

Construction of Optimal Portfolio using Sharpe's Single Index Model and Markowitz Model.

Portfolio Management and Investment Analysis

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Abstract

An attempt is made here to get an insight into the idea embedded in Markowitz and Sharpe's Single-Index Model (SIM) and constructing an optimal portfolio in Indian market. For this purpose, the daily closing prices of 30 companies listed in Bombay Stock Exchange (BSE) and 47 companies of Nifty 50 for the period of September 2015 to September 2020 have been considered.

The Markowitz model constructs an optimum portfolio consisting of fifteen stocks selected out of thirty stocks, gives a return of 20%. Markowitz approach is to select stocks on the basis of risk and return analysis. On the other hand, Sharpe's single-index model takes eight stocks out of forty-seven stocks to form an optimum portfolio, giving the return of 14.06%. This proposed method formulates a unique cut-off rate and selects those securities to construct an optimal portfolio whose excess return to beta ratio is greater than the cut-off rate. Then, proportion of investment in each of the selected securities is computed based on beta value, unsystematic risk, excess return to beta ratio and cut-off rate of each of the securities concerned.

The investor expects from the portfolio to maximize the return and minimize the risk. Therefore, a good combination of stocks in portfolio will give maximum return for a particular level of risk. Therefore, a good combination of stocks will give maximum return at a particular level of risk.

Keywords:

Optimum Portfolio, Markowitz model, Single-Index Model, Cut-off Rate, Systematic Risk, Unsystematic Risk, Diversification, Beta, Debt-to-Equity Ratio, Risk, Return and Variance, Excess Return-to-Beta Ratio, NSE Nifty 50 Index, BSE index.

Introduction

An investor needs to have a proper knowledge about security analysis and portfolio theory for making correct investment decisions. This can be done either through traditional or modern approaches to portfolio construction.

In the modern approach, the Markowitz model is used in selection of securities based on the risk and return analysis. Markowitz laid the foundation for quantifying risk and his contribution is popularly known as 'Modern Portfolio Theory'. He has provided analytical tools for analysis and selection of optimal portfolios. But William Sharpe extended the work done by Markowitz. He considered the market index while analyzing the portfolio. He simplified the amount and type of input data required to perform portfolio analysis. He made the numerous and complex computations easy which were essential to attain the optimal portfolio. He developed the Single Index Model to make these computations easy and construct an optimal portfolio. Till today, fund managers use this model in portfolio analysis and construction. According to his model, security's return is correlated to a single index which is usually a market index. All securities that are traded on the exchange will be a part of the market index.

The index movement will also indicate the changes in the security prices. Sharpe's Single Index Model (SIM) demands only $(3N + 2)$ bits of information whereas it is $\frac{[N(N+3)]}{2}$ bits of information in Markowitz model. In SIM, optimal portfolio is constructed by analyzing the reason behind inclusion of a particular stock based on the associated variables under consideration.

The present project seeks to construct an optimal portfolio using Markowitz and Sharpe's Single Index model and taking into consideration stocks of BSE Index and Nifty Fifty, respectively.

Objective of the project

1. To Understand the Portfolio theory– Sharpe Index Model & Markowitz Model.
2. To construct an optimal portfolio with risk-return using stocks listed in NIFTY 50 and BSE
3. To see the Comparison between Sharpe's Single Index Model (SIM) & Markowitz Model.
4. To know which Securities are performing well and which are performing low in the market.
5. To calculate the respective proportion for each selected stock to be invested in the portfolio for the time period September 2015 to September 2020.

Construction of Optimal Portfolio using Sharpe's Index Model and Markowitz:

Data has been collected from websites like www.nseindia.com and <https://in.finance.yahoo.com> For the assignment, Nifty 50 Index and BSE Index is taken into consideration as Market Index. Daily NSE INDICES and ^BSESN prices of all the stocks of Nifty 50 are taken for the period September 16, 2015 to September 15, 2020 for computing Daily return of each security. The daily price is calculated by using the formula:

$$R_i = (B_3/B_2) - 1$$

where

B₃ = Current closing price of stock ,

B₂ = Previous closing price of stock and R_i= return of stock.

The Daily mean return of all individual stock was calculated using Excel. The daily return is then converted into annual Return by using formula,

$$\text{i.e. Mean return} = \alpha_i + \beta_i R_m + e_i$$

For Risk free rate of return 10Y T-bills: 6.01% is taken from the RBI website and for Market risk variance of Nifty 50 and BSE is calculated.

Beta, Unsystematic Risk, Systematic Risk and Cut-off point is also calculated by using Excel. Securities with their 'Excess Return to Beta' is also being calculated. A number of financial and technical tools have been used for analysing data.

Sharpe's Single Index Model

The stocks to be included in an optimal portfolio are determined on the basis of their 'Excess return to beta ratio.' As per the rule of ranking, security having the highest 'excess return to beta ratio' will be placed in the first position, followed by the security with second highest beta ratio, and so on and so forth. Thereafter a cut-off point will be calculated and all the stock whose 'excess return to beta ratio' is above the cut-off point is included in the portfolio.

COMPANIES		
INFY.NS	GAIL.NS	M&M.NS
BAJAJFINSV.NS	BHARTIARTEL.NS	MARUTI.NS
AXISBANK.NS	BRITANNIA.NS	TITAN.NS
ICICIBANK.NS	IOC.NS	ZEEL.NS
BAJAJFINANCE.NS	SBIN.NS	RELIANCE.NS
LT.NS	ASIANPAINTS.NS	ULTRACEMCO.NS
HDFC.NS	KOTAKBANK.NS	ITC.NS
INFRATEL.NS	HCLTECH.NS	SUNPHARMA.NS
GRASIM.NS	COALINDIA.NS	SHREECHEM.NS
ADANIPTS.NS	POWERGRID.NS	NTPC.NS
BPCL.NS	TATAMOTORS.NS	NESTLEIND.NSS
EICHERMOT.NS	INDUSINDBK.NS	TECHM.NS
HDFCBANK.NS	TATASTEEL.NS	ONGC.NS
HINDUNILVR.NS	CIPLA.NS	WIPRO.NS
JSWSTEEL.NS	DRREDDY.NS	TCS.NS
BAJAJ-AUTO.NS	UPL.NS	

Source: NSE¹

The Steps in Constructing Optimal Portfolio are as follows:

- The Daily return of the Stocks are calculated by using Excel formula: $R_i = (B3/B2) - 1$

i.e. = Current closing price/ Previous closing price -1

R_i = Daily return of stock

The Daily return is then converted into annual Return by using Excel formula,

i.e. = Daily mean*250

Daily Mean = $\alpha_i + \beta_i R_m + e_i$

The variance of all the stocks are calculated by using the formula in Excel = VAR.S(Number1, Number2), where Number 1 and Number 2 are the range of the return of Stock 1 to Stock 47.

- Beta – It is a measure of a security's or portfolio's volatility, compared with rates of return on the market as a whole. Beta is a slope and can take any real value ($E[r]$). Here we have done regression to check the value of beta from the Data analysis tool present in Excel.

¹Sample companies: Companies are selected from NSE Nifty 50.

- Risk or variance of a security comprises of two components:

1. systematic risk, and $(\beta_i^2 \sigma_m^2)$
2. unsystematic risk. Variance due to index is the systematic risk and the residual variance is unsystematic risk. $(\sigma_i^2 - \text{Systematic Risk})$

where, $\sigma_i^2 = \text{Total variance of a security return}$

- The 'excess return to beta ratio' is calculated for each stock included in the project.
- The stocks are ranked in descending order as per the beta ratio $\frac{R_i - R_f}{\beta}$
- A cut off rate (C_i) is calculated for all the securities using the formula

$$C_i = \frac{\sigma_m^2 \frac{\sum_{i=1}^N (R_i - R_f) \beta}{\sigma_{ei}^2}}{1 + \sigma_m^2 \sum_{i=1}^N \frac{\beta_i^2}{\sigma_{ei}^2}}$$

- The proportion of each stock to be invested in portfolio is calculated using

the formula $W = \frac{Z_i}{\sum_{i=1}^N Z_i}$ where, $Z_i = \frac{\beta_i}{\sigma_{ei}^2} \left(\frac{R_i - R_f}{\beta_i} - C^* \right)$

Data Analysis and Interpretation

Constructing an optimal portfolio by SIM with various statistical data such as mean stock return (R_i), mean index return (R_m), variance of individual stock and market return, beta, systematic risk and unsystematic risk of all the 47 securities have been collected. Data inputs which have been collected are arranged in Table 1(in Annexure).

In Table 6 (in Annexure) let us determine the excess return to beta and cutoff to arrange the input data from highest to lowest and select stocks with optimal cutoff. The ranking of securities on the basis of their risk related returns and computation of cut-off point.

In Table 7 we have calculated the cutoff and found the optimal cutoff point (the highest of all) and selected all securities up to such cut-off point.

As per Sharpe's Index Model Securities with negative return are ignored in the selection portfolio. So, the securities that are selected are shown in Table 2(in Annexure).

Out of total 47 stocks considered for the construction, 23 securities have positive returns and now we are going to determine the best stocks for our portfolio using beta and debt to equity ratio.

In Table 7 (in Annexure) we are determining beta and debt to equity to select securities for constructing a portfolio. Out of 23 stocks only 8 stocks are eligible for inclusion in the optimal portfolio as shown in Table 4. Once the composition of stocks included in an optimal portfolio is determined, the final step is to find the proportion of each of these selected stocks in the portfolio.

As shown in Table 8 (in Annexure), there are eight stocks selected for constructing an optimal portfolio.

*INFY.NS, BAJAJFINSV.NS, AXISBANK.NS, ICICIBANK.NS, BAJAJFINANCE.NS, LT.NS,
HDFC.NS, INFRATEL.NS*

These stocks are selected on the basis of their cut-off, beta and debt to equity ratio. These securities show less risk with a good debt equity ratio which is close to 0 or approx.

Expected Return on Portfolio	14.06%
Variance of the Portfolio	0.289%
Expected Risk of the Portfolio	5.38%

Expected Return and Risk of the Portfolio constructed using
Sharpe Index Model

Sharpe index model (SIM) resolves the problems involved in the selection of securities to construct a portfolio of an investor with a high return.

Markowitz Model

In the year 1950, Markowitz Model stated that in order to obtain benefit investors must invest in more than one security, i.e. diversification helps to reduce the risk. The Markowitz Model is based on several assumptions

- Investors estimate risk on variability of return,
- Investors are based on expected return and variance of return,
- If we diversified, Risk will be decreased or reduced. Analysts have the opinion that if more and more portfolios are added, unsystematic risk can be reduced.

We have taken historical data of the Bombay Stock Exchange listed companies. We have considered 30 stocks randomly and then calculated the difference with beta (5Y) and beta (from regression) and shortlisted the stocks the 15 on the basis of debt to equity ratio.

It's an intricate task of selecting good investments by considering the trade-off between risk and return along with the combination of various types of investments for the investors. A rational investor always seeks to minimize risks and maximize returns on his investment in an optimal portfolio.

This is done through the construction of a portfolio of assets which is subject to the investor's portfolio. Therefore Risk-return analysis of the Selected 15 securities are calculated by the Help of Markowitz Model. To analyse the return and risk characteristic of the stocks, the annual mean return and standard deviation are calculated.

The Annual return on each stock is calculated as follows:

$R_i = (B_3/B_2) - 1$, where B_2 = current closing price of stock, B_1 = previous closing price of stock & R_i = return of stock

The portfolio return can be calculated with the help of the following formula: $R_p = \sum_{i=1}^N X_i R_i$

R_p – Return on Portfolio

X_i – portion of total portfolio invested in each security

R_i – Expected Return on security i

The total computation of Portfolio return and risk are

$$1. \text{ Portfolio Return } R_p = \sum_{i=1}^N X_i R_i$$

$$2. \text{ Portfolio Risk } \sigma_p = \sqrt{X_i^2 \sigma_i^2 + X_{i+1}^2 \sigma_{i+1}^2 + \dots + 2X_i X_{i+1} \dots (r_{i,i+1} \dots \sigma_i \sigma_{i+1} \dots)}$$

Data Analysis and Interpretation

Companies BSE			
J.K. Cement	Bosch	HDFC	L&T
Hexaware	Biocon	Colgate	Godrej
Tata	Birla	Coal	IGL
Cipla	Torrent	Infosys	Berger
Alok	Titan	Siemens	J.B Chem
Wipro	JSW Steel	NTPC	GAIL
HCL	Jyothy	Adani	
Reliance	JSW Energy	SBI	

Table 1 (source: BSE²)

Constructing an optimal portfolio by Markowitz with various statistical data such as expected return, standard deviation, mean stock return (R_i), mean index return (R_m), variance of individual stock and market return, beta, systematic and unsystematic risk of all the 30 securities that have been collected. Data inputs which have been collected are arranged in Table 1.

Now, in Table 2 Companies with Beta (5y) and Beta (calculated with regression). And securities with major differences in beta are eliminated.

Out of the total 30 stocks considered, we are going to determine the best stocks for our portfolio using beta and debt to equity ratio.

In Table 3, we are considering beta and debt to equity to select securities for constructing a portfolio. Out of 30 stocks only 15 stocks are eligible for inclusion in the optimal portfolio as shown in Table 4(pg. no. 12).

² Stocks are randomly selected from BSE

There are 15 stocks selected for constructing an optimal portfolio.

IGL.BO, BIOCON.BO, CIPLA.BO, COALINDIA.BO, HCLTECH.BO, JBCHEPHARM.BO, COLPAL.BO, GAIL.BO, INFY.BO, JYOTHYLAB.BO, TITAN.BO, TORNTPHARM.BO, WIPRO.BO

These stocks are selected on the basis of their beta and debt to equity ratio. These securities show less risk with a good debt equity ratio which is close to 0 or approx.

Companies BSE	Beta (5Y)	Beta (Calculated through Regression)	Difference
J.K. Cement	1.31	0.60	-0.71
Hexaware	1.07	0.81	-0.26
Tata	0.69	0.55	-0.14
Cipla	0.55	0.49	-0.06
Alok	0.73	0.70	-0.03
Wipro	0.56	0.53	-0.03
HCL	0.64	0.62	-0.02
Reliance	1.06	1.06	0.00
Bosch	0.88	0.88	0.00
Biocon	0.58	0.60	0.02
Birla	0.78	0.80	0.02
Torrent	0.45	0.47	0.02
Titan	0.79	0.89	0.10
JSW Steel	1.1	1.21	0.11
Jyothy	0.53	0.69	0.16
GAIL	0.58	0.74	0.16
JSW Energy	0.66	0.84	0.18
HDFC	0.77	0.96	0.19
Colgate	0.25	0.45	0.20
Coal	0.4	0.60	0.20
Infosys	0.53	0.75	0.22
Siemens	0.67	0.92	0.25
NTPC	0.41	0.66	0.25
Adani	1.09	1.38	0.29
SBI	0.97	1.29	0.32
L&T	0.72	1.05	0.33
Godrej	0.38	0.72	0.34
IGL	0.34	0.74	0.40
Berger	0.31	0.76	0.45
J.B Chem	0.06	0.60	0.54

Table 2 (source: Yahoo Finance³)

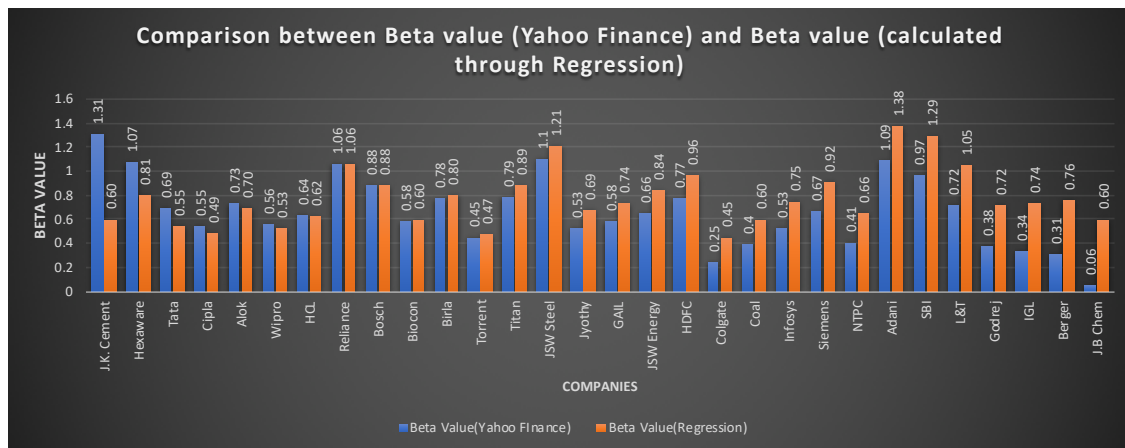


Figure 1

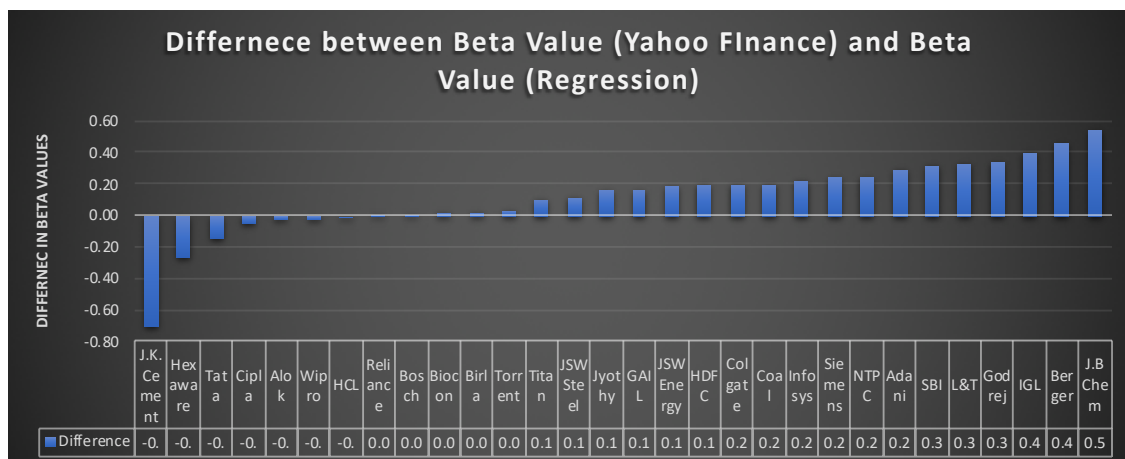


Figure 2

Companies shortlisted on the basis of Beta Value & Debt-to-Equity Ratio		
Companies	Debt-to-Equity Ratio	Beta Value
Indraprastha Gas Limited (IGL.BO)	0.018	0.74
Torrent Pharmaceuticals Limited (TORNTPHARM.BO)	1.214	0.60
Biocon Limited (BIOCON.BO)	0.370	0.84
Titan Company Limited (TITAN.BO)	0.534	0.70
Bosch Limited (BOSCHLTD.BO)	0.008	0.80
Cipla Limited (CIPLA.BO)	0.193	0.53
Coal India Limited (COALINDIA.BO)	0.198	0.55
HCL Technologies Limited (HCLTECH.BO)	0.137	0.69
J. B. Chemicals & Pharmaceuticals Limited (JBCHEPHARM.BO)	0.000	0.60
Wipro Limited (WIPRO.BO)	0.160	0.47
Jyothy Labs Limited (JYOTHYLAB.BO)	0.280	0.45
Colgate-Palmolive (India) Limited (COLPAL.BO)	0.064	0.60
GAIL (India) Limited (GAIL.BO)	0.134	0.89
Infosys Limited (INFY.BO)	0.068	0.88
Tata Consultancy Services Limited (TCS.BO)	0.091	0.62

Table 3

In Fig-1 & 2, 15 companies are shortlisted for construction of portfolio with beta and debt to equity ratio. Companies with low debt to equity ratio and appropriate Beta value the stocks were chosen to construct an optimal portfolio.

After selecting the companies, we will use the Markowitz Model to find the return and risk of the portfolio.

The portfolio risk and return using historical data is computed using the following formula:

$$\text{Portfolio return} = R_p = \sum_{i=1}^N X_i R_i$$

$$\text{Portfolio risk} = \sigma_p = \sqrt{X_i^2 \sigma_i^2 + X_{i+1}^2 \sigma_{i+1}^2 + \dots + 2X_i X_{i+1} \dots (r_{i,i+1} \dots \sigma_i \sigma_{i+1} \dots)}$$

Where

X_i = weights (percentage value)

r = return on the securities

The weights of the securities in the portfolio are represented by the variables X_i . Weights are the market values of the securities held by the investor.

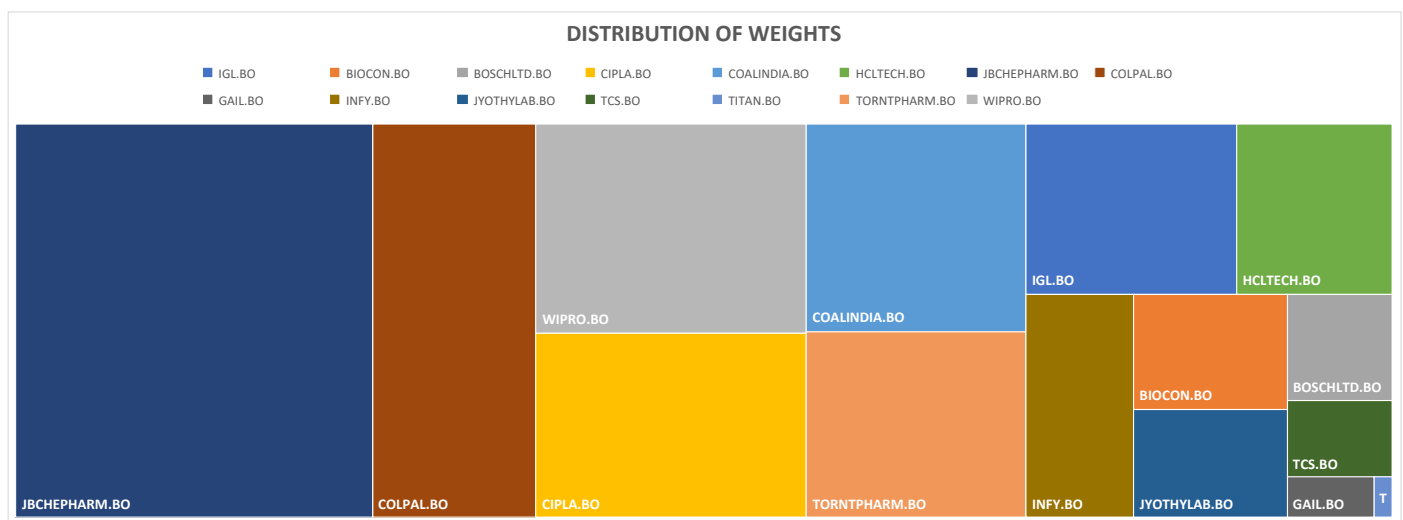


Figure 3

The total covariance is computed by considering the weight of each security in the portfolio. The square root of the variance gives the standard deviation of the portfolio, i.e., the risk of the portfolio. The following figure gives the computation of the standard deviation elaborately. The group of individual securities, n are related with each other to arrive at the covariance matrix.

Daily Expected Return and Expected Return (Annually)

	IGL.BO	BIOCON.BO	BOSCHLT.D.BO	CIPLA.BO	COALINDIA.B	HCLTECH.BO	JBCHEPHARM	COLPAL.BO	GAIL.BO	INFY.BO	JYOTHYLAB.B	TCS.BO	TITAN.BO	TORNTPHARM	WIPRO.BO
Expected Return	0.15%	0.18%	-0.02%	0.000256871	-0.04%	0.07%	0.13%	0.05%	0.06%	0.08%	0.03%	0.12%	0.13%	0.08%	0.04%
Variance	0.000381748	0.00047995	0.000365352	0.000293597	0.00030336	0.00030283	0.00041743	0.00020009	0.00056057	0.0003155	0.00045747	0.00127425	0.00045354	0.00036348	0.000243281
Standard Deviation	0.019538362	0.02190766	0.019114177	0.017134683	0.01741725	0.0174019	0.02043105	0.01414529	0.02367641	0.01776245	0.02138853	0.03569669	0.02129638	0.01906514	0.015597477
Annual Standard Deviation	30.89%	34.64%	30.22%	27.09%	27.54%	27.51%	32.30%	22.37%	37.44%	28.08%	33.82%	56.44%	33.67%	30.14%	24.66%

Table 4

Variance - Covariance Matrix

Companies	IGL.BO	BIOCON.BO	BOSCHLT.D.BO	CIPLA.BO	COALINDIA.BC	HCLTECH.BO	JBCHEPHARM	COLPAL.BO	GAIL.BO	INFY.BO	JYOTHYLAB.BC	TCS.BO	TITAN.BO	TORNTPHARM.BO	WIPRO.BO
IGL.BO	0.000381748	0.00010899	9.3785E-05	7.2968E-05	5.708E-05	6.7474E-05	0.00010315	5.2326E-05	0.00012477	6.4896E-05	9.9608E-05	3.09346E-05	0.00010437	7.39053E-05	4.7761E-05
BIOCON.BO	0.000108994	0.00047995	7.9064E-05	0.00011087	4.9532E-05	6.6999E-05	0.00011149	6.2095E-05	9.3096E-05	7.3205E-05	8.0946E-05	4.29935E-05	8.3285E-05	9.03769E-05	4.3617E-05
BOSCHLT.D.BO	9.37855E-05	7.9064E-05	0.00036535	5.4693E-05	8.5758E-05	7.7691E-05	8.5557E-05	5.9807E-05	9.2848E-05	7.128E-05	0.00010522	2.91418E-05	0.00012903	5.41019E-05	6.1518E-05
CIPLA.BO	7.29683E-05	0.00011087	5.4693E-05	0.0002936	6.0316E-05	3.9775E-05	6.0974E-05	4.5724E-05	7.1924E-05	4.6215E-05	6.8104E-05	3.73027E-05	7.191E-05	9.93524E-05	4.4186E-05
COALINDIA.BO	5.70797E-05	4.9532E-05	8.5758E-05	6.0316E-05	0.00030336	5.3247E-05	3.6082E-05	3.8906E-05	9.4863E-05	4.0177E-05	7.6533E-05	4.04693E-05	7.704E-05	4.0616E-05	5.7997E-05
HCLTECH.BO	6.74736E-05	6.6999E-05	7.7691E-05	3.9775E-05	5.3247E-05	0.00030283	5.6653E-05	4.6724E-05	7.4919E-05	0.00016634	5.7844E-05	0.000154989	6.8115E-05	5.13328E-05	0.00012251
JBCHEPHARM.BO	0.00010315	0.0001149	8.5557E-05	6.0974E-05	3.6082E-05	5.6653E-05	0.00041743	3.466E-05	8.2023E-05	7.6687E-05	9.8269E-05	2.27244E-05	8.6667E-05	6.9795E-05	4.5073E-05
COLPAL.BO	5.2326E-05	6.2095E-05	5.9807E-05	4.5724E-05	3.8906E-05	4.6724E-05	3.466E-05	0.00020009	5.6365E-05	4.2213E-05	6.0129E-05	3.34559E-05	7.1706E-05	6.31453E-05	4.7153E-05
GAIL.BO	0.000124774	9.3096E-05	9.2848E-05	7.1924E-05	9.4863E-05	7.4919E-05	8.2023E-05	5.6365E-05	0.00056057	6.2938E-05	8.8082E-05	0.000395948	9.2755E-05	7.33876E-05	7.248E-05
INFY.BO	6.48956E-05	7.3205E-05	7.128E-05	4.6215E-05	6.8104E-05	0.00016634	7.6687E-05	6.2938E-05	0.0003155	0.000395948	0.000153887	6.5286E-05	6.5286E-05	4.73668E-05	0.00012372
JYOTHYLAB.BO	9.96075E-05	8.0946E-05	0.00010522	6.8104E-05	7.6533E-05	5.7844E-05	9.8269E-05	6.0129E-05	8.8082E-05	5.5893E-05	0.00045747	7.26506E-05	0.00010762	5.60425E-05	5.1043E-05
TCS.BO	3.09346E-05	4.2993E-05	2.9142E-05	3.7303E-05	4.0469E-05	0.00015499	2.2724E-05	3.3456E-05	0.00039595	0.00015389	7.2651E-05	0.001274254	3.3379E-05	3.67717E-05	0.00010829
TITAN.BO	0.000104372	8.3285E-05	0.00012903	7.191E-05	7.704E-05	6.8115E-05	8.6667E-05	7.1706E-05	9.2755E-05	6.5286E-05	0.00010762	3.33785E-05	0.00045354	6.78253E-05	6.584E-05
TORNTPHARM.BO	7.39053E-05	9.0377E-05	5.4102E-05	9.9352E-05	4.0616E-05	5.1333E-05	6.9795E-05	6.3145E-05	7.3388E-05	4.7367E-05	5.6043E-05	3.67717E-05	6.7825E-05	0.00036348	3.6697E-05
WIPRO.BO	4.77607E-05	4.3617E-05	6.1518E-05	4.4186E-05	5.7997E-05	0.00012251	4.5073E-05	4.7153E-05	7.248E-05	0.00012372	5.1043E-05	0.000108287	6.584E-05	3.66967E-05	0.00024328

Table 5

In table 5, we were able to find out the variance of different companies and covariances with other companies.

Co-variance can also be measured in terms of the correlation coefficient. The correlation coefficient ranges between the value +1 and -1.

The variance and covariance matrix will help us to find the Return on portfolio and the variance of the portfolio.

	Annual
Expected Risk on the Portfolio	15.68%
Expected Return of Portfolio	20.00%

Expected Return and Risk of the Portfolio constructed using
Markowitz Model (mean-variance)

The weights play an important role to make the portfolio optimal. Hence, the optimisation of weights is done with the help of using Excel Add-in tool called Solver. It looks into every stocks' risk and return individually to distribute weight efficiently. This further helps us to make our portfolio optimal.

Limitation of the study

The above study is purely based on secondary data. The Historical data (5 years) is taken from Yahoo Finance. Hence, the results of the portfolio may not be universally applicable.

Out of 50 companies from Nifty Fifty 47 companies are chosen for Single Index Model and out of all companies in BSE only 30 companies are chosen without any bias.

Limitation of the Models

Although the Markowitz model is an acceptable and conceptually sound tool for analysing the risk and return of a portfolio. The problem in the Marowitz model is that a number of covariances have to be estimated. Also, Markowitz model is an old model to construct an optimal portfolio, whereas Sharpe Index Model is relatively new. Although, Sharpe Model also has some limitations like Sharpe ratios is backwards-looking and accounts for historical returns and volatility. Hence, the risk and returns that are calculated assume that all future performances will be similar to past.

Conclusion

The construction of an optimal portfolio was interesting but challenging. In this we attempted to construct an optimal portfolio taking 47 stocks of Nifty 50 Index and 30 of BSE. As shown above, only 8 stocks fulfil the Construction of Optimal Portfolio with Nifty 50 data using Sharpe's Single Index Model and further constructing one more portfolio with BSE 30 companies and calculating risk-return by using the Markowitz Model. Use of the cut-off rate played a vital role in constructing a simple index model portfolio. If the investor wants to earn a maximum return without considering the risk aspect then investment can be made on those securities which yield high returns. Even though the return is high, the risk involved in the stock return should be considered while taking investment decisions.

The risk can be reduced if the portfolio is diversified. The point of diversity is to achieve a given level of expected return while bearing the least possible risk.. The optimal portfolio analysis and risk return trade-off are determined by the challenging attitudes of investors towards a variety of economic, monetary, political and psychological forces prevailing in the stock market. Thus, the portfolio construction table would help an investor in investment decisions.

There is nothing called “*Good Portfolio*”. Good returns are subjective, but it is very important to not get greedy with high risk or lose out on great opportunities to meet financial goals by choosing a low-risk portfolio.

Table 6

TO DETERMINE CUTOFF												
Companies	β Beta	α Alpha	α + βR _m + ε Mean Return(R _i)	Risk Free Return (R _f)	β ² σ ² _ε Systematic Risk	σ ² _ε = σ ² _i - Sys ^{ematic} Risk	R _i - R _f Excess Return	R _i - R _f + β _i Excess Return Beta Ratio	$\frac{(R_i - R_f) \times \beta_i}{\sigma_{\epsilon}^2}$	$\frac{\beta_i^2}{\sigma_{\epsilon}^2}$	$\sum_{i=1}^n \frac{\beta_i^2}{\sigma_{\epsilon}^2}$	C _i cutoff
INFY.NS	1.43180325	0.00079727	0.153243264	0.0601	0.000262777	0.06349	0.093143264	7%	0.508501031	7.816705098	7.816705098	0%
BAJAJFINSV.NS	1.41702995	0.00138155	0.152254618	0.0601	0.000257383	0.02029	0.092154618	7%	1.006413691	7.656232237	15.47293734	0%
AXISBANK.NS	1.42982596	-0.0004169	0.151818609	0.0601	0.000262052	0.01744	0.091718609	6%	0.500031861	7.795130557	23.26806789	0%
ICICIBANK.NS	1.4007735	-0.0004908	0.148651408	0.0601	0.000251511	0.01919	0.088551408	6%	0.472955567	1.979401209	30.74964012	0%
BAJAJFINANCE.NS	1.27043734	0.00069756	0.135962716	0.0601	0.000206884	0.01756	0.075862716	6%	0.367484203	2.346885413	36.90372578	1%
LT.NS	1.28865059	-0.0007287	0.136475681	0.0601	0.000212859	0.02659	0.076375681	6%	0.375272994	2.722158407	43.23552875	1%
HDFC.NS	1.19330109	0.00040224	0.127454594	0.0601	0.000182525	0.01869	0.067354594	6%	0.306460352	3.028618758	48.66499467	1%
INFRATEL.NS	1.19940731	-0.0001477	0.127554841	0.0601	0.000184397	0.01927	0.067454841	6%	0.308486991	3.337105749	54.15016885	1%
GRASIM.NS	1.13962191	5.1647E-05	0.121388709	0.0601	0.000166473	0.01613	0.061288709	5%	0.266316639	3.603422388	59.10214587	1%
ADANIPTS.NS	1.10041749	-9.617E-05	0.117066746	0.0601	0.000155216	0.01895	0.05966746	5%	0.23902093	3.842443318	63.71927482	1%
BPCL.NS	1.05447329	0.00040263	0.112673807	0.0601	0.000142526	0.02062	0.052573807	5%	0.211379102	4.05382242	67.95890722	1%
EICHERMOT.NS	0.99344512	0.00024658	0.106020001	0.0601	0.000126505	0.01878	0.045920001	5%	0.173941339	4.227763759	71.72199891	1%
HDFCBANK.NS	0.96730323	0.00030969	0.103299751	0.0601	0.000119935	0.00944	0.043199751	4%	0.159331234	4.387094992	75.28964948	1%
HINDUNILVR.NS	0.89684709	0.00112394	0.096612441	0.0601	0.0001031	0.02614	0.036512441	4%	0.124857982	4.511952975	78.35650873	1%
JSWSTEEL.NS	0.87964687	1.3004E-05	0.093670178	0.0601	9.91832E-05	0.02327	0.033570178	4%	0.112594983	4.624547958	81.30686027	1%
BAJAJ-AUTO.NS	0.84763722	0.00012614	0.0903752	0.0601	9.20962E-05	0.01337	0.0302752	4%	0.09784848	4.722396438	84.04639666	1%
GAIL.NS	0.83942257	0.00014811	0.089522548	0.0601	9.03198E-05	0.01887	0.029422548	4%	0.094171169	4.816567607	86.7330914	1%
BHARTIARTEL.NS	0.80834829	0.00027459	0.086340511	0.0601	8.37565E-05	0.01932	0.026240511	3%	0.080877525	4.897445131	89.22455234	1%
BRITANNIA.NS	0.7877587	0.00064794	0.08452166	0.0601	7.95441E-05	0.01432	0.02442166	3%	0.073354274	4.970799405	91.59070879	1%
IOC.NS	0.78195739	0.00105966	0.084315704	0.0601	7.83768E-05	0.03256	0.024215704	3%	0.072200004	5.04299941	93.92214326	1%
SBIN.NS	0.77336417	-0.0002811	0.082060036	0.0601	7.66637E-05	0.02162	0.021960036	3%	0.064755124	5.107754533	96.20621732	1%
ASIANPAINTS.NS	0.76210746	0.00058592	0.081728516	0.0601	7.44481E-05	0.01357	0.021628516	3%	0.062849231	5.170603764	98.4717876	1%
KOTAKBANK.NS	0.72719135	-0.0005606	0.076864401	0.0601	6.77827E-05	0.02662	0.01764401	2%	0.04648296	5.217086724	100.4334843	1%
HCLTECH.NS	0.65565297	0.00044048	0.070248727	0.0601	5.51023E-05	0.01597	0.010148727	2%	0.025371296	5.24245802	102.072583	1%
COALINDIA.NS	0.65481676	-0.0006224	0.069096805	0.0601	5.49618E-05	0.01608	0.008996805	1%	0.022462865	5.264920885	103.7075034	1%
POWERGRID.NS	0.63728378	-0.0003211	0.067531388	0.0601	5.20586E-05	0.01588	0.007431388	1%	0.018057592	5.282978477	105.2560446	1%
TATAMOTORS.NS	0.58326659	-0.0007996	0.061301586	0.0601	4.36069E-05	0.01905	0.001201586	0%	0.002672262	5.285650739	106.5531977	1%
INDUSINDBK.NS	0.55816113	-0.0001584	0.059269732	0.0601	3.99338E-05	0.01372	-0.000830268	0%	-0.001766994	5.283883746	107.7410877	1%
TATASTEEL.NS	0.54281728	0.00061647	0.058410947	0.0601	3.77684E-05	0.02030	-0.001689053	0%	-0.003495859	5.280387886	108.8645652	1%
CIPLA.NS	0.53499463	0.0001224	0.057083995	0.0601	3.66877E-05	0.01633	-0.003016005	-1%	-0.006152314	5.274235572	109.9558946	1%
DRREDDY.NS	0.49322729	0.00026361	0.052778176	0.0601	3.11828E-05	0.01835	-0.007321824	-1%	-0.013769666	5.260465906	109.8834743	1%
UPL.NS	0.41806434	0.00056367	0.045075536	0.0601	2.24031E-05	0.03553	-0.015024464	-4%	-0.023949648	5.236516258	111.5498871	1%
M&M.NS	0.38182673	7.328E-05	0.04072688	0.0601	1.86876E-05	0.01919	-0.01937312	-5%	-0.028204794	5.208311464	112.1057781	2%
MARUTI.NS	0.37626694	0.00044524	0.040506881	0.0601	1.81474E-05	0.02800	-0.019593119	-5%	-0.02810973	5.180201734	112.6455984	2%
TITAN.NS	0.35374007	0.00112539	0.038788568	0.0601	1.60395E-05	0.03994	-0.021311432	-6%	-0.028744445	5.151457289	113.1221761	2%
ZEEL.NS	0.33839991	-9.627E-05	0.035933615	0.0601	1.46785E-05	0.01874	-0.024166385	-7%	-0.031181641	5.120275648	113.55935	2%
RELIANCE.NS	0.30652436	0.00144111	0.03407717	0.0601	1.20435E-05	0.03071	-0.02602283	-8%	-0.030414211	5.089861437	113.9176007	2%
ULTRACEMCO.NS	0.30610808	0.00029769	0.032889429	0.0601	1.20108E-05	0.02876	-0.027210571	-9%	-0.031759195	5.058102241	114.274879	2%
ITC.NS	0.30183486	-2.209E-05	0.032114674	0.0601	1.16778E-05	0.04367	-0.027985326	-9%	-0.032207484	5.025894757	114.6222518	2%
SUNPHARMA.NS	0.29518668	-0.0003078	0.03112112	0.0601	1.1169E-05	0.02334	-0.02897888	-10%	-0.032616353	4.993278404	114.9544908	2%
SHRECHEM.NS	0.2790177	0.00047424	0.030181623	0.0601	9.97895E-06	0.03169	-0.029918377	-11%	-0.03182928	4.961495124	115.2513295	2%
NTPC.NS	0.24112617	5.6257E-05	0.025729281	0.0601	7.45264E-06	2.29771	-0.034370719	-14%	-0.031600212	4.929848912	115.4730193	2%
NESTLEIND.NSS	0.20272798	0.00090889	0.022493611	0.0601	5.26804E-06	0.03084	-0.037606389	-19%	-0.029069152	4.900779759	115.6297249	2%
TECHM.NS	0.1741704	0.00042308	0.018967235	0.0601	3.88839E-06	0.01892	-0.041132765	-24%	-0.027316137	4.873463622	115.7453909	2%
ONGC.NS	0.17479505	-0.0002882	0.01832247	0.0601	3.91633E-06	0.21636	-0.04177753	-24%	-0.027843827	4.845619795	115.861888	2%
WIPRO.NS	0.08782066	0.0004114	0.00976178	0.0601	9.88587E-07	0.54262	-0.05033822	-57%	-0.016855886	4.828763909	115.891295	2%
TCS.NS	0.04750181	0.00076424	0.005821816	0.0601	2.89229E-07	0.03431	-0.054278184	-114%	-0.009830882	4.818933027	115.8998986	2%

Table 7

TO DETERMINE BEST STOCKS FOR OUR PORTFOLIO USING BETA & DEBT-TO-EQUITY RATIO									
Companies	β Beta	α Alpha	$\alpha + \beta R_m + e_i$ Mean Return	Risk Free Return (R_f)	$\beta^2 \sigma_m^2$ Systematic Risk	$e_i^2 = \sigma_i^2 - \text{Systematic Risk}$ Unsystematic Risk	$R_i - R_f$ Excess Return	$R_i - R_f \div \beta_i$ Excess Return Beta Ratio	D/E
INFY.NS	1.43180325	0.00079727	0.153243264	0.0601	0.000262777	0.06349	0.093143264	7%	0.068
BAJAJFINSV.	1.41702995	0.00138155	0.152254618	0.0601	0.000257383	0.02029	0.092154618	7%	2.0826
AXISBANK.NS	1.42982596	-0.0004169	0.151818609	0.0601	0.000262052	0.01744	0.091718609	6%	-
ICICIBANK.NS	1.4007735	-0.0004908	0.148651408	0.0601	0.000251511	0.01919	0.088551408	6%	-
BAJAJFINANCE.NS	1.27043734	0.00069756	0.135962716	0.0601	0.000206884	0.01756	0.075862716	6%	3.3635
LT.NS	1.28865059	-0.0007287	0.136475681	0.0601	0.000212859	0.02659	0.076375681	6%	1.8778
HDFC.NS	1.19330109	0.00040224	0.127454594	0.0601	0.000182525	0.01869	0.067354594	6%	-
INFRAELECT.NS	1.19940731	-0.0001477	0.127554841	0.0601	0.000184397	0.01927	0.067454841	6%	0.2447
GRASIM.NS	1.13962191	5.1647E-05	0.121388709	0.0601	0.000166473	0.01613	0.061288709	5%	0.7946
ADANI PORTS	1.10041749	-9.617E-05	0.117066746	0.0601	0.000155216	0.01895	0.056966746	5%	1.1872
BPCL.NS	1.05447329	0.00040263	0.112673807	0.0601	0.000142526	0.02062	0.052573807	5%	1.5976
EICHERMOT.	0.99344512	0.00024658	0.106020001	0.0601	0.000126505	0.01878	0.045920001	5%	0.0249
HDFCBANK.NS	0.96730323	0.00030969	0.103299751	0.0601	0.000119935	0.00944	0.043199751	4%	-
HINDUNILVR	0.89684709	0.00112394	0.096612441	0.0601	0.0001031	0.02614	0.036512441	4%	0.1021
JSWSTEEL.NS	0.87964687	1.3004E-05	0.093670178	0.0601	9.91832E-05	0.02327	0.033570178	4%	1.7367
BAJAJ-AUTO	0.84763722	0.00012614	0.0903752	0.0601	9.20962E-05	0.01337	0.0302752	4%	-
GAIL.NS	0.83942257	0.00014811	0.089522548	0.0601	9.03198E-05	0.01887	0.029422548	4%	0.1336
BHARTIARTL	0.80834829	0.00027459	0.086340511	0.0601	8.37565E-05	0.01932	0.026240511	3%	1.6959
BRITANNIA.NS	0.7877587	0.00064794	0.08452166	0.0601	7.95441E-05	0.01432	0.02442166	3%	0.35
IOC.NS	0.78195739	0.00105966	0.084315704	0.0601	7.83768E-05	0.03256	0.024215704	3%	1.3084
SBIN.NS	0.77336417	-0.0002811	0.082060036	0.0601	7.66637E-05	0.02162	0.021960036	3%	-
ASIANPAINT	0.76210746	0.00058592	0.081728516	0.0601	7.44481E-05	0.01357	0.021628516	3%	0.1062
KOTAKBANK	0.72719135	-0.0005606	0.076864401	0.0601	6.77827E-05	0.02662	0.016764401	2%	-

Table 8

STOCKS FOR OUR OPTIMAL PORTFOLIO															
Companies	β Beta	α Alpha	Mean Return	$\alpha + \beta R_m + e_i$ Risk Free Return	$\beta_i^2 \sigma_m^2$ Systematic Risk	$e_i^2 = \sigma_i^2 - \text{Systematic Risk}$ Unsystematic Risk	$R_i - R_f$ Excess Return	$R_i - R_f + \beta_i$ Excess Return Beta	D/E	C_i Cutoff	$z_i = \frac{\beta_i (R_i - R_f - C_i)}{\sigma_{e_i}^2}$	$W_i = \frac{z_i}{\sum_{i=1}^n z_i}$	$\alpha_i \times W_i$	$\beta_i \times W_i$	$e_i^2 X_i^2$
INFY.NS	1.43180325	0.00079727	0.153243264	0.0601	0.000262777	0.06349	0.093143264	7%	0.068	0%	0.349321463	4%	3.33034E-05	0.059809274	0.000110777
BAJAJFINSV.NS	1.41702995	0.00138155	0.152254618	0.0601	0.000257383	0.02029	0.092154618	7%	2.0826	0%	0.339964712	13%	0.000180648	0.185287448	0.000346929
AXISBANK.NS	1.42982596	-0.0004169	0.151818609	0.0601	0.000262052	0.01744	0.091718609	6%	-	0%	0.332402414	15%	-6.33764E-05	0.217378544	0.000403098
ICICIBANK.NS	1.4007735	-0.0004908	0.148651408	0.0601	0.000251511	0.01919	0.088551408	6%	-	0%	0.315232131	14%	-6.79839E-05	0.194025641	0.000368131
BAJAJFINANCE.NS	1.27043734	0.00069756	0.135962716	0.0601	0.000206884	0.01756	0.075862716	6%	3.3635	1%	0.264886761	15%	0.000106429	0.1938334861	0.000408812
LT.NS	1.28865059	-0.0007287	0.136475681	0.0601	0.000212859	0.02659	0.076375681	6%	1.8778	1%	0.262269039	10%	-7.33277E-05	0.129681206	0.000269306
HDFC.NS	1.19330109	0.00040224	0.127454594	0.0601	0.000182525	0.01869	0.067354594	6%	-	1%	0.226668811	14%	5.79498E-05	0.171916899	0.000387888
INFRATEL.NS	1.19940731	-0.0001477	0.127554841	0.0601	0.000184397	0.01927	0.067454841	6%	0.2447	1%	0.223500534	14%	-2.06197E-05	0.167498082	0.00037578
										TOTAL	2.314245865	1	0.000153023	1.319431957	0.00267072

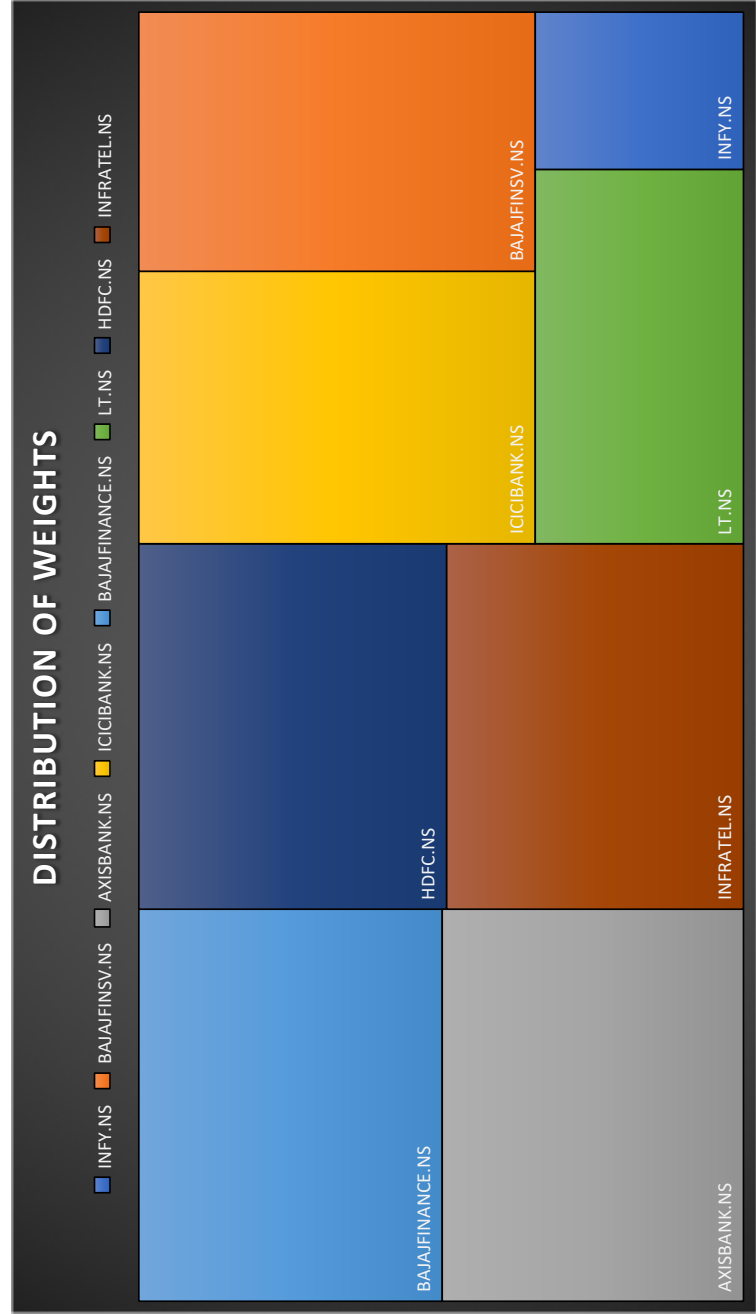


Figure 4

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