BILKENT UNIVERSITY ELECTRICAL AND ELECTRONICS ENGINEERING DEPARTMENT

2019 Summer School
EEE - 321: Signals and Systems
Lab 2 - Off-Lab Assignment
Lab Day/Time/Room: 04.07.2019 - 17:40-19:30 - EE 211
Report Submission Deadline: 05.07.2019 - 17:20

Please carefully study this off-lab assignment before coming to the laboratory. There will be a quiz at the beginning of the lab session; this may contain conceptual, analytical, and Matlab-based questions about the lab. Some of the exercises will be performed by hand and others by using Matlab. The report should be prepared as hardcopy and written by hand (except the Matlab based plots and the code segments). Include all the plots, codes, calculations, explanations etc. to your report in a readable format.

Part 1: Fourier Series Expansion

Suppose $x(t)$ is a continuous and periodic function with fundamental period $T = 2$ sec. Its one period is defined as

$$x(t) = \begin{cases} 4t & \text{if } 0 \leq t < 0.5 \\ 3 - t & \text{if } 0.5 \leq t < 1.5 \\ t & \text{if } 1.5 \leq t < 2 \end{cases} \tag{1}$$

1. Sketch three periods of $x(t)$ by hand.

2. Plot three periods of $x(t)$ for $t \in [0, 6]$ seconds in Matlab.

3. Compute the Fourier series coefficients of $x(t)$ analytically.

4. Let X_k be the Fouier series coefficient of the k^{th} harmonic. Find a positive integer N such that approximately 95% of the energy of the Fourier series coefficients is confined within the interval $k \in [-N, N]$. For this purpose, calculate the total energy of

the Fourier series coefficients analytically. That is, find N such that

$$0.95 \approx E_r = \frac{\sum\limits_{k=-N,\ k\neq 0}^{N} |X_k|^2}{\sum\limits_{k=-\infty,\ k\neq 0}^{\infty} |X_k|^2} \quad . \tag{2}$$

You can find N either analytically or using a Matlab code. (Hint: Use Parseval's relation to find the total energy, i.e. the denominator in above equation, of the Fourier series coefficients.)

5. For the value of N that you found in previous question, generate a stem plot of the magnitudes and the phases of X_ks for $k \in [-N, N]$. Put the labels and titles appropriately.

Part 2: Sum of Complex Exponentials

Consider the following sum of complex exponentials formula:

$$x_a(t) = \sum_{k=-N}^{N} X_k e^{j\frac{2\pi}{T}kt} \quad . \tag{3}$$

In this part, you will write a Matlab function which generates $x_a(t)$ by using X_k and T. That is, your function will look like

xa=FourierSum(Xk,T)

where $T \in \mathbb{R}^+$ determines the fundamental period and Xk is an array whose elements are the complex valued Fourier series coefficients in the form

$\mathbf{Xk} = [\mathbf{X_{-N}} \quad \mathbf{X_{-N+1}} \quad \mathbf{X_{-N+2}} \quad \ldots \quad \mathbf{X_{-1}} \quad \mathbf{X_0} \quad \mathbf{X_1} \quad \ldots \quad \mathbf{X_{N-2}} \quad \mathbf{X_{N-1}} \quad \mathbf{X_N}]\,; \quad .$

The output array of your function, xa, includes the time samples of three periods of $x_a(t)$ for $t \in [0, 3T)$. Choose time interval between time samples small enough such that the continuous nature of $x_a(t)$ will be reflected by **xa** and large enough such that the execution time of your code is not excessive and do not use memory inefficient. Your code should also generate the plot of **xa** as a function of time. Put the labels and title appropriately.

Part 3: Fourier Series Approximation

In this part, you will investigate the approximation to a signal by its Fourier series expansion. That is, you will find approximate signals to $x(t)$ given in Part 1 from its approximate Fourier series expansion.

1. Let N_1 be equal to N that you found in Part 1. Also, using Equation 2 find
   - N such that $E_r \approx 80\%$. Let this N be equal to N_2
   - N such that $E_r \approx 50\%$. Let this N be equal to N_3

- N such that $E_r \approx 35\%$. Let this N be equal to N_4
- N such that $E_r \approx 25\%$. Let this N be equal to N_5
- N such that $E_r \approx 10\%$. Let this N be equal to N_6.

2. For all N_is, $i \in \{1,2,3,4,5,6\}$, generate the plots of corresponding $x_a(t)$s by using the Matlab code that you wrote in the previous part. Discuss the approximatons for different N values to the original signal $x(t)$. Do you observe the Gibbs phenomena (You may read Section 3.4 from the course book for the further information for Gibbs phenomena).

3. For each $x_a(t)$ that corresponds to a different N_i, compute the approximate mean square error between the original signal and its approximation as

$$e = \frac{1}{T}\int_0^T |x_1(t) - x_a(t)|^2 \,\mathrm{d}t \approx \frac{\Delta t}{T}\sum_{i=0}^{T/\Delta t} |x_1(i\Delta t) - x_a(i\Delta t)|^2 \quad , \tag{4}$$

where Δt is the time interval between the successive entries of t, and it depends on your choice in Part 2. Discuss the effect of N in the mean square error.

Part 4: Time-shifting Property of Fourier Series Coefficients

Consider the signal $x_2(t) = x(t-7)$ and its Fourier series coefficients $X2_k$.

1. By using X_ks, compute $X2_k$s.
2. Using the new Fourier series coefficients, $X2_k$s, repeat Part 3.

Part 5: Derivative Property of Fourier Series Coefficients

Consider the signal $x_3(t) = \frac{\mathrm{d}x(3t)}{\mathrm{d}t}$ and its Fourier series coefficients $X3_k$.

1. By using X_ks, compute $X3_k$s.
2. Using the new Fourier series coefficients, $X3_k$s, repeat Part 3.