Construction of Magic Squares

Yusuf Seha Uysal

Chapters

1. Definiton
2. Siamese method for odd orders
3. Criss cross method for doubly even orders
4. Conway's LUX method for singly even orders

Definiton

A **magic square** is an nxn array containing the consecutive numbers from 1 to n^2. All its rows, columns and diagonals adds up to the same sum. This sum is called the **magic constant**. Number n which determines the size of the square is called the **order** of the magic square.

Definiton

A **magic square** is an nxn array containing the consecutive numbers from 1 to n^2. All its rows, columns and diagonals adds up to the same sum. This sum is called the **magic constant**. Number n which determines the size of the square is called the **order** of the magic square.

2	7	6	→15
9	5	1	→15
4	3	8	→15

15 15 15 15 15

Order: n=3

Magic constant: 15

Magic Constant

$$M = \frac{n(n^2 + 1)}{2}$$

1x1 and 2x2 magic squares

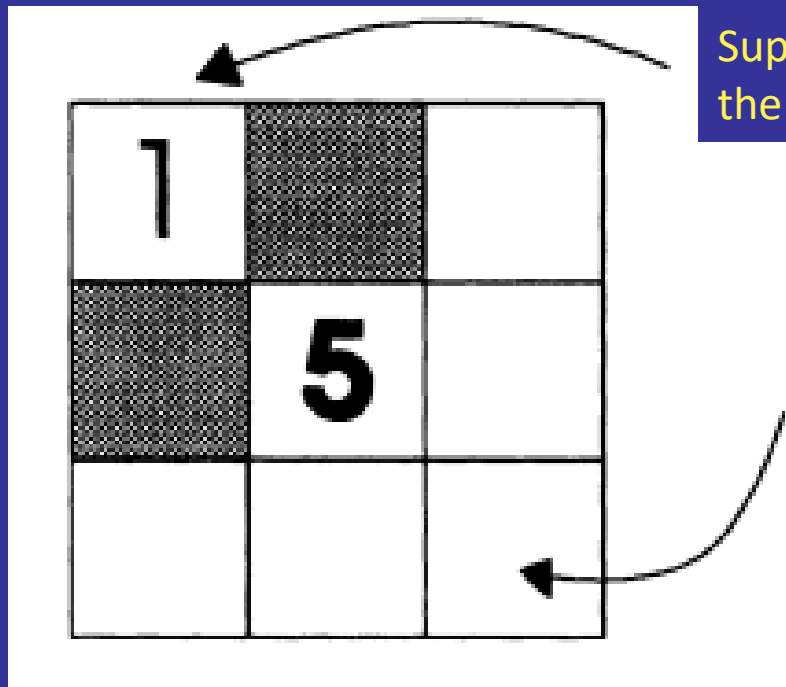| A	C
D	B
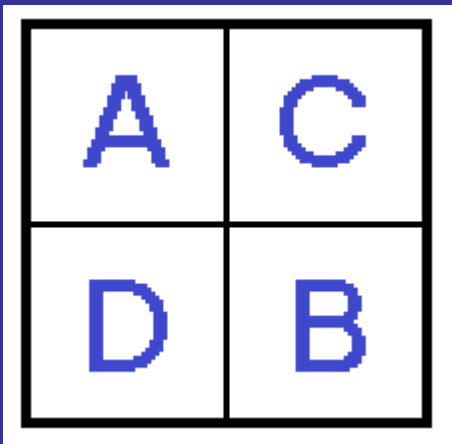
Uniqueness of 3x3

Uniqueness of 3x3
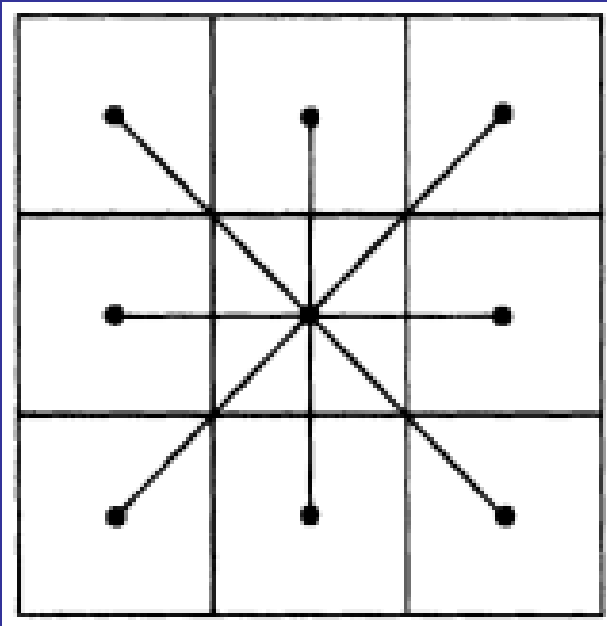
$$M = \frac{n(n^2 + 1)}{2}$$

The center number has to be 5

Uniqueness of 3x3

Suppose 1 is in the corner

The opposite corner has to be 9

No row or column that has 9 can include any numbers greater than 5. So, the numbers 6, 7, and 8 must all be placed in the two shaded squares. Contradiction. Then 1 is not in the corner.

Uniqueness of 3x3

4 and 2 has to be in the same row with 9. The rest follows easily.

Classification of Magic Squares

Classification of Magic Squares

Odd order magic squares (2k+1)

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

Classification of Magic Squares

Odd order magic squares (2k+1)

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

Doubly even order (4k)

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

Singly even order (4k+2)

35	1	6	26	19	24
3	32	7	21	23	25
31	9	2	22	27	20
8	28	33	17	10	15
30	5	34	12	14	16
4	36	29	13	18	11

Classification of Magic Squares

Odd order magic squares (2k+1)

Doubly even order (4k)   Singly even order (4k+2)

Classification of Magic Squares

Odd order magic squares (2k+1)

Siamese Method

Doubly even order (4k)

Criss Cross Method

Singly even order (4k+2)

LUX Method

Siamese method (2k+1)

Siamese method (2k+1)

Algorithm:

1. Start by placing 1 in the center of the first row of the square

Siamese method (2k+1)

Algorithm:

1. Start by placing 1 in the center of the first row of the square

Siamese method (2k+1)

Algorithm:

1. Start by placing 1 in the center of the first row of the square

2. Write the next number to the upper right of the previous one (↗)

Siamese method (2k+1)

Algorithm:

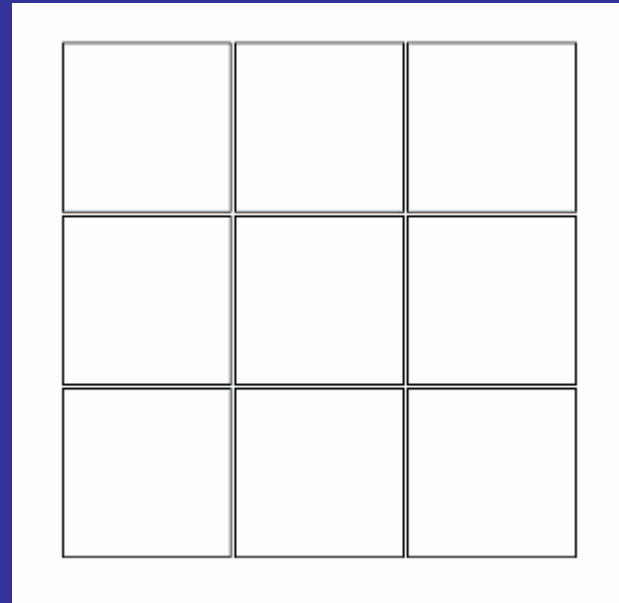1. Start by placing 1 in the center of the first row of the square

2. Write the next number to the upper right of the previous one (↗)

Siamese method (2k+1)

Algorithm:

1. Start by placing 1 in the center of the first row of the square

2. Write the next number to the upper right of the previous one (↗)

3. If the upper right is already filled, place it one square below the previous number (↓)

Siamese method (2k+1)

Algorithm:

1. Start by placing 1 in the center of the first row of the square

2. Write the next number to the upper right of the previous one (↗)

3. If the upper right is already filled, place it one square below the previous number (↓)

Siamese method (2k+1)

Algorithm:

1. Start by placing 1 in the center of the first row of the square

2. Write the next number to the upper right of the previous one (↗)

3. If the upper right is already filled, place it one square below the previous number (↓)

Siamese method (2k+1)

Siamese method (2k+1)

Criss cross method (4k)

Criss cross method (4k)

Algorithm:

1. Place the numbers from 1 to n^2 in consecutive order in the square

Criss cross method (4k)

Algorithm:

1. Place the numbers from 1 to n^2 in consecutive order in the square

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

Criss cross method (4k)

Algorithm:

1. Place the numbers from 1 to n^2 in consecutive order in the square
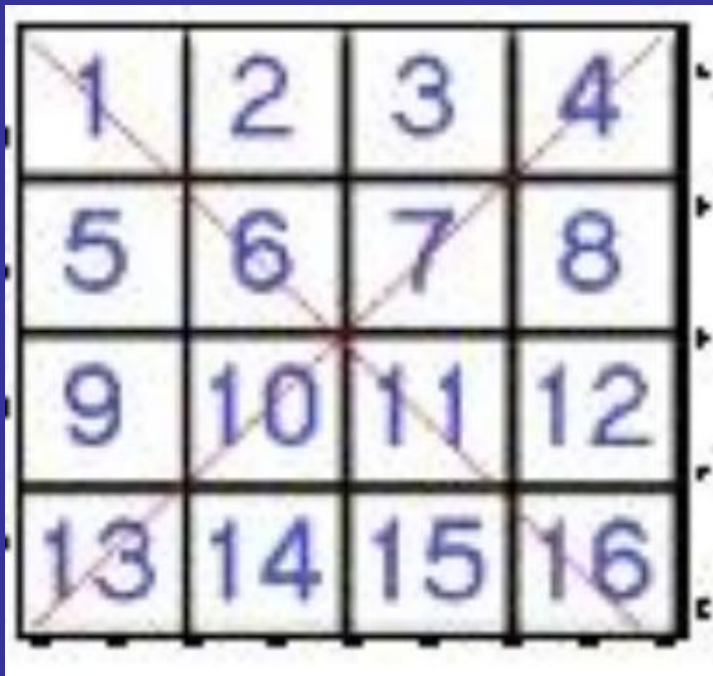
1	63	62	4	5	59	58	8
56	10	11	53	52	14	15	49
48	18	19	45	44	22	23	41
25	39	38	28	29	35	34	32
33	31	30	36	37	27	26	40
24	42	43	21	20	46	47	17
16	50	51	13	12	54	55	9
57	7	6	60	61	3	2	64

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

2. Draw an X to each 4x4 subsquare
3. Reverse the order of the entries that do not overlap with these X's

Criss cross method (4k)

Let's see an example:

Criss cross method (4k)

Let's see an example:

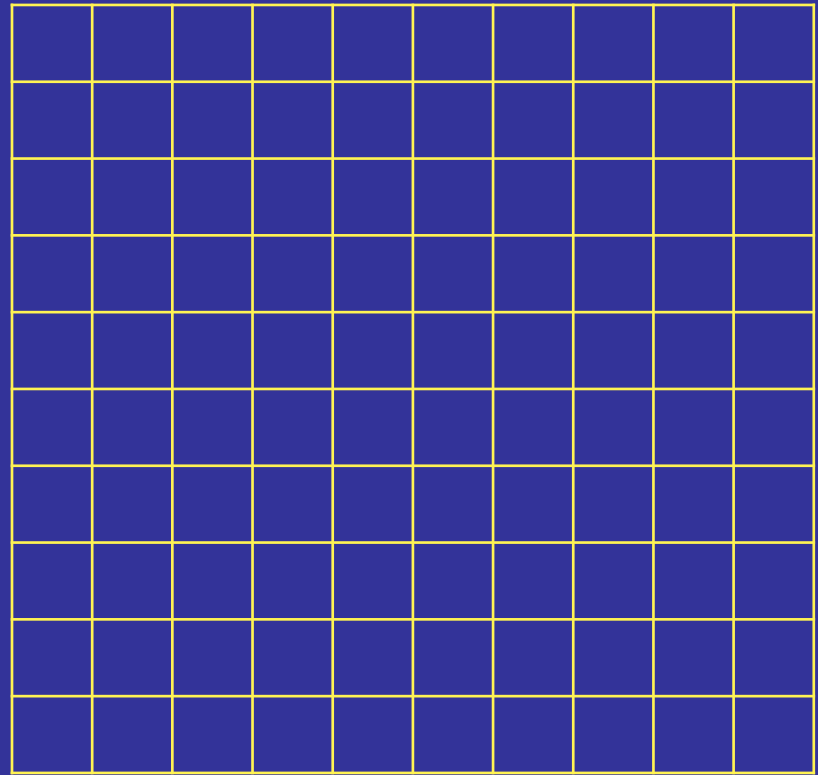# Conway's LUX method (4k+2)

Conway's LUX method (4k+2)
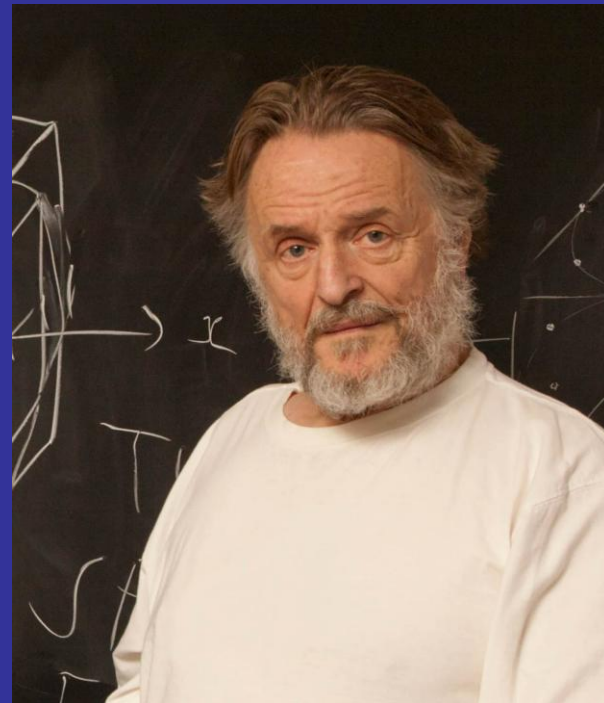
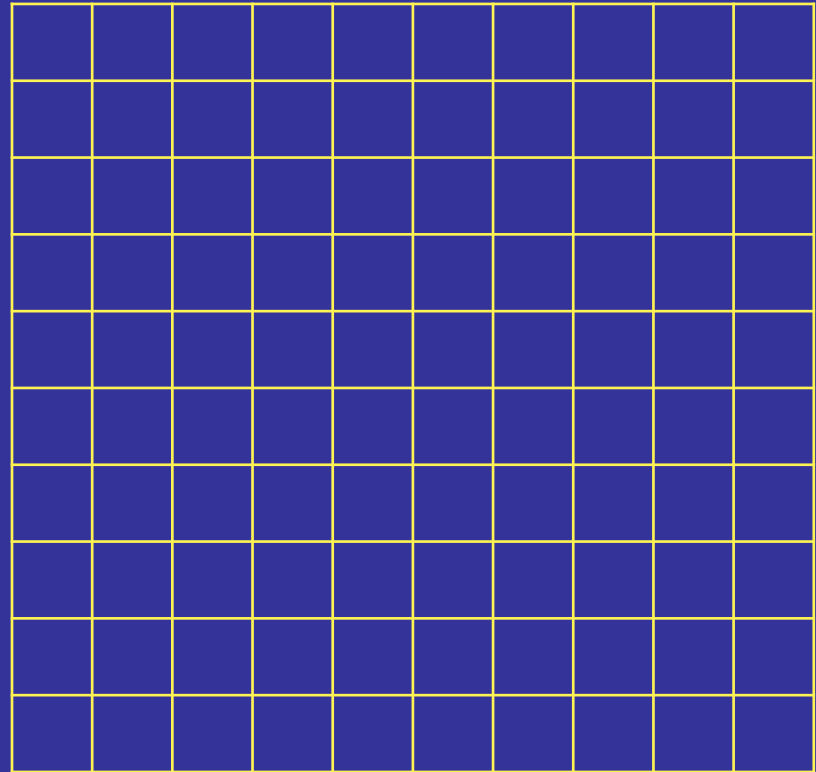John Horton Conway (1937–2020)

Conway's LUX method (4k+2)

10x10

Conway's LUX method (4k+2)

Algorithm:

1. Transform the given square into a (2k+1)x(2k+1) square consisting of 2x2 blocks

10x10
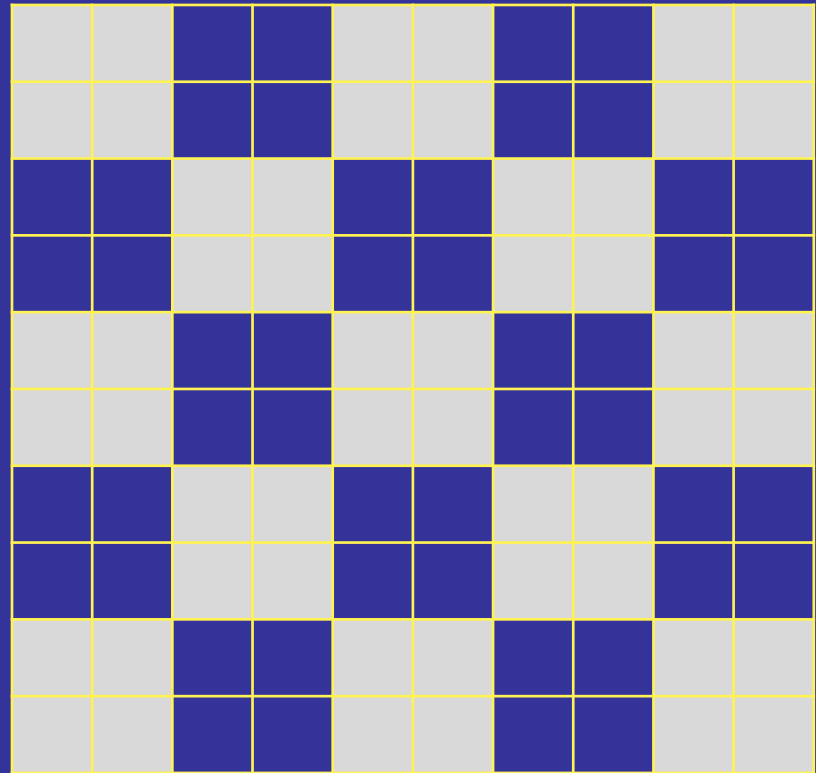
Conway's LUX method (4k+2)

Algorithm:

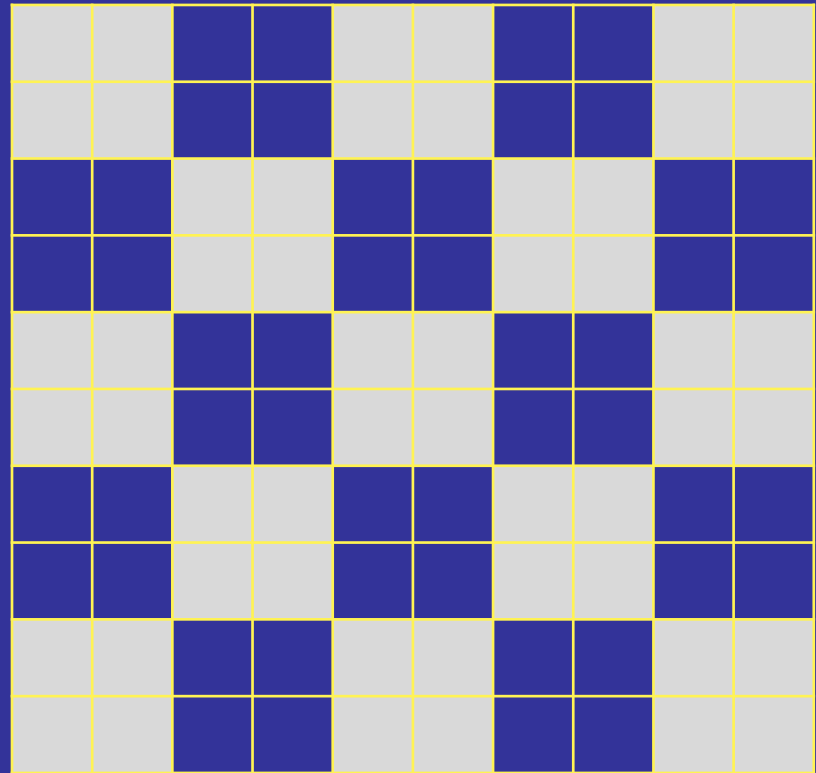1. Transform the given square into a (2k+1)x(2k+1) square consisting of 2x2 blocks

10x10 → 5x5

Conway's LUX method (4k+2)

Algorithm:

1. Transform the given square into a (2k+1)x(2k+1) square consisting of 2x2 blocks

2. Fill the,
   - First $k+1$ rows with **L**,
   - 1 row with **U**, and
   - The remaining $k-1$ rows with **X**

10x10 → 5x5

Conway's LUX method (4k+2)

Algorithm:

1. Transform the given square into a (2k+1)x(2k+1) square consisting of 2x2 blocks

2. Fill the,
   - First $k+1$ rows with **L**,
   - 1 row with **U**, and
   - The remaining k-1 rows with **X**

10x10 → 5x5

Conway's LUX method (4k+2)

Algorithm:

1. Transform the given square into a (2k+1)x(2k+1) square consisting of 2x2 blocks

2. Fill the,
   - First $k+1$ rows with **L**,
   - 1 row with **U**, and
   - The remaining k-1 rows with **X**

3. Swap the **U** in the center and the **L** above it

10x10 → 5x5

Conway's LUX method (4k+2)

Algorithm:

1. Transform the given square into a (2k+1)x(2k+1) square consisting of 2x2 blocks

2. Fill the,
   - First $k+1$ rows with **L**,
   - 1 row with **U**, and
   - The remaining k-1 rows with **X**

3. Swap the **U** in the center and the **L** above it

10x10 → 5x5

Conway's LUX method (4k+2)

Algorithm:

1. Transform the given square into a (2k+1)x(2k+1) square consisting of 2x2 blocks

2. Fill the,
   - First $k+1$ rows with **L**,
   - 1 row with **U**, and
   - The remaining k-1 rows with **X**

3. Swap the **U** in the center and the **L** above it

4. Number the resulting 2k+1 order square using the Siamese method we have seen earlier

5. Using this square, the 2x2 squares are each filled as follows

Conway's LUX method (4k+2)

$$L: \quad \begin{array}{cc} 4 & 1 \\ & \swarrow \\ 2 & \to 3 \end{array} \qquad U: \quad \begin{array}{cc} 1 & 4 \\ \downarrow & \uparrow \\ 2 & \to 3 \end{array} \qquad X: \quad \begin{array}{cc} 1 & 4 \\ \nwarrow\!\!\nearrow \\ 3 & 2 \end{array}$$

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

L	L	L	L	L
L	L	L	L	L
L	L	U	L	L
U	U	L	U	U
X	X	X	X	X

Conway's LUX method (4k+2)

$$\text{L}: \quad \begin{matrix} 4 & & 1 \\ & \swarrow & \\ 2 & \rightarrow & 3 \end{matrix} \qquad \text{U}: \quad \begin{matrix} 1 & & 4 \\ \downarrow & & \uparrow \\ 2 & \rightarrow & 3 \end{matrix} \qquad \text{X}: \quad \begin{matrix} 1 & & 4 \\ & \times & \\ 3 & & 2 \end{matrix}$$

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

L	L	L	L	L
L	L	L	L	L
L	L	U	L	L
U	U	L	U	U
X	X	X	X	X

Conway's LUX method (4k+2)

$$\mathrm{L}: \quad \begin{matrix} 4 & & 1 \\ & \swarrow & \\ 2 & \rightarrow & 3 \end{matrix} \qquad \mathrm{U}: \quad \begin{matrix} 1 & & 4 \\ \downarrow & & \uparrow \\ 2 & \rightarrow & 3 \end{matrix} \qquad \mathrm{X}: \quad \begin{matrix} 1 & & 4 \\ & \times & \\ 3 & & 2 \end{matrix}$$

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

L	L	L	L	L
L	L	L	L	L
L	L	U	L	L
U	U	L	U	U
X	X	X	X	X

Conway's LUX method (4k+2)

Conway's LUX method (4k+2)
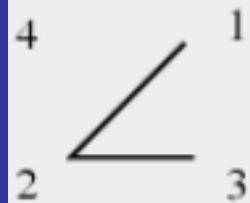
68	65	96	93	4	1	32	29	60	57
66	67	94	95	2	3	30	31	58	59
92	89	20	17	28	25	56	53	64	61
90	91	18	19	26	27	54	55	62	63
16	13	24	21	49	52	80	77	88	85
14	15	22	23	50	51	78	79	86	87
37	40	45	48	76	73	81	84	9	12
38	39	46	47	74	75	82	83	10	11
41	44	69	72	97	100	5	8	33	36
43	42	71	70	99	98	7	6	35	34

References

[1] Wikipedia contributors. "Magic square." *Wikipedia, The Free Encyclopedia.* *https://en.wikipedia.org/wiki/Magic_square*

[2] Mathematics in School, Vol. 24, No. 3 (May, 1995), p. 27

[3] **Jacob, G., & Murugan, A.** *On the Construction of Doubly Even Order Magic Squares*. Research and Development Centre, Bharathiar University, Coimbatore.

[4] Photo by Denise Applewhite, Princeton University Office of Communications.

[5] Delucchi, Emanuele. *Construction of Magic Squares Notes*.

[6] Wikipedia contributors. "Conway's LUX method for magic squares." *Wikipedia, The Free Encyclopedia.* *https://en.wikipedia.org/wiki/Conway%27s_LUX_method_for_magic_squares*

[7] Block and Tavares, *Before Sudoku: The World of Magic Squares*, OUP, 2009

[8] **Limpananont, S.** (2024). *Magic Squares: The Siamese Method* https://www.saranontlimpananont.com/magic-square-siamese-method/

[9] **Walkington, W.** (2012, March 9). *From the Magic Square to the Magic Torus*. *Magic Squares, Spheres and Tori*. Retrieved from https://carresmagiques.blogspot.com/2020/04/from-magic-square-to-magic-torus.html

Thank you for listening

We will now continue with the final part of our presentation