A

3.3 29, 33, 38, 41

29 a) Expected Value (mean value) of a discrete random variable X with set of possible values s and pmf $p(x)$ is $E(X) = \mu_x = \sum_{x \in S} x \cdot p(x)$

$E(X) = \sum_{x \in S} x \cdot p(x) = 1 \cdot 0.05 + 2 \cdot 0.1 + 4 \cdot 0.35 + 8 \cdot 0.4 + 16 \cdot 0.1 = 6.45$ GB

b) Variance of X, where X is a discrete random variable X with a set of possible values S and pmf $p(x)$, denoted $V(X)$ (σ_x^2 or σ^2) is

$V(X) = \sum_{x \in S} (x-\mu)^2 \cdot p(x) = E[(X-\mu)^2]$.

$V(X) = \sum_{x \in S} (x - E(X))^2 \cdot p(x) = (1-6.45)^2 \cdot 0.05 + (2-6.45)^2 \cdot 0.1 + (4-6.45)^2 \cdot 0.35 + (8-6.45)^2 \cdot 0.4$
$+ (16-6.45)^2 \cdot 0.1 = 15.6475$

c) Standard Deviation of X is $\sigma_x = \sqrt{\sigma^2_x}$

$\sigma_x = \sqrt{V(X)} = \sqrt{15.6475} = 3.956$ GB

d) Proposition: shortcut formula for $V(x)$:

$V(X) = \sigma^2 = \left[\sum_{x \in S} x^2 \cdot p(x)\right] - \mu^2 = E(X^2) - [E(X)]^2$

$E(X) = 6.45$, find $E(X^2)$

Expected Value (mean value) of any $g(X)$, where X is a discrete random variable X with set of possible values S and pmf $p(x)$, denoted as $E[g(X)]$ ($\mu_{g(x)}$), is

$E[g(X)] = \mu_{g(x)} = \sum_{x \in S} g(x) \cdot p(x)$.

In our case, $g(X) = X^2$, so $E(X^2) = \sum_{x \in S} g(x) \cdot p(x) = \sum_{x \in S} x^2 \cdot p(x) = 1^2 \cdot 0.05 + 2^2 \cdot 0.1 + 4^2 \cdot 0.35 + 8^2 \cdot 0.4$
$+ 16^2 \cdot 0.1 = 57.25$

$V(X) = E(X^2) - (E(x))^2 = 57.25 - (6.45)^2 = 15.6475$

33) Bernoulli random variable: any random variable which has only 2 possible values: 0 and 1, $p(0) = 1-p$ and $p(1) = p$

a) Expected value: $E[g(X)] = \mu_g(x) = \sum_{x \in S} g(x) \cdot p(x)$

$g(X) = X^2$, so $E(X^2) = \sum_{x \in S} x^2 \cdot p(x) = 0^2 \cdot (1-p) + 1^2 \cdot p = p$

b) $E(X) = \mu_x = \sum_{x \in S} x \cdot p(x) = 0 \cdot (1-p) + 1 \cdot p = p$

~~V(X) =~~ shortcut formula $V(X) = E(X^2) - [E(x)]^2 = p - [p]^2 = p - p^2 = p(1-p)$

c) $g(X) = X^{79}$

$E(X^{79}) = \sum_{x \in S} x^{79} \cdot p(x) = 0^{79} \cdot (1-p) + 1^{79} \cdot p = p$, in fact $E(X^n) = p$ for any non negative power n.

3B)

$\frac{1}{3.5} = \$0{,}29$

Expected value (mean value) : $E[g(X)] = \mu_{g(x)} = \sum_{x \in S} g(x) \cdot p(x)$.

$h(X) = \frac{1}{X}$

$E[h(X)] = E(\frac{1}{X}) = \sum_{x=1}^{6} (\frac{1}{x}) \cdot p(x) = \sum_{x=1}^{6} (\frac{1}{x}) \cdot \frac{1}{6} = \frac{1}{1} \cdot \frac{1}{6} + \frac{1}{2} \cdot \frac{1}{6} + \cdots + \frac{1}{6} \cdot \frac{1}{6} = \frac{49}{120} = \$0{,}41$

Expected value $E[h(X)]$ is bigger, so gamble

Note : In general, if $h(x)$ is concave up then $E[h(X)] > h(E(X))$, while $E[h(X)] < h(E(X))$ if $h(x)$ is concave down.

4.1) Variance of X : $V(X) = \sum_{x \in S} (x-\mu)^2 \cdot p(x) = E[(X-\mu)^2]$.

$V(aX+b) = E[((aX+b) - E(ax+b))^2] = \sum_{x \in S} [aX + b - E(aX+b)]^2 \cdot p(X)$

$= \sum_{x \in S} [aX + \cancel{b} - (aE(X) + \cancel{b})]^2 \cdot p(X) = \sum_{x \in S} [aX - a\mu]^2 \cdot p(X)$

$= a^2 \sum_{x \in S} (X - E(X))^2 \cdot p(X) = a^2 \cdot \sigma^2_X$