The Examination Paper of Jinan University

For Instructor Only	Academic Year : 2022–2023 Semester: 1st [✓] 2nd[ ] Course Title: Advanced Mathematics I Date of the Examination 26/12/2022 Instructor's Name Lianghui Xia	Course Type Compulsory[✓] Elective [ ] Form of the Examination Open-book [ ] Closed-book [✓] Paper A[ ] Paper B [✓] Total Pages 5
For Student Only	School/College International School Major Computer science and technology Name 蒋云翔 Student No. 2022102330 Mainland Student [✓] Non-mainland Student [ ]	

Section No.	I	II	III	IV	V	VI	VII	VIII	IX	X	Total Score
Score											

Score	Evaluator	
		Section I: Filling blanks (There are 8 questions, each question is 3 marks, the total score of this section is 24 marks)

1	2	3	4	5
2	$[-2,-1)\cup(1,2]$	$x=1$ and $x=2$	e^{-1}	$y=1$
6	7	8		
$\ln\lvert x\rvert-\sin x+\frac{1}{2}x^2-\arcsin x+2x+C$	-3	$x=0$		

Score	Evaluator	
		Section II: Choice questions (There are 10 questions, each question has four choices, but only one is true, the other three are false, choose the one which is true, each question is 2 marks, and the total score of this section is 20 marks)

1	2	3	4	5	6	7	8	9	10
C	B	C	D	D	B	C	B	~~A~~ C	B

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Score	Evaluator	Section III: Calculation (There are 7 questions, each question is 6 marks, the total score of this section is 42 marks)

1. Solution:

$$\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}=\lim_{x\to 9}\frac{\sqrt{x}-3}{(\sqrt{x}+3)(\sqrt{x}-3)}=\lim_{x\to 9}\frac{1}{\sqrt{x}+3}=\frac{1}{6}$$

2. Solution:

~~since~~: we know that: $\frac{1+2+\cdots+n}{n^2+n\pi}\le\frac{1}{n^2+\pi}+\frac{2}{n^2+2\pi}+\cdots+\frac{n}{n^2+n\pi}\le\frac{1+2+\cdots+n}{n^2+\pi}$

since: $\lim_{n\to\infty}\frac{1+2+\cdots+n}{n^2+n\pi}=\frac{\frac{1}{n^2}+\frac{2}{n^2}+\cdots+\frac{1}{n}}{1+\frac{\pi}{n}}=1$

$\lim_{n\to\infty}\frac{1+2+\cdots+n}{n^2+\pi}=\frac{\frac{1}{n^2}+\frac{2}{n^2}+\cdots+\frac{1}{n}}{1+\frac{\pi}{n^2}}=1$

By Sandwich Theorem: we know: $\lim_{x\to\infty}\left(\frac{1}{n^2+\pi}+\frac{2}{n^2+2\pi}+\cdots+\frac{n}{n^2+n\pi}\right)=1$

3. Solution:

$\cancel{h(x)=\left(\frac{1}{x^2}-5\right)^{-2}}$

so: $\cancel{h'(x)=-2\left(\frac{1}{x^2}-5\right)\cdot(-2x)}$

$\cancel{=4\left(\frac{1}{x}-5x\right)}$

$h(x)=\left(\frac{1}{x^2}-5\right)^{-2}$

so: $h'(x)=-2\left(\frac{1}{x^2}-5\right)^{-3}\cdot(-2x^{-3})$

$=4x^{-3}\left(\frac{1}{x^2}-5\right)^{-3}$

4. Solution:

$y-xy^2+x^2+1=0$

$y'-(y^2+x\cdot 2y\cdot y')+2x=0$

$y'(1-2xy)=y^2-2x$

$y'=\frac{y^2-2x}{1-2xy}$

5. Solution:

$$\lim_{x\to 1}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right)$$

$$=\lim_{x\to 1}\left(\frac{x\ln x-x+1}{(x-1)\ln x}\right)$$

$$=\lim_{x\to 1}\left(\frac{1+\ln x-1}{\ln x+1-\frac{1}{x}}\right)$$

$$=\lim_{x\to 1}\left(\frac{\frac{1}{x}}{\frac{1}{x}+\frac{1}{x^2}}\right)=\frac{1}{2}$$

6. Solution:

$$\int\frac{1}{1+e^x}dx$$

$$=\int\frac{(1+e^x)-e^x}{1+e^x}dx$$

$$=\int\left(1-\frac{e^x}{1+e^x}\right)dx$$

$$=\int 1\,dx-\int\frac{e^x}{1+e^x}dx$$

$$=x-\int\frac{1}{1+e^x}de^x+1+C$$

$$=x-\ln(1+e^x)+C$$

7. Solution:

$$\int x\arctan x\,dx$$

$$=\frac{1}{2}x^2\cdot\arctan x-\int\frac{x^2}{2(1+x^2)}dx$$

$$=\frac{1}{2}x^2\cdot\arctan x-\frac{1}{2}\int\frac{(1+x^2)-1}{1+x^2}dx$$

$$=\frac{1}{2}x^2\arctan x-\frac{1}{2}\left(\int 1\,dx-\int\frac{1}{1+x^2}dx\right)$$

$$=\frac{1}{2}x^2\arctan x-\frac{1}{2}(x-\arctan x+C)$$

$$=\frac{1}{2}x^2\arctan x-\frac{1}{2}x\cancel{\text{arc}}+\frac{1}{2}\arctan x+C$$

The Examination Paper (B) for Advanced Mathematics I of JNU

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Score	Evaluator	Section IV: Application problem (There are two questions, the first one is 8 marks, and the second one is 6 marks, the total score of this section is 14 marks.)

1. Solution:

For $f(x) = x^3(x+2)$

so: $f'(x) = 3x^2(x+2) + x^3 = 4x^3 + 6x^2 = 2x^2(2x+3)$

$f''(x) = 12x^2 + 12x = 12x(x+1)$

let $f'(x) = 0$, we~~x~~ know that $x_1 = 0$, $x_2 = -\frac{3}{2}$

let $f''(x) = 0$, we know that $x_3 = 0$, $x_4 = -1$

x	$(-\infty, -\frac{3}{2})$	$-\frac{3}{2}$	$(-\frac{3}{2}, -1)$	-1	$(-1, 0)$	0	$(0, +\infty)$
sign of $f'(x)$	$-$	0	$+$		$+$	0	$+$
sign of $f''(x)$	$+$		$+$	0	$-$	0	$+$
shape of $f(x)$							

So we know: (1) the critical point are $x_1 = 0$ and $x_2 = -\frac{3}{2}$

(2) $f(x)$ is increasing when $x \in (-\frac{3}{2}, +\infty)$

$f(x)$ is decreasing when $x \in (-\infty, -\frac{3}{2})$

$f(x)$ has a local minimum value when $x = -\frac{3}{2}$, and $f(-\frac{3}{2}) = -\frac{27}{16}$

$f(x)$ doesn't have local maximum value

(3) ~~f(x) con~~ the graph of $f(x)$ is concave up when $x \in (-\infty, -1) \cup (0, +\infty)$

the graph of $f(x)$ is concave down when $x \in (-1, 0)$

(4) the inflection points are $(-1, f(-1))$ and $(0, f(0))$

~~that is~~ they are: ~~(-1,-1)~~ $(-1, -1)$ and $(0, 0)$

2. Solution:

$V=\frac{1}{3}\pi x^2(3+y)$

since: $x^2+y^2=9$

we know: $x^2=9-y^2$

so: $V=\frac{1}{3}\pi(9-y^2)(3+y)$ $(y\in(0,3))$

then we know $V'=\frac{1}{3}\pi[-2y(3+y)+(9-y^2)]$

$=\frac{1}{3}\pi(-3y^2-6y+9)$

$=-\pi(y^2+2y-3)$

$=-\pi(y+3)(y-1)$

then we know:

$V'>0$. when $y\in(0,1)$, so V is increasing on $(0,1)$

$V'<0$ when $y\in(1,3)$, so V is decreasing on $(1,3)$

So we know, when $y=1$, V is the max.

so $V_{max}=\frac{1}{3}\pi(9-1)(3+1)$

$=\frac{32\pi}{3}$

so: the largest volume is $\frac{32}{3}\pi$.