6.1 (1, 8, 9, 13)

B

1.

a. estimator $= \bar{X}$, estimate $= \bar{x} = \frac{\Sigma x_i}{n} = \frac{219.8}{27} = 8.14$

b. estimator $= \bar{X}_{tr}$ (the strongest 50%), estimate $= \bar{x} = \frac{\Sigma x_i}{n} = \frac{122.3}{13} = 9.407$

c. estimator $= \sigma$, estimate $= \sigma = \sqrt{\sigma} = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n-1}} = 1.65$

d. estimator $= P$, estimate $= P = \frac{4}{27} = 0.148$

e. estimator $= \frac{\sigma}{\bar{x}}$, estimate $= \frac{1.65}{8.14} = 0.202$

8.

a. ~~$\bar{x}$ estimator:~~

$E(\hat{p}) = p = \frac{68}{80} = \frac{17}{20} = 0.85$

b. $P(\text{system work}) = p \cdot \frac{np-1}{n-1} = \frac{272}{380} = 0.716$

9.

a. $E(X)=\dfrac{0\times18+1\times37+2\times42+3\times30+4\times13+5\times7+6\times2+7\times1}{150}$

$=2.11=\mu$

$E(\bar{X})=\mu=2.11$

b.

$\sigma^2=\mu=2.11$

$\sigma_{\bar{x}}=\sqrt{\dfrac{\sigma^2}{n}}=0.119$

$\sigma_{\hat{\theta}}=\sigma_{\bar{x}}=0.119$

13.

$E(x)=\sum x_i f(x;\theta)$

$E(\bar{x})=\sum p\cdot x_i=\sum x_i f(x;\theta)$

$=E(X)$

$\hat{\theta}=3\bar{x}$

$E(\hat{\theta})=3E(\bar{x})=\theta$

b 2 (20, 21), 29, 32)

20.

a. $\hat{p} = \frac{X}{N} = \frac{3}{20} = 0.15$

b. $\hat{p} = \frac{X}{N}$, $E(\hat{p}) = E(\frac{X}{N}) = \frac{1}{n}E(X)$

$E(X) = np$

$E(\hat{p}) = p$

so $\hat{p} = \frac{X}{N}$ is an unbaised estimator

c. $\hat{p} = \frac{3}{20}$

$P(A) = (1-0.15)^5 = 0.4437$

21.

$E(X) = b\,\Gamma(1+\frac{1}{a})$

$V(X) = b^2\{\Gamma(1+\frac{2}{a}) - [\Gamma(1+\frac{1}{a})]^2\}$

$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

$S^2 = \frac{1}{n^2}\sum_{i=1}^{n} x_i - (\frac{1}{n}\sum_{i=1}^{n} x_i)^2$

$b = \frac{\bar{x}}{\Gamma(1+\frac{1}{\alpha})}$

$S^2 = (\frac{\bar{x}}{\Gamma(1+\frac{1}{\alpha})})^2 \{\Gamma(1+\frac{2}{a}) - [\Gamma(1+\frac{1}{a})]^2\}$

b. $n=20$, $\bar{x}=28$, $\Sigma x_i = 16900$

$$S^2 = \frac{1}{n}\sum_{i=1}^{n} x_i - \bar{x}^2 = 41$$

$$\Gamma(1+\frac{2}{\alpha}) - [\Gamma(1+\frac{1}{\alpha})]^2 = \frac{41}{\left(\frac{28}{\Gamma(1+\frac{1}{\alpha})}\right)^2}$$

$$\cancel{\Gamma(1+2\alpha) - [\Gamma(1+\alpha)]^2 = \frac{41\Gamma(1+\frac{1}{\alpha})^2}{784}}$$

$$\Gamma(1+2\frac{1}{\alpha}) - [\Gamma(1+\frac{1}{\alpha})]^2 = \frac{41\Gamma(1+\frac{1}{\alpha})}{784}, \quad a=5$$

$$b = \frac{\bar{x}}{\Gamma(1+\frac{1}{\alpha})} = 30.5$$

$$\hat{a} = 5 \qquad \hat{b} = 30.5$$

29.

a. $E(X) = \mu + \frac{1}{\lambda}$

$V(X) = \frac{1}{\lambda^2}$

$$\bar{x} = \mu + \frac{1}{\lambda}, \quad s^2 = \frac{1}{\lambda^2}$$

$$\lambda = \frac{1}{s}$$

So $\mu = \bar{x} - s$

$$\hat{\lambda} = \frac{1}{s}, \quad \hat{\mu} = \bar{x} - s$$

$$\hat{\lambda} = \frac{1}{s} = 0.152$$

$$\mu = \bar{x} - s = 21.44$$

b. $$s^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}\right)$$

$$= \frac{1}{19}\left(16990 - \frac{560^2}{20}\right)$$

$$= 43.15$$

32.

a. $F_Y(y) = P(Y \le y)$

$= (P(X \le y))^n$

$P(X_i \le y) = \frac{y}{\theta}$

$$F_Y(y) = \begin{cases} \left(\frac{y}{\theta}\right)^n & 0 < y \le \theta \\ 0 & \text{otherwise} \end{cases}$$

b. $E(Y) = \int_0^\theta y f_Y(y)\,dy$

$= \frac{n}{n+1}\theta$

$\theta_{unbiased} = \frac{n+1}{n} Y$

$E\left(\frac{n+1}{n} Y\right) = \frac{n+1}{n} E(Y)$

$= \theta$

so $\hat{\theta}$ is an unbiased estimator

MLE $\hat{\theta} = \max(X_i)$ is biased

$\frac{n+1}{n} \max(X_i)$ is unbiased