

## 0.13 Canonical Cover 2

you-said

This is the algorithm to find extraneous attributes in a set of functional dependencies  $F$ :  
 Given  $\rightarrow : \bullet$   $a$  is extraneous if  $(- a) +$  under  $F$   $\bullet$   $a$  is extraneous if  $a +$  under  $F' = (F - \{ \rightarrow \}) \cup \{ \rightarrow (- a) \}$   
 The algorithm of Closure of  $\rightarrow$  under  $F (+)$  is:  $result :=$   
 Do  $result\_previous = result$  For each  $\rightarrow$  in  $F$ : If  $(- result)$ :  $result := result \cup$  While  $(result\_previous \neq result)$  Return  $result$   
 Show if  $E$  is an extraneous attribute in  $\{C, E\} \rightarrow \{A\}$  for functional dependencies  $F = \{ \{C, E\} \rightarrow \{A\}, \{C\} \rightarrow \{D\}, \{A\} \rightarrow \{B\}, \{D\} \rightarrow \{B, E\}, \{B\} \rightarrow \{F\}, \{A, D\} \rightarrow \{C, F\} \}$ .

ChatGPT said:

To determine whether  $EE$  is an extraneous attribute in  $\{C, E\} \rightarrow \{A\}$  for the given set of functional dependencies  $FF$ , we will follow the steps outlined in the algorithm for finding extraneous attributes.

step-1-checking-if-e-in- $\{C, E\}$  is extraneous on the left-hand side

**Step 1: Checking if  $E \in \{C, E\}$  is extraneous on the left-hand side**  
**1: Checking if  $E \in \{C, E\}$  is extraneous on the left-hand side**

To check whether  $EE$  is extraneous from the left-hand side of the dependency  $\{C, E\} \rightarrow \{A\}$ , we need to determine if  $A \subseteq \{C\}^+ A$  (i.e., whether we can still derive  $A$  using only  $CC$ ) under  $FF$ .

step-1.1-calculate-the-closure-of- $\{C\}$  under  $FF$  denoted as  $CC$ .

Given functional dependencies:

- $\{C, E\} \rightarrow \{A\}$
- $\{C\} \rightarrow \{D\}$
- $\{A\} \rightarrow \{B\}$
- $\{D\} \rightarrow \{B, E\}$
- $\{B\} \rightarrow \{F\}$
- $\{A, D\} \rightarrow \{C, F\}$

Let's compute the closure  $\{C\}^+$  under  $FF$ .

1. Start with  $\{C\}^+ = \{C\}$ .

2. Using  $\{C\} \rightarrow \{D\}$   $\{C\} \rightarrow \{D\}$ , add  $DD$  to the closure:  $\{C\}^+ = \{C, D\}$   $\{C\}^+ = \{C, D\}$ .
3. Using  $\{D\} \rightarrow \{B, E\}$   $\{D\} \rightarrow \{B, E\}$ , add  $BB$  and  $EE$ :  $\{C\}^+ = \{C, D, B, E\}$   $\{C\}^+ = \{C, D, B, E\}$ .
4. Using  $\{B\} \rightarrow \{F\}$   $\{B\} \rightarrow \{F\}$ , add  $FF$ :  $\{C\}^+ = \{C, D, B, E, F\}$   $\{C\}^+ = \{C, D, B, E, F\}$ .
5. There are no other dependencies that apply at this point.

At this point,  $\{C\}^+ = \{C, D, B, E, F\}$   $\{C\}^+ = \{C, D, B, E, F\}$ , but  $AA$  is not included in  $\{C\}^+ \{C\}^+$ .

conclusion-for-step-1

Since  $A \notin \{C\}^+ A / \{C\}^+$ ,  $EE$  is **not extraneous** on the left-hand side of the dependency  $\{C, E\} \rightarrow \{A\}$   $\{C, E\} \rightarrow \{A\}$ .

step-2-checking-if-a-in-aaa-is-extraneous-on-the-right-hand-side

**Step 2: Checking if  $A \in \{A\} A \{A\}$  is extraneous on the right-hand side**  
**Checking if  $A \in \{A\} A \{A\}$  is extraneous on the right-hand side**

Since  $AA$  is the only attribute on the right-hand side of the dependency  $\{C, E\} \rightarrow \{A\}$   $\{C, E\} \rightarrow \{A\}$ , it cannot be extraneous on the right-hand side.

final-conclusion

**Final Conclusion:**

$EE$  is **not extraneous** in the functional dependency  $\{C, E\} \rightarrow \{A\}$   $\{C, E\} \rightarrow \{A\}$  under the given set of functional dependencies  $FF$ .

you-said-1

This is the algorithm to find extraneous attributes in a set of functional dependencies  $F$ : Given  $\rightarrow : \bullet a$  is extraneous if  $(- a)^+$  under  $F \bullet a$  is extraneous if  $a^+ +$  under  $F' = (F - \{ \rightarrow \}) \cup \{ \rightarrow (- a) \}$  The algorithm of Closure of under  $F (+)$  is: result := Do result\_previous = result For each  $\rightarrow$  in  $F$ : If ( result): result := result Union While (result\_previous  $\neq$  result) Return result Show if  $F$  is an extraneous attribute in  $\{A, D\} \rightarrow \{C, F\}$  for functional dependencies  $F = \{\{C, E\} \rightarrow \{A\}, \{C\} \rightarrow \{D\}, \{A\} \rightarrow \{B\}, \{D\} \rightarrow \{B, E\}, \{B\} \rightarrow \{F\}, \{A, D\} \rightarrow \{C, F\}\}$ .

ChatGPT said:

To determine whether  $FF$  is an extraneous attribute in  $\{A, D\} \rightarrow \{C, F\}$   $\{A, D\} \rightarrow \{C, F\}$  for the given set of functional dependencies  $FF$ , we will follow the steps from the algorithm to identify extraneous attributes.

step-1-checking-if-f-in-ffcf-is-extraneous-on-the-right-hand-side

**Step 1: Checking if  $F \in \{C, F\}^+ F\{C, F\}$  is extraneous on the right-hand side**  
**1: Checking if  $F \in \{C, F\}^+ F\{C, F\}$  is extraneous on the right-hand side**

To check whether  $FF$  is extraneous from the right-hand side of the dependency  $\{A, D\} \rightarrow \{C, F\}$  we need to see if  $F \in \{A, D\}^+ F\{A, D\}^+$  under the modified set of functional dependencies  $F' = (F - \{\{A, D\} \rightarrow \{C, F\}\}) \cup \{\{A, D\} \rightarrow \{C\}\}$ .

step-1.1-create-the-modified-set-fprimef

Remove  $\{A, D\} \rightarrow \{C, F\}$  and add  $\{A, D\} \rightarrow \{C\}$ . The new set  $F'$  is:

- $\{C, E\} \rightarrow \{A\}$
- $\{C\} \rightarrow \{D\}$
- $\{A\} \rightarrow \{B\}$
- $\{D\} \rightarrow \{B, E\}$
- $\{B\} \rightarrow \{F\}$
- $\{A, D\} \rightarrow \{C\}$

step-1.2-calculate-the-closure-adad-under-fprimef

1. Start with  $\{A, D\}^+ = \{A, D\}$ .
2. Using  $\{A, D\} \rightarrow \{C\}$ , add  $CC$ :  $\{A, D\}^+ = \{A, D, C\}$ .
3. Using  $\{A\} \rightarrow \{B\}$ , add  $BB$ :  $\{A, D\}^+ = \{A, D, C, B\}$ .
4. Using  $\{B\} \rightarrow \{F\}$ , add  $FF$ :  $\{A, D\}^+ = \{A, D, C, B, F\}$ .

conclusion-for-step-1-1

Since  $F \in \{A, D\}^+ F\{A, D\}^+$ , we can still derive  $FF$  without it explicitly being on the right-hand side of the functional dependency. Therefore,  $FF$  is **extraneous** on the right-hand side of  $\{A, D\} \rightarrow \{C, F\}$ .

final-conclusion-1

**Final Conclusion:**

$FF$  is **extraneous** in the functional dependency  $\{A, D\} \rightarrow \{C, F\}$   $\{A, D\} \rightarrow \{C, F\}$  under the given set of functional dependencies  $FF$ .

you-said-2

Replace all functional dependencies of type  $\rightarrow 1$  and  $\rightarrow 2$  by  $\rightarrow 1 \cup 2$  in  $F = \{\{C, E\} \rightarrow \{A\}, \{C\} \rightarrow \{D\}, \{A\} \rightarrow \{B\}, \{D\} \rightarrow \{B, E\}, \{B\} \rightarrow \{F\}, \{A, D\} \rightarrow \{C, F\}\}$

ChatGPT said:

To replace functional dependencies of the form  $\alpha \rightarrow \beta_1 \rightarrow 1$  and  $\alpha \rightarrow \beta_2 \rightarrow 2$  with  $\alpha \rightarrow \beta_1 \cup \beta_2 \rightarrow 1$  2, we need to identify where the same left-hand side  $\alpha$  appears in multiple functional dependencies and combine the corresponding right-hand sides.

Given the set of functional dependencies:

$F = \{\{C, E\} \rightarrow \{A\}, \{C\} \rightarrow \{D\}, \{A\} \rightarrow \{B\}, \{D\} \rightarrow \{B, E\}, \{B\} \rightarrow \{F\}, \{A, D\} \rightarrow \{C, F\}\}$   $F = \{\{C, E\} \rightarrow \{A\}, \{C\} \rightarrow \{D\}, \{A\} \rightarrow \{B\}, \{D\} \rightarrow \{B, E\}, \{B\} \rightarrow \{F\}, \{A, D\} \rightarrow \{C, F\}\}$

We'll group and merge the dependencies where the left-hand side is the same.

step-1-identify-functional-dependencies-with-the-same-left-hand-side

**Step 1: Identify functional dependencies with the same left-hand side**

- $\{C, E\} \rightarrow \{A\}$   $\{C, E\} \rightarrow \{A\}$  (no need to merge, unique left-hand side).
- $\{C\} \rightarrow \{D\}$   $\{C\} \rightarrow \{D\}$  (no need to merge, unique left-hand side).
- $\{A\} \rightarrow \{B\}$   $\{A\} \rightarrow \{B\}$  (no need to merge, unique left-hand side).
- $\{D\} \rightarrow \{B, E\}$   $\{D\} \rightarrow \{B, E\}$  (no need to merge, already combined).
- $\{B\} \rightarrow \{F\}$   $\{B\} \rightarrow \{F\}$  (no need to merge, unique left-hand side).
- $\{A, D\} \rightarrow \{C, F\}$   $\{A, D\} \rightarrow \{C, F\}$  (no need to merge, unique left-hand side).

step-2-combine-the-dependencies-if-necessary

**Step 2: Combine the dependencies (if necessary)**

In this case, there are no instances where the same left-hand side appears in different functional dependencies, except for  $\{D\} \rightarrow \{B\}$   $\{D\} \rightarrow \{B\}$  and  $\{D\} \rightarrow \{E\}$   $\{D\} \rightarrow \{E\}$ , which is already combined into  $\{D\} \rightarrow \{B, E\}$   $\{D\} \rightarrow \{B, E\}$ .

final-result

**Final result:**

The set of functional dependencies after applying the merging rule remains the same, as all the relevant dependencies are already in the merged form:

$$F = \{\{C, E\} \rightarrow \{A\}, \{C\} \rightarrow \{D\}, \{A\} \rightarrow \{B\}, \{D\} \rightarrow \{B, E\}, \{B\} \rightarrow \{F\}, \{A, D\} \rightarrow \{C, F\}\}$$

$$F = \{\{C, E\} \rightarrow \{A\}, \{C\} \rightarrow \{D\}, \{A\} \rightarrow \{B\}, \{D\} \rightarrow \{B, E\}, \{B\} \rightarrow \{F\}, \{A, D\} \rightarrow \{C, F\}\}$$

you-said-3

is C an extraneous attribute in  $\{C, E\} \rightarrow \{A\}$  using the algorithm

2/2

ChatGPT said:

To determine whether  $CC$  is an extraneous attribute in  $\{C, E\} \rightarrow \{A\}$  using the algorithm, we need to follow the steps outlined for finding extraneous attributes.

step-1-checking-if-c-in-ccc-is-extraneous-on-the-left-hand-side

**Step 1: Checking if  $C \in \{C, E\}$  is extraneous on the left-hand side**

**1: Checking if  $C \in \{C, E\}$  is extraneous on the left-hand side**

To check whether  $CC$  is extraneous from the left-hand side of the dependency  $\{C, E\} \rightarrow \{A\}$ , we need to determine if  $A \subseteq \{E\}^+ A$  (i.e., whether we can still derive  $AA$  using only  $EE$ ) under the set of functional dependencies  $FF$ .

given-set-of-functional-dependencies

**Given set of functional dependencies:**

$$F = \{\{C, E\} \rightarrow \{A\}, \{C\} \rightarrow \{D\}, \{A\} \rightarrow \{B\}, \{D\} \rightarrow \{B, E\}, \{B\} \rightarrow \{F\}, \{A, D\} \rightarrow \{C, F\}\}$$

$$F = \{\{C, E\} \rightarrow \{A\}, \{C\} \rightarrow \{D\}, \{A\} \rightarrow \{B\}, \{D\} \rightarrow \{B, E\}, \{B\} \rightarrow \{F\}, \{A, D\} \rightarrow \{C, F\}\}$$

step-1.1-calculate-the-closure-of-ee-under-ff-denoted-as-ee

**Step 1.1: Calculate the closure of  $\{E\}$  under  $FF$  (denoted as  $\{E\}^+$ )**

**1.1: Calculate the closure of  $\{E\}$  under  $FF$  (denoted as  $\{E\}^+$ )**

1. Start with  $\{E\}^+ = \{E\}$ .
2. No functional dependencies directly involve  $EE$  alone on the left-hand side.
3. The only dependency that involves  $EE$  in the left-hand side is  $\{C, E\} \rightarrow \{A\}$ , but this requires both  $CC$  and  $EE$  to derive  $AA$ .
4. No other functional dependencies apply.

Thus,  $\{E\}^+ = \{E\}$ .

Since  $A \notin \{E\}^+ A$ , we cannot derive  $AA$  from  $EE$  alone.

conclusion-for-step-1-2

**Conclusion for Step 1:**

Since  $A \notin \{E\}^+A/\{E\}^+$ ,  $CC$  is **not extraneous** on the left-hand side of the dependency  $\{C, E\} \rightarrow \{A\} \{C, E\} \rightarrow \{A\}$ .

final-conclusion-2

**Final Conclusion:**

$CC$  is **not extraneous** in the functional dependency  $\{C, E\} \rightarrow \{A\} \{C, E\} \rightarrow \{A\}$  under the given set of functional dependencies  $FF$ .

you-said-4

is  $E$  an extraneous attribute in  $\{C, E\} \rightarrow \{A\}$  using the algorithm for functional dependencies  $F = \{\{C, E\} \rightarrow \{A\}, \{C\} \rightarrow \{D\}, \{A\} \rightarrow \{B\}, \{D\} \rightarrow \{B, E\}, \{B\} \rightarrow \{F\}, \{A, D\} \rightarrow \{C\}\}$

ChatGPT said:

To determine whether  $EE$  is an extraneous attribute in  $\{C, E\} \rightarrow \{A\} \{C, E\} \rightarrow \{A\}$  using the algorithm, we will follow the steps for checking extraneous attributes in the left-hand side of a functional dependency.

step-1-checking-if-e-in-cc-ee-is-extraneous-on-the-left-hand-side-1

**Step 1: Checking if  $E \in \{C, E\} \{C, E\}$  is extraneous on the left-hand side**  
**1: Checking if  $E \in \{C, E\} \{C, E\}$  is extraneous on the left-hand side**

To check whether  $EE$  is extraneous from the left-hand side of the dependency  $\{C, E\} \rightarrow \{A\} \{C, E\} \rightarrow \{A\}$ , we need to determine if  $A \subseteq \{C\}^+A/\{C\}^+$  (i.e., whether we can derive  $AA$  using only  $CC$ ) under the set of functional dependencies  $FF$ .

given-set-of-functional-dependencies-1

**Given set of functional dependencies:**

$$F = \{\{C, E\} \rightarrow \{A\}, \{C\} \rightarrow \{D\}, \{A\} \rightarrow \{B\}, \{D\} \rightarrow \{B, E\}, \{B\} \rightarrow \{F\}, \{A, D\} \rightarrow \{C\}\}$$

$$F = \{\{C, E\} \rightarrow \{A\}, \{C\} \rightarrow \{D\}, \{A\} \rightarrow \{B\}, \{D\} \rightarrow \{B, E\}, \{B\} \rightarrow \{F\}, \{A, D\} \rightarrow \{C\}\}$$

step-1.1-calculate-the-closure-of-cc-under-ff-denoted-as-cc

**Step 1.1: Calculate the closure of  $\{C\} \{C\}$  under  $FF$  (denoted as  $\{C\}^+ \{C\}^+$ )****1.1: Calculate the closure of  $\{C\} \{C\}$  under  $FF$  (denoted as  $\{C\}^+ \{C\}^+$ )**

1. Start with  $\{C\}^+ = \{C\} \{C\}^+ = \{C\}$ .
2. Using  $\{C\} \rightarrow \{D\} \{C\} \rightarrow \{D\}$ , add  $DD$  to the closure:  $\{C\}^+ = \{C, D\} \{C\}^+ = \{C, D\}$ .
3. Using  $\{D\} \rightarrow \{B, E\} \{D\} \rightarrow \{B, E\}$ , add  $BB$  and  $EE$ :  $\{C\}^+ = \{C, D, B, E\} \{C\}^+ = \{C, D, B, E\}$ .
4. Using  $\{B\} \rightarrow \{F\} \{B\} \rightarrow \{F\}$ , add  $FF$ :  $\{C\}^+ = \{C, D, B, E, F\} \{C\}^+ = \{C, D, B, E, F\}$ .

5. There are no other applicable functional dependencies at this point.

At this point,  $\{C\}^+ = \{C, D, B, E, F\}$ , but **AA is not in**  $\{C\}^+ \{C\}^+$ .  
conclusion-for-step-1-3

### Conclusion for Step 1:

Since  $A \notin \{C\}^+ A / \{C\}^+$ , we cannot derive **AA** from **CC** alone. Therefore, **EE** is **not extraneous** on the left-hand side of the dependency  $\{C, E\} \rightarrow \{A\}$ .  
final-conclusion-3

### Final Conclusion:

**EE** is **not extraneous** in the functional dependency  $\{C, E\} \rightarrow \{A\}$  under the given set of functional dependencies **FF**.  
you-said-5

At this point,  $\{C\}^+ = \{C, D, B, E, F\}$ , using  $\{C, E\} \rightarrow \{A\}$  will add **A** in the closure

ChatGPT said:

You are correct! Let's correct the approach.

We are checking if **EE** is an extraneous attribute in the functional dependency  $\{C, E\} \rightarrow \{A\}$ . Here's how to proceed step-by-step:

step-1-checking-if-e-in-ee-is-extraneous-on-the-left-hand-side-2

### Step 1: Checking if $E \in \{C, E\}$ is extraneous on the left-hand side

#### 1: Checking if $E \in \{C, E\}$ is extraneous on the left-hand side

To check whether **EE** is extraneous from the left-hand side of the dependency  $\{C, E\} \rightarrow \{A\}$ , we need to determine if  $A \in \{C\}^+ A$  (i.e., whether we can derive **AA** using only **CC**) under the set of functional dependencies **FF**.

given-functional-dependencies

### Given functional dependencies:

$$F = \{\{C, E\} \rightarrow \{A\}, \{C\} \rightarrow \{D\}, \{A\} \rightarrow \{B\}, \{D\} \rightarrow \{B, E\}, \{B\} \rightarrow \{F\}, \{A, D\} \rightarrow \{C\}\}$$

$$F = \{\{C, E\} \rightarrow \{A\}, \{C\} \rightarrow \{D\}, \{A\} \rightarrow \{B\}, \{D\} \rightarrow \{B, E\}, \{B\} \rightarrow \{F\}, \{A, D\} \rightarrow \{C\}\}$$

step-1.1-calculate-the-closure-of-cc-under-ff

### Step 1.1: Calculate the closure of $\{C\}$ under **FF**

#### closure of $\{C\}$ under **FF**

We will compute the closure  $\{C\}^+ \{C\}^+$  to check if  $A \in \{C\}^+ A \{C\}^+$ .

1. Start with  $\{C\}^+ = \{C\}$ .

2. Using  $\{C\} \rightarrow \{D\}$   $\{C\} \rightarrow \{D\}$ , add  $DD$  to the closure:  $\{C\}^+ = \{C, D\}$   $\{C\}^+ = \{C, D\}$ .
3. Using  $\{D\} \rightarrow \{B, E\}$   $\{D\} \rightarrow \{B, E\}$ , add  $BB$  and  $EE$ :  $\{C\}^+ = \{C, D, B, E\}$   $\{C\}^+ = \{C, D, B, E\}$ .
4. Using  $\{B\} \rightarrow \{F\}$   $\{B\} \rightarrow \{F\}$ , add  $FF$ :  $\{C\}^+ = \{C, D, B, E, F\}$   $\{C\}^+ = \{C, D, B, E, F\}$ .
5. Now, since  $\{C, E\} \rightarrow \{A\}$   $\{C, E\} \rightarrow \{A\}$  is applicable (with  $EE$  already in the closure), add  $AA$ :  $\{C\}^+ = \{C, D, B, E, F, A\}$   $\{C\}^+ = \{C, D, B, E, F, A\}$ .

Thus,  $A \in \{C\}^+ A \{C\}^+$ .  
 conclusion-for-step-1-4

### Conclusion for Step 1:

Since  $A \in \{C\}^+ A \{C\}^+$ , we can derive  $AA$  from  $CC$  alone, without needing  $EE$ . Therefore,  $EE$  is **extraneous** on the left-hand side of the dependency  $\{C, E\} \rightarrow \{A\}$   $\{C, E\} \rightarrow \{A\}$ .  
 final-conclusion-4

### Final Conclusion:

$EE$  is **extraneous** in the functional dependency  $\{C, E\} \rightarrow \{A\}$   $\{C, E\} \rightarrow \{A\}$  under the given set of functional dependencies  $FF$ .

you-said-6

now what are the functional dependencies now after removing all the extraneous attributes

ChatGPT said:

After identifying that  $EE$  is extraneous in the functional dependency  $\{C, E\} \rightarrow \{A\}$   $\{C, E\} \rightarrow \{A\}$ , we can remove  $EE$  from that dependency. The new functional dependency will be  $\{C\} \rightarrow \{A\}$   $\{C\} \rightarrow \{A\}$ .

Now, let's update the set of functional dependencies after removing the extraneous attribute:

original-set-of-functional-dependencies

### Original set of functional dependencies:

$$F = \{\{C, E\} \rightarrow \{A\}, \{C\} \rightarrow \{D\}, \{A\} \rightarrow \{B\}, \{D\} \rightarrow \{B, E\}, \{B\} \rightarrow \{F\}, \{A, D\} \rightarrow \{C\}\}$$

$$F = \{\{C, E\} \rightarrow \{A\}, \{C\} \rightarrow \{D\}, \{A\} \rightarrow \{B\}, \{D\} \rightarrow \{B, E\}, \{B\} \rightarrow \{F\}, \{A, D\} \rightarrow \{C\}\}$$

updated-functional-dependencies

### Updated functional dependencies:

- We replace  $\{C, E\} \rightarrow \{A\}$   $\{C, E\} \rightarrow \{A\}$  with  $\{C\} \rightarrow \{A\}$   $\{C\} \rightarrow \{A\}$ , as  $EE$  is extraneous.



- The rest of the functional dependencies remain unchanged.

Thus, the updated set of functional dependencies is:

$$F' = \{\{C\} \rightarrow \{A, D\}, \{A\} \rightarrow \{B\}, \{D\} \rightarrow \{B, E\}, \{B\} \rightarrow \{F\}, \{A, D\} \rightarrow \{C\}\}$$

$$F = \{\{C\} \rightarrow \{A, D\}, \{A\} \rightarrow \{B\}, \{D\} \rightarrow \{B, E\}, \{B\} \rightarrow \{F\}, \{A, D\} \rightarrow \{C\}\}$$

Now we have the set  $F'F$ , with all extraneous attributes removed.

You've hit the Free plan limit for GPT-4o.

Responses will use another model until your limit resets after 11:08 PM.

Get Plus  
prompt-textarea

composer-background