Homework
Section 2.4: 46, 50, 58, 63

A+

高6ft人中找篮球选手难,
但篮球运动员中选高于6ft的易

46. Suppose an individual is randomly selected from the population of all adult males living in the United States. Let A be the event that the selected individual is over 6 ft in height, and let B be the event that the selected individual is a professional basketball player. Which do you think is larger, $P(A \mid B)$ or $P(B \mid A)$? Why?

event A: the selected individual is over 6 ft in height.
event B: the selected individual is a professional basketball player.

$P(A|B)$ means that when we know the individual is a professional basketball player, then the probability of the individual being over 6ft in height.

$P(B|A)$ means the probability of the individual being a professional basketball player when knowing that the individual is more than 6 ft in height. $P(A|B)$ will be larger. since most of the professional basketball players are tall, so in the smalle sample space of the individual whose is a professional basketball player, the proportion of individual who 6ft in height is large.

50. A department store sells sport shirts in three sizes (small, medium, and large), three patterns (plaid, print, and stripe), and two sleeve lengths (long and short). The accompanying tables give the proportions of shirts sold in the various category combinations.

Short-sleeved

	Pattern		
Size	Pl	Pr	St
S	.04	.02	.05
M	.08	.07	.12
L	.03	.07	.08

Long-sleeved

	Pattern		
Size	Pl	Pr	St
S	.03	.02	.03
M	.10	.05	.07
L	.04	.02	.08

a. What is the probability that the next shirt sold is a medium, long-sleeved, print shirt?
b. What is the probability that the next shirt sold is a medium print shirt?
c. What is the probability that the next shirt sold is a short-sleeved shirt? A long-sleeved shirt?
d. What is the probability that the size of the next shirt sold is medium? That the pattern of the next shirt sold is a print?
e. Given that the shirt just sold was a short-sleeved plaid, what is the probability that its size was medium?
f. Given that the shirt just sold was a medium plaid, what is the probability that it was short-sleeved? Long-sleeved?

a) use the given characters to represent the event (i.e M means medium, LS means Long sleeve. Pr means print
$P(M \cap LS \cap Pr) = 0.05$ get from the table.

b.) $P(M \cap Pr) = P(M \cap LS \cap Pr) + P(M \cap SS \cap Pr) = 0.05 + 0.07 = 0.12$

c) the probability next shirt sold is a short-sleeved shirt.
$P(SS) = 0.56$ (sum in the short sleeve table)

... a long-sleeve shirt.
$P(LS) = 1 - 0.56 = 0.44$

d) we can get from the table.
$P(M) = 0.49$

$P(Pr) = 0.25$

e) the probability of the question:
$P(M|SS \cap PL) = \dfrac{P(M \cap SS \cap PL)}{P(SS \cap PL)} = \dfrac{0.08}{0.04+0.08+0.03} \approx 0.533$

f) $P(SS|M \cap PL) = \dfrac{P(SS \cap M \cap PL)}{P(M \cap PL)} = \dfrac{0.08}{0.08+0.1} = 0.444$

$P(LS|M \cap PL) = \dfrac{P(LS \cap M \cap PL)}{P(M \cap PL)} = 0.556$

Homework
Section 2.4: 46, 50, 58, 63

58. Show that for any three events A, B, and C with $P(C) > 0$,
$P(A \cup B \mid C) = P(A \mid C) + P(B \mid C) - P(A \cap B \mid C)$.

Proof:

$$P(A \cup B \mid C) = \frac{P((A\cup B)\cap C)}{P(C)} = \frac{P((A\cap C)\cup(B\cap C))}{P(C)} = \frac{P(A\cap C)+P(B\cap C)-P(A\cap B\cap C)}{P(C)}$$

$$= P(A\mid C)+P(B\mid C)-P(A\cap B\mid C)$$

63. For customers purchasing a refrigerator at a certain appliance store, let A be the event that the refrigerator was manufactured in the U.S., B be the event that the refrigerator had an icemaker, and C be the event that the customer purchased an extended warranty. Relevant probabilities are

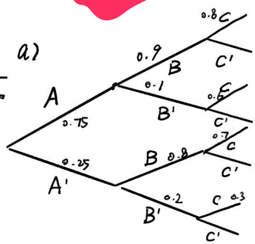
$P(A) = .75 \quad P(B \mid A) = .9 \quad P(B \mid A') = .8$

$P(C \mid A \cap B) = .8 \quad P(C \mid A \cap B') = .6$

$P(C \mid A' \cap B) = .7 \quad P(C \mid A' \cap B') = .3$

a. Construct a tree diagram consisting of first-, second-, and third-generation branches, and place an event label and appropriate probability next to each branch.
b. Compute $P(A \cap B \cap C)$.
c. Compute $P(B \cap C)$.
d. Compute $P(C)$.
e. Compute $P(A \mid B \cap C)$, the probability of a U.S. purchase given that an icemaker and extended warranty are also purchased.

a)



b) get from the graph.
$P(A\cap B\cap C)$
$= 0.75 \times 0.9 \times 0.8 = 0.54$

c) $P(B\cap C) = P(A\cap B\cap C) + P(A'\cap B\cap C)$
$= 0.54 + 0.25 \times 0.8 \times 0.7 = 0.68.$

d) $P(C) = P(A\cap B\cap C) + P(A'\cap B\cap C) + P(A\cap B'\cap C) + P(A'\cap B'\cap C)$
$= 0.74$

e) $P(A\mid B\cap C) = \dfrac{P(A\cap(B\cap C))}{P(B\cap C)} = \dfrac{0.54}{0.68} \approx 0.794$

Section 2.5

A

71. An oil exploration company currently has two active projects, one in Asia and the other in Europe. Let A be the event that the Asian project is successful and B be the event that the European project is successful. Suppose that A and B are independent events with $P(A) = .4$ and $P(B) = .7$.

a. If the Asian project is not successful, what is the probability that the European project is also not successful? Explain your reasoning.

b. What is the probability that at least one of the two projects will be successful?

c. Given that at least one of the two projects is successful, what is the probability that only the Asian project is successful?

event A: Asian project is successful.

event B: European project is successful.

a) since A and B are independent.

$$P(B'|A') = \frac{P(B'\cap A')}{P(A')} = P(B') = 1-0.7 = 0.3$$

b) $P(A\cup B) = P(A)+P(B) - P(A\cap B) = 0.4+0.7-0.4\times0.7 = 0.82$

c) $$P(A\cap B'|A\cup B) = \frac{P(A\cap B')\cap(A\cup B)}{P(A\cup B)} = \frac{P(A\cap B')}{P(A\cup B)} = \frac{P(A)\cdot P(B')}{P(A\cup B)} = \frac{0.12}{0.82} \approx 0.146$$

72. In Exercise 13, is any A_i independent of any other A_j? Answer using the multiplication property for independent events.

13. A computer consulting firm presently has bids out on three projects. Let $A_i = \{\text{awarded project } i\}$, for $i = 1, 2, 3$, and suppose that $P(A_1) = .22$, $P(A_2) = .25$, $P(A_3) = .28$, $P(A_1 \cap A_2) = .11$, $P(A_1 \cap A_3) = .05$, $P(A_2 \cap A_3) = .07$, $P(A_1 \cap A_2 \cap A_3) = .01$. Express in words each of the following events, and compute the probability of each event:

a. $A_1 \cup A_2$

b. $A_1' \cap A_2'$ [*Hint:* $(A_1 \cup A_2)' = A_1' \cap A_2'$]

c. $A_1 \cup A_2 \cup A_3$

d. $A_1' \cap A_2' \cap A_3'$

e. $A_1' \cap A_2' \cap A_3$

f. $(A_1' \cap A_2') \cup A_3$

from exercise 13.

① $P(A_1\cap A_2) = 0.11$

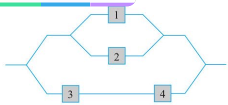
$P(A_1)\cdot P(A_2) = 0.22\times0.25 = 0.055$

$P(A_1\cap A_2) \neq P(A_1)\cdot P(A_2)$ so A_1 and A_2 are not independent.

② $P(A_1\cap A_3) = 0.05 \neq P(A_1)\cdot P(A_3) = 0.0616$ so A_1 and A_3 are not independent.

③ $P(A_2\cap A_3) = 0.07$, $P(A_2)P(A_3) = 0.07$ so A_2 and A_3 are independent

80. Consider the system of components connected as in the accompanying picture. Components 1 and 2 are connected in parallel, so that subsystem works iff either 1 or 2 works; since 3 and 4 are connected in series, that subsystem works iff both 3 and 4 work. If components work independently of



one another and $P(\text{component works}) = .9$, calculate $P(\text{system works})$.

Let A_i represent the ith component work $(i=1,2,3,4)$

$P(\text{system works}) = P(A_1 \cup A_2) \cup (A_3 \cap A_4)$

$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = 0.99$

$P(A_3 \cap A_4) = P(A_3) \cdot P(A_4) = 0.81$ since they're independent.

$P(\text{system works}) = P(A_1 \cup A_2) + P(A_3 \cap A_4) - P[(A_1 \cup A_2) \cap (A_3 \cap A_4)]$ since $A_1 \cup A_2$ and $A_3 \cap A_4$ are also independent.

$= 0.99 + 0.81 - (0.99) \cdot (0.81) = 0.9981$

84. Seventy percent of all vehicles examined at a certain emissions inspection station pass the inspection. Assuming that successive vehicles pass or fail independently of one another, calculate the following probabilities:

a. P(all of the next three vehicles inspected pass)
b. P(at least one of the next three inspected fails)
c. P(exactly one of the next three inspected passes)
d. P(at most one of the next three vehicles inspected passes)
e. Given that at least one of the next three vehicles passes inspection, what is the probability that all three pass (a conditional probability)?

since the event that the ith vehicle passes is independent, $(i=1,2,3)$, Let A_i denote this event, $P(A_i) = 0.7$

a) P(all of the next three vehicles inspected pass)

$= P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P(A_2) \cdot P(A_3) = (0.7)^3 = 0.343.$

b) at least one of the next three inspected fails
it is the complement of the event that all of them pass.

so P(at least one of the next three inspected fails) $= 1 - 0.343 = 0.657$

c) P(exactly one of the next three inspected passes) $= P[(A_1 \cap A_2' \cap A_3') \cup (A_1' \cap A_2 \cap A_3') \cup (A_1' \cap A_2' \cap A_3)]$

the complement of independence event is independent)

$= [(0.7) \times (0.3) \times (0.3)] + [0.3 \times 0.7 \times 0.3] + [0.3 \times 0.3 \times 0.7]$

$= 0.189$ (they're disjoint)

d) P(at most one of the next three vehicles inspected passes)

$= P(A_1' \cap A_2' \cap A_3') + P$(exactly one passes)

$= 0.027 + 0.189 = 0.216$

e) $P(A_1 \cap A_2 \cap A_3 \mid$ at least one of next three inspected passes$)$, $P(A_1 \cup A_2 \cup A_3) = 1 - P(A_1' \cap A_2' \cap A_3')$

$= 0.973$

$= P(A_1 \cap A_2 \cap A_3 \mid A_1 \cup A_2 \cup A_3)$

$$= \frac{P(A_1 \cap A_2 \cap A_3) \cap (A_1 \cup A_2 \cup A_3)}{P(A_1 \cup A_2 \cup A_3)} = \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1 \cup A_2 \cup A_3)} = \frac{0.343}{0.973} \approx 0.3525$$

Section 3.1
(... 8. 10)

Section 3.2
(12. 23. 25)

4. Let X = the number of nonzero digits in a randomly selected zip code. What are the possible values of X? Give three possible outcomes and their associated X values.

X = the number of nonzero digits in a randomly selected zip code.

we know that a zip code has 5 digits. and it can not be all zero or 4 zero

so X = 2, 3, 4, 5

for example, 12345, X = 5
23401, X = 4
34005, X = 3

5. If the sample space $\mathcal{S}$ is an infinite set, does this necessarily imply that any rv X defined from $\mathcal{S}$ will have an infinite set of possible values? If yes, say why. If no, give an example.

No, rv is a function of S. it can be finite.

For example, S = { the coin is flipped until it hits heads }

then Let X means it just flip at most 3 times, X = 1
otherwise X = 0. then the rv is finite

连续3次成功

8. Each time a component is tested, the trial is a success (*S*) or failure (*F*). Suppose the component is tested repeatedly until a success occurs on three *consecutive* trials. Let *Y* denote the number of trials necessary to achieve this. List all outcomes corresponding to the five smallest possible values of *Y*, and state which *Y* value is associated with each one.

$Y = 3, 4, 5 \ldots$

$Y=3$, SSS

$Y=4$, FSSS

$Y=5$, FFSSS, SFSSS

$Y=6$, FFFSSS, SFFSSS, SSFSSS, FSFSSS

$Y=7$, FFFFSSS, SFFFSSS, FSFFSSS, FFSFSSS, SSFFSSS, SFSFSSS, FSSFSSS.

10. The number of pumps in use at both a six-pump station and a four-pump station will be determined. Give the possible values for each of the following random variables:

a. T = the total number of pumps in use

b. X = the difference between the numbers in use at stations 1 and 2

c. U = the maximum number of pumps in use at either station

d. Z = the number of stations having exactly two pumps in use

a. T = the total number of pumps in use.

the possible value of it: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

b. X = the difference between the numbers in use at stations 1 and 2, the possible value: −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, 6.

c. the possible value of U: 0, 1, 2, 3, 4, 5, 6.

d. the possible value of Z: 0, 1, 2.

(12, 23, 25)

12. Airlines sometimes overbook flights. Suppose that for a plane with 50 seats, 55 passengers have tickets. Define the random variable Y as the number of ticketed passengers who actually show up for the flight. The probability mass function of Y appears in the accompanying table.

y	45	46	47	48	49	50	51	52	53	54	55
$p(y)$	.05	.10	.12	.14	.25	.17	.06	.05	.03	.02	.01

a. What is the probability that the flight will accommodate all ticketed passengers who show up?

b. What is the probability that not all ticketed passengers who show up can be accommodated?

c. If you are the first person on the standby list (which means you will be the first one to get on the plane if there are any seats available after all ticketed passengers have been accommodated), what is the probability that you will be able to take the flight? What is this probability if you are the third person on the standby list?

a) there are 50 seats, so the flight can accomodate all ticketed passengers who show up when they no more than 50.

$P(Y \le 50) = 0.05 + 0.10 + 0.12 + 0.14 + 0.25 + 0.17$
$= 0.83$

b) the probability that not all ticketed passengers who show up can be accommodated.

$P(Y > 50) = 1 - P(Y \le 50) = 0.17$

c) first standby, there should be no more than 49 people to show up, $P(Y \le 49) = 0.05 + 0.10 + 0.12 + 0.14 + 0.25 = 0.66$.

third standby, the possibility that you can accommodate $P(Y \le 47) = 0.05 + 0.10 + 0.12 = 0.2$

23. A consumer organization that evaluates new automobiles customarily reports the number of major defects in each car examined. Let X denote the number of major defects in a randomly selected car of a certain type. The cdf of X is as follows:

$F(x) = P(X \le x)$ 累积

$$F(x) = \begin{cases} 0 & x < 0 \\ .06 & 0 \le x < 1 \\ .19 & 1 \le x < 2 \\ .39 & 2 \le x < 3 \\ .67 & 3 \le x < 4 \\ .92 & 4 \le x < 5 \\ .97 & 5 \le x < 6 \\ 1 & 6 \le x \end{cases}$$

Calculate the following probabilities directly from the cdf:

a. $p(2)$, that is, $P(X = 2)$ b. $P(X > 3)$
c. $P(2 \le X \le 5)$ d. $P(2 < X < 5)$

a). $p(2) = P(X=2) = F(2) - F(2-1)$
$= 0.39 - 0.19 = 0.20$.

b) $P(X > 3) = 1 - P(X \le 3) = 1 - F(3) = 1 - 0.67 = 0.33$

c) $P(2 \le X \le 5) = F(5) - F(2-1) = 0.92 - 0.19$

d) $P(2 < X < 5) = 0.92 - 0.39 = 0.53$.

25. In Example 3.12, let Y = the number of girls born before the experiment terminates. With $p = P(B)$ and $1 - p = P(G)$, what is the pmf of Y? [*Hint:* First list the possible values of Y, starting with the smallest, and proceed until you see a general formula.]

the possible values of Y: 0, 1, 2 ....

$P(0) = P(Y=0) = p$.
$p(1) = P(Y=1) = (1-p) \cdot p$
$p(2) = P(Y=2) = (1-p)^2 \cdot p$ (that there are 2 girls birth before the boy)

Then $P(y) = P$(y girls and then a boy) $= (1-p)^y \cdot p$ for $y = 0, 1, 2 \ldots$