Section 3.3

A

Ex 29

a. $E(x) = \sum_{x=1}^{16} x p(x) = 1\times0.05 + 2\times0.1 + 4\times0.35 + 8\times0.4 + 16\times0.1 = 6.45$

b. $V(x) = \sum_{x=1}^{16} (x-\mu)^2 p(x) = (1-6.45)^2\times0.05 + (2-6.45)^2\times0.1 + (4-6.45)^2\times0.35 + (8-6.45)^2\times0.4 + (16-6.45)^2\times0.1$
$= 15.6748$

c. $\sigma = \sqrt{\sigma^2} = \sqrt{V(x)} = 3.959$

d. ~~$V(x) = E(x^2) - E(x)^2$~~

$E(x^2) = 1\times0.05 + 4\times0.1 + 16\times0.35 + 64\times0.4 + 256\times0.1 = 57.25$; $E(x)^2 = 41.6025$

$V(x) = E(x^2) - E(x)^2 = 57.25 - 41.60 = 15.65$

Ex 33. a.

X	0	1
$p(x)$	$1-p$	p

$E(x) = p$

$E(x^2) = 1^2\cdot p + 0^2(1-p) = p$

b. $V(x) = E(x^2) - E(x)^2 = p - p^2 = p(1-p)$, by the short cut method.

c. $E(x^n) = 1^n\cdot p + 0^n p = p$.

Ex 38.

X	1	2	3	4	5	6
p	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$E(\frac{1}{X}) = \frac{1}{6}\times(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}) = \frac{49}{120}$

~~assume we roll the die six times, then we get for the first way:~~

~~total $= 3.5\times6 = \frac{1}{7}$~~

~~for the second way:~~ given that the guaranteed amount is $\frac{1}{3.5}$

~~$6\times\frac{1}{3.5}$~~ and $\frac{49}{120} > \frac{1}{3.5}$

So the gamble one is more acceptable.

Ex 41.

$V(ax+b) = V(h(x)) = E(h(x)^2) - E(h(x))^2 = E((ax+b)^2) - E(ax+b)^2$

given that $E((ax+b)^2) = E(a^2x^2 + 2abx + b^2) = a^2E(x^2) + 2ab\,E(x) + b^2$,

and $E(ax+b)^2 = (aE(x)+b)^2 = a^2E(x)^2 + 2abE(x) + b^2$,

So $V(ax+b) = a^2E(x^2) + 2abE(x) + b^2 - [a^2E(x)^2 + 2abE(x) + b^2] = a^2E(x^2) - a^2E(x)^2 = a^2\sigma_x^2$

Section 7.4

Ex 46. a. $p = \binom{8}{3} 0.35^3 (1-0.35)^5 = 0.2786$

b. $p = \binom{8}{5} 0.6^5 (1-0.6)^3 = 0.2787$

c. $P(3 \le X \le 5) = \cancel{P(X\le 5) - P(X \le 2)}\ P(X=3)+P(X=4)+P(X=5) = \cancel{13}$

$\binom{7}{3} 0.6^3 (1-0.6)^4 + \binom{7}{4} 0.6^4 (1-0.6)^3 + \binom{7}{5} 0.6^5 (1-0.6)^2 = 0.1935 + 0.2903 + 0.2613 = 0.7451$

d. $P(1 \le X) = 1 - P(X=0) = 1 - 0.9^8 = 0.6128$ ~~$P(X=0)+P(X=1) = (1-0.1)^8 + \binom{8}{1} 0.1 (1-0.1)^7 = 0.3874 + 0.3874 = 0.7748$~~

Ex 47 a. $P(X \le 4) = 0.515$

b. $P(X=4) = P(X \le 4) - P(X \le 3) = 0.515 - 0.297 = 0.218$

c. $P(X=6) = P(X \le 6) - P(X \le 5) = 0.015 - 0.004 = 0.01$

d. $P(2 \le X \le 4) = P(X \le 4) - P(X \le 1) = 0.515 - 0.127 = 0.388$

e. $P(2 \le X) = P(X \le 15) - P(X \le 1) = 1 - 0.035 = 0.965$

f. $P(X \le 1) = 0.0$.

g. $P(2 < X < 6) = P(2 \le X \le 6) - P(X=2) - P(X=6) = P(X \le 6) - P(X \le 1) - [P(X \le 2) - P(X \le 1)] - [P(X \le 6) - P(X \le 5)]$

$= P(X \le 5) - P(X \le 2) = 0.722 - 0.127$

$= 0.595$

Ex 48.

a. $P(X \le 2) = 0.873$, from the Appendix Tables

b. $P(X \ge 5) = 1 - P(X \le 4) = 1 - 0.993 = 0.007$, from the ~~Addend~~ Appendix Tables.

c. $P(1 \le X \le 4) = \cancel{1 - P(X \ge 5) - P(X=0)}\ P(X \le 4) - P(X=0) = 0.993 - 0.277 = 0.716$, by the Tables

d. $P(X=0) = 0.277$, ~~from the~~ from the Appendix Tables.

e. $E(X) = np = 25 \times 0.05 = 1.25$

$\sigma = \sqrt{\sigma^2} = \sqrt{V(X)} = \sqrt{np(1-p)} = \sqrt{25 \times 0.05 \times 0.95} = 1.087$

a, we know that: $X \sim b(10, 0.6)$. $p(X\ge 6) = 1 - p(X\le 5) = 1 - 0.367 = 0.633$ (from table)

b. $E(X) = np = 10\times 0.6 = 6$, $\sigma = \sqrt{\sigma^2} = \sqrt{V(X)} = \sqrt{np(1-p)} = \sqrt{10\times 0.6\times 0.4} = 1.54$

the domain of the mean value of the standard deviation is $[4.46, 7.54]$

$p(5\le X\le 7) = p(X\le 7) - p(X\le 4) = 0.833 - 0.166 = 0.667$

c. $p(3\le X\le 7) = p(X\le 7) - p(X\le 2) = 0.833 - 0.012 = 0.821$

Section 3.4

Ex 68

a. ~~$X\sim B(20, 0.18)$~~ $X \sim B(6, 0.18)$

b. $p(X=2) = \binom{6}{2}0.18^2(1-0.18)^4 = 0.2197$

$p(X\le 2) = p(X=0) + p(X=1) + p(X=2) = (1-0.18)^6 + \binom{6}{1}0.18\times(1-0.18)^5 + \binom{6}{2}0.18^2(1-0.18)^4 = 0.9241$

$p(X\ge 2) = 1 - p(X=0) - p(X=1) = 1 - 0.3040 - 0.4004 = 0.2956$

c. $E(X) = np = 6\times 0.18 = 1.08$

$V(X) = np(1-p) = 6\times 0.18\times 0.82 = 0.8856$

Ex69.

a. $p(X=5) = \frac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}} = \frac{105}{924}$

b. $p(X\le 4) = 1 - p(X>4) = 1 - [p(X=5) + p(X=6)] = 1 - \frac{\binom{7}{5}\binom{5}{1}}{\binom{12}{6}} + \frac{\binom{7}{6}\binom{5}{0}}{\binom{12}{6}} = 0.879$

c. $E(X) = n\frac{M}{N} = 6\times\frac{7}{12} = 3.5$

$V(X) = \left(\frac{12-6}{12-1}\right)\times 6\times\frac{7}{12}\times\left(1-\frac{7}{12}\right) = 0.89$

$p(X>\mu+\sigma) = p(X>4.392) = p(X=5) + p(X=6) = 0.121$

Ex72. a. $h(x; 6, 4, 11) = \frac{\binom{4}{x}\binom{7}{6-x}}{\binom{11}{6}} =$

b. $E(X) = n\frac{M}{N} = 6\times\frac{4}{11} = 2.18$

5. a. $p(X=x) = nb(x; 2, 0.5) = \binom{x+2-1}{2-1}\times(0.5)^2\times(1-0.5)^x = (x+1)0.5^{x+2}$

b. p(4 children) = p(2 males) = $p(X=2) = nb(2; 2, .5) = (2+1)\times 0.5^4 = 0.188$

c. p(at most 4 children) = $p(X\le 2) = \sum_{x=0}^{2} nb(x; 2, 0.5) = 0.688$