$$\Omega^0 \xleftarrow{d} \Omega^1 \xleftarrow{d} \Omega^2$$

$$B^1 \subset \operatorname{Ker} d = B^1 \oplus H^1$$

$$\beta^1 = \delta(\Omega^2) \subset \operatorname{Ker} \delta = \zeta^1 = H^1 \oplus \beta^1$$