Forced spring-mass simulation and estimation

Yusuf Oluwatoki yusuf

May 3, 2020

The forced spring-mass system is a second order differential equation equation defined by

$$\frac{d^2x(t)}{dt^2} + \gamma \frac{dx(t)}{dt} + v^2 x(t) = w(t) \tag{1}$$

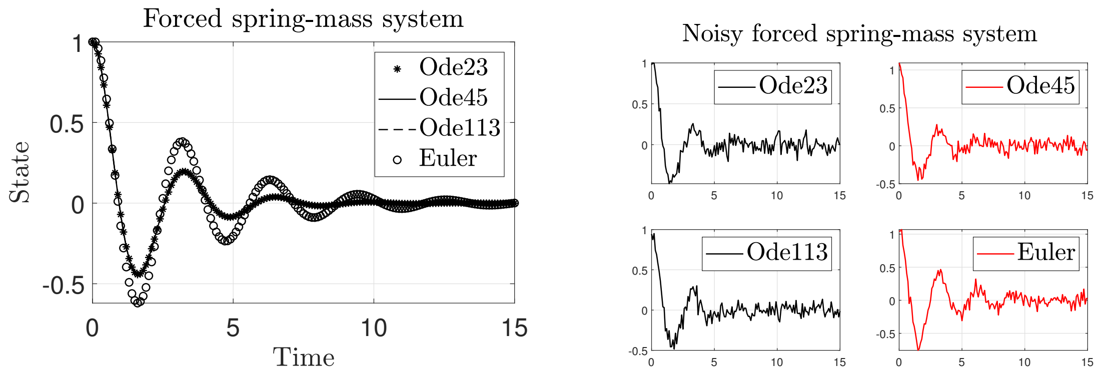
where γ and v are constants that determine the resonant angular velocity and damping of the spring. The force $w(t)$ is some given function that may or may not depend on time. The spring-mass system can hence be described as a system of first order linear equation

$$\begin{pmatrix} dx_1(t)/dt \\ dx_2(t)/dt \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -v^2 & -\gamma \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} w(t). \tag{2}$$

The system was solved with different ODE solvers in matlab with $\gamma = 1$ and $v^2 = 4$. The solutions were consistent at $t \in [0\ 15]$ except for the Euler method which deviates from the true solution when $\Delta t = 0.1$. Some white noise were added to the solutions to have a noise system which can be used for the MCMC simulation.

An optimized fit of the model was obtained with fminsearch in matlab to recover the true parameter from the noisy system using the mean squared residual. The parameter values obtained were random depending on the randomness of the noise but very close to the true parameters ($\gamma = 1 \pm 0.05$ and $v^2 = 4 \pm 0.1$).

The adaptive Metropolis algorithm (AM) and delayed-rejection with adaption (DRAM) were used to simulate the noisy system to obtain true parameter of the system. Since



(a) Forced spring-mass system with 4 method of solution. The Euler method has large error with time step $\Delta t = 0.1$. (b) White noise added to the four solution method.

Figure 1. Forced spring-mass system with and without noise.

the Euler method solution is a bit different from others, this and ODE45 were used for the two MCMC techniques but at a smaller time step $\Delta t = 0.05$. Each MCMC run was 10000 single chain with the first 2000 chain removed as burn-in. The chain mean for the γ parameter with ODE45 and AM was 1.23 which was far from the true value of $\gamma = 1$. This consequently made the predictive envelope deviates from the optimized fit of the system. However, the Euler method with AM fits closely with the true parameters having the chain mean of $\gamma = 1.04$ and $v^2 = 4.09$, hence the predictive envelope also fits closely with the optimized parameter fit.

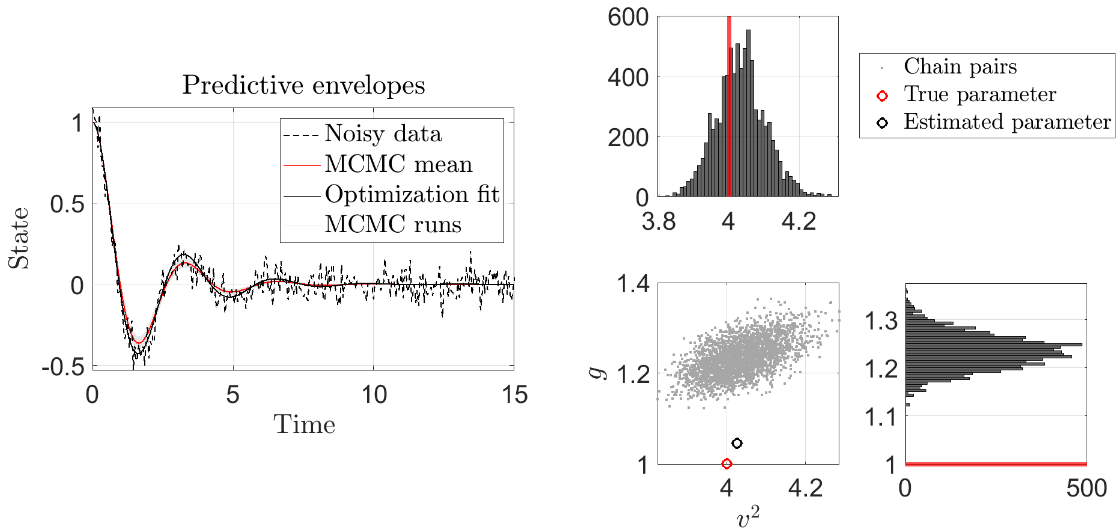


Figure 2. MCMC analysis plot for forced spring-mass system with ODE45 and AM.

There was no significant difference with the two MCMC techniques. The chains statis-

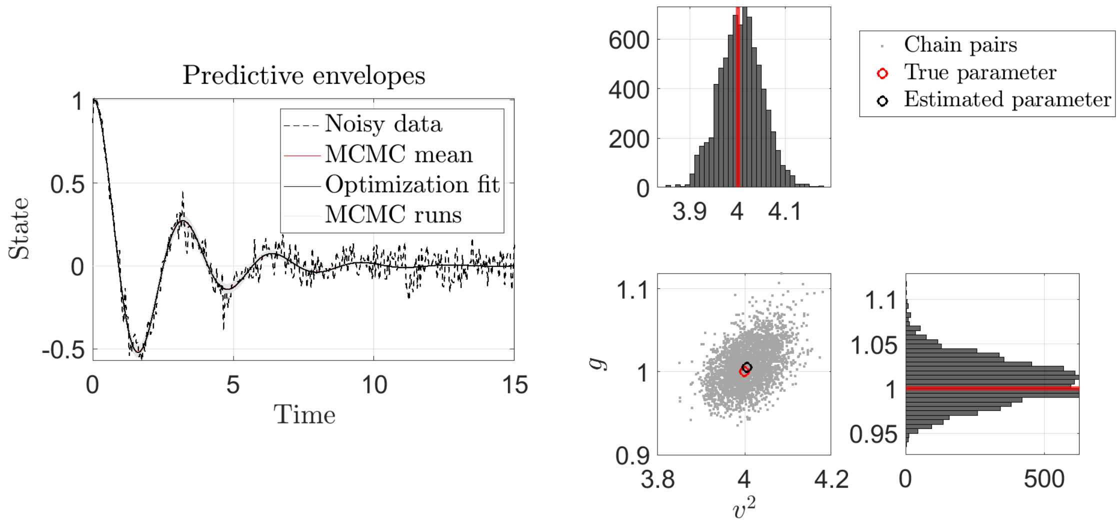
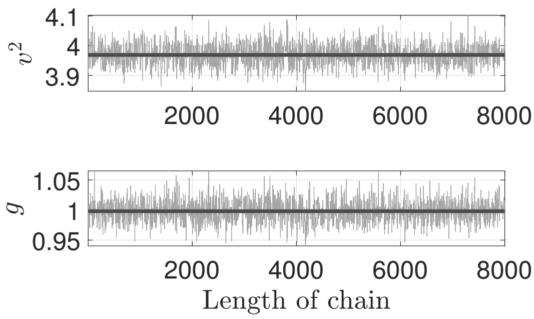
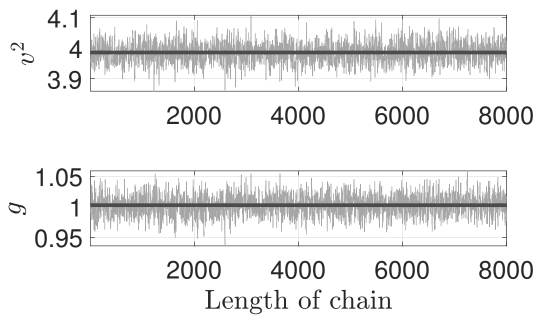


Figure 3. MCMC analysis plot for forced spring-mass system with Euler method and AM.



(a) AM chain with Euler method.

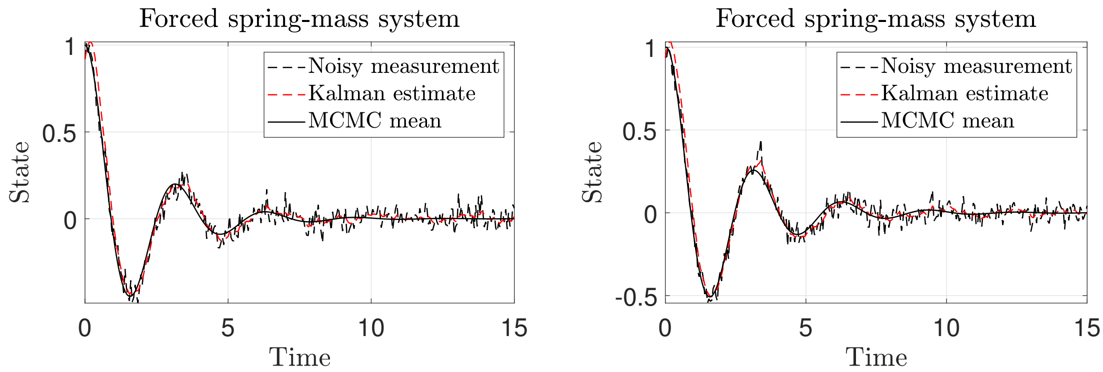


(b) Dram chain with Euler method.

Figure 4. MCMC chain plot.

tics shows they both sampled parameter values from the range of the system solution. However, both techniques did not give good estimate for the solution with the ODE45.

The Kalman filter was used to estimate the noisy system using the AM MCMC solution from Euler method and ODE45 as the ground truth. It was found that the Kalman filter also estimates some of the noise in the system. The Kalman filter compared to the ground truth from Euler method has a root mean squared error (RMSE) of 0.038 with large part of the error from the deviation at the beginning of the system. The RMSE from the Kalman estimates and ODE45 solution is 0.037 which is smaller the that from the Euler method. This values are reported for the case where random number generator is set to default and used for both methods.

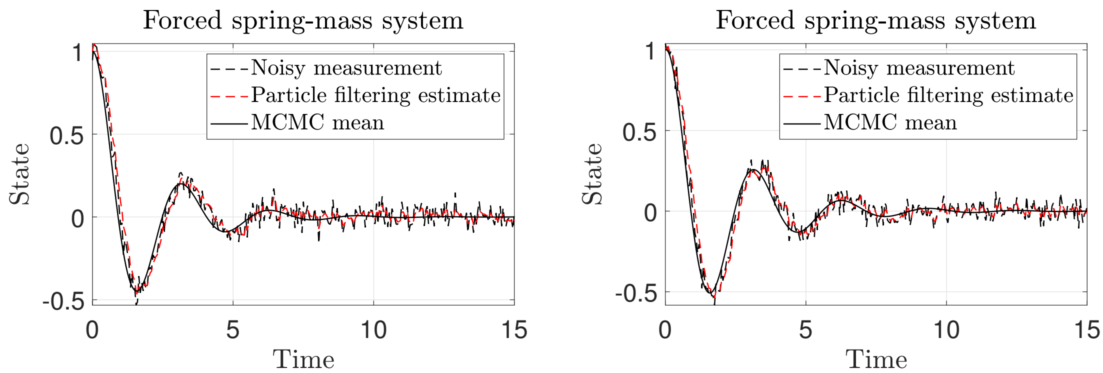


(a) ODE45 solution with AM compared with Kalman filter estimates.

(b) Euler method solution with AM compared with Kalman filter estimates.

Figure 5. Kalman filter estimate compared with ODE45 solution and Euler method solution from AM algorithm.

A similar comparison was done with the particle filtering method. Here more of the noise in the system was estimated by the filter for both solutions. The RMSE from the Euler method and the particle filter is 0.049 while that from the ODE45 and particle filter is 0.044. This is as seen with the Kalman filter where the errors from the ODE45 were smaller than those of Euler method.



(a) ODE45 solution with AM compared with particle filter estimates.

(b) Euler method solution with AM compared with particle filter estimates.

Figure 6. Particle filter estimate compared with ODE45 solution and Euler method solution from AM algorithm.