





DFT+U hands-on

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Eastern Africa School on Electronic Structure Methods and Applications

Addis Ababa, Ethiopia 2 July 2019





Outline

Exercise 1: Calculation of *U*eff and DFT+*U* study of LiCoO2

Exercise 2: Calculation of *U*eff and DFT+*U* study of FeO

Exercise 3: Calculation of *U*eff and DFT+*U* study of NiO

Outline

Exercise 1: Calculation of *U*eff and DFT+*U* study of LiCoO2

Exercise 2: Calculation of Ueff and DFT+U study of FeO

Exercise 3: Calculation of *Ueff* and DFT+*U* study of NiO

DFT+U

The DFT+*U* total energy:

$$E_{\text{DFT}+U} = E_{\text{DFT}} + E_{U}$$

The Hubbard correction energy:

$$E_{U} = \frac{1}{2} \sum_{I,m,m',\sigma} \underbrace{\left(U^{I} - J^{I}\right)}_{U_{\text{eff}}^{I}} \left(\delta_{mm'} - n_{mm'}^{I\sigma}\right) n_{m'm}^{I\sigma}$$

effective Hubbard parameter

The occupation matrix:

$$n_{mm'}^{I\sigma} = \sum_{v,\mathbf{k}} f_{v\mathbf{k}}^{\sigma} \langle \psi_{v\mathbf{k}}^{\sigma} | \varphi_{m'}^{I} \rangle \langle \varphi_{m}^{I} | \psi_{v\mathbf{k}}^{\sigma} \rangle$$

The total occupation of localized states (*d* or *f*) at site *I* : $n^I = \sum n^{I\sigma}_{mm}$

$$n^I = \sum_{m,\sigma} n^{I\sigma}_{mm}$$

The Kohn-Sham equation:
$$\left[-\frac{1}{2} \nabla^2 + \hat{V}_{\mathrm{KS}}^{\sigma} + \hat{V}_{\boldsymbol{U}}^{\sigma} \right] \psi_{v\mathbf{k}}^{\sigma}(\mathbf{r}) = \varepsilon_{v\mathbf{k}}^{\sigma} \, \psi_{v\mathbf{k}}^{\sigma}(\mathbf{r})$$

Exersice 1: LiCoO₂

Go to the directory with the input files:

In this directory you will find:

- README
- LiCoO2.pw.in
- LiCoO2.hp.in
- LiCoO2.vcrelax.in
- pseudopotentials
- reference
- tmp

- File describing how to do the exercise
- Input file for the DFT ground-state calculation
- Input file for the DFPT calculation of Ueff
- Input file for the structural optimization
- Directory with the pseudopotentials
- Directory with the reference results
- Directory for temporary files

Exersice 1: LiCoO2

Task 1

Perform a DFT ground-state calculation

Perform a linear-response (DFPT) calculation of the effective Hubbard parameter

Analyze the input and output files

```
&control
                                                     DFT self-consistent (scf) ground-state calculation
   calculation='scf'
   restart_mode='from scratch'.
   prefix='LiCoO2'
   pseudo dir = './pseudopotentials/'
                                                      Rhombohedral cell and its lattice parameters
   outdir='./tmp/'
   verbosity='high'
                                                             (experimental lattice parameters)
&system
   ibrav = 5.
   celldm(1) = 9.3705,
   celldm(4) = 0.83874,
   nat = 4.
   ntvp = 3.
   ecutwfc = 30.0
   ecutrho = 240.0
   lda plus u = .true.,
   lda plus u kind = 0.
   U projection type = 'atomic',
   Hubbard U(1) = 1.d-8
 &electrons
   conv_thr = 1.d-12
   mixing beta = 0.7
ATOMIC SPECIES
Co 59.0 co pbesol v1.2.uspp.F.UPF
   16.0 O.pbesol-n-kjpaw psl.0.1.UPF
        li pbesol v1.4.uspp.F.UPF
Li 7.0
ATOMIC POSITIONS {crystal}
 Co 0.0000000000
                  0.0000000000
                                 0.0000000000
                                                           Atomic positions in crystal units
    0.2604885000
                0.2604885000
                                 0.2604885000
                0.7395115000
    0.7395115000
                                 0.7395115000
                                                           (experimental atomic positions)
 Li 0.5000000000 0.5000000000
                                0.5000000000
K POINTS {automatic}
 2 2 2 0 0 0
```

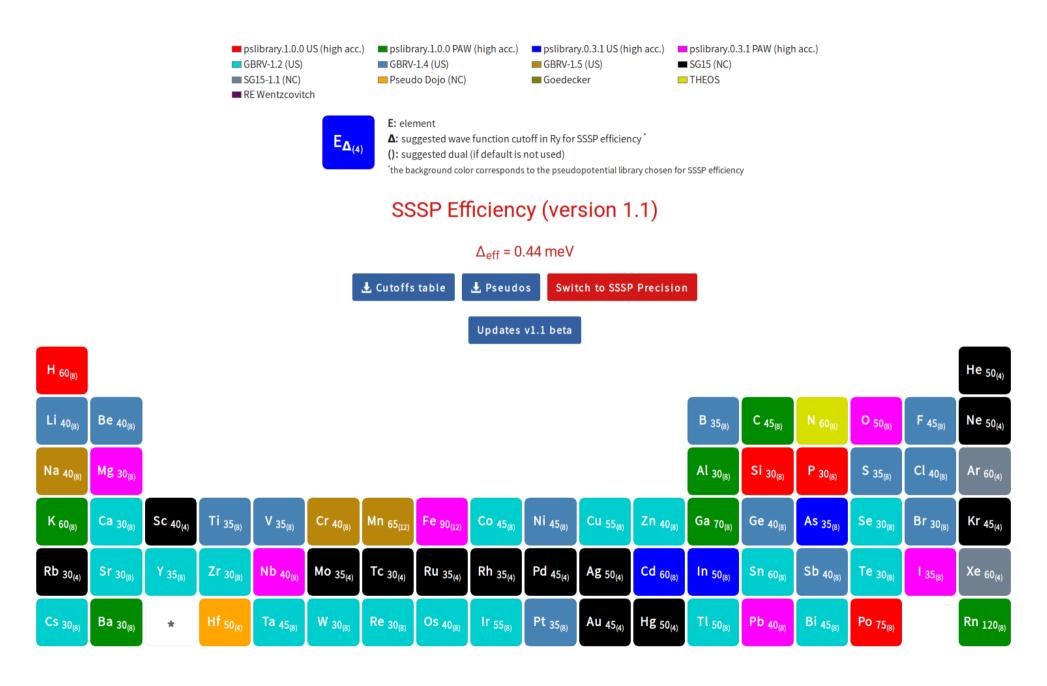
```
&control
   calculation='scf'
    restart mode='from scratch',
    prefix='LiCoO2'
    pseudo dir = './pseudopotentials/'
    outdir='./tmp/'
    verbosity='high'
 &system
   ibrav = 5.
   celldm(1) = 9.3705,
    celldm(4) = 0.83874,
   nat = 4.
   ntvp = 3.
    ecutwfc = 30.0
    ecutrho = 240.0
   lda plus u = .true..
    lda plus u kind = 0.
    U projection type = 'atomic',
    Hubbard U(1) = 1.d-8
 &electrons
   conv_thr = 1.d-12
    mixing beta = 0.7
ATOMIC SPECIES
Co 59.0 co pbesol v1.2.uspp.F.UPF
  16.0 O.pbesol-n-kjpaw_psl.0.1.UPF
Li 7.0 li pbesol v1.4.uspp.F.UPF
ATOMIC POSITIONS {crystal}
Co 0.0000000000
                   0.0000000000
                                  0.0000000000
    0.2604885000 0.2604885000
                                  0.2604885000
                   0.7395115000
                                  0.7395115000
    0.7395115000
Li 0.5000000000
                 0.5000000000
                                  0.5000000000
K POINTS {automatic}
2 2 2 0 0 0
```

Kinetic-energy cutoffs:

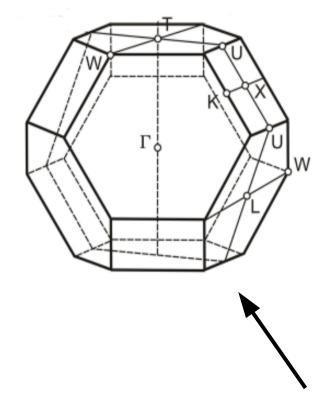
- for wavefunctions (ecutwfc)
- for density and potentials (ecutrho)

Ultrasoft (uspp) and PAW pseudopotentials PBEsol functional

Selecting pseudopotentials from the SSSP library



```
&control
    calculation='scf'
    restart mode='from scratch',
    prefix='LiCo02'
    pseudo dir = './pseudopotentials/'
    outdir='./tmp/'
    verbosity='high'
 &system
   ibrav = 5.
    celldm(1) = 9.3705,
    celldm(4) = 0.83874,
    nat = 4.
    ntyp = 3,
    ecutwfc = 30.0
    ecutrho = 240.0
    lda plus u = .true..
    lda plus u kind = 0,
    U projection type = 'atomic',
    Hubbard U(1) = 1.d-8
 &electrons
    conv_thr = 1.d-12
    mixing beta = 0.7
ATOMIC SPECIES
Co 59.0 co pbesol v1.2.uspp.F.UPF
  16.0 O.pbesol-n-kjpaw_psl.0.1.UPF
Li 7.0 li pbesol v1.4.uspp.F.UPF
ATOMIC POSITIONS {crystal}
 Co 0.0000000000 0.0000000000
                                  0.0000000000
    0.2604885000 0.2604885000
                                0.2604885000
    0.7395115000 0.7395115000
                                  0.7395115000
Li 0.5000000000
                 0.5000000000
                                0.5000000000
K POINTS {automatic}
 2 2 2 0 0 0
```



k points sampling of the Brillouin zone

```
&control
    calculation='scf'
    restart mode='from scratch',
    prefix='LiCoO2'
    pseudo dir = './pseudopotentials/'
    outdir='./tmp/'
    verbosity='high'
 &system
    ibrav = 5.
    celldm(1) = 9.3705,
    celldm(4) = 0.83874
    nat = 4.
    ntvp = 3.
    ecutwfc = 30.0
    ecutrho = 240.0
    lda plus u = .true..
    lda plus u kind = 0,
    U projection type = 'atomic',
    Hubbard U(1) = 1.d-8
 &electrons
    conv thr = 1.d-12
    mixing beta = 0.7
ATOMIC SPECIES
Co 59.0 co pbesol v1.2.uspp.F.UPF
0 16.0 0.pbesol-n-kjpaw_psl.0.1.UPF
Li 7.0 li pbesol v1.4.uspp.F.UPF
ATOMIC POSITIONS {crystal}
 Co 0.0000000000 0.0000000000
                                  0.0000000000
    0.2604885000 0.2604885000
                                  0.2604885000
    0.7395115000 0.7395115000
                                  0.7395115000
 Li 0.5000000000
                 0.5000000000
                                  0.5000000000
K POINTS {automatic}
 2 2 2 0 0 0
```

DFT+*U* specific variables

lda plus u logical keyword

It means that we want to perform a DFT+*U* calculation

Note: it is not necessarily LDA+U, it refers also to GGA+U or any other functional +U.

This name is for historical reasons.

```
&control
    calculation='scf'
    restart mode='from scratch',
    prefix='LiCoO2'
    pseudo dir = './pseudopotentials/'
    outdir='./tmp/'
    verbosity='high'
 &system
    ibrav = 5.
    celldm(1) = 9.3705,
    celldm(4) = 0.83874,
    nat = 4.
   ntyp = 3,
    ecutwfc = 30.0
    ecutrho = 240.0
    lda plus u = .true..
    lda plus u kind = 0.
    U projection type = 'atomic',
    Hubbard U(1) = 1.d-8
 &electrons
    conv thr = 1.d-12
    mixing beta = 0.7
ATOMIC SPECIES
Co 59.0 co pbesol v1.2.uspp.F.UPF
   16.0 O.pbesol-n-kjpaw psl.0.1.UPF
         li pbesol v1.4.uspp.F.UPF
Li 7.0
ATOMIC POSITIONS {crystal}
 Co 0.0000000000
                   0.0000000000
                                   0.0000000000
    0.2604885000 0.2604885000
                                   0.2604885000
                   0.7395115000
                                   0.7395115000
     0.7395115000
 Li 0.5000000000
                  0.5000000000
                                   0.5000000000
K POINTS {automatic}
 2 2 2 0 0 0
```

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lda_plus_u_kind=0 means a simplified rotationally-invariant formulation of DFT+*U* according to [1].

[1] S.L. Dudarev et al., Phys. Rev. B **57**, 1505 (1998)

```
&control
    calculation='scf'
    restart mode='from scratch',
    prefix='LiCoO2'
    pseudo dir = './pseudopotentials/'
    outdir='./tmp/'
    verbosity='high'
 &system
    ibrav = 5.
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    celldm(4) = 0.83874,
    nat = 4.
    ntyp = 3,
    ecutwfc = 30.0
    ecutrho = 240.0
    lda plus u = .true.
    lda plus u kind = 0.
    U projection type = 'atomic',
    Hubbard U(1) = 1.d-8
 &electrons
    conv thr = 1.d-12
    mixing beta = 0.7
ATOMIC SPECIES
Co 59.0 co pbesol v1.2.uspp.F.UPF
  16.0 O.pbesol-n-kjpaw psl.0.1.UPF
Li 7.0 li pbesol v1.4.uspp.F.UPF
ATOMIC POSITIONS {crystal}
 Co 0.0000000000
                   0.0000000000
                                   0.0000000000
    0.2604885000 0.2604885000
                                   0.2604885000
                   0.7395115000
                                  0.7395115000
     0.7395115000
 Li 0.5000000000
                 0.5000000000
                                   0.5000000000
K POINTS {automatic}
 2 2 2 0 0 0
```

DFT+*U* specific variables

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It means that we want to perform a DFT+*U* calculation

Note: it is not necessarily LDA+U, it refers also to GGA+U or any other functional +U.

This name is for historical reasons.

lda_plus_u_kind=0 means a simplified rotationally-invariant formulation of DFT+*U* according to [1].

U_projection_type selects the type of localized orbitals which we want to use in projectors.

$$n_{mm'}^{I\sigma} = \sum_{v,\mathbf{k}} f_{v\mathbf{k}}^{\sigma} \langle \psi_{v\mathbf{k}}^{\sigma} | \varphi_{m'}^{I} \rangle \langle \varphi_{m}^{I} | \psi_{v\mathbf{k}}^{\sigma} \rangle$$

[1] S.L. Dudarev et al., Phys. Rev. B **57**, 1505 (1998)

```
&control
    calculation='scf'
    restart mode='from scratch',
    prefix='LiCoO2'
    pseudo dir = './pseudopotentials/'
    outdir='./tmp/'
    verbosity='high'
 &system
    ibrav = 5.
    celldm(1) = 9.3705,
    celldm(4) = 0.83874,
    nat = 4.
    ntvp = 3.
    ecutwfc = 30.0
    ecutrho = 240.0
    lda plus u = .true..
    lda plus u kind = 0.
    U projection type = 'atomic',
    Hubbard U(1) = 1.d-8
 &electrons
    conv_thr = 1.d-12
    mixing beta = 0.7
ATOMIC SPECIES
Co 59.0 co pbesol v1.2.uspp.F.UPF
  16.0 O.pbesol-n-kjpaw psl.0.1.UPF
         li pbesol v1.4.uspp.F.UPF
Li 7.0
ATOMIC POSITIONS {crystal}
 Co 0.0000000000
                    0.0000000000
                                   0.0000000000
     0.2604885000
                    0.2604885000
                                   0.2604885000
                    0.7395115000
     0.7395115000
                                   0.7395115000
                   0.5000000000
 Li 0.5000000000
                                   0.5000000000
K POINTS {automatic}
 2 2 2 0 0 0
```

DFT+*U* specific variables

lda plus u logical keyword

It means that we want to perform a DFT+*U* calculation

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U_projection_type selects the type of localized orbitals which we want to use in projectors.

$$n_{mm'}^{I\sigma} = \sum_{v,\mathbf{k}} f_{v\mathbf{k}}^{\sigma} \langle \psi_{v\mathbf{k}}^{\sigma} | \varphi_{m'}^{I} \rangle \langle \varphi_{m}^{I} | \psi_{v\mathbf{k}}^{\sigma} \rangle$$

Value of the Hubbard *U*eff parameter.

Here we have essentially 0. We put this very small value in order just to let the code know which element we want to consider as a Hubbard atom.

[1] S.L. Dudarev et al., Phys. Rev. B **57**, 1505 (1998)

```
DFT+U specific variables
&control
   calculation='scf'
   restart mode='from scratch',
                                                  lda plus u logical keyword
   prefix='LiCoO2'
   pseudo dir = './pseudopotentials/'
                                                  It means that we want to perform a DET+LL calculation
   outdir='./tmp/'
   verbo
                                                                                                 Iso to GGA+U
 &system
   ibrav
   celld
               Detailed documentation can be found here
                                                                                                sons.
   celld
   nat :
   ntyp
               quantum-espresso/PW/Doc/INPUT PW.txt
   ecuty
                                                                                                bnally-invariant
   ecuti
                                                                                                cording to [1].
   lda p
               or here
   lda p
   U pro
   Hubba
                                                                                                of localized
               https://www.quantum-espresso.org/Doc/INPUT PW.html
 &electro
   conv
   mixir
                                                                                                 |\psi_{v\mathbf{k}}^{\sigma}\rangle
ATOMIC SE
Co 59.0
   16.0
         U.pdesot-n-Kjpaw pst.U.1.UPF
   7.0
          li pbesol v1.4.uspp.F.UPF
                                                   Value of the Hubbard Ueff parameter.
ATOMIC POSITIONS {crystal}
   0.0000000000
                  0.0000000000
                                0.0000000000
                                                   Here we have essentially 0. We put this very small
    0.2604885000
                  0.2604885000
                                0.2604885000
                                                   value in order just to let the code know which element
                  0.7395115000
    0.7395115000
                                0.7395115000
                                                   we want to consider as a Hubbard atom.
                  0.5000000000
 Li 0.5000000000
                                0.5000000000
K POINTS {automatic}
```

[1] S.L. Dudarev et al., Phys. Rev. B **57**, 1505 (1998)

2 2 2 0 0 0

Localized-states manifold

How do we know to what states of a Hubbard atom the U_{eff} correction will be applied?



Currently, this is hard-coded (in Quantum ESPRESSO 6.4.1)



Check two subroutines in quantum-espresso/Modules:





set hubbard n.f90

principal quantum number "n"

For Co: n = 3





orbital quantum number "I"

For Co : I = 2



Apply the Hubbard *U*eff correction to the *3d* electrons of Co

DFPT for *U*eff

Solve (self-consistently, iteratively) the linear-response Kohn-Sham equations for every \mathbf{q} :

$$\left[-\frac{1}{2} \left[\nabla + i (\mathbf{k} + \mathbf{q}) \right]^2 + \hat{V}_{\mathrm{KS}, \mathbf{k} + \mathbf{q}}^{\sigma \circ} + \hat{V}_{U, \mathbf{k} + \mathbf{q}}^{\sigma \circ} - \varepsilon_{v \mathbf{k}}^{\sigma \circ} \right] \Delta_{\mathbf{q}}^{s} u_{v \mathbf{k}}^{\sigma}(\mathbf{r}) = -\hat{\mathcal{P}}_{\mathbf{k} + \mathbf{q}}^{\sigma} \left[\Delta_{\mathbf{q}}^{s} V_{\mathrm{Hxc}}^{\sigma} + \sum_{m} |\phi_{m \mathbf{k} + \mathbf{q}}^{s}\rangle \langle \phi_{m \mathbf{k}}^{s}| \right] u_{v \mathbf{k}}^{\sigma \circ}(\mathbf{r})$$



Compute linear-response occupation matrices:

$$\Delta_{\mathbf{q}}^{s'} n_{mm'}^{s\sigma} = \frac{1}{N} \sum_{v,\mathbf{k}} \left[\langle u_{v\mathbf{k}}^{\sigma \circ} | \phi_{m'\mathbf{k}}^{s} \rangle \langle \phi_{m\mathbf{k}+\mathbf{q}}^{s} | \Delta_{\mathbf{q}}^{s'} u_{v\mathbf{k}}^{\sigma} \rangle + \langle u_{v\mathbf{k}}^{\sigma \circ} | \phi_{m\mathbf{k}}^{s} \rangle \langle \phi_{m'\mathbf{k}+\mathbf{q}}^{s} | \Delta_{\mathbf{q}}^{s'} u_{v\mathbf{k}}^{\sigma} \rangle \right]$$



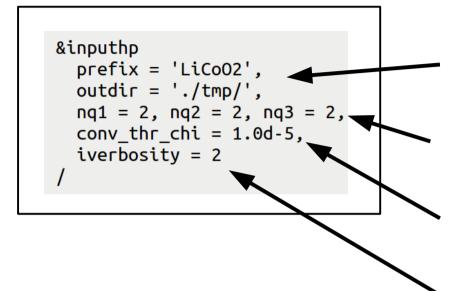
Sum up over **q** and compute the response matrices:

$$\chi_{IJ} = \sum_{m,\sigma} \frac{dn_{mm}^{I\sigma}}{d\alpha_J}, \qquad \chi_{IJ}^0 = \sum_{m,\sigma} \frac{dn_{mm}^{0\,I\sigma}}{d\alpha_J}$$



Compute Hubbard *U***eff**:

$$U_{\text{eff}}^{I} = ((\chi^{0})^{-1} - \chi^{-1})_{II}$$



Prefix and **outdir** must be exactly the same as in the input file LiCoO2.pw.in

Size of the **q** point mesh: 2x2x2 in this case

Convergence threshold for the calculation of the response matrices χ_{IJ}

Controls how many details of the calculation is printed in the output file

Detailed documentation can be found here

quantum-espresso/HP/Doc/INPUT HP.txt

or here

https://www.quantum-espresso.org/Doc/INPUT_HP.html

Exersice 1: LiCoO₂

Perform a DFT ground-state calculation using the **pw.x** code:

mpirun -np 2 \$PATH/pw.x < LiCoO2.pw.in > LiCoO2.pw.out

\$PATH=/home/max/codes/q-e-qe-6.4.1/bin

Perform a DFPT calculation using the hp.x code (Hubbard Parameters \rightarrow HP):

mpirun -np 2 \$PATH/hp.x < LiCoO2.hp.in > LiCoO2.hp.out

\$PATH=/home/max/codes/q-e-qe-6.4.1/bin

Analyze the output files LiCoO2.pw.out, LiCoO2.hp.out, and LiCoO2.Hubbard_parameters.dat

Exersice 1: LiCoO2

Task 2

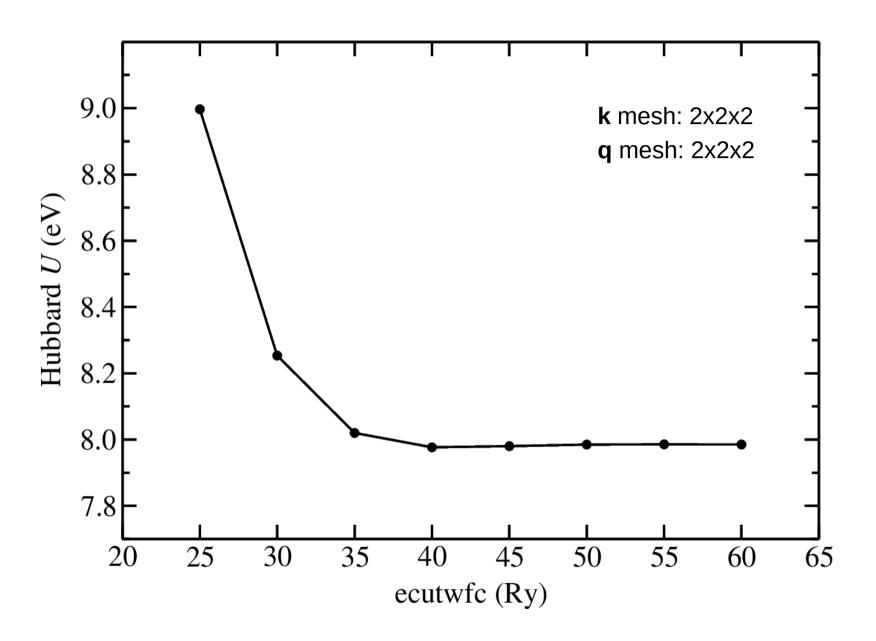
Perform convergence tests of *U*_{eff} with respect to the kinetic-energy cutoff.

Hint: try these values of ecutwfc = 25, 30, 35, 40, 45, 50, 55, 60 (Ry).

Keep the ratio ecutrho / ecutwfc = 8 in this exercise.

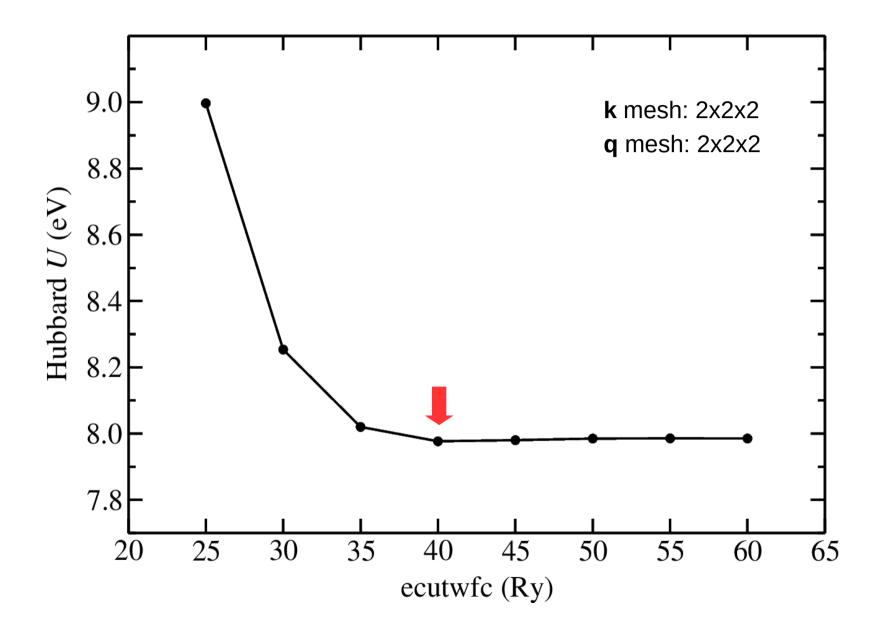
Use \mathbf{k} point mesh 2x2x2 and \mathbf{q} point mesh 2x2x2.

Convergence of Ueff with respect to cutoff



ecutrho = 8 * ecutwfc

Convergence of Ueff with respect to cutoff



Let us use the cutoff ecutwfc = 40 (Ry): U_{eff} is converged with the accuracy of ~0.01 (eV).

Convergence of Ueff with respect to cutoff

In the SSSP library it is recommended to use such cutoffs:

Li : ecutwfc = 40 (Ry), ecutrho = 320 (Ry)

Co : ecutwfc = 45 (Ry), ecutrho = 360 (Ry)

O: ecutwfc = 50 (Ry), ecutrho = 400 (Ry)

The rule is that when we mix these elements, we need to select the highest cutoff (i.e. ecutwfc = 50 (Ry), ecutrho = 400 (Ry)) and use it for all elements in the compound.

However, from our convergence tests of $U_{\rm eff}$ for Co-3d states in LiCoO2 we found that already at ecutwfc = 40 (Ry) and ecutrho = 320 (Ry) the value of $U_{\rm eff}$ is converged very good (with the accuracy of 0.01 (Ry)).



This is not a universal finding! One has to check carefully the convergence of *U*eff with respect to the cutoff for every compound.

Exersice 1: LiCoO₂

Task 3

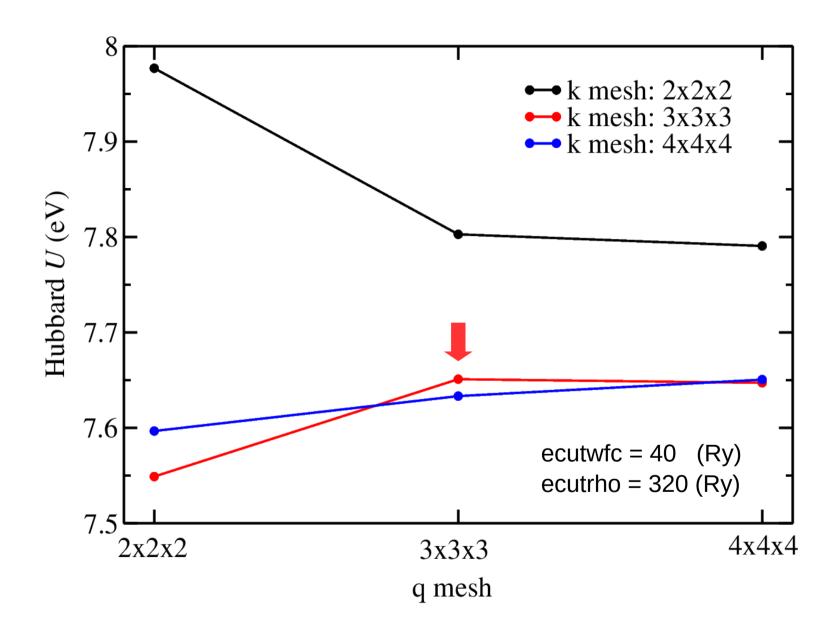
Perform convergence tests of U_{eff} with respect to the **k** mesh and **q** mesh.

Hint: use the converged value of the cutoffs obtained in the previous step.

Try these combinations:

k mesh	q mesh
2x2x2	2x2x2
2x2x2	3x3x3
2x2x2	4x4x4
3x3x3	2x2x2
3x3x3	3x3x3
3x3x3	4x4x4
4x4x4	2x2x2
4x4x4	3x3x3
4x4x4	4x4x4

Convergence of Ueff with respect to k and q



Let us use the **k** mesh 3x3x3 and **q** mesh 3x3x3: Ueff is converged with the accuracy of ~ 0.01 (eV).

Exersice 1: LiCoO₂

Task 4

Perform a self-consistent calculation of *U*eff (including a structural optimization)

Hint: use the convergence parameters obtained in previous steps.

So far we have performed a "one-shot" calculation of *U*eff using the experimental geometry.

After convergence tests we obtained the value of *U*eff for Co-3*d* which is 7.63 (eV) with the accuracy of 0.01 (eV).

This value was obtained using:

ecutwfc = 40 (Ry)

ecutrho = 320 (Ry)

k mesh: 3x3x3

q mesh: 3x3x3

So far we have performed a "one-shot" calculation of *U*eff using the experimental geometry.

After convergence tests we obtained the value of Ueff for Co-3d which is 7.63 (eV) with the accuracy of 0.01 (eV).

This value was obtained using:

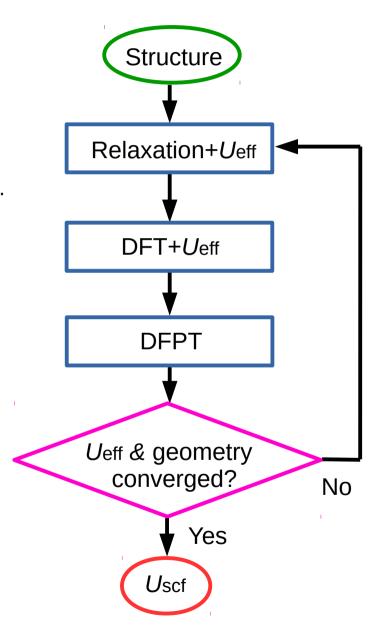
ecutwfc = 40 (Ry)

ecutrho = 320 (Ry)

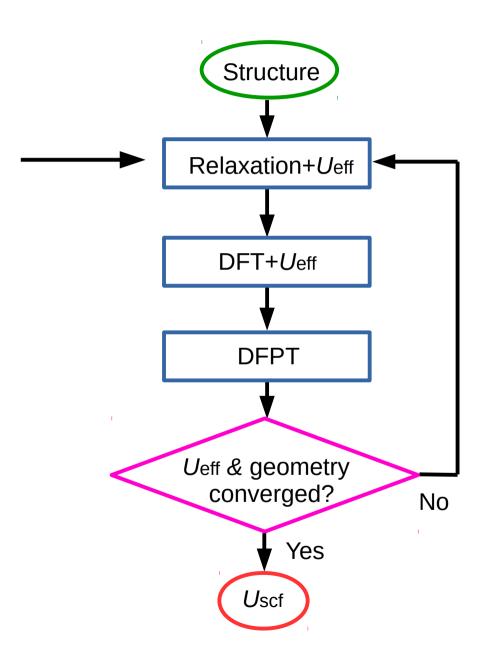
k mesh: 3x3x3

q mesh: 3x3x3

Now let us compute *U*eff in a self-consistent way as shown on the schema on the right-hand side.

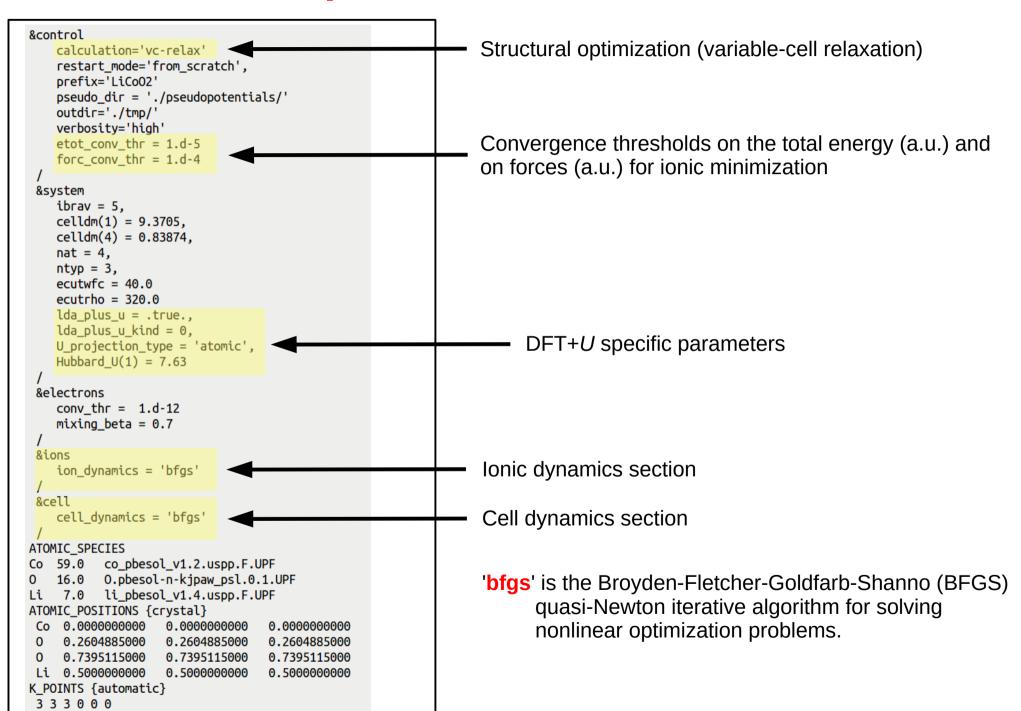


Our next step is to perform a variable-cell relaxation of the structure using the Hubbard correction with Ueff = 7.63 (eV).



Now let us compute *U*eff in a self-consistent way as shown on the schema on the right-hand side.

Input file LiCoO2.vcrelax.in



Structural optimization

Input

```
ibrav = 5,
celldm(1) = 9.3705,
celldm(4) = 0.83874,
```

```
ATOMIC_POSITIONS {crystal}
Co 0.000000000 0.000000000 0.0000000000
0 0.2604885000 0.2604885000 0.2604885000
0 0.7395115000 0.7395115000 0.7395115000
Li 0.5000000000 0.5000000000 0.5000000000
```

Output

```
CELL_PARAMETERS (alat= 9.37050000)
0.281280268 -0.162397239 0.930675561
0.000000000 0.324794477 0.930675561
-0.281280268 -0.162397239 0.930675561
```

It is possible to convert the data above for cell parameters to ibrav, celldm(1), and celldm(4) using a simple program cell2ibrav.x located in quantum-espresso/PW/tools.

Conversion of structural parameters

Output

```
CELL_PARAMETERS (alat= 9.37050000)
0.281280268 -0.162397239 0.930675561
0.000000000 0.324794477 0.930675561
-0.281280268 -0.162397239 0.930675561
```



By using **cell2ibrav.x** we obtain the following:

```
Enter the lattice vectors (one per line)
0.281280268, -0.162397239, 0.930675561
0.000000000, 0.324794477, 0.930675561
-0.281280268, -0.162397239, 0.930675561
Enter alat (or 'x' to skip if axis are in bohr)
9.3705
ibrav = 5
celldm(1) = 9.23671081
celldm(4) = 0.83714565
```

Note: This conversion is optional. You can still set up the input for pw.x using cell parameters as will be shown on next slides.

Structural optimization

Input

```
ibrav = 5,
celldm(1) = 9.3705,
celldm(4) = 0.83874,
```

```
ATOMIC_POSITIONS {crystal}
Co 0.000000000 0.000000000 0.000000000
0 0.2604885000 0.2604885000 0.2604885000
0 0.7395115000 0.7395115000 0.7395115000
Li 0.5000000000 0.5000000000 0.5000000000
```

Output

```
ibrav = 5,
celldm(1) = 9.2367,
celldm(4) = 0.83715,
```

After a structural optimization at the DFT+U level the atomic positions and the lattice parametershave changed. Now we need to recompute Ueff using this new geometry.

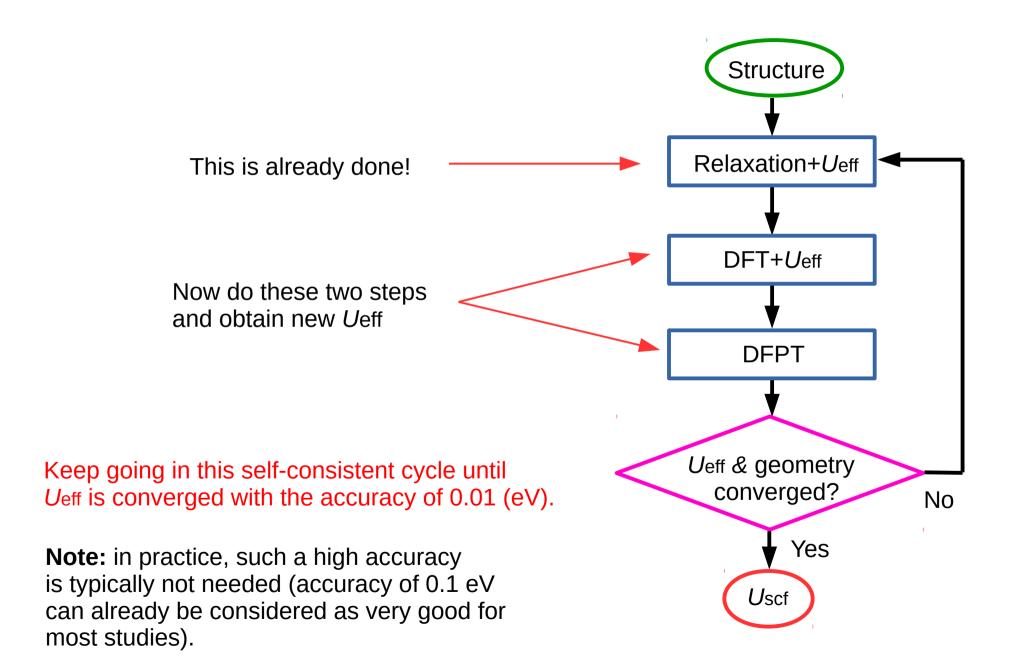
Two ways of setting up a new input file LiCoO2.pw.in

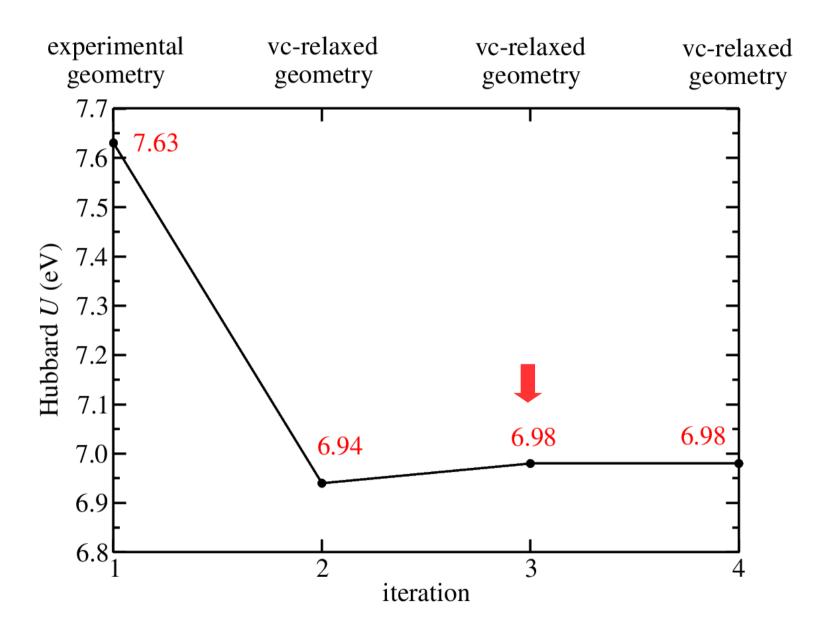
Option 1:

&control calculation='scf' restart mode='from scratch'. prefix='LiCoO2' pseudo dir = './pseudopotentials/' outdir='./tmp/' verbositv='hiah' &system ibrav = 5. celldm(1) = 9.2367, - new celldm(4) = 0.83715, nat = 4.ntvp = 3. ecutwfc = 40.0ecutrho = 320.0lda plus u = .true.,lda plus u kind = 0, U projection type = 'atomic', Hubbard U(1) = 7.63&electrons conv thr = 1.d-12mixing beta = 0.7ATOMIC SPECIES Co 59.0 co pbesol v1.2.uspp.F.UPF 16.0 O.pbesol-n-kjpaw_psl.0.1.UPF Li 7.0 li pbesol v1.4.uspp.F.UPF ATOMIC POSITIONS {crystal} Co 0.000000000 0.000000000 0.000000000 0 0.259303480 0.259303480 0.259303480 0.740696520 0.740696520 0.740696520 0.500000000 0.500000000 0.500000000 K POINTS {automatic} 3 3 3 0 0 0

Option 2:

```
&control
    calculation='scf'
    restart mode='from scratch',
    prefix='LiCo02'
    pseudo dir = './pseudopotentials/'
    outdir='./tmp/'
    verbosity='high'
 &system
    ibrav = 0.
    celldm(1) = 9.3705,
    nat = 4.
    ntyp = 3.
    ecutwfc = 40.0
    ecutrho = 320.0
   lda plus u = .true.,
    lda plus u kind = 0.
    U_projection_type = 'atomic',
    Hubbard U(1) = 7.63
 &electrons
    conv_thr = 1.d-12
    mixing beta = 0.7
ATOMIC SPECIES
Co 59.0 co_pbesol_v1.2.uspp.F.UPF
0 16.0 0.pbesol-n-kjpaw_psl.0.1.UPF
Li 7.0 li pbesol v1.4.uspp.F.UPF
CELL PARAMETERS {alat}
   0.281280268 -0.162397239
                              0.930675561
   0.000000000 0.324794477
                             0.930675561
  -0.281280268 -0.162397239
                              0.930675561
ATOMIC_POSITIONS {crystal}
 Co
          0.000000000
                       0.000000000
                                     0.000000000
 0
         0.259303480
                       0.259303480
                                     0.259303480
         0.740696520
                       0.740696520
                                     0.740696520
         0.500000000
                       0.500000000
                                     0.500000000
K_POINTS {automatic}
 3 3 3 0 0 0
```





Only 3 iterations is enough to converge U_{eff} with the accuracy of 0.01 (eV) for LiCoO₂.

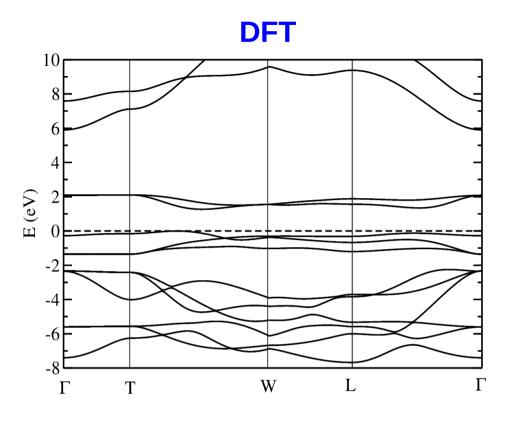
Exersice 1: LiCoO₂

Task 5

Perform calculations of the band structure and of the projected density of states (PDOS) using DFT+U with the value of the self-consistent Ueff which you obtained in previous steps.

What is the value of a band gap? How does it compare with the experimental gap: 2.7 ± 0.3 (eV)?

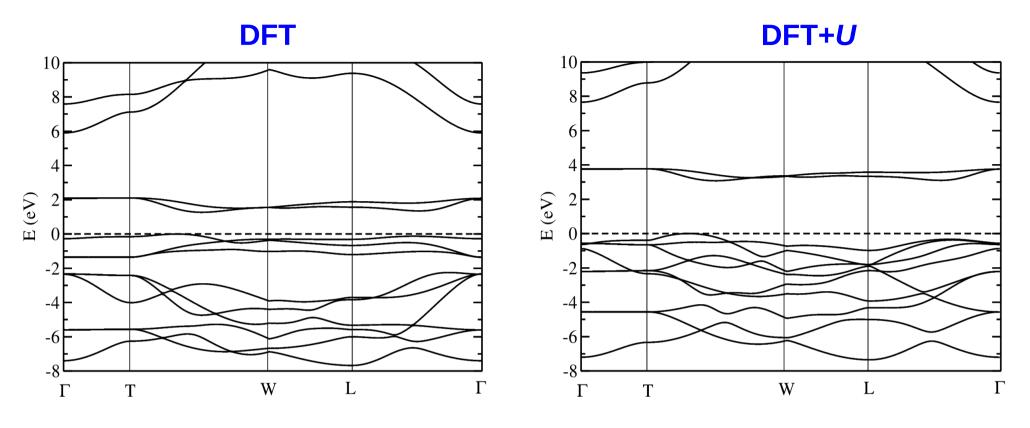
What is the difference in PDOS obtained at the DFT and DFT+U levels of theory? In particular, plot Co-3d and O-2p PDOS and check their relative location. Which conclusions can you make?



Band gap =
$$1.14$$
 (eV)

Experimental band gap = 2.7 ± 0.3 (eV)

DFT with the PBEsol functional underestimates the band gap.

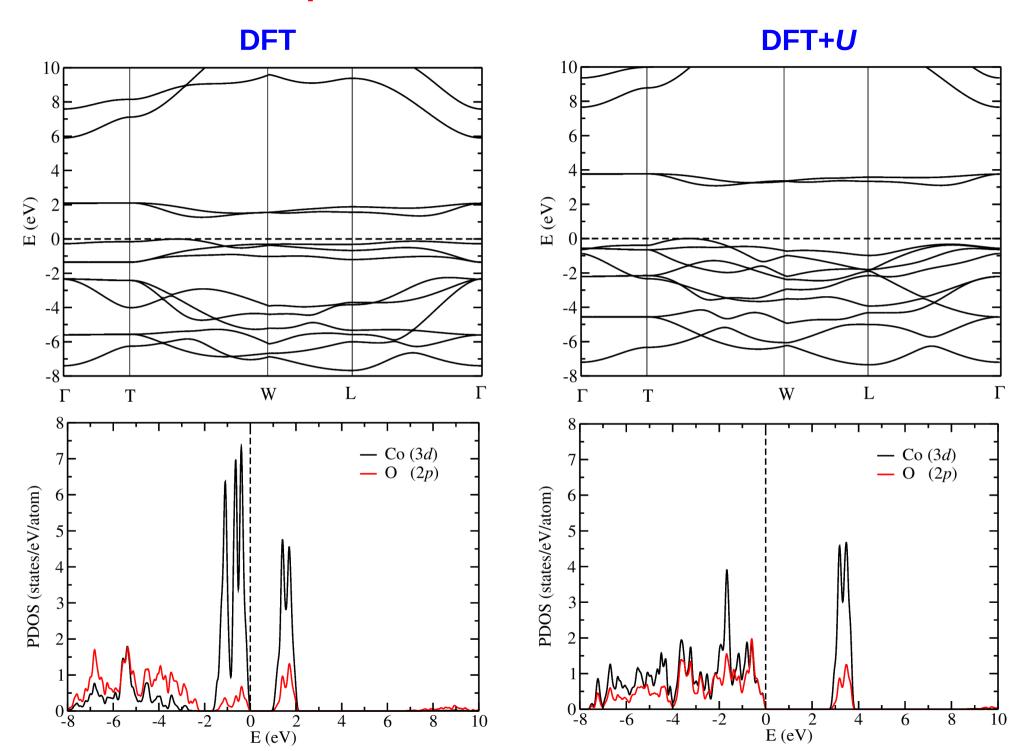


Band gap =
$$1.14$$
 (eV)

Band gap =
$$2.94$$
 (eV)

Experimental band gap = 2.7 ± 0.3 (eV)

Application of the Hubbard correction to Co-3*d* significantly improves the band gap.



Outline

Exercise 1: Calculation of *U*eff and DFT+*U* study of LiCoO2

Exercise 2: Calculation of *U*eff and DFT+*U* study of FeO

Exercise 3: Calculation of *U*eff and DFT+*U* study of NiO

Exersice 2: FeO

Go to the directory with the input files:

In this directory you will find:

•	README	

- FeO.pw.in
- FeO.hp.in
- FeO.pw bands.in
- FeO.pw nscf.in
- FeO.pdos.in
- FeO.plotband.in
- pseudopotentials
- reference
- tmp

- File describing how to do the exercise
- Input file for the DFT ground-state calculation
- Input file for the DFPT calculation of Ueff
- Input file for the band structure calculation
- Input file for the non-SCF calculation
- Input file for the PDOS calculation
- Input file for the band structure plot
- Directory with the pseudopotentials
- Directory with the reference results
- Directory for temporary files

Input file FeO.pw.in

```
&control
    calculation='scf'
   restart mode='from scratch',
    prefix='Fe0'
   pseudo dir = './pseudopotentials/'
    outdir='./tmp/'
 &system
    ibrav = 0.
    celldm(1) = 8.19.
    nat = 4,
    ntvp = 3.
    ecutwfc = 30.0.
    ecutrho = 240.0,
    occupations = 'smearing',
    smearing = 'mv'.
    degauss = 0.02.
   nspin = 2,
   starting_magnetization(1) = 0.5,
   starting magnetization(2) = -0.5
    lda plus u = .true..
   lda_plus_u_kind = 0,
   U_projection_type = 'atomic',
    Hubbard U(1) = 1.d-8
   Hubbard_U(2) = 1.d-8
 &electrons
    conv_thr = 1.d-9
    mixing beta = 0.3
ATOMIC SPECIES
Fe1 55.845 Fe.pbesol-spn-kjpaw_psl.0.2.1.UPF
Fe2 55.845 Fe.pbesol-spn-kjpaw psl.0.2.1.UPF
            O.pbesol-n-kjpaw psl.0.1.UPF
CELL_PARAMETERS {alat}
 0.50 0.50 1.00
 0.50 1.00 0.50
1.00 0.50 0.50
ATOMIC_POSITIONS {crystal}
Fe1 0.00 0.00 0.00
Fe2 0.50 0.50 0.50
     0.25 0.25 0.25
     0.75 0.75 0.75
K POINTS {automatic}
3 3 3 0 0 0
```

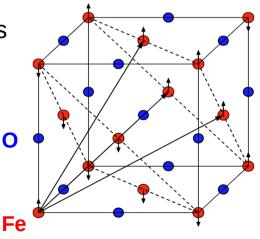
Experimental lattice parameter

'MV' smearing for metals (FeO is metallic @PBEsol)

Spin-polarized (collinear) calculation (antiferromagnetic ordering)

Initialize Hubbard parameters for two types of Fe

Two sublattices of Fe atoms



Exersice 2: FeO

Task 1

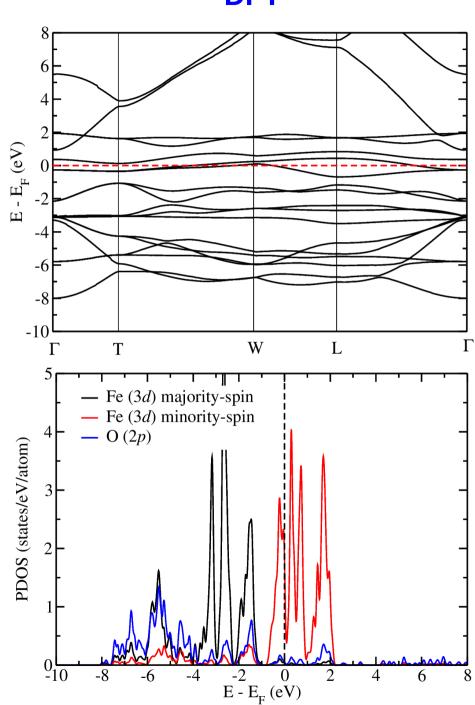
Perform a ground-state calculation

Perform a "one-shot" calculation of the effective Hubbard parameter *U*eff.

Task 2

Perform a calculation of the band structure and PDOS using DFT+U and Ueff which you obtained in the previous step.



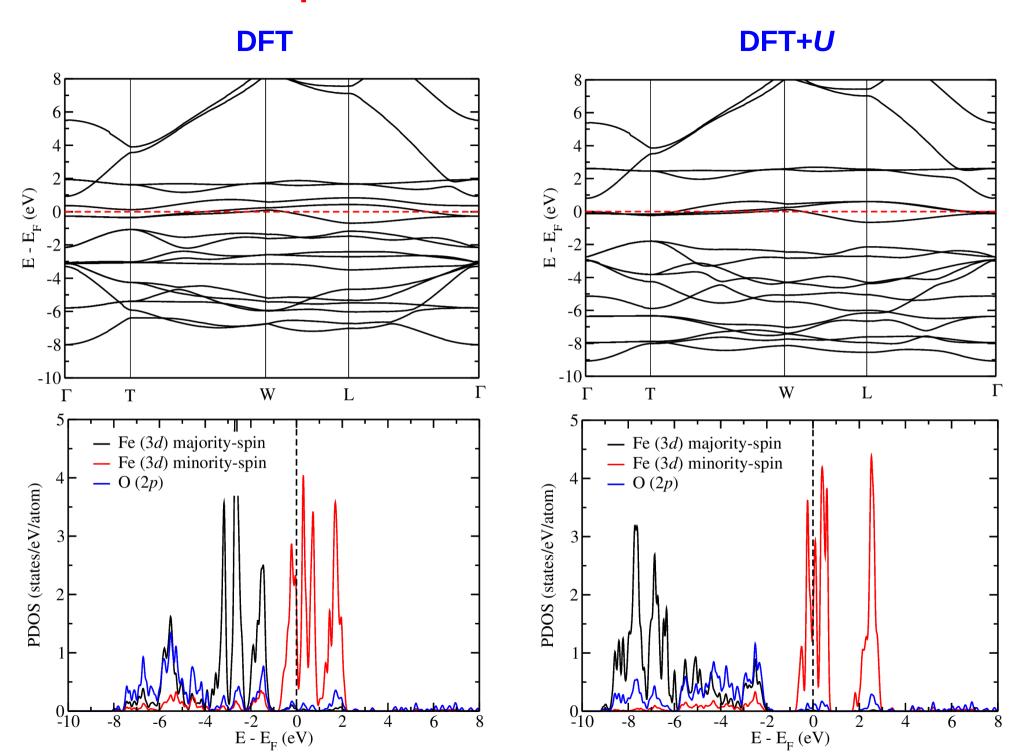


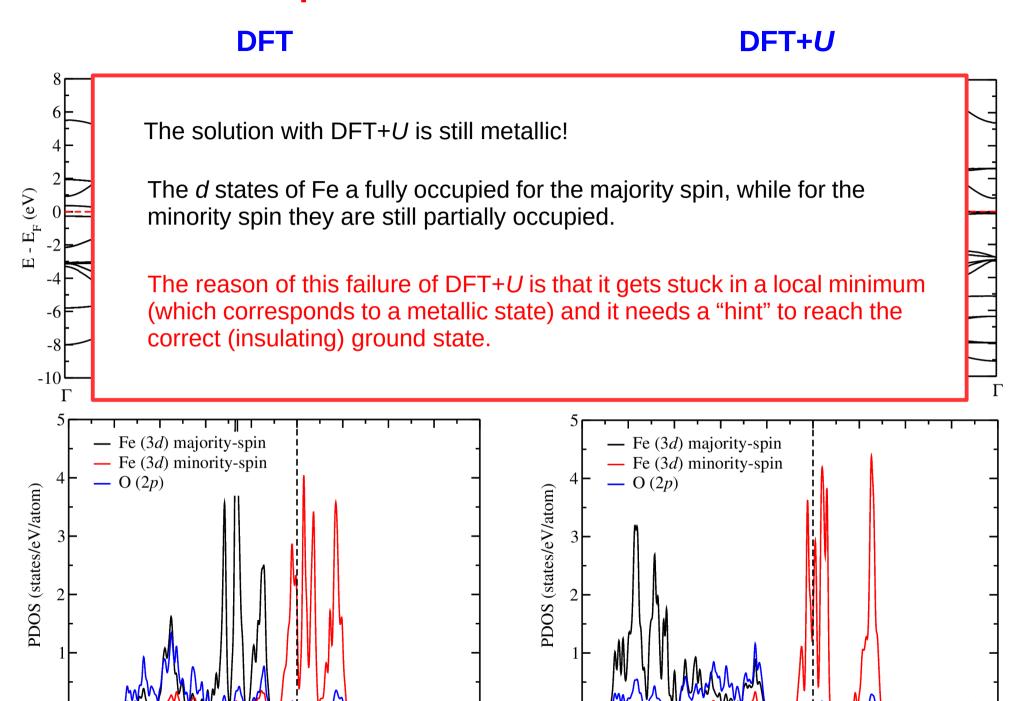
DFT-PBEsol predicts FeO to be metallic (but this is wrong)

Experimentally FeO is know to be insulating

First weak peak in the absorption spectrum is reported between 0.5 and 2 eV and a stronger absorption peak occurs at 2.4 eV

M. Cococcioni and S. de Gironcoli, Phys. Rev. B 71, 035105 (2005)





 $E - E_F(eV)$

 $E - E_F(eV)$

Exersice 2: FeO

Task 3

Perform a calculation of the band structure and PDOS using DFT+U and Ueff which you obtained in the previous step, but this time use the input parameter starting ns eigenvalue.

A "hint" for DFT+U where to look for a correct solution

```
&control
    calculation='scf'
    restart mode='from scratch'.
    prefix='Fe0'
   pseudo dir = './pseudopotentials/'
    outdir='./tmp/'
 &system
    ibrav = 0,
    celldm(1) = 8.19
    nat = 4.
    ntvp = 3.
    ecutwfc = 30.0,
    ecutrho = 240.0,
    occupations = 'smearing',
    smearing = 'mv',
    degauss = 0.02,
    nspin = 2.
   starting magnetization(1) = 0.5,
    starting magnetization(2) = -0.5
   lda_plus_u = .true.,
    lda_plus_u_kind = 0,
   U projection type = 'atomic',
   Hubbard\ U(1) = 5.2
   Hubbard U(2) = 5.2
    starting ns eigenvalue(5,2,1) = 1.0
    starting ns eigenvalue(5,1,2) = 1.0
 &electrons
    conv_thr = 1.d-9
    mixing beta = 0.3
ATOMIC_SPECIES
Fe1 55.845 Fe.pbesol-spn-kjpaw_psl.0.2.1.UPF
Fe2 55.845 Fe.pbesol-spn-kjpaw_psl.0.2.1.UPF
      16.0
             O.pbesol-n-kjpaw psl.0.1.UPF
CELL PARAMETERS {alat}
 0.50 0.50 1.00
 0.50 1.00 0.50
 1.00 0.50 0.50
ATOMIC_POSITIONS {crystal}
 Fe1 0.00 0.00 0.00
 Fe2 0.50 0.50 0.50
      0.25 0.25 0.25
      0.75 0.75 0.75
K_POINTS {automatic}
 3 3 3 0 0 0
```

```
starting_ns_eigenvalue(m,ispin,ityp)
```

In the first iteration of a DFT+*U* run it overwrites the m-th eigenvalue of the ns occupation matrix for the ispin spin-component of atomic species of type ityp.

Why? This is useful to suggest the desired orbital occupations when the default choice takes another path.

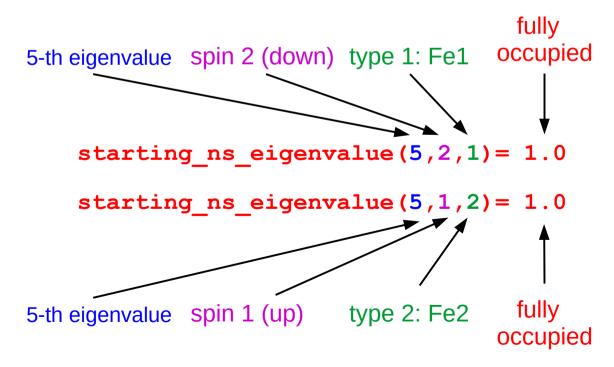
A "hint" for DFT+U where to look for a correct solution

```
&control
    calculation='scf'
    restart mode='from scratch',
    prefix='Fe0'
    pseudo dir = './pseudopotentials/'
    outdir='./tmp/'
 &system
    ibrav = 0.
    celldm(1) = 8.19,
    nat = 4,
    ntvp = 3.
    ecutwfc = 30.0,
    ecutrho = 240.0.
    occupations = 'smearing',
    smearing = 'mv'.
    degauss = 0.02.
    nspin = 2,
    starting_magnetization(1) = 0.5,
    starting magnetization(2) = -0.5
   lda plus u = .true.,
   lda_plus_u_kind = 0,
   U_projection_type = 'atomic',
   Hubbard U(1) = 5.2
    Hubbard U(2) = 5.2
    starting ns eigenvalue(5,2,1) = 1.0
    starting ns eigenvalue(5.1.2) = 1.0
 &electrons
    conv_thr = 1.d-9
    mixing beta = 0.3
ATOMIC SPECIES
Fe1 55.845 Fe.pbesol-spn-kjpaw psl.0.2.1.UPF
Fe2 55.845 Fe.pbesol-spn-kjpaw psl.0.2.1.UPF
            O.pbesol-n-kjpaw psl.0.1.UPF
      16.0
CELL_PARAMETERS {alat}
 0.50 0.50 1.00
 0.50 1.00 0.50
 1.00 0.50 0.50
ATOMIC POSITIONS {crystal}
 Fe1 0.00 0.00
 Fe2 0.50 0.50 0.50
      0.25 0.25 0.25
      0.75 0.75 0.75
K_POINTS {automatic}
 3 3 3 0 0 0
```

```
starting_ns_eigenvalue(m,ispin,ityp)
```

In the first iteration of a DFT+*U* run it overwrites the m-th eigenvalue of the ns occupation matrix for the ispin spin-component of atomic species of type ityp.

Why? This is useful to suggest the desired orbital occupations when the default choice takes another path.



Understanding the setup of starting_ns_eigenvalue

Check the output file FeO.pw.out from the DFT+*U* calculation (without starting_ns_eigenvalue):

After the 1-st iteration we have:

```
Tr[ns(na)] (up, down, total) = 5.00634 1.09448 6.10082
Fe1
       atom
          spin 1
           eigenvalues:
         1.000 1.000 1.002 1.002 1.002
          spin 2
           eigenvalues:
         0.129 0.129 0.270 0.270 0.296
Fe2
                  Tr[ns(na)] (up, down, total) = 1.09429 5.00635 6.10063
       atom
          spin 1
           eigenvalues:
         0.129 0.129 0.270 0.270 0.296
          spin 2
          eigenvalues:
         1.000 1.000 1.002 1.002 1.002
```

Understanding the setup of starting_ns_eigenvalue

Check the output file FeO.pw.out from the DFT+*U* calculation (without starting_ns_eigenvalue):

After the 1-st iteration we have:

```
Fe1
                  Tr[ns(na)] (up, down, total) = 5.00634 1.09448 6.10082
       atom
          spin 1
           eigenvalues:
         1.000 1.000 1.002 1.002 1.002
                                              starting ns eigenvalue (5,2,1) = 1.0
          spin 2
           eigenvalues:
                            0.270 0.296
         0.129 0.129 0.270
Fe2
                  Tr[ns(na)] (up, down, total) = 1.09429 5.00635 6.10063
       atom
          spin 1
           eigenvalues:
         0.129 0.129 0.270 0.270 0.296
          spin 2
                                              starting ns eigenvalue (5,1,2) = 1.0
          eigenvalues:
         1.000 1.000 1.002 1.002 1.002
```

We force/suggest the occupancy of the 5-th state to be 1.0 instead of 0.296!

Why 5-th state? Because it is the one which is non-degenerate and if occupied fully could lead to an insulating result.

Understanding the setup of starting_ns_eigenvalue

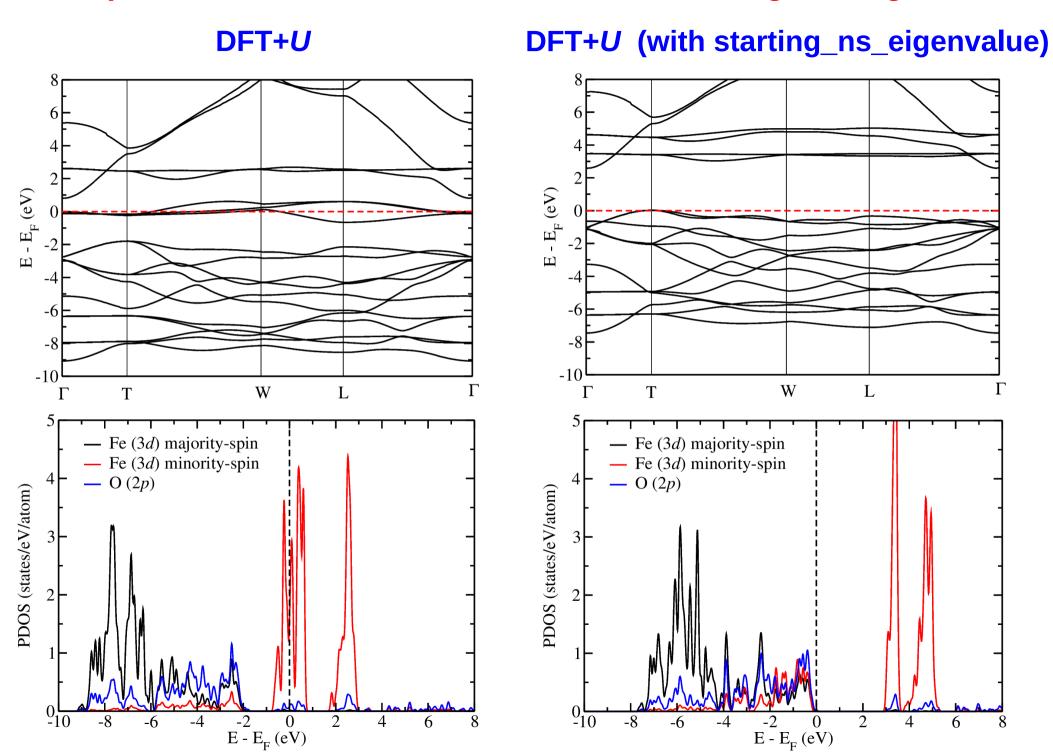
Check the output file FeO.pw.out from the DFT+*U* calculation (with starting_ns_eigenvalue):

After the 1-st iteration (i.e. when we forced the 1.0 occupancy of the 5-th state) we have:

```
Modify starting ns matrices according to input values
Fe1
                Tr[ns(na)] (up. down, total) = 5.00634 1.79838 6.80472
     atom
        spin 1
         eigenvalues:
       1.000 1.000 1.002 1.002 1.002
        spin 2
         eigenvalues:
       0.129 0.129 0.270
                          0.270
                                 1.000
     atom
                Tr[ns(na)] (up, down, total) = 1.79825 5.00635 6.80460
        spin 1
         eigenvalues:
       0.129 0.129 0.270 0.270
                                1.000
        spin 2
         eigenvalues:
       1.000 1.000 1.002 1.002 1.002
```

After this the DFT+*U* calculation converges to an insulating ground state with lower energy than the previous one.

Comparison of DFT+*U* results w/o and w/ starting_ns_eigenvalue



Outline

Exercise 1: Calculation of *U*eff and DFT+*U* study of LiCoO2

Exercise 2: Calculation of Ueff and DFT+U study of FeO

Exercise 3: Calculation of *U*eff and DFT+*U* study of NiO

Exersice 3: NiO

Go to the directory with the input files:

In this directory you will find:

- README
- NiO.pw.in
- NiO.hp.in
- pseudopotentials
- reference
- tmp

- File describ ing how to do the exercise
- Input file for the DFT ground-state calculation
- Input file for the DFPT calculation of Ueff
- Directory with the pseudopotentials
- Directory with the reference results
- Directory for temporary files

Input file NiO.pw1.in

```
&control
    calculation='scf'
    restart mode='from scratch',
    prefix='Ni0'
    pseudo dir = './pseudopotentials/'
    outdir='./tmp/'
 &system
    ibrav = 0.
    celldm(1) = 7.88.
    nat = 4.
    ntyp = 3,
    ecutwfc = 30.0,
    ecutrho = 240.0.
    occupations = 'smearing'.
    smearing = 'mv'.
    degauss = 0.005,
   nspin = 2,
   starting_magnetization(1) =
   starting_magnetization(2) = -0.5
    lda plus u = .true..
    lda plus u kind = 0,
    U projection type = 'atomic',
    Hubbard U(1) = 1.d-8
    Hubbard U(2) = 1.d-8
 &electrons
    conv thr = 1.d-9
    mixing beta = 0.3
ATOMIC SPECIES
Ni1 58.693 ni pbesol v1.4.uspp.F.UPF
Ni2 58.693 ni pbesol v1.4.uspp.F.UPF
            O.pbesol-n-kjpaw psl.0.1.UPF
CELL_PARAMETERS {alat}
 0.50 0.50 1.00
 0.50 1.00 0.50
 1.00 0.50 0.50
ATOMIC POSITIONS {crystal}
 Ni1 0.00 0.00 0.00
 Ni2 0.50 0.50 0.50
      0.25 0.25 0.25
      0.75 0.75 0.75
K POINTS {automatic}
 3 3 3 0 0 0
```

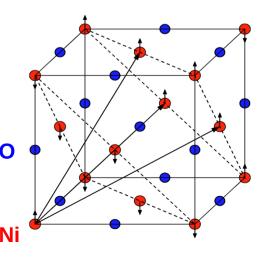
Experimental lattice parameter

'MV' smearing (NiO is insulating @PBEsol) (needed because we want to use starting_magnetization - this is a technical issue)

Spin-polarized (collinear) calculation (antiferromagnetic ordering)

Initialize Hubbard parameters for two types of Ni

Two sublattices of Ni atoms



Input file NiO.pw2.in

```
&control
    calculation='scf'
    restart mode='from scratch'.
    prefix='Ni0'
    pseudo_dir = './pseudopotentials/'
    outdir='./tmp/'
 &system
    ibrav = 0.
    celldm(1) = 7.88.
    nat = 4.
    ntyp = 3,
    ecutwfc = 30.0,
    ecutrho = 240.0.
    occupations = 'fixed'.
    nspin = 2,
    tot magnetization = 0.0
    lda plus u = .true..
    lda plus u kind = 0,
    U_projection_type = 'atomic',
    Hubbard U(1) = 1.d-8
    Hubbard U(2) = 1.d-8
    nbnd = 29
 &electrons
    conv thr = 1.d-9
    mixing_beta = 0.3
    startingpot = 'file'
    startingwfc = 'file'
ATOMIC_SPECIES
Ni1 58.693 ni_pbesol_v1.4.uspp.F.UPF
Ni2 58.693 ni_pbesol_v1.4.uspp.F.UPF
             0.pbesol-n-kjpaw_psl.0.1.UPF
      16.0
CELL PARAMETERS {alat}
 0.50 0.50 1.00
 0.50 1.00 0.50
 1.00 0.50 0.50
ATOMIC_POSITIONS {crystal}
 Ni1 0.00 0.00 0.00
 Ni2 0.50 0.50 0.50
      0.25 0.25 0.25
      0.75 0.75 0.75
K POINTS {automatic}
 3 3 3 0 0 0
```

Now we treat the system as a real insulator (i.e. no "fake" smearing).

Now we use the value of the total magnetization, which we need to extract from the output of the previous calculation. In this case it is zero, because it is antiferromagnetic. But for other systems we may not know what is the value of the total magnetization, so the first step was needed.

Keep the number of bands (states) as in the previous calculation.

Read the density/potential and wavefunctions from the previous converged calculation. This would be a very good start for this iterative calculation.

DFPT for magnetic insulators

The two-step procedure in the ground-state calculation for magnetic insulators is needed to avoid singularity problems in DFPT.

More precisely: in DFPT for metals there is a devision by the density of states at the Fermi level, and the latter quantity is extremely small if we put a fake smearing in an insulator – so this fake smearing must be eliminated once the magnetization of the system is determined.

Task 1

Perform a "one-shot" calculation of *U*eff by following this procedure.

Task 2

Perform convergence tests for U_{eff} and compute it self-consistenly.