

Notes on Decision Theory and Practice

02.229 - Decision Theory and Practice, 2019 Jan-April

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1 Course Information

1.1 Instructor

Zsombor Zoltan Meder

1.2 Office Hours

- Nothing fixed, just email him / go to his office
 - Note: He doesn't like email / respond fast to email

1.2.1 Zsombor's Classes, can find him after

Tue: 9-10 / 2.404???? Thu: 15-17 / 1.508 Fri: 11.30-13.30 / 1.508

1.3 Textbook

Loads, but main is Peterson, M.: An introduction to decision theory. Cambridge University Press, 2017.

1.4 Grading

Percentage	Component Name	Notes
35%	Midterm	W8; tested on W1-W6
35%	Case Study	Proposal 25/3, 2350; Due 28/4, 2359
20%	Reading Summaries	Weekly (12 total); lowest 3 dropped
10%	Short surprise quizzes	6 total; lowest 1 dropped

1.4.1 Reading Summaries

- Due after class every Monday
- Roughly 1 page (a little more or less is k)
- Grade $\in \{ 0, 1, 2 \}$

1.4.2 Case Study

3k-4k words, max 8k

Can be based on fiction

Feedback till 31/3

2 TODO Current Questions to Ask

2.1 TODO For Zsombor

2.1.1 TODO Can you trust someone to act against your interest?

Probs yes, but I wanna be sure

2.2 TODO For Not Zsombor

2.2.1 TODO What was Zsombor's Paradigm/Theory distinction thing?

2.2.2 TODO What's completeness again?

3 Normative Decision Theory

3.1 W1: Introduction. Decision trees.

3.1.1 Peterson CH1: Introduction

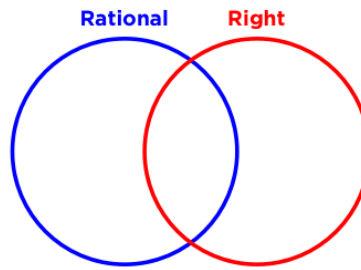
Term	Definition/Explanation	Notes
Risk	$\mathbb{P}(\text{outcomes})$ known	
Ignorance	$\mathbb{P}(\text{outcomes})$ unknown	
Uncertainty	$(\text{Ignorance}) \vee (\text{Risk} \cup \text{Ignorance})$	Context-dependent :(
Rational	Most reasonable outcome (ex ante)	I assume reasonability is based on available info
Right	Outcome is (at least weakly) pareto dominant. (ex post)	
Social Choice Theory	More than one decision maker	Some collective entities can be reduced to single decision makers (\therefore not SCT)

3.1.1.1 Terms

3.1.1.1.1 Self-Explanatory

- Decision-maker
- Set of alternatives
- True state of the world
- Outcome
- Principle of maximizing expected value

3.1.1.2 Right vs Rational Due to imperfect information, rationality does not necessarily correspond to rightness.



3.1.1.2.1 Irrational Right Story: Battle of Narva

1. Context

- 20 November 1700
- Sweden vs Russia, on border of Estonia
 - Note: Estonia did not exist back then
- King Carl of Sweden: 8,000 troops
- Tsar Peter the Great (Russia): ~80,000 troops
- No strategic reason for Sweden to attack (little to gain)

2. Story

- Sweden attacked (irrationally)
- Unexpected blizzard blinded Russian army
- Swedes won
- Battle ended <2h
- Swedes lost 667 men
- Russians lost ~15,000 men

3.1.1.3 Pragmatically, Normative Decision Theory > Descriptive Decision Theory Author claims that people behave rationally most of the time as they have good lives. Possibly flimsy argument (I don't like it).

3.1.1.3.1 My Problems with it

- **Rational \neq right**, as seen above. E.g. maybe good lives are due to instinct rather than rationally correct behaviour
- People aren't living close to their best possible lives. Most of their lives suck. IMO people operate on habit more than reason

3.1.1.4 Instrumental Rationality Presupposes an aim (which is external to decision theory)

3.1.1.4.1 Is this aim always rational?

- Widely thought that single aims cannot be evaluated in terms of rationality (though sets of aims can be irrational, e.g. inconsistent)
- John Rawls argues some aims are irrational (e.g. counting blades of grass on a courthouse lawn is too unimportant to be rational)
 - IMO, problematic argument. Importance varies according to values, values vary between people and even within the same person they are temporally inconsistent

3.1.1.5 Jean-Jacques Rousseau's Stag Hunt

	stag	rabbit
stag	5 , 5	0 , 3
rabbit	3 , 0	3 , 3

- Tension between risk minimization and outcome maximization
- Rational choice is solely and directly dependent on **trust**

3.1.1.6 History of Decision Theory

3.1.1.6.1 Period 1: Old Period (Ancient Greece)

- Normative decision examples instead of rules
- Followed by 1500 years of decision theory stagnation

3.1.1.6.2 Period 2: Pioneering Period (>1650s)

- Probability theory developed (Pascal and Fermat through letter correspondence)
- Some resistance by Catholic Church in normative moral theory (of course)
- 1738, **moral value** (now known as utility) was coined

3.1.1.6.3 Period 3: Axiomatic Period (>1920s)

- Attempt to make axioms from principles of rational decision making
- 1950s was a golden age for decision theory
 - Still highly relevant to today

3.1.2 Peterson CH2: The Decision Matrix

Notation	Term	Definition/Explanation
(square in decision tree)	Choice Node	-
(circle in decision tree)	Chance Node	-
π	<i>Formal decision problem</i> , $\pi = \langle A, S, O \rangle$	$\langle A, S, O \rangle =$ Acts, States, Outcomes
$t(\pi) \succeq \pi$	-	$t(\pi)$ is <i>at least</i> as reasonable as π
$t(\pi) \sim \pi$	-	$t(\pi)$ is <i>exactly</i> as reasonable as π
$a \circ b$	-	$(a \circ b)(\pi) = b(a(\pi))$
-	Transformative decision rule	Decision rule that modifies formalization of a decision problem
-	Effective decision rule	Filter that singles out some acts to produce a set of recommended acts
-	Rival formalizations	≥ 2 formalizations of same problem that are both 1. equally reasonable and 2. strictly better than other formalizations

3.1.2.1 Terms

3.1.2.1.1 Scales

Scale	Strictly increasing	Difference information	Ratio information	Allowed Transforms
Ordinal	Yes	No	No	+ve Monotone
Cardinal: Interval	Yes	Yes	No	+ve Linear
Cardinal: Ratio	Yes	Yes	Yes	+ve Scalar

By information we mean $h(f(a), f(b)) = h(f(c), f(d))$, where h is whatever information (e.g. difference)

1. **TODO** A scale is not a function, it s a collection of functions. [4/2/19] Zsombor sending definition soon... Need to read and understand

3.1.2.2 3 Transformative Decision Rules

1. Order-Independence (OI) If OI-condition holds for all $\pi \in \Pi$:
 - $(u \circ t)(\pi) = (t \circ u)(\pi)$
2. The Principle of Insufficient Reason (ir) If state probabilities are unknwon, π may be transformed into π' in which *equal probabilities are assigned to all states*
3. Merger of states (ms) If ≥ 2 states yield identical outcomes under all acts, they can be collapsed into one (with probabilities summed up, if known)

3.1.3 Gilboa CH1: Feasibility and Desirability

Can (feasibility) vs want (desirability)

3.1.3.1 No direct causal link Usually, straightforwardly independent

3.1.3.1.1 Zen and the Absurd (as in Camus Absurd) Under some cases (e.g. mathematicians who like challenges), feasibility itself appears to have direct negative causal link with desirability.

Author argues that the act of challenge is sought rather than the state of infeasibility, and thus the causal link still does not exist.

Violates Occam's razor, but makes sense

3.1.3.2 Uncertainty and Feasibility Feasibility of states need not propagate to feasibility of states

3.1.3.2.1 Example You can certainly perform the act try to solve a math problem for 2h, without being certain about whether the state of having it solved is indeed achievable)

3.1.3.3 Link is mediated by information

1. Example

- Desire: Buy strawberries
- Situation: End of day; 1 box of strawberries left over (∴ feasible)
- Thought process: Why did no one buy that last box? Does it suck?
- Decision: Don't buy, even though feasible, as feasibility

3.1.4 Class

Less Wrong

3.1.4.1 Decision Theory vs Social Choice Theory vs Game Theory (Zsombor's Distinctions)
Decision Theory is about single decision makers making decisions

SCT is about decision makers acting as a collective agent.

Game Theory is about players acting strategically (oppositionally almost)

3.1.4.2 Distinctions

3.1.4.2.1 For States

- Certainty / Uncertainty
- Possible / Impossible
- Desirable / Undesirable

3.1.4.2.2 For Acts

- Feasible / Infeasible

3.2 W2: Decision rules under uncertainty.

3.2.1 Peterson CH3: Decisions Under Ignorance

Term	Definition/Explanation	Notes
Dominance	(just pareto stuff)	Strong/weak dominance distinction

3.2.1.1 Effective Decision Rules

3.2.1.1.1 Maximin and Maximax (ordinal) They're the same, just at different extreme ends.

1. Maximin

Informal Principle of choosing the act with the largest minimal outcome obtainable

Formal $a_i \succeq a_j \iff \min(a_i) \geq \min(a_j)$

2. Maximax

Informal: Principle of choosing the act with the largest maximal outcome obtainable

Formal: $a_i \succeq a_j \iff \max(a_i) \geq \max(a_j)$

3.2.1.1.2 Further Constraints: Leximin and Leximin Modifications (ordinal) Below is for leximin. Leximax is just the opposite

1. Explanation

Description: Essentially a way to filter the dominance space from **maximin** (also an **effective decision rule**)

Procedure: Iteratively compare the next minimal outcomes under the states until you find a difference. Remove the act(s) with a lower outcome

Equivalence: The only remaining equivalent acts are acts which are equivalent in every state

2. Formal Definition

$a_i \succ a_j \iff$ there exists some positive integer n such that $\min^n(a_i) > \min^n(a_j)$ and $\min^m(a_i) = \min^m(a_j)$ for all $m < n$

3.2.1.1.3 Combination: Optimism-Pessimism Rule (a.k.a alpha-index rule) (cardinal) **Informal:** Weighted combination of maximin and maximax. Weight parameter α reflects optimism

Formal: $a_i \succeq a_j \iff \alpha \cdot \max(a_i) + (1 - \alpha) \cdot \min(a_i) \geq \alpha \cdot \max(a_j) + (1 - \alpha) \cdot \min(a_j)$

1. Random Thought You can probably combine with leximin/leximax to some degree too, to get an even better framework

3.2.1.1.4 Problem: Relevance of non-extreme values [all] Particularly easy to see when the mins are close. E.g. below.

Applying the maximin principle selects option a_2 , but intuitively a_1 seems far better.

Note: Example formulated for **maximin/leximin**. Just flip the logic for **maximax**

	s_1	s_2	s_3	s_4	s_5	s_6	s_7
a_1	1	0.99	99999	99999	99999	99999	99999
a_2	1	1	1	1	1	1	1

Pedantic Note: you don't know the probability distribution of the state space. Maybe a_2 is better after all...

3.2.1.1.5 Problem: Unintuitive equivalence [Minimax, Maximax] Note: Example formulated for **maximin**. Just flip the logic for **maximax**

	s_1	s_2
a_1	1	99999
a_2	1	1

Under vanilla **maximin**, both acts are equally reasonable. Obviously, this is weird

3.2.1.1.6 Minimax Regret

1. Explanation

- Essentially an attempt to formalize the concept of **regret**

2. Procedure (won't formally describe)

Before	a_1	12	8	20	20
	a_2	10	15	16	8
	a_3	30	6	25	14
	a_4	20	4	30	10
<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <div style="color: blue; font-weight: bold; font-size: 1.2em;">-30</div> <div style="color: blue; font-size: 2em;">↓</div> </div> <div style="text-align: center;"> <div style="color: blue; font-weight: bold; font-size: 1.2em;">-15</div> <div style="color: blue; font-size: 2em;">↓</div> </div> <div style="text-align: center;"> <div style="color: blue; font-weight: bold; font-size: 1.2em;">-30</div> <div style="color: blue; font-size: 2em;">↓</div> </div> <div style="text-align: center;"> <div style="color: blue; font-weight: bold; font-size: 1.2em;">-20</div> <div style="color: blue; font-size: 2em;">↓</div> </div> </div>					
After	a_1	-18	-7	-10	0
	a_2	-20	0	-14	-12
	a_3	0	-9	-5	-6
	a_4	-10	-11	0	-10

3. Note It's not globally accepted that this concept is relevant to rational decision making. But a substantial number of theorists think it is.

4. Problem: Argument from irrelevant alternatives

- Ranking can be altered by adding a non-optimal alternative
- Breaks intuition about how a "normatively plausible decision rule must not be sensitive to the addition of irrelevant alternatives"

(a) Example: Addition of a_5

Table 3.14

a_1	12	8	20	20
a_2	10	15	16	8
a_3	30	6	25	14
a_4	20	4	30	10
a_5	-10	10	10	39

Table 3.15

a_1	-18	-7	-10	-19
a_2	-20	0	-14	-31
a_2	0	-9	-5	-25
a_3	-10	-11	0	-29
a_5	-40	-5	-20	0

(b) Counter

- Prima facie intuition is wrong. It's "rational to compare alternatives with the entire set of alternatives".

3.2.1.2 Transformative Decision Rules

3.2.1.2.1 Principle of Insufficient Reason Pro: Decision under ignorance \rightarrow Decision under risk

1. Problem: Which states should be considered? Modeling problem

- More states means lower probability for each state (direct influence on choice strategy)
- Choosing relevant states is often not easy
- Traditional argument for ir is from symmetric states (e.g. dice sides). Many problems have no such symmetry

2. Problem: Uniform probability assumption seems arbitrary Under ignorance, any probability distribution *seems to be* equally justifiable as any other. Assumption of equality seems arbitrary

(a) Counter: Symmetry

- Assume every probability distribution is equally justifiable
- Use lens of toy 2-state case $S = \{s_1, s_2\}$
- Every probability distribution has a symmetric partner (e.g. $\{p_{s_1} = 0.6, p_{s_2} = 0.4\}$ has $\{p_{s_1} = 0.4, p_{s_2} = 0.6\}$
 - Exception: Uniform distribution. Suggests uniform distribution is a collapsed state of (in this case) 2 identical probability distributions. Making it multiplicatively more reasonable as any other case (in this case, 2x more reasonable)

i. Problem

- Beautiful argument, but the assumption of the uniform distribution being an additively collapsed one is a bit dubious imo
3. Problem: Practically, it's often not complete ignorance You generally know some things or at least have a sense of ordinal ranking for the probabilities of some of the states. Why not use it?
4. Problem: Further transformations

3.2.1.2.2 Randomized Acts

1. Procedure

- Create a new act with expected values as outcomes
- If your decision making strategy selects random act, then randomly choose one of those initial acts
- Note: Choosing the random act is not a choice on its own, but a procedure to arrive at an actual choice

(a) Example

Introduce random act a_3

Table 3.18

a_1	1	0
a_2	0	1

Table 3.19

a_1	1	0
a_2	0	1
a_3	1/2	1/2

2. Potential Problem

I'm assuming the random choice doesn't have to be uniformly distributed. But this opens up a whole can of worms by allowing you to tweak the probability distribution of the random function to bias it towards whatever choice you irrationally want.

3.2.1.3 Axiomatic Analysis of the Decision Rules Taken directly and shamelessly from the text-book

3.2.1.3.1 Descriptions of Axioms

1. **Ordering:** \succeq is transitive and complete. (See Chapter 5.)
2. **Symmetry:** The ordering imposed by \succeq is independent of the labeling of acts and states, so any two rows or columns in the decision matrix could be swapped.
3. **Strict Dominance:** If the outcome of one act is strictly better than the outcome of another under every state, then the former act is ranked above the latter.
4. **Continuity:** If one act weakly dominates another in a sequence of decision problems under ignorance, then this holds true also in the limit decision problem under ignorance.
5. **Interval scale:** The ordering imposed by \succeq remains unaffected by a positive linear transformation of the values assigned to outcomes.
6. **Irrelevant alternatives:** The ordering between old alternatives does not change if new alternatives are added to the decision problem.
7. **Column linearity:** The ordering imposed by \succeq does not change if a constant is added to a column.
8. **Column duplication:** The ordering imposed by \succeq does not change if an identical state (column) is added.
9. **Randomisation:** If two acts are equally valuable, then every randomisation between the two acts is also equally valuable.
10. **Special row adjunction:** Adding a weakly dominated act does not change the ordering of old acts.

3.2.1.3.2 Axiomatic Analysis

	Maximin	Optimism– pessimism	Minimax regret	Insufficient reason
1. Ordering	⊗	⊗	⊗	⊗
2. Symmetry	⊗	⊗	⊗	⊗
3. Strict dominance	⊗	⊗	⊗	⊗
4. Continuity	⊗	⊗	⊗	⊗
5. Interval scale	×	⊗	×	×
6. Irrelevant alternatives	⊗	⊗	–	⊗
7. Column linearity	–	–	⊗	⊗
8. Column duplication	⊗	⊗	⊗	–
9. Randomisation	⊗	–	⊗	×
10. Special row adjunction	×	×	⊗	×

Symbol	Meaning
–	Incompatible with decision rule
×	Compatible with decision rule
⊗	Necessary and sufficient for decision rule

3.2.2 Class

Term	Definition/Explanation	Notes
State	Event outside of DM's Control	Causally independent from acts
Act	A mapping of a state to an outcome	Under this definition, you may need a sequence of choices to constitute the act

3.2.2.1 Terms

3.2.2.1.1 Distinctions

- Preference (states)
- Dominance (acts)

Both use the \succeq signs

3.2.2.2 Why necessary untruths are sometimes included as states E.g. having two states: one for $2^3 = 8$ and one for $2^3 = 9$.

It's necessary sometimes because the actor may not have information about whether it's true or false

3.2.2.3 Misc

- Four Color Theorem: you can color any plane (e.g. a map) into any number of contiguous regions that

3.2.2.3.1 Some possible units for decision matrix

- \$
- Utility
- Value

3.3 W3: Probability theory and Bayes' Rule. Expected value maximization.

3.3.1 Lack of Summary of Notes

To be frank, I already know this material (mostly from the course 40.001 Probability). I did input a lot of it into anki, but I'd rather not expend the effort to summarize basic probability rules if that's okay.

I'm fine dropping this summary as one of my lowest scores. I don't know any of the other topics well enough that I'd want to skip doing the summary.

3.3.2 Class

3.3.2.1 John Rawles' Theory of Justice

- Social setups should be decided under the *veil of ignorance* (meaning you strip away your personal identity and could very well be anybody else)
 - He essentially leximin'd it in order to make society as good as possible for the worst stratum

3.3.2.2 Strategies for Guessing the Initial Prior (Bayes application)

- Guess
 - Optimism/Pessimism
 - Pivot
 - * Bets
- Comparable Events
- Principle of Insufficient Reason
- Previous Evidence

3.3.2.3 Value of Information

3.4 W4: Preferences and utility. Expected utility maximization.

3.4.1 Terminology

Term	Definition/Explanation	Notes
Terminal node	Decision tree, the leafs where payoffs are attached	
Payoff function	Function that maintains transitivity of pre-existing preference relations	
Degenerate lottery	A deterministic situation (probably pointlessly) modelled as a lottery	Probabilities of states arising based off chosen actions would be 1 or 0
Intensity	Essentially, magnitude	E.g. intensity of preference is not invariant to positive monotonic transforms (it's cardinal, not ordinal)
Risk neutral	Indifferent between a lottery and sure situation given that the expected payout is the same	
Risk averse	Strictly prefers a sure situation to a lottery, given that the expected payout is the same	
Risk loving	Strictly prefers a lottery to a sure situation, given that the expected payout is the same	Not as ridiculous as it seems <i>prima facie</i> . In situations where you're wealthy/have nothing substantial to lose, risk is fun.
Risk neutral	Indifferent between a lottery and sure situation given that the expected payout is the same	

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Term	Definition/Explanation	Notes
Degenerate lottery	A deterministic situation (probably pointlessly) modelled as a lottery	Probabilities of states arising based off chosen actions would be 1 or 0

3.4.2 Tadelis Ch1

3.4.2.1 Completeness Axiom in Preference Relations Complete iff any two outcomes can be ranked by preference relation

3.4.2.2 Rational Choice Paradigm

3.4.2.2.1 Assumptions Full understanding of the following:

1. All possible acts
2. All possible outcomes
3. Mappings for every state/outcome pair
4. His rational preferences (payoffs) over outcomes

3.4.2.2.2 Meaning Rational if payoff-maximizing.

Manner of maximization is unclear in the text (is it expectation maximization?) but various methods were covered in Week 2.

3.4.2.2.3 Homo Economicus Means "economic man". Essentially, the perfect rational agent.

1. Fun Facts

- Wikipedia describes this as a "caricature" and "mythical species", although in a way that seems to be a tool, rather than pejoratively
- Oppositionally, see homo reciprocans ("reciprocating man").
 - "cooperative actors who are motivated by improving their environment."
- Sometimes used by non-economists to critique economic approaches (distilled down to the "people aren't rational" argument)

3.4.3 Tadelis Ch2.1-2.3 + Peterson CH4.1 - 4.3, 4.6

3.4.3.1 Simple and Compound Lotteries

3.4.3.1.1 Meaning of Simple Lottery

- Stochastic state distribution
- Probability of states are contingent upon performed acts, which suggest that acts are not necessarily seen as independent of states. This is different from what we learnt in class (assumption of causal independence)

3.4.3.1.2 Compound Lotteries

- A simple lottery after a simple lottery (multi-level decision tree)
- Can be transformed into a simple lottery (take each path as a single decision, then consider terminal nodes only)

3.4.3.2 Backward Induction It's easiest to work backwards then dealing with multi-stage decisions trees.

3.4.3.2.1 Relation to Dynamic Programming It's one strategy for it, but certainly not the only one (e.g. you could just as easily work forwards in shortest path problems)

3.4.3.3 Maximizing Expectation

3.4.3.3.1 Units

Unit	Explanation/Notes
Monetary value	\$\$\$
Value	Monetary value, adjusted for diminishing marginal 'value' of money to the agent
Utility	Yes. Not well defined, but POV is important. Allows for non-monetary evaluation

Utility, value, monetary value, payoff

3.4.3.3.2 Keynes's Objection "In the long run we are all dead".

Essentially, a criticism of relying on LLN.

3.4.3.4 Axiomatic Approach Arrives at the principle of maximizing utility without using LLN (thereby sidestepping Keynes' problem).

3.4.3.4.1 Axioms

1. If all outcomes of an act have utility u , then the utility of the act is u
2. For acts, utility relations follow dominance relations (e.g. strict dominance implies higher utility)
3. Every decision probable can be transformed into a rival formalization with equiprobable (possibly repeated) states, which maintain the utilities of all acts
4. Setup: 1) two equiprobable outcomes 2) The better outcome is made slightly worse Claim) Overall utility of act can be preserved by adding some amount of utility to the other outcome

These 4 axioms lead to the following conclusion: the utility of an act = its expected utility

3.4.3.5 Misc

3.4.3.5.1 de minimis Principle as a Solution to the St Petersburg Paradox Roughly put, sufficiently improbable outcomes should be ignored

3.4.3.5.2 Two-Envelope Paradox (infinite swapping reasoning)

- No established solution exists.
- This is **not** the same as the Monty Hall problem

1. Set Up

- Two seemingly identical envelopes (A and B)
- 1 envelope contains twice as much as the other
- You don't know which is which, can only choose 1

2. Choice

- Make a decision
- Given chance to swap decision

3. The paradox

- $\mathbb{E}[swap] = \frac{1}{2} \cdot 2x + \frac{1}{2} \cdot \frac{x}{2} = \frac{5}{4}x$, so logically, swap
- Given choice again just before picking
- $\mathbb{E}[swap] = \frac{1}{2} \cdot 2y + \frac{1}{2} \cdot \frac{y}{2} = \frac{5}{4}y$, so logically, swap
- Given choice again, just before picking
- Repeat ad infinitum

- 4. Comments The problem is not to find an alternate formulation of the given situation. The problem is to find the issue with this current formulation, which gives nonsensical results.

4 Behavioural Decision Theory

4.1 W5: Discounting the future. The value of information. Biased choice.

4.1.1 Discounting the Future

4.1.1.1 Misc Problem is known as choosing consumption over time. It's a formalization of why greedy approaches don't work.

4.1.1.1.1 Example case Budgeting. You've got \$X to last you n time. Obviously, you can't just spend it straight away because you'll be screwed in the long run (almost immediate run, in this case)

4.1.1.2 Strategy Essentially, do the following where:

- discount factor $\delta \in [0, 1]$
- x_1 represents the act of spending $\$x_1$
- $\$X$ is the total amount of money you have

$$\max_{x_1} u(x_1) + \delta u(X - x_1)$$

4.1.1.2.1 Quickly calculating optimal act If we assume u is concave, we just find the max point and solve for x_1 , depending on the specifics of what your u function was in the first place. This is:

$$u'(x_1) = \delta u'(K - x_1)$$

1. **TODO** ??? Why can we assume concavity? Idk yet.

4.1.2 The Value of Information

1. Take the act that gives the max expected value without information
2. Take the act that gives the max expected value with information
3. 2 - 1 gives you the value of information.

Note: Value can indeed be negative (maybe having the info would bias you to act badly)

4.1.3 Biased Choice

Microecons stuff, basically.

4.1.3.1 Possible Fallacies Note that these are not definitely fallacies and often can be argued either way.

4.1.3.1.1 Framing effects The effect the frame/representation has on decision

4.1.3.1.2 Endowment effect Tendency to value what we ave more than what we do not yet have

Possible Reasons

- Information (more info once we own it)
- Transaction costs
- Habit formation (linked to transaction costs)

4.1.3.1.3 Sunk Costs Yes.

4.1.3.2 Heuristics Central idea is that in order to deal with overwhelming info, we resort to heuristics (which may give rise to the 'fallacies') (idea from Kahneman and Tversky)

4.1.3.2.1 Representative Heuristic

- You have a representation of a thing, and use this to govern your estimation of a probability distribution
- Leads to problems where $\mathbb{P}(AB)$ is seen as more likely than $\mathbb{P}(A)$ on its own if you think B is more in line with the 'representation'

4.1.3.2.2 Availability Heuristic

- If you can think of an example of it happening, you assume it's more likely
- Leads to problems of anecdotal evidence/whatever example you have the brainpower to conjure up being used

4.1.4 Multiple Selves as a Framework to Understand Irrational Behavior (Ainslie)

4.1.4.1 General Argumentation Strategy Ainslie is essentially proposing a new psychological, reductionist framework of interpreting irrational behaviour: modelling each individual as multiple, internally rational selves. He contrasts this with other reductionist attempts from psychology which he claims have failed.

4.1.4.2 Subtle vs Visceral Rewards This is his terminology. I don't think it's standard. Also, was a little tough to decipher so I'm not sure I'm correct in this.

Visceral: basal, biological/survival rewards (visceral)

Subtle: other rewards (emotional etc)

4.1.4.3 Status Quo: The "Shopping List" Definition of Value Essentially, "that which a person will pay money for is good"

- Also being turned to by psychologists as a method of defining reward
- Glaring problem (in depth below) is that people often behave irrationally

4.1.4.4 Problems

4.1.4.4.1 Many rewards function as punishments as well

4.1.4.4.2 Abstemoniousness in situations where we could very well reward ourselves

4.1.4.4.3 Events serving as rewards are changeable E.g. diminishing returns, fashion, etc

4.1.4.4.4 Many apparent rewards cannot be produced by direct effort Described as "states which usually occur as by-products", and "lost in any systematic attempt to attain them".

Examples given are:

- sleep
- laughter
- happiness
- dignity

Personally, I see no problem here and disagree that systematic attempts are not possible. I don't see the issue with a sequence of steps being needed in order to achieve these states. Particularly since we've seen that multi-level decision trees can be linearized into a decision matrix.

4.1.4.5 Failed reductionist resolution attempts using the field of psychology

4.1.4.5.1 Classical Conditioning as a bridging mechanism (between 'subtle' and 'visceral' rewards) Atomic units here are the visceral rewards, which are composed with each other in complex manners.

- Attempts to model subtle rewards as a composition of visceral ones.
- Problematic as the modelling complexity and verification of the model are pretty untenable
 - e.g. do you like computer science because someone told you you're good at computers when you were a kid? Not easy to prove it, and not necessarily correct either

4.1.4.5.2 Elementary Drives for Game-like Rewards as a bridging mechanism (between 'subtle' and 'visceral' rewards) Atomic units here are the elementary drives for game-like rewards, which are composed with each other in complex manners.

- Attempt in the 60s by Fowler (1967) and Hunt (1963)
- Problem: Tough to extricate properties of situation needed to satisfy these drives
- Declared a failure by Coombs and Avrunin (1977)

4.1.4.5.3 Empirically, people often fail to max expected income/min costs

- Value of money varies a lot
 - Different people value money differently
 - People treat money differently in different situations (e.g. you're more okay losing \$100 when spending 1,000,000 on a house than you would be normally)

4.1.4.5.4 People consent to undergo pain

4.1.4.5.5 Empirically, discount factor seems to be very, very high

4.1.4.6 Resolution using multiple selves (demonstrated by intrapersonal bargaining) Atomic unit here is multiple, internally-rational selves for each person, which interact through **intrapersonal bargaining** (basically the cartoon trope of angel/devil on your shoulders arguing over what you should do)

This is the crux of what the chapter is talking about. The author thinks it's very promising. IMO it's a great and promising approach, but he hasn't developed it well.

4.1.4.6.1 General rule of thumb Agent will try to obtain "situations which permit a stable compromise between his long- and short-term interest"

1. Devices that aid compromise Note that this is supposedly descriptive. These devices supposedly act in concert to produce a prima facie irrationality.
 - Extrapsychic devices (physical/social constraints which limit future choice space)
 - Control of attention (philosophy of "out of sight -> out of mind")
 - Control of emotions (e.g. "positive thinking")
 - Private rules (grouping temptations into sets, to up the ante of breaking rules about giving in to any single temptation)

4.2 W6: Paradoxes of choice. Prospect theory. Biases in probabilistic reasoning.

4.3 W8: Two system-theories. Behavioral design: nudging and fast-and-frugal heuristics.

4.4 W9: Social preferences and choice.

5 Philosophy and Decision Theory

5.1 W10: Interpretations of probability. The problem of induction.

5.2 W11: Causal, evidential and functional decision theory.

5.3 W12: Applications: I. Pascal's Wager. II. Discounting. III. The value of life.

5.4 W13: Superintelligence and the AI alignment problem.