b. $b(4; 15, 0.3) = 0.515 - 0.297 = 0.218$

c. $b(6; 15, 0.7) = 0.15 - 0.04 = 0.11$

d. $P(2 \le x \le 4)$ when $x \sim Bin(15, 0.3) = 0.515 - 0.35 = 0.480$

e. $P(2 \le x)$ when $x \sim Bin(15, 0.3) = 1 - 0.35 = 0.965$

f. $P(x \le 1)$ when $x \sim Bin(15, 0.7) = 0$

g. $P(2 < x < 6)$ when $x \sim Bin(15, 0.3) = 0.722 - 0.127 = 0.595$

48. $x \sim Bin(25, 0.05)$

a. $P(x \le 2) = 0.873$

b. $P(X \ge 5) = 1 - 0.993 = 0.07$

c. $P(1 \le x \le 4) = 0.993 - 0.277 = 0.716$

d. $P(x = 0) = 0.277$

e. $E(X) = np = 25 \cdot 0.05 = 1.25$

$\delta = \sqrt{npq} = \sqrt{25 \cdot 0.05 \cdot 0.95} = 1.09$

54. $x \sim Bin(10, 0.6)$

a. $P(x \ge 6) = 1 - 0.367 = 0.633$

b. $E(x) = 10 \cdot 0.6 = 0.6$

$\delta = \sqrt{10 \cdot 0.6 \cdot 0.4} = 1.5492$

$E(x) - \delta \le x \le E(x) + \delta$

$\hookrightarrow 6 - 1.5492 \le x \le 6 + 1.5492$

$P(4.4508 \le x \le 7.5492) = P(5 \le x \le 7)$

$= 0.833 - 0.166$

$= 0.667$

C?

Hypergeometric distribution

$n=6$, $M=12$, $N=20$

b. $P(X=2) = h(2;6,12,20) = \dfrac{\binom{12}{2}\binom{20-12}{6-2}}{\binom{20}{6}} = \dfrac{66 \cdot 70}{38760} = 0.12$

$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$
$= 0.0007 + 0.0173 + 0.12$
$= 0.138$

$P(X \ge 2) = 1 - P(X=0) - P(X=1)$
$= 1 - 0.018$
$= 0.982$

c. $E(X) = n \cdot \frac{M}{N} = 6 \cdot \frac{12}{20} = 3.6$

$\sigma = \sqrt{V(X)} = \sqrt{\left(\frac{N-n}{N-1}\right) \cdot n \cdot \frac{M}{N} \cdot \left(1-\frac{M}{N}\right)}$
$= \sqrt{\left(\frac{20-6}{20-1}\right) \cdot 6 \cdot \frac{12}{20} \cdot \left(1-\frac{12}{20}\right)}$
$= 1.03$

69. $n=6$, $M=7$, $N=12$

a. $P(X=5) = \dfrac{\binom{7}{5}\binom{12-7}{6-5}}{\binom{12}{6}} = 0.1136$

b. $P(X \le 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$
$= \frac{1}{132} + \frac{5}{44} + \frac{25}{66} + \frac{25}{66}$
$= 0.8788$

c. $E(X) = 6 \cdot \frac{7}{12} = 3.5$

$\delta = \sqrt{\left(\frac{12-6}{12-1}\right) \cdot 6 \cdot \frac{7}{12} \cdot \left(1-\frac{7}{12}\right)} = 0.8919$

$P(X \ge 4.3919) = P(X \ge 5) = 1 - 0.8788 = 0.1212$

d. $n = 15$ $\quad p = \frac{40}{400} = 0.1$
$P(X \le 5) = 0.998$

72. $n=4$, $M=6$, $N=11$

a. $h(x;4,6,11) = P(X=x) = \dfrac{\binom{6}{x}\binom{11-6}{4-x}}{\binom{11}{4}}$

$P(X=0) = 0.0152$
$P(X=1) = 0.1819$
$P(X=2) = 0.4546$
$P(X=3) = 0.3030$
$P(X=4) = 0.0455$

b. $E(X) = 4 \cdot \frac{6}{11} = 2.18$

We can expect two of the top four candidates to be interviewed on the first day.

$= 2$

$, 0.5) = \binom{x+2-1}{2-1} 0.5^2 (1-0.5)^x$

$= 0.25(x+1)(1-0.5)^x$

b. $P(x=4) = 0.25(4+1)(1-0.5)^4 = 0.0781$

c. $P(x \le 4) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$

$= 0.25 + 0.25 + 0.1875 + 0.125 + 0.0781$

$= 0.8906$

d. $E(x) = \dfrac{r(1-p)}{p} = \dfrac{2 \cdot 0.5}{0.5} = 2$

We expect the family to have 2 male children. Since the family needs to have 2 female children, we expect the family to have $2+2=4$ children in total.

79. a. $P(x \le 8) = 0.932$

b. $P(x=8) = 0.932 - 0.867 = 0.065$

c. $P(9 \le x) = 1 - 0.932 = 0.068$

d. $P(5 \le x \le 8) = 0.932 - 0.440 = 0.492$

e. $P(5 < x < 8) = 0.867 - 0.616 = 0.251$

84. a. $E(x) = 10000 \times 0.001 = 10$

$\delta = \sqrt{\lambda} = \sqrt{10} = 3.162$

b. $P(x > 10) = 1 - P(x \le 10) = 1 - 0.583 = 0.417$

c. $P(x=0) = 0.000045$

86. $\lambda = 5$

a. $P(x=4) = \dfrac{5^4 e^{-5}}{4!} = 0.1755$

b. $P(x \ge 4) = 1 - P(x \le 3)$

$= 1 - 0.265$

$= 0.735$

c. $5 \cdot \dfrac{45}{60} = 3.75$

87. a. $\lambda = \alpha t = 4 \cdot 2 = 8$

$P(x=10) = \dfrac{8^{10} e^{-8}}{10!} = 0.099$

b. $\lambda = 4 \cdot \frac{1}{2} = 2$

$P(x=0) = \dfrac{2^0 e^{-2}}{0!} = 0.1353$

c. $E(x) = 4 \cdot \frac{1}{2} = 2$