θを計算したい

次のc_γ,s_γ,c_δ,s_δとtan^{-1}を使ってθ_eを計算したい。

$$
\begin{aligned}
c_\gamma &= 2A(-KL_m \sin(2\theta_\gamma)\sin(\theta_e) + (L_i - L_m\cos(2\theta_\gamma))\cos(\theta_e)) \\
s_\gamma &= 2A(KL_m \sin(2\theta_\gamma)\cos(\theta_e) + (L_i - L_m\cos(2\theta_\gamma))\sin(\theta_e)) \\
c_\delta &= 2A(-L_m \sin(2\theta_\gamma)\sin(\theta_e) - (KL_i + KL_m\cos(2\theta_\gamma))\cos(\theta_e)) \\
s_\delta &= 2A(-L_m \sin(2\theta_\gamma)\cos(\theta_e) + (KL_i + KL_m\cos(2\theta_\gamma))\sin(\theta_e))
\end{aligned}
$$

条件を付与すると計算できた。

K=0のとき

$$
\begin{aligned}
c_\gamma &= 2A((L_i - L_m\cos(2\theta_\gamma))\cos(\theta_e)) \\
s_\gamma &= 2A((L_i - L_m\cos(2\theta_\gamma))\sin(\theta_e)) \\
\theta_e &= \tan^{-1}\frac{s_\gamma}{c_\gamma}
\end{aligned}
$$

K=1のとき

$$
\begin{aligned}
c_\gamma &= 2A(-L_m \sin(2\theta_\gamma)\sin(\theta_e) + (L_i - L_m\cos(2\theta_\gamma))\cos(\theta_e)) \\
s_\gamma &= 2A(L_m \sin(2\theta_\gamma)\cos(\theta_e) + (L_i - L_m\cos(2\theta_\gamma))\sin(\theta_e)) \\
c_\delta &= 2A(-L_m \sin(2\theta_\gamma)\sin(\theta_e) - (L_i + L_m\cos(2\theta_\gamma))\cos(\theta_e)) \\
s_\delta &= 2A(-L_m \sin(2\theta_\gamma)\cos(\theta_e) + (L_i + L_m\cos(2\theta_\gamma))\sin(\theta_e)) \\
c_\gamma - c_\delta &= 2A((2L_i)\cos(\theta_e)) \\
s_\gamma + s_\delta &= 2A((2L_i)\sin(\theta_e)) \\
\theta_e &= \tan^{-1}\frac{s_\gamma + s_\delta}{c_\gamma - c_\delta}
\end{aligned}
$$

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なるほど、回転の変換の積が作用してるように見えるのか！書き直してみる。

$$
\begin{aligned}
c_\gamma &= 2A((L_i - L_m\cos(2\theta_\gamma))\cos(\theta_e) + KL_m\sin(2\theta_\gamma)(-\sin(\theta_e))) \\
s_\gamma &= 2A((L_i - L_m\cos(2\theta_\gamma))\sin(\theta_e) + KL_m\sin(2\theta_\gamma)\cos(\theta_e)) \\
c_\delta &= 2A(-K(L_i + L_m\cos(2\theta_\gamma))\cos(\theta_e) - L_m\sin(2\theta_\gamma)\sin(\theta_e)) \\
s_\delta &= 2A(-K(L_i + L_m\cos(2\theta_\gamma))(-\sin(\theta_e)) - L_m\sin(2\theta_\gamma)\cos(\theta_e))
\end{aligned}
$$

$$
\begin{aligned}
\gamma = \frac{1}{2A}(c_\gamma + js_\gamma) &= (L_i - L_m\cos(2\theta_\gamma) + jKL_m\sin(2\theta_\gamma))e^{i\theta_e} \\
\delta = \frac{1}{2A}(c_\delta + js_\delta) &= (-K(L_i + L_m\cos(2\theta_\gamma)) - jL_m\sin(2\theta_\gamma))e^{-i\theta_e}
\end{aligned}
$$

共役

$$
\begin{aligned}
\overline{\gamma} =& (L_i - L_m \cos{(2\theta_\gamma)} - jKL_m \sin{(2\theta_\gamma)})e^{-i\theta_e} \\
\overline{\delta} =& (-K(L_i + L_m \cos{(2\theta_\gamma)}) + jL_m \sin{(2\theta_\gamma)})e^{i\theta_e}
\end{aligned}
$$

$$
\begin{aligned}
\gamma - \overline{\delta} =& (L_i - L_m \cos{(2\theta_\gamma)} + jKL_m \sin{(2\theta_\gamma)})e^{i\theta_e} \\
& + (K(L_i + L_m \cos{(2\theta_\gamma)}) - jL_m \sin{(2\theta_\gamma)})e^{i\theta_e} \\
=& ((K+1)L_i + (K-1)L_m(\cos{2\theta_\gamma} + j\sin{2\theta_\gamma}))e^{i\theta_e} \\
=& ((K+1)L_i + (K-1)L_m e^{i2\theta_\gamma})e^{i\theta_e}
\end{aligned}
$$

$$
\begin{aligned}
\overline{\gamma} + \delta =& (L_i - L_m \cos{(2\theta_\gamma)} - jKL_m \sin{(2\theta_\gamma)})e^{-i\theta_e} \\
& + (-K(L_i + L_m \cos{(2\theta_\gamma)}) - jL_m \sin{(2\theta_\gamma)})e^{-i\theta_e} \\
=& ((-K+1)L_i - (K+1)L_m(\cos{2\theta_\gamma} + j\sin{2\theta_\gamma}))e^{-i\theta_e} \\
=& ((-K+1)L_i - (K+1)L_m e^{i2\theta_\gamma})e^{-i\theta_e}
\end{aligned}
$$

$L_m(e^{i2\theta_\gamma})$を除去する。

(再掲)

$$
\begin{aligned}
\gamma - \overline{\delta} =& ((K+1)L_i + (K-1)L_m e^{i2\theta_\gamma})e^{i\theta_e} \\
\overline{\gamma} + \delta =& ((-K+1)L_i - (K+1)L_m e^{i2\theta_\gamma})e^{-i\theta_e}
\end{aligned}
$$

より、

$$
\begin{aligned}
(K+1)(\gamma - \overline{\delta}) =& ((K+1)^2 L_i + (K+1)(K-1)L_m e^{i2\theta_\gamma})e^{i\theta_e} \\
(K-1)(\overline{\gamma} + \delta) =& (-(K-1)^2 L_i - (K+1)(K-1)L_m e^{i2\theta_\gamma})e^{-i\theta_e}
\end{aligned}
$$

$$
(K+1)(\gamma - \overline{\delta}) + (K-1)(\overline{\gamma} + \delta) = (K+1)^2 L_i e^{i\theta_e} - (K-1)^2 L_i e^{-i\theta_e} + (K+1)(K-1)L_m e^{i2\theta_\gamma}(e^{i\theta_e} - e^{-i\theta_e})
$$

再計算

$$
\begin{aligned}
(K+1)(\gamma - \overline{\delta})e^{-i\theta_e} =& ((K+1)^2 L_i + (K+1)(K-1)L_m e^{i2\theta_\gamma}) \\
(K-1)(\overline{\gamma} + \delta)e^{i\theta_e} =& (-(K-1)^2 L_i - (K+1)(K-1)L_m e^{i2\theta_\gamma})
\end{aligned}
$$

$$
(K+1)(\gamma - \overline{\delta})e^{-i\theta_e} + (K-1)(\overline{\gamma} + \delta)e^{i\theta_e} = (K+1)^2 L_i - (K-1)^2 L_i = 4KL_i
$$

この式は実部しか持たない事が分かる。展開する。

$$
\begin{aligned}
\gamma &= \frac{1}{2A}(c_\gamma + js_\gamma) \\
\delta &= \frac{1}{2A}(c_\delta + js_\delta) \\
\overline{\gamma} &= \frac{1}{2A}(c_\gamma - js_\gamma) \\
\overline{\delta} &= \frac{1}{2A}(c_\delta - js_\delta)
\end{aligned}
$$

より、

$$
\begin{aligned}
&(K+1)(\gamma - \overline{\delta})e^{-i\theta_e} + (K-1)(\overline{\gamma} + \delta)e^{i\theta_e} \\
&= \frac{1}{2A}(K+1)((c_\gamma - c_\delta) + j(s_\gamma + s_\delta))e^{-i\theta_e} + \frac{1}{2A}(K-1)((c_\gamma + c_\delta) + j(s_\delta - s_\gamma))e^{i\theta_e} \\
&= \frac{1}{2A}(K+1)((c_\gamma - c_\delta) + j(s_\gamma + s_\delta))(\cos\theta_e - j\sin\theta_e) + \frac{1}{2A}(K-1)((c_\gamma + c_\delta) + j(s_\delta - s_\gamma))(\cos\theta_e + j\sin\theta_e) \\
&= \frac{1}{2A}(K+1)(((c_\gamma - c_\delta)\cos\theta_e + (s_\gamma + s_\delta)\sin\theta_e) + j((s_\gamma + s_\delta)\cos\theta_e - (c_\gamma - c_\delta)\sin\theta_e)) \\
&+ \frac{1}{2A}(K-1)(((c_\gamma + c_\delta)\cos\theta_e - (s_\gamma - s_\delta)\sin\theta_e) + j((s_\gamma - s_\delta)\cos\theta_e + (c_\gamma + c_\delta)\sin\theta_e))
\end{aligned}
$$

$$
\begin{aligned}
&\frac{1}{2A}(K+1)(j((s_\gamma + s_\delta)\cos\theta_e - (c_\gamma - c_\delta)\sin\theta_e)) \\
&+\frac{1}{2A}(K-1)(j((s_\gamma - s_\delta)\cos\theta_e + (c_\gamma + c_\delta)\sin\theta_e)) = 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{A}j(Ks_\gamma + s_\delta)\cos\theta_e + \frac{1}{A}j(Kc_\delta - c_\gamma)\sin\theta_e = 0 \\
\frac{\sin\theta_e}{\cos\theta_e} = -\frac{Ks_\gamma + s_\delta}{Kc_\delta - c_\gamma}
\end{aligned}
$$