BILKENT UNIVERSITY ELECTRICAL AND ELECTRONICS ENGINEERING DEPARTMENT

2019 Summer School
EEE - 321 Signals and Systems
Lab 1 - Off-Lab Assignment
Lab Day/Time/Room: June 27, 2019 - 17:40-19:30 - EE-211
Report Submission Deadline: June 28, 2019 - 17:20

Please carefully study this off-lab assignment before coming to the laboratory. There will be a quiz at the beginning of the lab session; this may contain conceptual, analytical and Matlab-based questions about the lab. Some of the exercises will be performed by hand and others by using Matlab. The report should be prepared as hardcopy and written by hand (except the Matlab based plots and the code segments). Include all the plots, codes, calculations, explanations etc. to your report in a readable format.

Part 1: Discrete-time Cosine Signal

Consider the most general form of a discrete-time cosine signal

$$x_d[n] = A_d \cos\left(\hat{\omega} n + \phi_d\right) \quad , \tag{1}$$

where A_d is the amplitude, $\hat{\omega}$ is the normalized frequency in radians, n is the integer valued discrete index and ϕ_d is the phase.

1. Let A_d be the penultimate digit (the second last digit) of your school ID number (you may choose A_d as 8 if the penultimate digit of your school ID is 0), $\hat{\omega}$ be $2\pi/(10A_d)$ and ϕ_d be $2\pi/A_d$. Also, let $n \in [0,\ N-1]$.
   a) Analytically find N such that $x_d[n]$ includes exactly 6 periods of the cosine signal.
   b) For the value of N that you found in the previous part, generate $x_d[n]$ and name this array as $xd1$.
   c) Generate a stem plot of $xd1$. Clearly label the plot and put an appropriate title.

2. Let A_d be the last digit of your school ID number (you may choose A_d as 4 if the last digit of your school ID is 0). If the last and the second last digits of your ID are the same, then take A_d as the last digit minus 1. Let $\hat{\omega}$ be $2\pi/(10A_d)$ and ϕ_d be $2\pi/A_d$. Also, let $n \in [0,\ N-1]$.

   a) Analytically find N such that $x_d[n]$ includes exactly 6 periods of the cosine signal.

   b) For the value of N that you found in the previous part, generate $x_d[n]$ and name this array as $xd2$.

   c) Generate a stem plot of $xd2$. Clearly label the plot and put an appropriate title.

   d) Compare the stem plots of $xd1$ and $xd2$ by generating them in the same figure. Find a method such that these two signals are well separated even if the stem plot is printed by a black-and-white printer. Clearly label the plot and put an appropriate title. Include a legend, as well.

3. Consider a decaying discrete-time cosine signal, defined as

$$x_b[n] = \begin{cases} e^{-bn} x_d[n] & \text{if } n \geq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

   where $b \in \mathbb{R}^+$.

   a) Analytically find b such that, at the first index of the sixth period of $xd1$, it drops to one sixth of its maximum.

   b) For the value of b that you found in the previous part, generate $x_b[n]$ and name this array as $xb1$.

   c) Generate a stem plot of $xb1$. Clearly label the plot and put an appropriate title.

Part 2: Continuous-time Cosine Signal

Consider the most general form of a continuous-time cosine signal

$$x_c(t) = A_c \cos(\omega t + \phi_c) \tag{3}$$

where A_c is the amplitude, ω is the frequency in radians, t is the time and ϕ_c is the phase. In this part, you may assume A_c as 1.

Since the computers have a discrete nature, it is not possible to generate a continuous cosine signal in Matlab. However, for a given ω, by choosing the time index with high resolution, it is possible to generate an approximate continuous cosine signal.

1. First go to the website http://www.phy.mtu.edu/~suits/notefreqs.html. Then choose a note such that its frequency is between 300 Hz and 1 KHz. Take the frequency of this note as f, where $\omega = 2\pi f$. Please note that, a pure musical note is a signal that is exactly generated as $x_c(t)$.

2. Now assume that, for this value of f, you would like to listen to this note 20 seconds. First generate t array such that it starts from 0 sec. and it includes 100 sample points at each period for the chosen value of ω. That is, you will write the command $\mathbf{t} = [\mathbf{0} : \Delta\mathbf{t} : \mathbf{20}]$;, where Δt is computed such that at each period of the cosine signal, you will have 100 sample points. Clearly describe how you compute Δt.

3. For the generated **t** and for $\phi_c = 0$, generate the approximate continuous cosine signal and name it as $xc1$. Listen to $xc1$ using the *soundsc* command. In order to listen to the cosine signal appropriately, you need to compute the sampling rate, f_s, which will be given as an input to the *soundsc* command, as well. Describe how you compute this sampling rate.

4. Repeat the previous item for $\phi_c = \pi/4,\ \pi/2,\ \pi$ rad.

5. Do you recognize a difference between the sounds that you listened for different ϕ_c values ? What is the effect of the phase on the sound that you hear.

6. Plot three periods of $xc1$ for $\phi_c = 0$ as if it is a continuous signal. Clearly label the plot and put an appropriate title.

7. Repeat items 1 to 6 by choosing a different f from the same website, again between 300 Hz and 1 Khz. Name the new signal as $xc2$. What is the effect of the frequency on the sound that you hear?

8. Consider a decaying continuous cosine signal defined for nonnegative values of t,

$$x_a(t) = e^{-at} x_c(t) \quad , \tag{4}$$

   for $\phi_c = 0$ and $A_c = 1$. You may choose an ω among two ωs that you used.

   a) Calculate a such that $x_a(t)$ drops to half of its maximum at 6^{th} sec.
   b) For the value of a that you calculated, listen to $x_a(t)$ and make your comments.
   c) Plot $x_a(t)$ as if it is a continuous signal. Clearly label the plot and put an appropriate title.

Part 3: Complex Sinusoids

A continuous time complex sinusoid is defined as

$$x_s(t) = A_s e^{j\omega t} \quad , \tag{5}$$

where A_s is a complex valued number and determines the complex amplitude of the sinusoid. Euler expansion states that $x_s(t)$ can be written as

$$x_s(t) = A_s e^{j\omega t} = |A_s|\, e^{j\angle A_s} e^{j\omega t} = |A_s|\, e^{j(\omega t + \angle A_s)} = |A_s| \left[\cos(\omega t + \angle A_s) + j \sin(\omega t + \angle A_s)\right] \quad . \tag{6}$$

1. For the two ω values that you choose in the previous part, that are ω_1 and ω_2, and for Δt that you found in the first item of previous part, compute $x_l(t) = e^{j\omega_1 t} + e^{j\omega_2 t}$.

2. Plot the real and imaginary parts of $x_l(t)$ as well as its complex amplitude and phase. The plots should be located in the same figure window, but at separate positions within the window (Hint: use the *subplot* command). Clearly label the plots and put an appropriate title.

3. By looking at the plots, can you separate two different complex sinusoids ? In other words, would you infer that the plots that you look include two different complex sinusoidal signals ?

Part 4: Periodic Signals

A continuous signal $x(t)$ is said to be periodic with $T \in \mathbb{R}^+$ if it satisfies

$$x(t) = x(t+T) \tag{7}$$

for all t. The smallest T that satisfies the periodicity condition is called as the *fundamental period.* If a signal is periodic with T, it is also periodic with kT, for any positive integer valued k.

A discrete signal $x[n]$ is said to be periodic with $N \in \mathbb{Z}^+$ if it satisfies

$$x[n] = x[n+N] \tag{8}$$

for all n. The smallest N that satisfies the periodicity condition is again called as the *fundamental period.* If a discrete signal is periodic with N, it is also periodic with kN, for any positive integer valued k. Please note that, for a discrete signal to be periodic, N should be positive integer, whereas, for the continuous signals, T can take any positive real number.

1. By using the definition of periodicity for the continuous signals, show that whether

$$x(t) = \cos(\omega_c t) \tag{9}$$

   is a periodic function for an arbitrary ω_c. Is there constraint on ω_c for $x(t)$ to be a periodic signal.

2. By using the definition of the periodicity for the discrete signals, show that whether

$$x[n] = \cos[\hat{\omega} n] \tag{10}$$

   is a periodic function for an arbitrary $\hat{\omega}$. Is there constraint on $\hat{\omega}$ for $x[n]$ to be a periodic signal.

3. Are the following signals periodic or not? If it is periodic, find the fundamental period. Show your work. (The time indexes t and n determine whether the corresponding signal is continuous or discrete).

a) $\cos(0.4\pi t)$

b) $\cos[0.4\pi n]$

c) $\cos(0.4t)$

d) $\cos[0.4n]$

e) $\cos(0.4et)$

f) $\cos[0.4en]$

g) $\cos(0.2\pi t) + \sin(0.4\pi t)$

h) $\cos[0.4\pi n] + \sin[0.2\pi n]$

i) $\sin[0.2n] + \sin[0.4\pi n]$

j) $\cos(0.2t) + \cos(0.2et)$

4. Is $\cos\left(\alpha t^2\right)$ is a periodic function for an arbitrary α. Is there any α that makes $\cos\left(\alpha t^2\right)$ a periodic function. If yes, find the set of α values that makes $\cos\left(\alpha t^2\right)$ a periodic function.

5. Is $\cos\left[\alpha n^2\right]$ is a periodic function for an arbitrary α. Is there any α that makes $\cos\left[\alpha n^2\right]$ a periodic function. If yes, find the set of α values that makes $\cos\left[\alpha n^2\right]$ a periodic function.