Q1 - 2024 (04 Apr Shift 1)

If the solution $y=y(x)$ of the differential equation $\left(x^4+2x^3+3x^2+2x+2\right)\mathrm{d}y-\left(2x^2+2x+3\right)\mathrm{d}x=0$ satisfies $y(-1)=-\frac{\pi}{4}$, then $y(0)$ is equal to :

(1) $\frac{\pi}{2}$

(2) $-\frac{\pi}{2}$

(3) 0

(4) $\frac{\pi}{4}$

Q2 - 2024 (04 Apr Shift 1)

Let the solution $y=y(x)$ of the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x}-y=1+4\sin x$ satisfy $y(\pi)=1$. Then $y\left(\frac{\pi}{2}\right)+10$ is equal to ______

Q3 - 2024 (04 Apr Shift 2)

Let $y=y(x)$ be the solution of the differential equation $\left(x^2+4\right)^2dy+\left(2x^3y+8xy-2\right)dx=0$. If $y(0)=0$, then $y(2)$ is equal to

(1) $\frac{\pi}{32}$

(2) 2π

(3) $\frac{\pi}{8}$

(4) $\frac{\pi}{16}$

Q4 - 2024 (04 Apr Shift 2)

Let $y=y(x)$ be the solution of the differential equation $(x+y+2)^2dx=dy, y(0)=-2$. Let the maximum and minimum values of the function $y=y(x)$ in $\left[0,\frac{\pi}{3}\right]$ be α and β, respectively. If $(3\alpha+\pi)^2+\beta^2=\gamma+\delta\sqrt{3}, \gamma, \delta\in\mathbb{Z}$, then $\gamma+\delta$ equals ______

Q5 - 2024 (05 Apr Shift 1)

If $y=y(x)$ is the solution of the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x}+2y=\sin(2x), y(0)=\frac{3}{4}$, then $y\left(\frac{\pi}{8}\right)$ is equal to:

(1) $\mathrm{e}^{\pi/8}$

(2) $\mathrm{e}^{\pi/4}$

(3) $\mathrm{e}^{-\pi/4}$

(4) $\mathrm{e}^{-\pi/8}$

Q6 - 2024 (05 Apr Shift 2)

The differential equation of the family of circles passing through the origin and having centre at the line $y = x$ is :

(1) $\left(x^2 - y^2 + 2xy\right) \mathrm{d}x = \left(x^2 - y^2 - 2xy\right) \mathrm{d}y$

(2) $\left(x^2 + y^2 + 2xy\right) \mathrm{d}x = \left(x^2 + y^2 - 2xy\right) \mathrm{d}y$

(3) $\left(x^2 + y^2 - 2xy\right) \mathrm{d}x = \left(x^2 + y^2 + 2xy\right) \mathrm{d}y$

(4) $\left(x^2 - y^2 + 2xy\right) \mathrm{d}x = \left(x^2 - y^2 + 2xy\right) \mathrm{d}y$

Q7 - 2024 (05 Apr Shift 2)

Let $y = y(x)$ be the solution of the differential equation

$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2x}{(1+x^2)^2} y = x\mathrm{e}^{\frac{1}{(1+x^2)}}; y(0) = 0.$

Then the area enclosed by the curve $f(x) = y(x)\mathrm{e}^{-\frac{1}{(1+x^2)}}$ and the line $y - x = 4$ is__________

Q8 - 2024 (06 Apr Shift 1)

Let $y = y(x)$ be the solution of the differential equation $\left(1 + x^2\right) \frac{dy}{dx} + y = e^{\tan^{-1} x}$, $y(1) = 0$. Then $y(0)$ is

(1) $\frac{1}{2}\left(e^{\pi/2} - 1\right)$

(2) $\frac{1}{2}\left(1 - e^{\pi/2}\right)$

(3) $\frac{1}{4}\left(1 - e^{\pi/2}\right)$

(4) $\frac{1}{4}\left(e^{\pi/2} - 1\right)$

Q9 - 2024 (06 Apr Shift 1)

Let $y=y(x)$ be the solution of the differential equation $\left(2 x \log _e x\right) \frac{d y}{d x}+2 y=\frac{3}{x} \log _e x, x>0$ and $y\left(e^{-1}\right)=0$. Then, $y(e)$ is equal to

(1) $-\frac{3}{\mathrm{e}}$

(2) $-\frac{3}{2 e}$

(3) $-\frac{2}{3 e}$

(4) $-\frac{2}{\mathrm{e}}$

Q10 - 2024 (06 Apr Shift 2)

Suppose the solution of the differential equation $\frac{d y}{d x}=\frac{(2+\alpha) x-\beta y+2}{\beta x-2 \alpha y-(\beta \gamma-4 \alpha)}$ represents a circle passing through origin. Then the radius of this circle is :

(1) 2

(2) $\sqrt{17}$

(3) $\frac{1}{2}$

(4) $\frac{\sqrt{17}}{2}$

Q11 - 2024 (06 Apr Shift 2)

If the solution $y(x)$ of the given differential equation $\left(\mathrm{e}^y+1\right) \cos x \mathrm{~d} x+\mathrm{e}^y \sin x \mathrm{~d} y=0$ passes through the point $\left(\frac{\pi}{2}, 0\right)$, then the value of $\mathrm{e}^{y\left(\frac{\pi}{6}\right)}$ is equal to________

Q12 - 2024 (08 Apr Shift 1)

Let $y=y(x)$ be the solution of the differential equation

$\left(1+y^2\right) e^{\tan x} d x+\cos ^2 x\left(1+e^{2 \tan x}\right) d y=0, y(0)=1$. Then $y\left(\frac{\pi}{4}\right)$ is equal to

(1) $\frac{2}{e}$

(2) $\frac{2}{e^2}$

(3) $\frac{1}{e}$

(4) $\frac{1}{e^2}$

Q13 - 2024 (08 Apr Shift 2)

Let $y=y(x)$ be the solution curve of the differential equation $\sec y \frac{\mathrm{d}y}{\mathrm{d}x}+2x\sin y=x^3\cos y, y(1)=0$. Then $y(\sqrt{3})$ is equal to :

(1) $\frac{\pi}{3}$

(2) $\frac{\pi}{6}$

(3) $\frac{\pi}{12}$

(4) $\frac{\pi}{4}$

Q14 - 2024 (08 Apr Shift 2)

Let $\alpha|x|=|y|\mathrm{e}^{xy-\beta}, \alpha, \beta \in \mathbf{N}$ be the solution of the differential equation $x\,\mathrm{d}y-y\,\mathrm{d}x+xy(x\,\mathrm{d}y+y\,\mathrm{d}x)=0, y(1)=2$. Then $\alpha+\beta$ is equal to ________

Q15 - 2024 (09 Apr Shift 1)

The solution curve, of the differential equation $2y\frac{\mathrm{d}y}{\mathrm{d}x}+3=5\frac{\mathrm{d}y}{\mathrm{d}x}$, passing through the point $(0,1)$ is a conic, whose vertex lies on the line:

(1) $2x+3y=9$

(2) $2x+3y=-9$

(3) $2x+3y=-6$

(4) $2x+3y=6$

Q16 - 2024 (09 Apr Shift 1)

The solution of the differential equation $\left(x^2+y^2\right)\mathrm{d}x-5xy\,\mathrm{d}y=0, y(1)=0$, is :

(1) $\left|x^2-2y^2\right|^6=x$

(2) $\left|x^2-4y^2\right|^6=x$

(3) $\left|x^2-4y^2\right|^5=x^2$

(4) $\left|x^2-2y^2\right|^5=x^2$

Q17 - 2024 (09 Apr Shift 2)

For a differentiable function $f : \mathbb{R} \to \mathbb{R}$, suppose $f'(x) = 3f(x) + \alpha$, where $\alpha \in \mathbb{R}$, $f(0) = 1$ and $\lim_{x \to -\infty} f(x) = 7$. Then $9f(-\log_e 3)$ is equal to_________

Answer Key

Q1 (4) **Q2** (7) **Q3** (1) **Q4** (31)

Q5 (3) **Q6** (1) **Q7** (18) **Q8** (2)

Q9 (1) **Q10** (4) **Q11** (3) **Q12** (3)

Q13 (4) **Q14** (4) **Q15** (1) **Q16** (3)

Q17 (61)

Q1

$$\int dy=\int \frac{\left(2 x^2+2 x+3\right)}{x^4+2 x^3+3 x^2+2 x+2} d x$$

$$y=\int \frac{\left(2 x^2+2 x+3\right)}{\left(x^2+1\right)\left(x^2+2 x+2\right)} d x$$

$$y=\int \frac{d x}{x^2+2 x+2}+\int \frac{d x}{x^2+1}$$

$$y=\tan ^{-1}(x+1)+\tan ^{-1} x+C$$

$$y(-1)=\frac{-\pi}{4}$$

$$\frac{-\pi}{4}=0-\frac{\pi}{4}+C \Rightarrow C=0$$

$$\Rightarrow y=\tan ^{-1}(x+1)+\tan ^{-1} x$$

$$y(0)=\tan ^{-1} 1=\frac{\pi}{4}$$

Q2

$$y e^{-x}=\int\left(e^{-x}+4 e^{-x} \sin x\right) d x$$

$$y e^{-x}=-e^{-x}-2\left(e^{-x} \sin x e^{-x} \cos x\right)+C$$

$$y=-1-2(\sin x+\cos x)+c e^x$$

$$\because y(\pi)=1 \Rightarrow c=0$$

$$y(\pi / 2)=-1-2=-3$$

$$\text { Ans }=10-3=7$$

Q3

$$\frac{dy}{dx}+y\left(\frac{2x^3+8x}{\left(x^2+4\right)^2}\right)=\frac{2}{\left(x^2+4\right)^2}$$

$$\frac{dy}{dx}+y\left(\frac{2x}{x^2+4}\right)=\frac{2}{\left(x^2+4\right)^2}$$

$$\text{IF }=e^{\int\frac{2x}{x^2+4}dx}$$

$$\text{IF }=x^2+4$$

$$y\times\left(x^2+4\right)=\int\frac{2}{\left(x^2+4\right)^2}\times\left(x^2+4\right)$$

$$y\left(x^2+4\right)=2\int\frac{dx}{x^2+2^2}$$

$$y\left(x^2+4\right)=\frac{2}{2}\tan^{-1}\left(\frac{x}{2}\right)+c$$

$$0=0+c=c=0$$

$$y\left(x^2+4\right)=\tan^{-1}\left(\frac{x}{2}\right)$$

y at $x=2$

$$y(4+4)=\tan^{-1}(1)$$

$$y(2)=\frac{\pi}{32}$$

Q4

$\frac{dy}{dx}=(x+y+2)^2$ ...(1)

$y(0)=-2$

Let $\mathrm{x}+\mathrm{y}+2=\mathrm{v}$

$1+\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{\mathrm{dv}}{\mathrm{dx}}$

from (1) $\frac{dv}{dx}=1+v^2$

$$\int\frac{dv}{1+v^2}=\int dx$$

$$\tan^{-1}(v)=x+C$$

$$\tan^{-1}(x+y+2)=x+C$$

at $x=0y=-2\Rightarrow C=0$

$\Rightarrow\tan^{-1}(\mathrm{x}+\mathrm{y}+2)=\mathrm{x}$

$\mathrm{y}=\tan\mathrm{x}-\mathrm{x}-2$

$$\mathrm{f}(\mathrm{x})=\tan\mathrm{x}-\mathrm{x}-2,\mathrm{x}\in\left[0,\frac{\pi}{3}\right]$$

$$f'(x)=\sec^2x-1>0\Rightarrow f(x)\uparrow$$

$$f_{\min}=f(0)=-2=\beta$$

$$f_{\max}=f\left(\frac{\pi}{3}\right)=\sqrt{3}-\frac{\pi}{3}-2=\alpha$$

now $(3\alpha+\pi)^2+\beta^2=\gamma+\delta\sqrt{3}$

$\Rightarrow(3\alpha+\pi)^2+\beta^2=(3\sqrt{3}-6)^2+4$

$\gamma+\delta\sqrt{3}=67-36\sqrt{3}$

$\Rightarrow\gamma=67$ and $\delta=-36\Rightarrow\gamma+\delta=31$

Q5

$\frac{dy}{dx}+2y=\sin 2x, y(0)=\frac{3}{4}$

I.F $=e^{\int 2dx}=e^{2x}$

$y\cdot e^{2x}=\int e^{2x}\sin 2x dx$

$y\cdot e^{2x}=\frac{e^{2x}(2\sin 2x-2\cos 2x)}{4+4}+C$

$x=0, y=\frac{3}{4}\Rightarrow\frac{3}{4}\cdot 1=\frac{1(0-2)}{8}+C$

$\frac{3}{4}=-\frac{1}{4}+C$

$1=C$

$y=\frac{2\sin 2x-2\cos 2x}{8}+1\cdot e^{-2x}$

$x=\frac{\pi}{8}, y=\frac{1}{8}\left(2\sin\frac{\pi}{4}-2\cos\frac{\pi}{4}\right)+e^{-2\left(\frac{\pi}{8}\right)}$

$y=0+e^{-\frac{\pi}{4}}$

Q6

$C\equiv x^2+y^2+gx+gy=0\ldots.(1)$

$2x+2yy'+g+gy'=0$

$g=-\left(\frac{2x+2yy'}{1+y'}\right)$

Put in (1)

$x^2+y^2-\left(\frac{2x+2yy'}{1+y'}\right)(x+y)=0$

$\left(x^2-y^2-2xy\right)y'=x^2-y^2+2xy$

Q7

$IF = e^{\int \frac{2x}{(1+x^2)^2} dx} = e^{\frac{-1}{1+x^2}}$

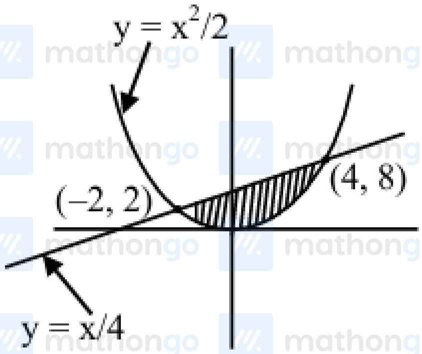
$y \cdot e^{\frac{-1}{1+x^2}} = \int x \cdot e^{\frac{1}{1+x^2}} \cdot e^{\frac{-1}{1+x^2}} dx$

$y \cdot e^{\frac{-1}{1+x^2}} = \frac{x^2}{2} + c$

$(0,0) \Rightarrow C = 0$

$y(x) = \frac{x^2}{2} e^{\frac{1}{1+x^2}}$

$f(x) = \frac{x^2}{2}$



$A = \int_{-2}^{4} (x+4) - \frac{x^2}{2} dx = 18$

Q8

$$\frac{dy}{dx}+\frac{y}{1+x^2}=\frac{e^{\tan^{-1}x}}{1+x^2}$$

$$\text{I.F. }=e^{\int\frac{1}{1+x^2}dx}=e^{\tan^{-1}x}$$

$$y\cdot e^{\tan^{-1}x}=\int\left(\frac{e^{\tan^{-1}x}}{1+x^2}\right)e^{\tan^{-1}x}\cdot dx$$

$$\text{Let }\tan^{-1}x=z\quad\therefore\frac{dx}{1+x^2}=dz$$

$$\therefore y\cdot e^z=\int e^{2z}dz=\frac{e^{2z}}{2}+C$$

$$y\cdot e^{\tan^{-1}x}=\frac{e^{2\tan^{-1}x}}{2}+C$$

$$\Rightarrow y=\frac{e^{\tan^{-1}x}}{2}+\frac{C}{e^{\tan^{-1}x}}$$

$$\because \mathrm{y}(1)=0\Rightarrow 0=\frac{\mathrm{e}^{\pi/4}}{2}+\frac{\mathrm{C}}{\mathrm{e}^{\pi/4}}\Rightarrow \mathrm{C}=\frac{-\mathrm{e}^{\pi/2}}{2}$$

$$\therefore \mathrm{y}=\frac{\mathrm{e}^{\tan^{-1}\mathrm{x}}}{2}-\frac{\mathrm{e}^{\pi/2}}{2\mathrm{e}^{\tan^{-1}\mathrm{x}}}$$

$$\therefore \mathrm{y}(0)=\frac{1-\mathrm{e}^{\pi/2}}{2}$$

Q9

$$\frac{dy}{dx}+\frac{y}{x\ell\ln x}=\frac{3}{2x^2}$$

$$\therefore\ \text{I.F. }=e^{\int\frac{1}{x\ln x}dx}=e^{\ln(\ln(x))}=\ln x$$

$$\therefore y\ell\ln x=\int\frac{3\ln x}{2x^2}dx$$

$$=\frac{3\ln x}{2}\int x^{-2}dx-\int\left(\frac{3}{2x}\cdot\int x^{-2}dx\right)dx$$

$$=\frac{3\ln x}{2}\left(-\frac{1}{x}\right)-\int\frac{3}{2x}\left(-\frac{1}{x}\right)dx$$

$$\text{y. }\ln x=\frac{-3\ell nx}{2x}-\frac{3}{2x}+C$$

$$\because \mathrm{y}\left(\mathrm{e}^{-1}\right)=0$$

$$\therefore 0(-1)=\frac{3\mathrm{e}}{2}-\frac{3\mathrm{e}}{2}+\mathrm{C}\Rightarrow \mathrm{C}=0$$

$$\therefore \mathrm{y}=\frac{-3\ell\mathrm{nx}}{2\mathrm{x}}-\frac{3}{2\mathrm{x}}$$

$$\therefore \mathrm{y}(\mathrm{e})=\frac{-3}{2\mathrm{e}}-\frac{3}{2\mathrm{e}}=\frac{-3}{\mathrm{e}}$$

Q10

$$\frac{dy}{dx}=\frac{(2+\alpha)x-\beta y+2}{\beta x-y(2\alpha+\beta)+4\alpha}$$

$\beta xdy-(2\alpha+\beta)ydy+4\alpha dy=(2+\alpha)xdx-\beta ydx+2dx$

$\beta(xdy+ydx)-(2\alpha+\beta)ydy+4\alpha dy=(2+\alpha)xdx+2dx$

$\beta xy-\frac{(2\alpha+\beta)y^2}{2}+4\alpha y=\frac{(2+\alpha)x^2}{2}$

$\Rightarrow \beta=0$ for this to be circle

$(2+\alpha)\frac{x^2}{2}+\alpha y^2+2x-4\alpha y=0$

coeff. of $x^2=y^2$ $\Rightarrow$ $2+\mathrm{a}=2\mathrm{a}$

$\Rightarrow \boxed{\alpha=2}$

i.e. $2x^2+2y^2+2x-8y=0$

$x^2+y^2+x-4y=0$

$rd=\sqrt{\frac{1}{4}+4}=\frac{\sqrt{17}}{2}$

Q11

$(\mathrm{e}^{\mathrm{y}}+1)\cos\mathrm{x}\,\mathrm{dx}+\mathrm{e}^{\mathrm{y}}\sin\mathrm{x}\,\mathrm{dy}=0$

$\Rightarrow \mathrm{d}\left((\mathrm{e}^{\mathrm{y}}+1)\sin\mathrm{x}\right)=0$

$(\mathrm{e}^{\mathrm{y}}+1)\sin\mathrm{x}=\mathrm{C}$

It passes through $\left(\frac{\pi}{2},0\right)$

$\Rightarrow \mathrm{c}=2$

Now, $x=\frac{\pi}{6}$

$\Rightarrow \mathrm{e}^{\mathrm{y}}=3$

Q12

$\left(1+y^2\right)e^{\tan x}dx+\cos^2 x\left(1+e^{2\tan x}\right)dy=0$

$\int\frac{\sec^2 xe^{\tan x}}{1+e^{2\tan x}}dx+\int\frac{dy}{1+y^2}=C$

$\Rightarrow \tan^{-1}\left(e^{\tan x}\right)+\tan^{-1}y=C$

for $x=0, y=1, \tan^{-1}(1)+\tan^{-1}1=C$

$\mathrm{C}=\frac{\pi}{2}$

$\tan^{-1}(e^{\tan x}) + \tan^{-1} y = \frac{\pi}{2}$

Put $x = \pi, \tan^{-1} e + \tan^{-1} y = \frac{\pi}{2}$

$\tan^{-1} y = \cot^{-1} e$

$y = \frac{1}{e}$

Q13

$$\sec^2 y \frac{dy}{dx} + 2x \sin y \sec y = x^3 \cos y \sec y$$

$$\sec^2 y \frac{dy}{dx} + 2x \tan y = x^3$$

$$\tan y = t \Rightarrow \sec^2 y \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} + 2xt = x^3, \text{ If } = e^{\int 2x dx} = e^{x^2}$$

$$tx^{x^2} = \int x^3 \cdot e^{x^2} dx + c$$

$$x^2 = Z \Rightarrow t \cdot e^Z = \frac{1}{2} \int e^Z \cdot Z dZ = \frac{1}{2}\left[e^Z \cdot Z - e^Z\right] + c$$

$$2 \tan y = \left(x^2 - 1\right) + 2e^{-x^2}$$

$$y(1) = 0 \Rightarrow c = 0 \Rightarrow y(\sqrt{3}) = \frac{\pi}{4}$$

Q14

$a|x| = |y|e^{yx-\beta}, a, b \in N$

$xdy - ydx + xy(xdy + ydx) = 0$

$\frac{dy}{y} - \frac{dx}{x} + (xdy + ydx) = 0$

$\ell n|y| - \ell n|x| + xy = c$

$y(1) = 2$

$\ell n|2| - 0 + 2 = c$

$c = 2 + \ell n 2$

$\ell n|y| - \ell n|x| + xy = 2 + \ell n 2$

$\ell n|x| = \ell n\left|\frac{y}{2}\right| - 2 + xy$

$|x| = \left|\frac{y}{2}\right| e^{xy-2}$

$2|x| = |y|e^{xy-2}$

$\alpha = 2 \quad \beta = 2 \quad \alpha + \beta = 4$

Q15

$$(2y-5)\frac{dy}{dx}=-3$$

$$(2y-5)dy=-3dx$$

$$2\cdot\frac{y^2}{2}-5y=-3x+\lambda$$

$\because$ Curve passes through $(0,1)$

$\Rightarrow \lambda=-4$

$\because$ Curve will be

$$\left(y-\frac{5}{2}\right)^2=-3\left(x-\frac{3}{4}\right)$$

$\therefore$ Vertex of parabola will be $\left(\frac{3}{4},\frac{5}{2}\right)$

$\because 2x+3y=9$

Q16

$$\left(x^2+y^2\right)dx=5xydy$$

$$\Rightarrow \frac{dy}{dx}=\frac{x^2+y^2}{5xy}$$

Put $y=Vx$

$$\Rightarrow \mathrm{V}+\mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}=\frac{1+\mathrm{V}^2}{5\ \mathrm{V}}$$

$$\Rightarrow \frac{\mathrm{xdv}}{\mathrm{dx}}=\frac{1-4\ \mathrm{V}^2}{5\ \mathrm{V}}$$

$$\Rightarrow \int\frac{\mathrm{V}}{1-4\ \mathrm{V}^2}\mathrm{dV}=\int\frac{\mathrm{dx}}{5\mathrm{x}}$$

Let $1-4\ \mathrm{V}^2=\mathrm{t}$

$\Rightarrow -8\,\mathrm{VdV} = \mathrm{dt}$

$\Rightarrow \int \frac{\mathrm{dt}}{(-8)(\mathrm{t})} = \int \frac{\mathrm{dx}}{5\mathrm{x}}$

$\Rightarrow \frac{-1}{8} \ln|\mathrm{t}| = \frac{1}{5} \ln|\mathrm{x}| + \ln \mathrm{C}$

$\Rightarrow -5 \ln|\mathrm{t}| = 8 \ln|\mathrm{x}| + \ln \mathrm{K}$

$\Rightarrow \ln \mathrm{x}^8 + \ln\left|\mathrm{t}^5\right| + \ln \mathrm{K} = 0$

$\Rightarrow \mathrm{x}^8 \left|\mathrm{t}^5\right| = \mathrm{C}$

$\Rightarrow \mathrm{x}^8 \left|1 - 4\,\mathrm{V}^2\right|^5 = \mathrm{C}$

$\Rightarrow \mathrm{x}^8 \left|\frac{\mathrm{x}^2 - 4\mathrm{y}^2}{\mathrm{x}^2}\right|^5 = \mathrm{C}$

$\Rightarrow \left|\mathrm{x}^2 - 4\mathrm{y}^2\right|^5 = \mathrm{Cxx}^2$

given $\mathrm{y}(1) = 0$

$\Rightarrow |1|^5 = \mathrm{C} \Rightarrow \mathrm{C} = 1$

$\Rightarrow \left|\mathrm{x}^2 - 4\mathrm{y}^2\right|^5 = \mathrm{x}^2$

Q17

$\frac{dy}{dx} - 3y = \alpha$

If $= e^{\int -3dx} = e^{-3x}$

$\therefore \quad y - e^{-3x} = \int e^{-3x} \cdot \alpha dx$

$ye^{-3x} = \frac{\alpha e^{-3x}}{-3} + c$

$(*e^{3x})$

$y = \frac{\alpha}{-3} + C \cdot e^{3x}$

on substituting $\mathrm{x} = 0, \mathrm{y} = 1$

$x \rightarrow -\infty, y = 7$

we get $y = 7 - 6e^{3x}$

$9f\left(-\log_e 3\right) = 61$