

0.15 Canonical Cover Calculation

you-said

Relation r2 (A,B ,C, D,E ,G ,H ,K) ; Functional Dependencies $F = \{\{A, B, H\} \rightarrow C\}, \{A\} \rightarrow \{D, E\}, \{B, G, H\} \rightarrow \{K\}, \{K\} \rightarrow \{A, D, H\}, \{B, H\} \rightarrow \{G, E\}$. use this algorithm to find canonical cover: result := F Do Replace FDs in result of the form $\alpha \rightarrow \beta_1$ and $\alpha \rightarrow \beta_2$ by $\alpha \rightarrow \beta_1 \cup \beta_2$ If $\alpha \rightarrow \beta$ in result has extraneous attribute a in the left side (right side): Remove a from α (β) While result does not change Return result

ChatGPT said:

We can proceed step by step to compute the canonical cover of the given functional dependencies FF.

step-1-combine-functional-dependencies-with-the-same-left-side

Step 1: Combine Functional Dependencies with the Same Left Side

The given set of functional dependencies is:

$$F = \{\{A, B, H\} \rightarrow C, \{A\} \rightarrow D, E, \{B, G, H\} \rightarrow K, \{K\} \rightarrow A, D, H, \{B, H\} \rightarrow G, E\}$$

$$F = \{\{A, B, H\} \rightarrow C, \{A\} \rightarrow D, E, \{B, G, H\} \rightarrow K, \{K\} \rightarrow A, D, H, \{B, H\} \rightarrow G, E\}$$

We first combine dependencies that have the same left-hand side.

- $\{A\} \rightarrow D$ and $\{A\} \rightarrow E$ can be combined into $\{A\} \rightarrow \{D, E\}$.
- $\{B, H\} \rightarrow G$ and $\{B, H\} \rightarrow E$ can be combined into $\{B, H\} \rightarrow \{G, E\}$.

After combining, we get the following set of dependencies:

$$F = \{\{A, B, H\} \rightarrow C, \{A\} \rightarrow \{D, E\}, \{B, G, H\} \rightarrow K, \{K\} \rightarrow \{A, D, H\}, \{B, H\} \rightarrow \{G, E\}\}$$

$$F = \{\{A, B, H\} \rightarrow C, \{A\} \rightarrow \{D, E\}, \{B, G, H\} \rightarrow K, \{K\} \rightarrow \{A, D, H\}, \{B, H\} \rightarrow \{G, E\}\}$$

step-2-remove-extraneous-attributes

Step 2: Remove Extraneous Attributes

Now, we check for any extraneous attributes on the left-hand side of the functional dependencies.

$$1. \{A, B, H\} \rightarrow C \quad \{\mathbf{A}, \mathbf{B}, \mathbf{H}\} \rightarrow \mathbf{C}:$$

- Check if AA , BB , or HH is extraneous.
- Remove one at a time and check if the closure of the remaining attributes still determines CC .
- If $\{A, H\}^+ \{A, H\}^+$ does not determine CC , BB is not extraneous.
- If $\{B, H\}^+ \{B, H\}^+$ does not determine CC , AA is not extraneous.
- If $\{A, B\}^+ \{A, B\}^+$ does not determine CC , HH is not extraneous.
- Thus, no extraneous attributes are found for this FD.

$$2. \{A\} \rightarrow \{D, E\} \quad \{\mathbf{A}\} \rightarrow \{\mathbf{D}, \mathbf{E}\}:$$

- This has only one attribute on the left-hand side, so no extraneous attributes are present.

$$3. \{B, G, H\} \rightarrow K \quad \{\mathbf{B}, \mathbf{G}, \mathbf{H}\} \rightarrow \mathbf{K}:$$

- Check if BB , GG , or HH is extraneous.
- If $\{G, H\}^+ \{G, H\}^+$ does not determine KK , BB is not extraneous.
- If $\{B, H\}^+ \{B, H\}^+$ does not determine KK , GG is not extraneous.
- If $\{B, G\}^+ \{B, G\}^+$ does not determine KK , HH is not extraneous.
- Thus, no extraneous attributes are found for this FD.

$$4. \{K\} \rightarrow \{A, D, H\} \quad \{\mathbf{K}\} \rightarrow \{\mathbf{A}, \mathbf{D}, \mathbf{H}\}:$$

- This has only one attribute on the left-hand side, so no extraneous attributes are present.

$$5. \{B, H\} \rightarrow \{G, E\} \quad \{\mathbf{B}, \mathbf{H}\} \rightarrow \{\mathbf{G}, \mathbf{E}\}:$$

- Check if BB or HH is extraneous.

- If $\{H\}^+\{H\}^+$ does not determine GG or EE , BB is not extraneous.
- If $\{B\}^+\{B\}^+$ does not determine GG or EE , HH is not extraneous.
- Thus, no extraneous attributes are found for this FD.

step-3-remove-extraneous-attributes-from-the-right-side

Step 3: Remove Extraneous Attributes from the Right Side

Next, we check for extraneous attributes on the right-hand side of the functional dependencies.

1. $\{A, B, H\} \rightarrow C$ **$\{A, B, H\} \rightarrow C$:**

- The right side has only one attribute, so no extraneous attributes are present.

2. $\{A\} \rightarrow \{D, E\}$ **$\{A\} \rightarrow \{D, E\}$:**

- Check if DD or EE is extraneous.
- Neither DD nor EE can be determined by the other.
- Thus, no extraneous attributes are present.

3. $\{B, G, H\} \rightarrow K$ **$\{B, G, H\} \rightarrow K$:**

- The right side has only one attribute, so no extraneous attributes are present.

4. $\{K\} \rightarrow \{A, D, H\}$ **$\{K\} \rightarrow \{A, D, H\}$:**

- Check if AA , DD , or HH is extraneous.
- Neither AA , DD , nor HH can be determined by the other.
- Thus, no extraneous attributes are present.

5. $\{B, H\} \rightarrow \{G, E\}$ **$\{B, H\} \rightarrow \{G, E\}$:**

- Check if GG or EE is extraneous.
- Neither GG nor EE can be determined by the other.
- Thus, no extraneous attributes are present.

final-canonical-cover

Final Canonical Cover

After applying the algorithm, the final canonical cover is:

$$\{\{A, B, H\} \rightarrow C, \{A\} \rightarrow \{D, E\}, \{B, G, H\} \rightarrow K, \{K\} \rightarrow \{A, D, H\}, \{B, H\} \rightarrow \{G, E\}\}$$

$$\{\{A, B, H\} \rightarrow C, \{A\} \rightarrow \{D, E\}, \{B, G, H\} \rightarrow K, \{K\} \rightarrow \{A, D, H\}, \{B, H\} \rightarrow \{G, E\}\}$$

This is the minimal set of functional dependencies representing the original set FF .

you-said-1

Algorithm for canonical cover is: result := F Do Replace FDs in result of the form $\alpha \rightarrow \beta_1$ and $\alpha \rightarrow \beta_2$ by $\alpha \rightarrow \beta_1 \cup \beta_2$ If $\alpha \rightarrow \beta$ in result has extraneous attribute a in the left side (right side): Remove a from α (β) While result does not change Return result conditions for extraneous attributes are: Given $\alpha \rightarrow \beta$: • a belongs to α is extraneous if $\beta \subseteq (\alpha - a)^+$ under F • a belongs to β is extraneous if a belongs to α^+ under $F' = (F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - a)\}$ find canonical cover for For $R(A, B, C)$; $FD = \{A \rightarrow BC, B \rightarrow C, A \rightarrow B, AB \rightarrow C\}$.

ChatGPT said:

Let's apply the canonical cover algorithm step by step to the functional dependencies $F = \{A \rightarrow BC, B \rightarrow C, A \rightarrow B, AB \rightarrow C\}$ for the relation $R(A, B, C)$.

step-1-combine-fds-with-the-same-left-side

Step 1: Combine FDs with the Same Left Side

First, we look for functional dependencies with the same left-hand side and combine them if necessary. In this case:

- The FDs $A \rightarrow BC$ and $A \rightarrow B$ have the same left-hand side, so we combine them: $A \rightarrow BC$.
- The FD $AB \rightarrow C$ doesn't have a matching left-hand side with other FDs.

After combining, we have:

$$F = \{A \rightarrow BC, B \rightarrow C, AB \rightarrow C\}$$

$$F = \{A \rightarrow BC, B \rightarrow C, AB \rightarrow C\}$$

step-2-remove-extraneous-attributes-from-the-left-side

Step 2: Remove Extraneous Attributes from the Left Side

We now check for extraneous attributes on the left side of the functional dependencies.

1. FD $A \rightarrow BC$ $A \rightarrow BC$:

- We check if AA is extraneous in $A \rightarrow BC$ $A \rightarrow BC$. To do this, compute the closure of A , which is empty. Thus, AA is not extraneous.

2. FD $B \rightarrow C$ $B \rightarrow C$:

- The left side BB has only one attribute, so no extraneous attributes are possible here.

3. FD $AB \rightarrow C$ $AB \rightarrow C$:

- Check if AA is extraneous in $AB \rightarrow C$ $AB \rightarrow C$. To check, compute the closure of B under F :
 - From $B \rightarrow C$ $B \rightarrow C$, we get $\{B\}^+ = \{B, C\}$ $\{B\}^+ = \{B, C\}$, but CC cannot determine AA .
 - Therefore, AA is not extraneous.
- Check if BB is extraneous in $AB \rightarrow C$ $AB \rightarrow C$. To check, compute the closure of A under F :
 - From $A \rightarrow BC$ $A \rightarrow BC$, we get $\{A\}^+ = \{A, B, C\}$ $\{A\}^+ = \{A, B, C\}$, which includes CC .
 - Therefore, BB is extraneous in this FD.
 - We can simplify $AB \rightarrow C$ $AB \rightarrow C$ to $A \rightarrow C$ $A \rightarrow C$.

Now the set of dependencies is:

$$F = \{A \rightarrow BC, B \rightarrow C, A \rightarrow C\}$$

$$F = \{A \rightarrow BC, B \rightarrow C, A \rightarrow C\}$$

step-3-remove-extraneous-attributes-from-the-right-side-1

Step 3: Remove Extraneous Attributes from the Right Side

Next, we check if there are extraneous attributes on the right side of the functional dependencies.

1. FD $A \rightarrow BC$ $\mathbf{A \rightarrow BC}$:

- Check if BB is extraneous in $A \rightarrow BC$ $\mathbf{A \rightarrow BC}$. To do this, compute the closure of AA under $F' = F - \{A \rightarrow BC\} + \{A \rightarrow C\}$ $F = F \{A \rightarrow BC\} + \{A \rightarrow C\}$:
 - From $A \rightarrow C$ $\mathbf{A \rightarrow C}$, we get $\{A\}^+ = \{A, C\}$ $\{A\}^+ = \{A, C\}$, but we do not get BB .
 - Therefore, BB is not extraneous.
- Check if CC is extraneous in $A \rightarrow BC$ $\mathbf{A \rightarrow BC}$. Compute the closure of AA under $F' = F - \{A \rightarrow BC\} + \{A \rightarrow B\}$ $F = F \{A \rightarrow BC\} + \{A \rightarrow B\}$:
 - From $A \rightarrow B$ $\mathbf{A \rightarrow B}$, we get $\{A\}^+ = \{A, B\}$ $\{A\}^+ = \{A, B\}$, but we do not get CC .
 - Therefore, CC is not extraneous.

2. FD $B \rightarrow C$ $\mathbf{B \rightarrow C}$:

- The right side CC has only one attribute, so no extraneous attributes are possible.

3. FD $A \rightarrow C$ $\mathbf{A \rightarrow C}$:

- The right side CC has only one attribute, so no extraneous attributes are possible.

step-4-final-canonical-cover

Step 4: Final Canonical Cover

The final canonical cover is:

$$\{A \rightarrow BC, B \rightarrow C, A \rightarrow C\}$$

$$\{A \rightarrow BC, B \rightarrow C, A \rightarrow C\}$$

This is the minimal set of functional dependencies representing the original set.
you-said-2

C is extraneous in $A \rightarrow BC$ using $A \rightarrow B$ closure has AB then from $B \rightarrow C$ the closure becomes ABC thus C is extraneous

ChatGPT said:

You're absolutely correct! I missed that key point when checking for extraneous attributes in the right side of the dependency $A \rightarrow BC$. Let's correct that step and go over the solution again.

step-by-step-application-of-the-canonical-cover-algorithm

Step-by-Step Application of the Canonical Cover Algorithm

We are working with the functional dependencies:

$$F = \{A \rightarrow BC, B \rightarrow C, A \rightarrow B, AB \rightarrow C\}$$

$$F = \{A \rightarrow BC, B \rightarrow C, A \rightarrow B, AB \rightarrow C\}$$

step-1-combine-fds-with-the-same-left-side-1

Step 1: Combine FDs with the Same Left Side

There are two functional dependencies with the same left-hand side:

- $A \rightarrow BC$ and $A \rightarrow B$ can be combined into $A \rightarrow BC$ (since $BCBC$ already includes BB).

So, we combine them, leaving us with the set of functional dependencies:

$$F = \{A \rightarrow BC, B \rightarrow C, AB \rightarrow C\}$$

$$F = \{A \rightarrow BC, B \rightarrow C, AB \rightarrow C\}$$

step-2-remove-extraneous-attributes-from-the-left-side-1

Step 2: Remove Extraneous Attributes from the Left Side

Next, we check for extraneous attributes on the left-hand side of each functional dependency.

1. FD $A \rightarrow BC$:

- The left-hand side only has one attribute A , so no extraneous attributes are present.

2. **FD** $B \rightarrow C \mathbf{B} \rightarrow \mathbf{C}$:

- The left-hand side only has one attribute BB , so no extraneous attributes are present.

3. **FD** $AB \rightarrow C \mathbf{A} \mathbf{B} \rightarrow \mathbf{C}$:

- Check if AA is extraneous. To check this, we compute the closure of BB under the current set of dependencies:
 - From $B \rightarrow C \mathbf{B} \rightarrow \mathbf{C}$, we get $\{B\}^+ = \{B, C\}$, but we do not get AA .
 - Therefore, AA is not extraneous.
- Check if BB is extraneous. To check this, compute the closure of AA under the current set of dependencies:
 - From $A \rightarrow BC \mathbf{A} \rightarrow \mathbf{BC}$, we get $\{A\}^+ = \{A, B, C\}$, which already includes CC .
 - Therefore, BB is extraneous.
- We can simplify $AB \rightarrow C \mathbf{A} \mathbf{B} \rightarrow \mathbf{C}$ to $A \rightarrow C \mathbf{A} \rightarrow \mathbf{C}$.

Now the set of functional dependencies becomes:

$$F = \{A \rightarrow BC, B \rightarrow C, A \rightarrow C\}$$

$$F = \{A \rightarrow BC, B \rightarrow C, A \rightarrow C\}$$

step-3-remove-extraneous-attributes-from-the-right-side-2

Step 3: Remove Extraneous Attributes from the Right Side

Now, we check for extraneous attributes on the right-hand side of the functional dependencies.

1. **FD** $A \rightarrow BC \mathbf{A} \rightarrow \mathbf{BC}$:

- Check if CC is extraneous in $A \rightarrow BC \mathbf{A} \rightarrow \mathbf{BC}$. To check this, compute the closure of AA under the current set of dependencies (without $A \rightarrow BC \mathbf{A} \rightarrow \mathbf{BC}$):

- From $A \rightarrow B$ and $B \rightarrow C$, we get $\{A\}^+ = \{A, B, C\}$.
- Therefore, C is extraneous, and we can simplify $A \rightarrow BC$ to $A \rightarrow B$.

2. **FD** $B \rightarrow C$:

- The right side has only one attribute C , so no extraneous attributes are present.

3. **FD** $A \rightarrow C$:

- The right side has only one attribute C , so no extraneous attributes are present.

step-4-final-canonical-cover-1

Step 4: Final Canonical Cover

After applying the algorithm, the final canonical cover is:

$$\{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$$

$\{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$

This is the minimal set of functional dependencies representing the original set F .

you-said-3

are you sure this is the end of algorithm?

ChatGPT said:

You're right to ask. The canonical cover algorithm involves iterating over the functional dependencies to ensure no extraneous attributes remain in both the left and right sides of the dependencies. While the steps I've described earlier brought us close, we should ensure the process is fully complete.

Let's review where we are and see if any further optimization can be done.

current-fds-after-simplification

Current FDs After Simplification:

After applying the simplifications mentioned:

$$F = \{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$$

$F = \{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$

We need to check again for extraneous attributes and ensure the set is minimal.

step-1-check-for-extraneous-attributes-in-left-hand-side

Step 1: Check for Extraneous Attributes in Left-Hand Side

Let's review the current set of FDs to see if we can further simplify them.

1. **FD** $A \rightarrow B$ **A**→**B**:

- The left-hand side only has one attribute, so no extraneous attributes can be removed.

2. **FD** $B \rightarrow C$ **B**→**C**:

- The left-hand side only has one attribute, so no extraneous attributes can be removed.

3. **FD** $A \rightarrow C$ **A**→**C**:

- The left-hand side only has one attribute, so no extraneous attributes can be removed.

step-2-check-for-extraneous-attributes-in-right-hand-side

Step 2: Check for Extraneous Attributes in Right-Hand Side

Next, we check the right-hand side of the current FDs:

1. **FD** $A \rightarrow B$ **A**→**B**:

- The right-hand side has only one attribute, so no extraneous attributes can be removed.

2. **FD** $B \rightarrow C$ **B**→**C**:

- The right-hand side has only one attribute, so no extraneous attributes can be removed.

3. **FD** $A \rightarrow C$ **A**→**C**:

- The right-hand side has only one attribute, so no extraneous attributes can be removed.

step-3-check-for-redundant-dependencies

Step 3: Check for Redundant Dependencies

The final step is to check if any of the dependencies are implied by others, meaning they are redundant.

1. Is $A \rightarrow C$ implied by $A \rightarrow B$ and $B \rightarrow C$?

- Compute the closure of A using $A \rightarrow B$ and $B \rightarrow C$:
 - From $A \rightarrow B$, we get $\{A\}^+ = \{A, B\}$.
 - From $B \rightarrow C$, we get $\{A\}^+ = \{A, B, C\}$.
 - Since $A \rightarrow C$ can be derived from $A \rightarrow B$ and $B \rightarrow C$, $A \rightarrow C$ is redundant.

Thus, we can **remove** $A \rightarrow C$ because it is implied by the other two dependencies.

Final Canonical Cover

The final, minimal set of functional dependencies is:

$$F = \{A \rightarrow B, B \rightarrow C\}$$

$F = \{A \rightarrow B, B \rightarrow C\}$

This is the canonical cover for the given functional dependencies.

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