Construction of Magic Squares

Yusuf Seha Uysal

Chapters

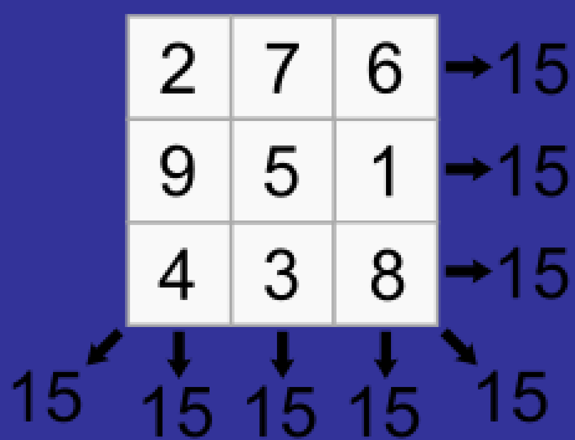
1. Definiton
2. Siamese method for odd orders
3. Criss cross method for doubly even orders
4. Conway’s LUX method for singly even orders

Definiton

A magic square is an nxn array containing the consecutive numbers from 1 to n^2. All its rows, columns and diagonals adds up to the same sum. This sum is called the magic constant. Number n which determines the size of the square is called the order of the magic square.

Definiton

A magic square is an nxn array containing the consecutive numbers from 1 to n^2. All its rows, columns and diagonals adds up to the same sum. This sum is called the magic constant. Number n which determines the size of the square is called the order of the magic square.

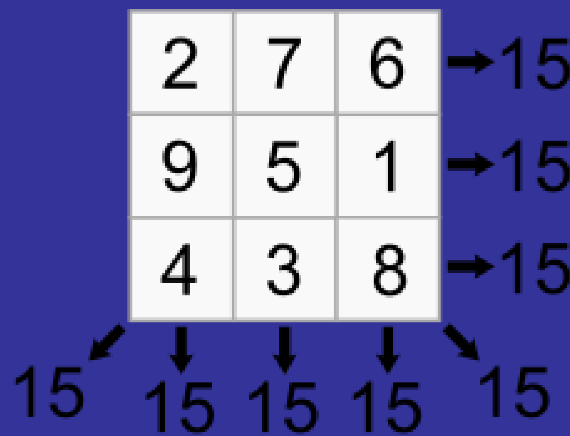


Order: n=3

Magic constant: 15

Magic Constant

$$M = \frac{n(n^2 + 1)}{2}$$



1x1 and 2x2 magic squares

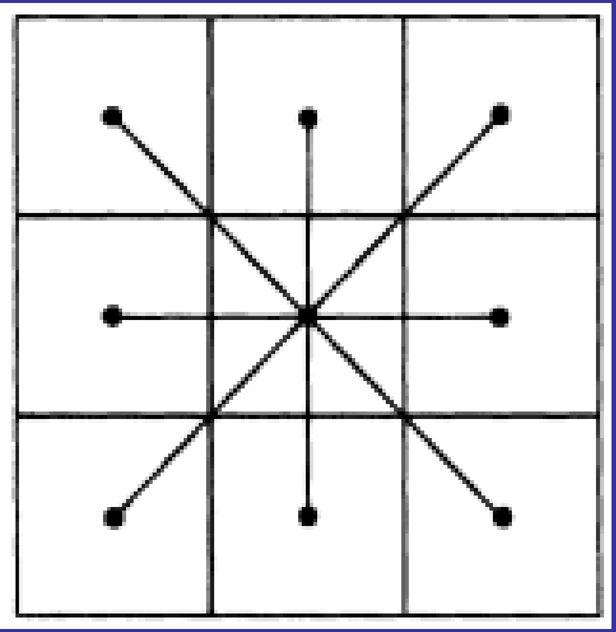
1

A	C
D	B

Uniqueness of 3x3

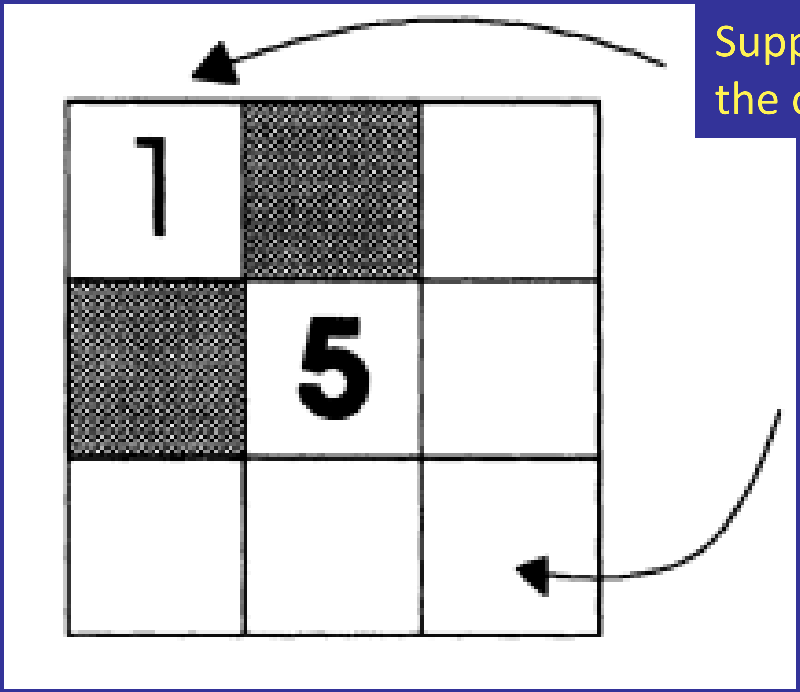
Uniqueness of 3x3

$$M = \frac{n(n^2 + 1)}{2}$$



The center number has to be 5

Uniqueness of 3x3

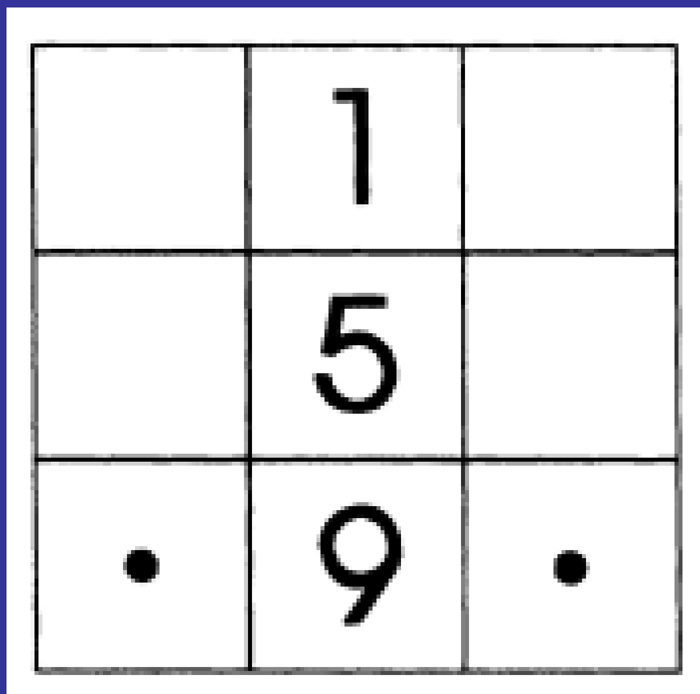


Suppose 1 is in the corner

The opposite corner has to be 9

No row or column that has 9 can include any numbers greater than 5. So, the numbers 6, 7, and 8 must all be placed in the two shaded squares. Contradiction. Then 1 is not in the corner.

Uniqueness of 3x3



4 and 2 has to be in the same row with 9. The rest follows easily.

Classification of Magic Squares

Classification of Magic Squares

Odd order magic squares (2k+1)

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

Classification of Magic Squares

Odd order magic squares (2k+1)

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

Doubly even order (4k)

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

Singly even order (4k+2)

35	1	6	26	19	24
3	32	7	21	23	25
31	9	2	22	27	20
8	28	33	17	10	15
30	5	34	12	14	16
4	36	29	13	18	11

Classification of Magic Squares

Odd order magic squares (2k+1)

Doubly even order (4k)

Singly even order (4k+2)

Classification of Magic Squares

Odd order magic squares (2k+1)

Siamese Method

Doubly even order (4k)

Criss Cross Method

Singly even order (4k+2)

LUX Method

Siamese method (2k+1)

Siamese method (2k+1)

Algorithm:

1. Start by placing 1 in the center of the first row of the square

Siamese method (2k+1)

Algorithm:

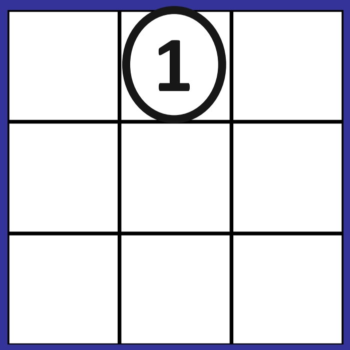
1. Start by placing 1 in the center of the first row of the square

	1	

Siamese method (2k+1)

Algorithm:

1. Start by placing 1 in the center of the first row of the square
2. Write the next number to the upper right of the previous one (↗)



Siamese method (2k+1)

Algorithm:

1. Start by placing 1 in the center of the first row of the square
2. Write the next number to the upper right of the previous one (↗)

	1	
		2

Siamese method (2k+1)

Algorithm:

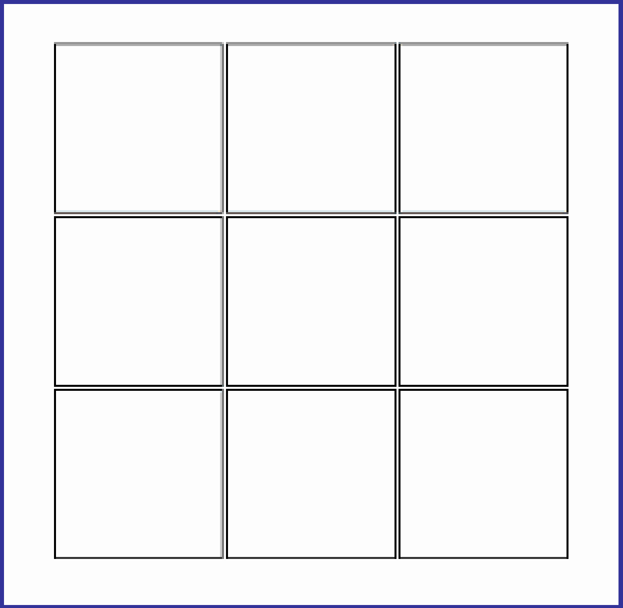
1. Start by placing 1 in the center of the first row of the square
2. Write the next number to the upper right of the previous one (↗)
3. If the upper right is already filled, place it one square below the previous number (↓)

	1	
3		
		2

Siamese method (2k+1)

Algorithm:

1. Start by placing 1 in the center of the first row of the square
2. Write the next number to the upper right of the previous one (↗)
3. If the upper right is already filled, place it one square below the previous number (↓)



Siamese method (2k+1)

Algorithm:

1. Start by placing 1 in the center of the first row of the square
2. Write the next number to the upper right of the previous one (↗)
3. If the upper right is already filled, place it one square below the previous number (↓)

8	1	6
3	5	7
4	9	2

Siamese method (2k+1)

๑๕๕	๑๗๔	๑๙๓	๒๑๒	๒๓๑	๒๕๐	๒๖๙	๒๘๘	๑	๒๐	๓๙	๕๘	๗๗	๙๖	๑๑๕	๑๓๔	๑๕๓
๑๗๓	๑๙๒	๒๑๑	๒๓๐	๒๔๙	๒๖๘	๒๘๗	๑๗	๑๙	๓๘	๕๗	๗๖	๙๕	๑๑๔	๑๓๓	๑๕๒	๑๕๔
๑๙๑	๒๑๐	๒๒๙	๒๔๘	๒๖๗	๒๘๖	๑๖	๑๘	๓๗	๕๖	๗๕	๙๔	๑๑๓	๑๓๒	๑๕๑	๑๗๐	๑๗๒
๒๐๙	๒๒๘	๒๔๗	๒๖๖	๒๘๕	๑๕	๓๔	๓๖	๕๕	๗๔	๙๓	๑๑๒	๑๓๑	๑๕๐	๑๖๙	๑๗๑	๑๙๐
๒๒๗	๒๔๖	๒๖๕	๒๘๔	๑๔	๓๓	๓๕	๕๔	๗๓	๙๒	๑๑๑	๑๓๐	๑๔๙	๑๖๘	๑๘๗	๑๘๙	๒๐๘
๒๔๕	๒๖๔	๒๘๓	๑๓	๓๒	๕๑	๕๓	๗๒	๙๑	๑๑๐	๑๒๙	๑๔๘	๑๖๗	๑๘๖	๑๘๘	๒๐๗	๒๒๖
๒๖๓	๒๘๒	๑๒	๓๑	๕๐	๕๒	๗๑	๙๐	๑๐๙	๑๒๘	๑๔๗	๑๖๖	๑๘๕	๒๐๔	๒๐๖	๒๒๕	๒๔๔
๒๘๑	๑๑	๓๐	๔๙	๖๘	๗๐	๘๙	๑๐๘	๑๒๗	๑๔๖	๑๖๕	๑๘๔	๒๐๓	๒๐๕	๒๒๔	๒๔๓	๒๖๒
๑๐	๒๙	๔๘	๖๗	๖๙	๘๘	๑๐๗	๑๒๖	๑๔๕	๑๖๔	๑๘๓	๒๐๒	๒๒๑	๒๒๓	๒๔๒	๒๖๑	๒๘๐
๒๘	๔๗	๖๖	๘๕	๘๗	๑๐๖	๑๒๕	๑๔๔	๑๖๓	๑๘๒	๒๐๑	๒๒๐	๒๒๒	๒๔๑	๒๖๐	๒๗๙	๙
๔๖	๖๕	๘๔	๘๖	๑๐๕	๑๒๔	๑๔๓	๑๖๒	๑๘๑	๒๐๐	๒๑๙	๒๓๘	๒๔๐	๒๕๙	๒๗๘	๘	๒๗
๖๔	๘๓	๑๐๒	๑๐๔	๑๒๓	๑๔๒	๑๖๑	๑๘๐	๑๙๙	๒๑๘	๒๓๗	๒๓๙	๒๕๘	๒๗๗	๗	๒๖	๔๕
๘๒	๑๐๑	๑๐๓	๑๒๒	๑๔๑	๑๖๐	๑๗๙	๑๙๘	๒๑๗	๒๓๖	๒๕๕	๒๕๗	๒๗๖	๖	๒๕	๔๔	๖๓
๑๐๐	๑๑๙	๑๒๑	๑๔๐	๑๕๙	๑๗๘	๑๙๗	๒๑๖	๒๓๕	๒๕๔	๒๕๖	๒๗๕	๕	๒๔	๔๓	๖๒	๘๑
๑๑๘	๑๒๐	๑๓๙	๑๕๘	๑๗๗	๑๙๖	๒๑๕	๒๓๔	๒๕๓	๒๗๒	๒๗๔	๔	๒๓	๔๒	๖๑	๘๐	๙๙
๑๓๖	๑๓๘	๑๕๗	๑๗๖	๑๙๕	๒๑๔	๒๓๓	๒๕๒	๒๗๑	๒๗๓	๓	๒๒	๔๑	๖๐	๗๙	๙๘	๑๑๗
๑๓๗	๑๕๖	๑๗๕	๑๙๔	๒๑๓	๒๓๒	๒๕๑	๒๗๐	๒๘๙	๒	๒๑	๔๐	๕๙	๗๘	๙๗	๑๑๖	๑๓๕

Siamese method (2k+1)

Criss cross method (4k)

Criss cross method (4k)

Algorithm:

1. Place the numbers from 1 to n^2 in consecutive order in the square

Criss cross method (4k)

Algorithm:

1. Place the numbers from 1 to n^2 in consecutive order in the square

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

Criss cross method (4k)

Algorithm:

1. Place the numbers from 1 to n^2 in consecutive order in the square

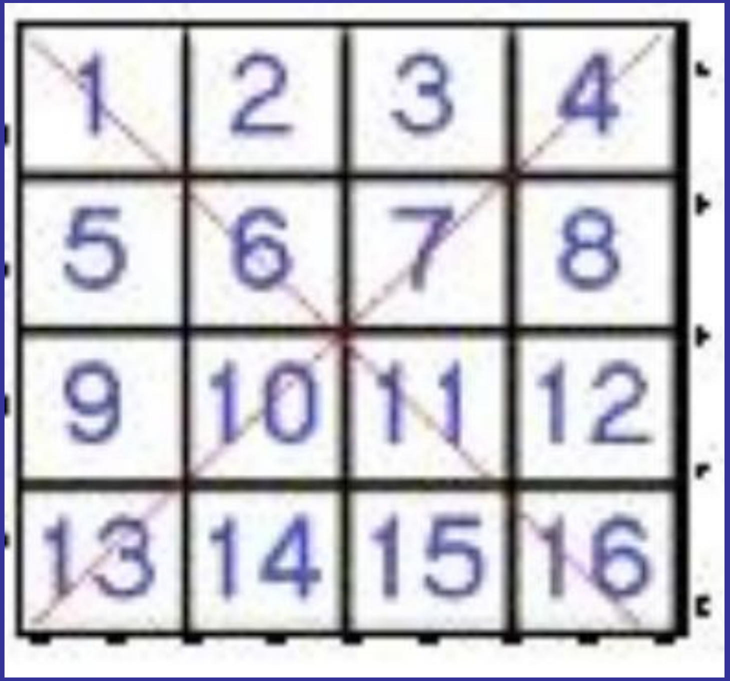
1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

1	63	62	4	5	59	58	8
56	10	11	53	52	14	15	49
48	18	19	45	44	22	23	41
25	39	38	28	29	35	34	32
33	31	30	36	37	27	26	40
24	42	43	21	20	46	47	17
16	50	51	13	12	54	55	9
57	7	6	60	61	3	2	64

2. Draw an X to each 4x4 subsquare
3. Reverse the order of the entries that do not overlap with these X's

Criss cross method (4k)

Let’s see an example:



Criss cross method (4k)

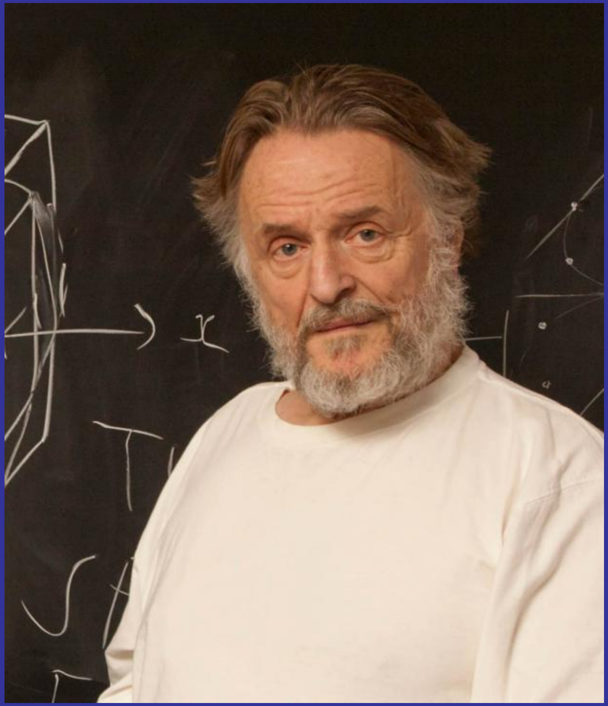
Let’s see an example:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

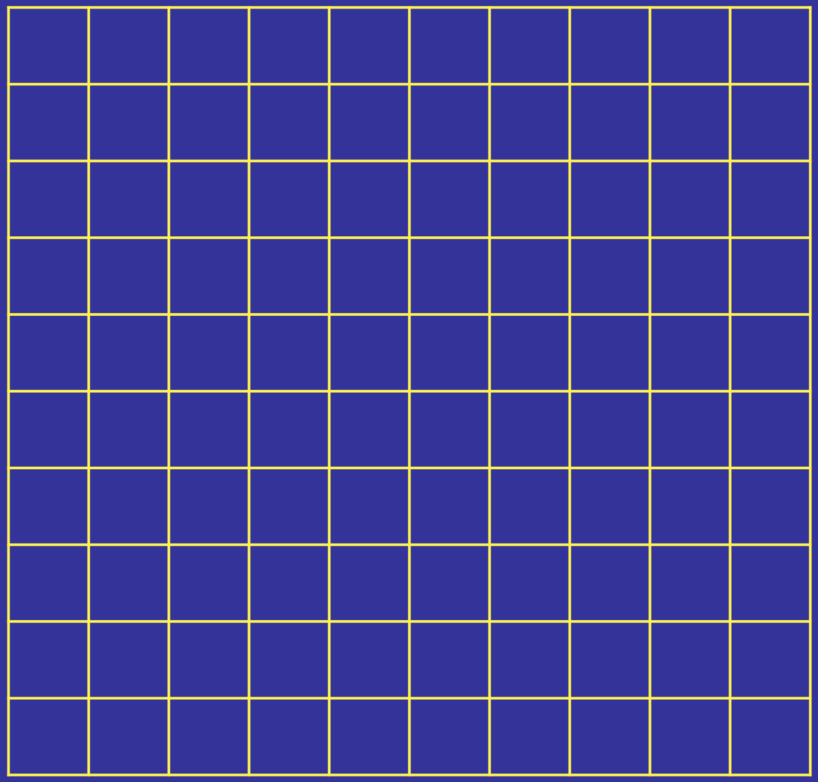
Conway's LUX method (4k+2)

Conway’s LUX method (4k+2)



John Horton Conway (1937–2020)

Conway's LUX method (4k+2)

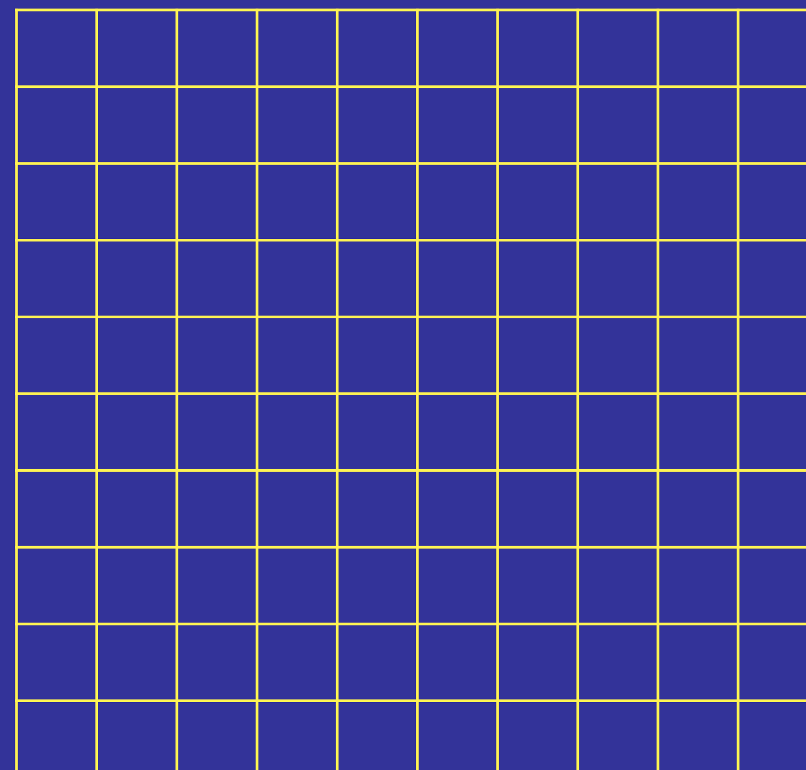


10x10

Conway’s LUX method (4k+2)

Algorithm:

1. Transform the given square into a (2k+1)x(2k+1) square consisting of 2x2 blocks

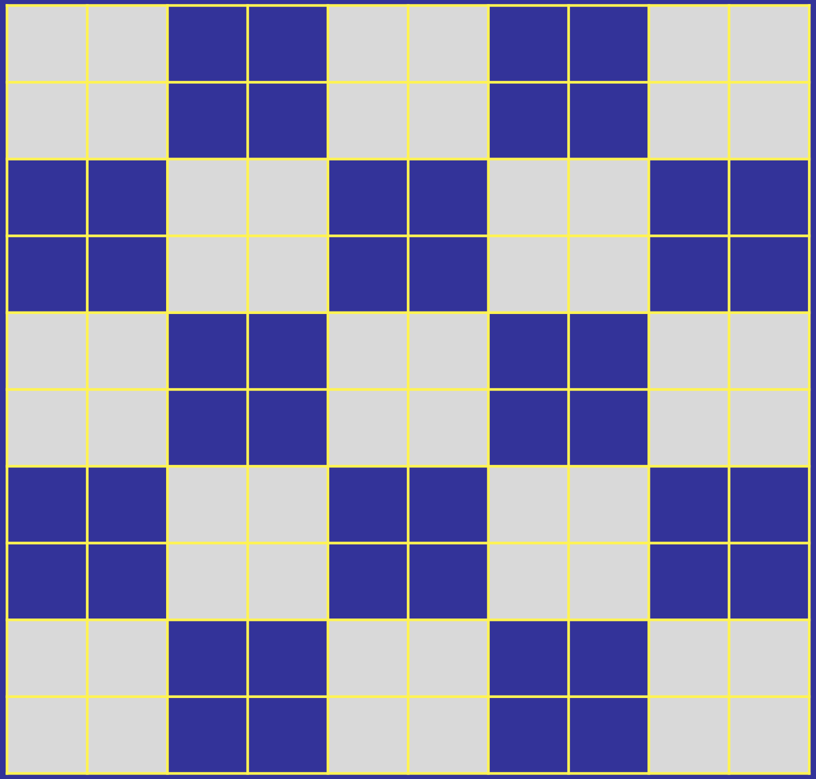


10x10

Conway's LUX method (4k+2)

Algorithm:

1. Transform the given square into a (2k+1)x(2k+1) square consisting of 2x2 blocks

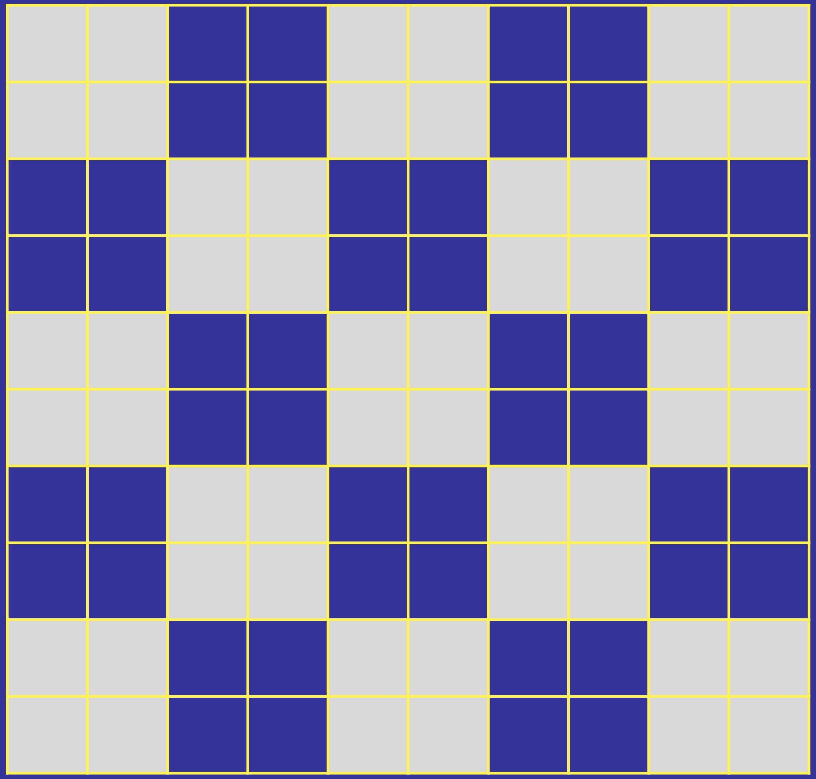


10x10 → 5x5

Conway's LUX method (4k+2)

Algorithm:

1. Transform the given square into a (2k+1)x(2k+1) square consisting of 2x2 blocks
2. Fill the,
   - First k+1 rows with **L**,
   - 1 row with **U**, and
   - The remaining k-1 rows with **X**

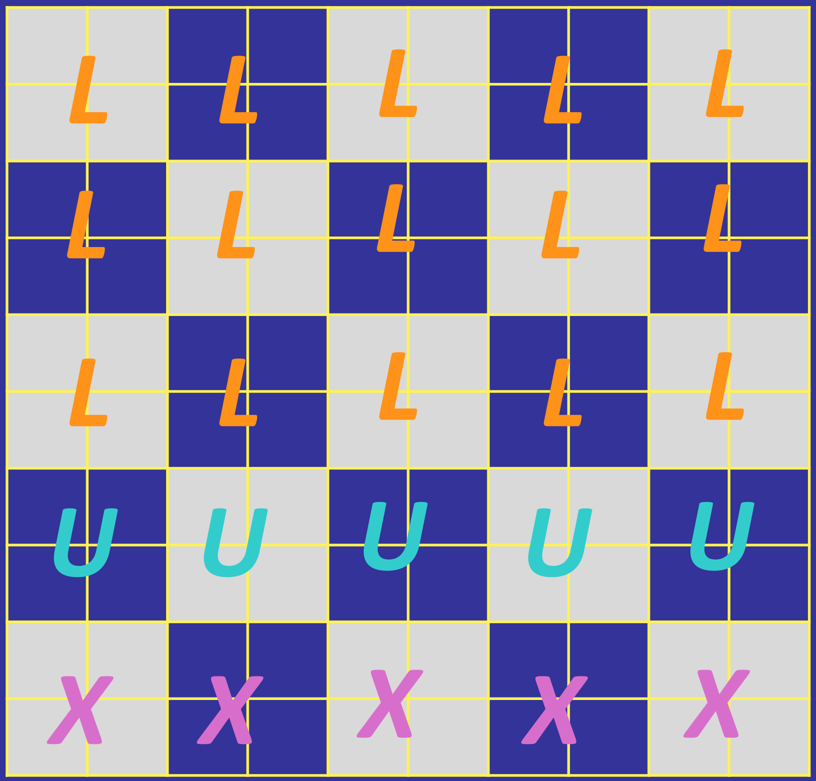


10x10 → 5x5

Conway's LUX method (4k+2)

Algorithm:

1. Transform the given square into a (2k+1)x(2k+1) square consisting of 2x2 blocks
2. Fill the,
   - First *k*+1 rows with **L**,
   - 1 row with **U**, and
   - The remaining *k*-1 rows with **X**

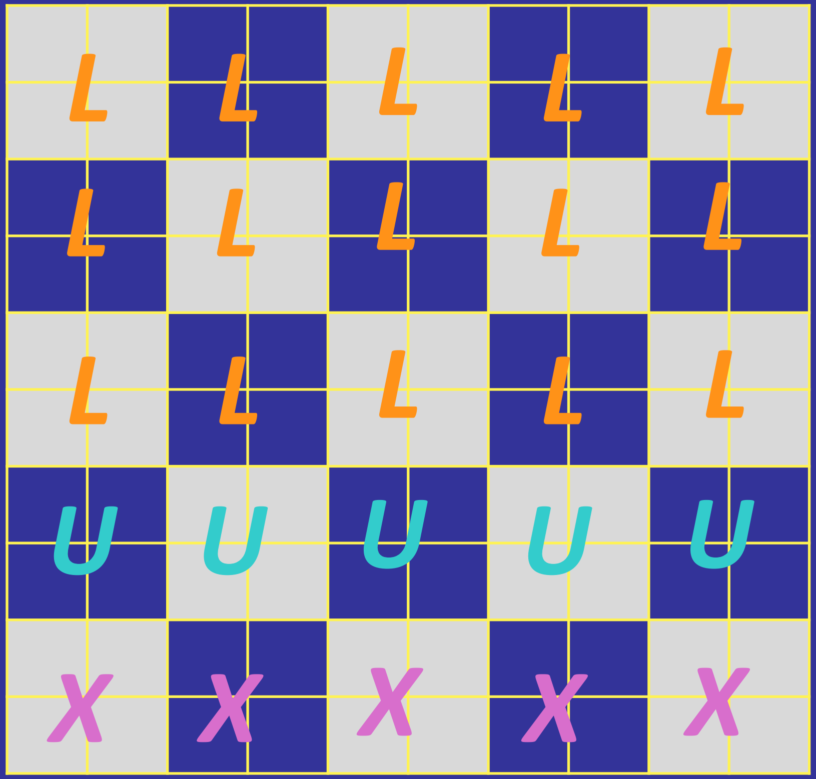


10x10 → 5x5

Conway’s LUX method (4k+2)

Algorithm:

1. Transform the given square into a (2k+1)x(2k+1) square consisting of 2x2 blocks
2. Fill the,
   - First k+1 rows with **L**,
   - 1 row with **U**, and
   - The remaining k-1 rows with **X**
3. Swap the **U** in the center and the **L** above it

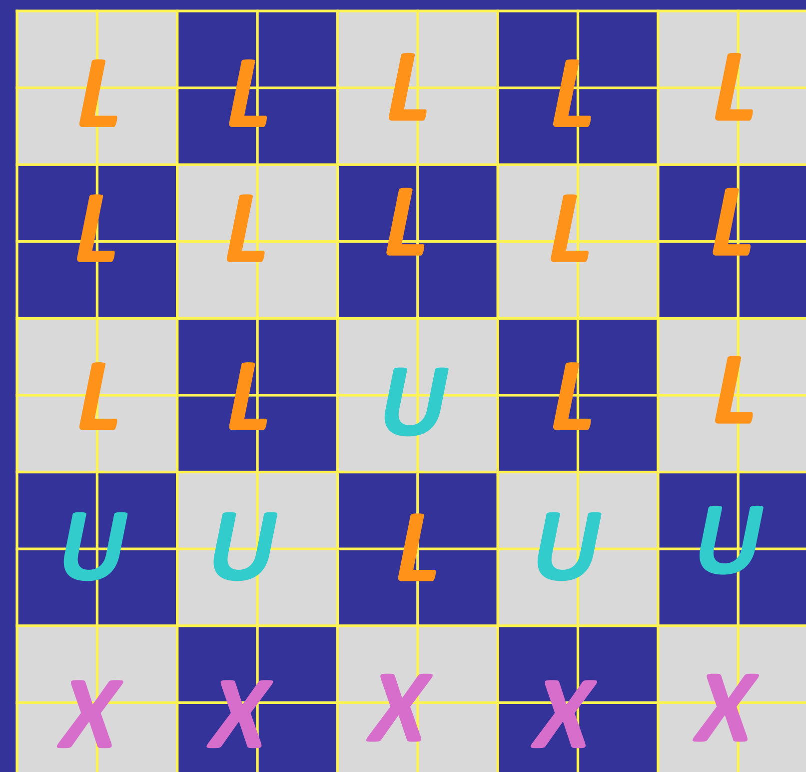


10x10 → 5x5

Conway's LUX method (4k+2)

Algorithm:

1. Transform the given square into a (2k+1)x(2k+1) square consisting of 2x2 blocks
2. Fill the,
   - First *k*+1 rows with **L**,
   - 1 row with **U**, and
   - The remaining *k*-1 rows with **X**
3. Swap the **U** in the center and the **L** above it



10x10 → 5x5

Conway’s LUX method (4k+2)

Algorithm:

1. Transform the given square into a (2k+1)x(2k+1) square consisting of 2x2 blocks
2. Fill the,
   - First k+1 rows with **L**,
   - 1 row with **U**, and
   - The remaining k-1 rows with **X**
3. Swap the **U** in the center and the **L** above it
4. Number the resulting 2k+1 order square using the Siamese method we have seen earlier
5. Using this square, the 2x2 squares are each filled as follows

L	L	L	L	L
L	L	L	L	L
L	L	U	L	L
U	U	L	U	U
X	X	X	X	X

Conway's LUX method (4k+2)

$$L: \begin{array}{ccc} 4 & & 1 \\ & \swarrow & \\ 2 & \rightarrow & 3 \end{array} \qquad U: \begin{array}{ccc} 1 & & 4 \\ \downarrow & & \uparrow \\ 2 & \rightarrow & 3 \end{array} \qquad X: \begin{array}{ccc} 1 & & 4 \\ & \nearrow\!\!\!\searrow & \\ 3 & & 2 \end{array}$$

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

L	L	L	L	L
L	L	L	L	L
L	L	U	L	L
U	U	L	U	U
X	X	X	X	X

Conway's LUX method (4k+2)

$$\mathrm{L}: \begin{array}{ccc} 4 & & 1 \\ & \swarrow & \\ 2 & \rightarrow & 3 \end{array} \qquad \mathrm{U}: \begin{array}{ccc} 1 & & 4 \\ \downarrow & & \uparrow \\ 2 & \rightarrow & 3 \end{array} \qquad \mathrm{X}: \begin{array}{ccc} 1 & & 4 \\ & \nearrow\!\!\!\searrow & \\ 3 & & 2 \end{array}$$

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

L	L	L	L	L
L	L	L	L	L
L	L	U	L	L
U	U	L	U	U
X	X	X	X	X

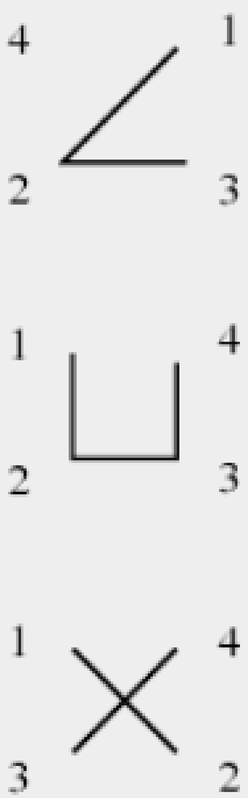
Conway’s LUX method (4k+2)

$$\mathrm{L}: \quad \begin{array}{ccc} 4 & & 1 \\ & \swarrow & \\ 2 & \rightarrow & 3 \end{array} \qquad \mathrm{U}: \quad \begin{array}{ccc} 1 & & 4 \\ \downarrow & & \uparrow \\ 2 & \rightarrow & 3 \end{array} \qquad \mathrm{X}: \quad \begin{array}{ccc} 1 & & 4 \\ & \nearrow\!\!\!\!\!\searrow & \\ 3 & & 2 \end{array}$$

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

L	L	L	L	L
L	L	L	L	L
L	L	U	L	L
U	U	L	U	U
X	X	X	X	X

Conway’s LUX method (4k+2)



68	65	96	93	4	1	32	29	60	57
L		L		L		L		L	
66	67	94	95	2	3	30	31	58	59
92	89	20	17	28	25	56	53	64	61
L		L		L		L		L	
90	91	18	19	26	27	54	55	62	63
16	13	24	21	49	52	80	77	88	85
L		L		U		L		L	
14	15	22	23	50	51	78	79	86	87
37	40	45	48	76	73	81	84	9	12
U		U		L		U		U	
38	39	46	47	74	75	82	83	10	11
41	44	69	72	97	100	5	8	33	36
X		X		X		X		X	
43	42	71	70	99	98	7	6	35	34

Conway's LUX method (4k+2)

68	65	96	93	4	1	32	29	60	57
66	67	94	95	2	3	30	31	58	59
92	89	20	17	28	25	56	53	64	61
90	91	18	19	26	27	54	55	62	63
16	13	24	21	49	52	80	77	88	85
14	15	22	23	50	51	78	79	86	87
37	40	45	48	76	73	81	84	9	12
38	39	46	47	74	75	82	83	10	11
41	44	69	72	97	100	5	8	33	36
43	42	71	70	99	98	7	6	35	34

References

[1] Wikipedia contributors. "Magic square." *Wikipedia, The Free Encyclopedia. https://en.wikipedia.org/wiki/Magic_square*

[2] Mathematics in School, Vol. 24, No. 3 (May, 1995), p. 27

[3] **Jacob, G., & Murugan, A.** *On the Construction of Doubly Even Order Magic Squares*. Research and Development Centre, Bharathiar University, Coimbatore.

[4] Photo by Denise Applewhite, Princeton University Office of Communications.

[5] Delucchi, Emanuele. *Construction of Magic Squares Notes*.

[6] Wikipedia contributors. "Conway's LUX method for magic squares." *Wikipedia, The Free Encyclopedia. https://en.wikipedia.org/wiki/Conway%27s_LUX_method_for_magic_squares*

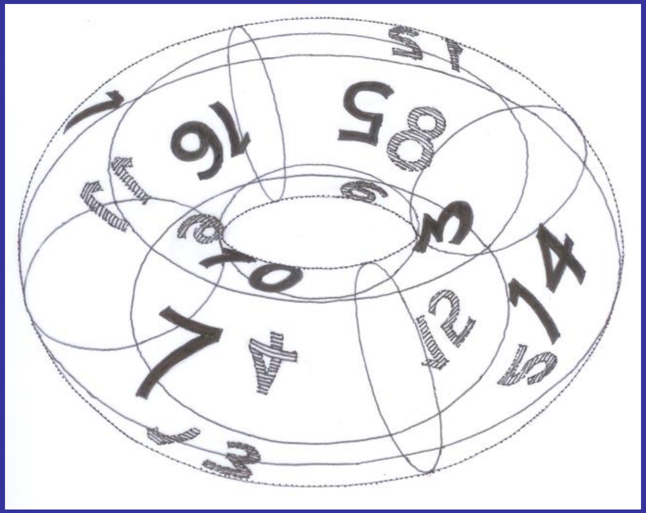
[7] Block and Tavares, *Before Sudoku: The World of Magic Squares*, OUP, 2009

[8] **Limpananont, S.** (2024). *Magic Squares: The Siamese Method* https://www.saranontlimpananont.com/magic-square-siamese-method/

[9] **Walkington, W.** (2012, March 9). *From the Magic Square to the Magic Torus*. *Magic Squares, Spheres and Tori*. Retrieved from https://carresmagiques.blogspot.com/2020/04/from-magic-square-to-magic-torus.html

Thank you for listening

16	255	2	241	14	253	4	243	12	251	6	245	10	249	8	247
1	242	15	256	3	244	13	254	5	246	11	252	7	248	9	250
240	31	226	17	238	29	228	19	236	27	230	21	234	25	232	23
225	18	239	32	227	20	237	30	229	22	235	28	231	24	233	26
223	48	209	34	221	46	211	36	219	44	213	38	217	42	215	40
210	33	224	47	212	35	222	45	214	37	220	43	216	39	218	41
63	208	49	194	61	206	51	196	59	204	53	198	57	202	55	200
50	193	64	207	52	195	62	205	54	197	60	203	56	199	58	201
80	191	66	177	78	189	68	179	76	187	70	181	74	185	72	183
65	178	79	192	67	180	77	190	69	182	75	188	71	184	73	186
176	95	162	81	174	93	164	83	172	91	166	85	170	89	168	87
161	82	175	96	163	84	173	94	165	86	171	92	167	88	169	90
159	112	145	98	157	110	147	100	155	108	149	102	153	106	151	104
146	97	160	111	148	99	158	109	150	101	156	107	152	103	154	105
127	144	113	130	125	142	115	132	123	140	117	134	121	138	119	136
114	129	128	143	116	131	126	141	118	133	124	139	120	135	122	137



We will now continue with the final part of our presentation