Kyber_Estimation_512

April 12, 2024

1 Kyber Estimation

Estimate the drop of bit security caused by multiple information leaked from power side channel.

```
[1]: !ls
     Kyber1024_bikz.pdf
                           Kyber_Estimation.ipynb
                                                             modular_result512.txt
     Kyber1024_qbit.pdf
                           Kyber_Estimation_512.ipynb
                                                             modular_result768.txt
     Kyber512_bikz.pdf
                           Kyber_Estimation_512_768.ipynb
                                                             stdout.txt
                          'Kyber_Estimation_64*3.ipynb'
     Kyber512_qbit.pdf
                                                             untitled.txt
     Kyber768_bikz.pdf
                           PlotDrops.ipynb
                                                             untitled1.txt
     Kyber768_qbit.pdf
                           modular_result1024.txt
    load("../framework/instance_gen.sage")
[3]: build_centered_binomial_law(3)
[3]: \{-3: 0.01562500000000000,
      -2: 0.09375000000000000,
      -1: 0.234375000000000,
      0: 0.312500000000000,
      1: 0.234375000000000,
      2: 0.09375000000000000,
      3: 0.01562500000000000}
[4]: ## LWattackstance initilization
     KYBER_K = 2
      ## default as 768
     ntt_n = 256
     n = KYBER_K *ntt_n
     m = n
     q = 3329
     # D_e = build_centered_binomial?
```

```
bit_security_constant = 0.292
     if KYBER_K ==3 or KYBER_K ==4:
         D_e = build_centered_binomial_law(2)
         # D_e = {-2: 0.0625, -1: 0.25, 0: 0.375, 1: 0.25, 2: 0.0625}
     elif KYBER K == 2 or KYBER K ==1 :
         D_e = build_centered_binomial_law(3)
         # D e = build centered binomial law(3)
         \# D_{\underline{e}} = \{-2:0.093754, -1:0.2343754, 2:0.093754, 1:0.2343754, 0:0.31255, -3: \underline{\Box}\}
      → (1- 0.31255 -0.2343754*2 -0.093754 *2)/2, 3: (1- 0.31255 -0.2343754*2 -0.
      →093754 *2)/2}
     else:
         assert("The input KYBER_N must be in {2,3,4} corresponding to {Kyber_512, □
      9768, 1024")
     D_s = D_e
     A, b, dbdd = initialize_from_LWE_instance(DBDD_predict, n, q, m, D_e, D_s)
     # _ = dbdd.integrate_q_vectors(q, report_every=20)
     beta, delta = dbdd.estimate_attack()
          Build DBDD from LWE
     n=512 m=512 q=3329
           Attack Estimation
     dim=1025
                =1.003945
                                      =405.53
[5]: beta*0.292
[5]: 118.415201129387
[6]: R = IntegerModRing(3329)
     V = VectorSpace(R,ntt n)
     import numpy as np
     def bit_reverse(x):return 2*int( "0b" + bin(x)[2:].rjust(7,'0')[::-1] ,2)+1
     NTT_matrix = []
     def add (x,y) : return x +y
     for x in range(ntt_n/2):
         NTT_matrix.append(V(reduce(add, [[(17)^(x*bit_reverse(i)),0] for i in_
      \rightarrowrange(ntt_n/2)])))
         NTT_matrix.append(V(reduce(add, [[0,(17)^(x*bit_reverse(i))] for i in_
      \rightarrowrange(ntt n/2)])))
```

```
NTT_matrix = matrix(NTT_matrix)
      inv_NTT_matrix = NTT_matrix^-1
 []:
 [7]: s = [[0 for i in range(ntt_n)] for j in range(KYBER_K)]
      for i in range(KYBER_K):
          for j in range(ntt_n):
              v0 = [0 \text{ for i in } range(m + n)]
              v0[i*ntt_n+j]=1
              s[i][j] = dbdd.leak(v0)
 [8]: s?
     Type:
                      [[1, 0, -1, -1, 1, 0, 1, 1, 2, 2, 1, -1, 1, -3, 2, 1, 0, -2, -1, ]
     String form:
      _{\circ}0, -2, 2, 0, -1, 1, -1, 0, 1, 0, <...> 1, -1, 1, 2, 0, 2, 0, -1, 0, 0, -1, 0, 2, _{\sqcup}
      90, -1, 0, -1, 1, -1, -1, -1, 0, -1, -1, 1, -1, -1, -1]
     Length:
     File:
     Docstring:
     Built-in mutable sequence.
     If no argument is given, the constructor creates a new empty list. The
     argument must be an iterable if specified.
     Init docstring: Initialize self. See help(type(self)) for accurate signature.
 [9]: s_hat_list = [V(i)*NTT_matrix for i in s]
[10]: # Hint_list
      s_{relation_list} = [R(s_{hat}[2*i])/R(s_{hat}[2*i+1]) for i in range(ntt_n/2)]_{location_list}

¬for s_hat in s_hat_list]
      v_list = []
      v_list_1 = []
      for j in range(KYBER_K):
          for i in range(ntt_n/2):
              s_relation = s_relation_list[j]
              k = s relation[i]
              v = list((NTT_matrix.column(2*i) - k*NTT_matrix.column(2*i+1)))
              v = [0]*int(ntt_n*j) + v + [0]*int(ntt_n*(KYBER_K-j-1))
              v_{prime} = [int(i) \text{ for } i \text{ in } list(v) + [0]*(m)]
              v_prime= vec(v_prime)
              v_list.append(v_prime)
```

```
v_list_1.append(v)
              # print(dbdd.leak(v_prime)%3329)
[11]: from tqdm import tqdm,tqdm_notebook
[12]: \# for i in tqdm(range(10)):
            sleep(1)
[13]: # import logging
      # import sys
      # import datetime
      # def init_logger(filename, logger_name):
      #
      #
            @brief:
                initialize logger that redirect info to a file just in case we lost
       ⇔connection to the notebook
      #
            Oparams:
      #
                filename: to which file should we log all the info
                logger_name: an alias to the logger
            111
            # get current timestamp
      #
            timestamp = datetime.datetime.utcnow().strftime('%Y%m%d %H-%M-%S')
      #
            logging.basicConfig(
      #
                level=logging.INFO,
                format='[%(asctime)s] %(name)s {%(filename)s:%(lineno)d}__
       →%(levelname)s - %(message)s',
                handlers=[
      #
                    logging.FileHandler(filename=filename),
                    logging.StreamHandler(sys.stdout)
                7
      #
            # Test
      #
            logger = logging.getLogger(logger_name)
            logger.info('### Init. Logger {} ###'.format(logger_name))
            return logger
      # # Initialize
      # my_logger = init_logger("./ml_notebook.log", "ml_logger")
```

```
[14]:  # sys.stdout = open('stdout_512.txt','w')
```

```
for v0 in tqdm(v_list):
    a=dbdd.leak(v0)%3329
    if a!=0:
        assert("a!=0")
    dbdd.integrate_modular_hint(v0,0,3329)
 0%|
| 0/256 [00:00<?, ?it/s]
integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00395149, =404.55
 0%1
| 1/256 [00:04<21:09, 4.98s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00395807, =403.58
 1%|
| 2/256 [00:10<22:26, 5.30s/it]
integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00396465, =402.61
 1%1
| 3/256 [00:14<20:33, 4.88s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00397123, =401.65
 2%1
| 4/256 [00:20<21:31, 5.12s/it]
integrate modular hint (smooth)
                                      Worthy hint !
 dim=1025, =1.00397780, =400.68
 2%1
| 5/256 [00:26<22:15, 5.32s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00398437, =399.72
 2%1
| 6/256 [00:31<22:21, 5.37s/it]
integrate modular hint
                                      Worthy hint!
                         (smooth)
 dim=1025, =1.00399094, =398.77
 3%|
| 7/256 [00:36<21:24, 5.16s/it]
```

```
integrate modular hint
                        (smooth)
                                      Worthy hint!
 dim=1025, =1.00399751, =397.81
 3%1
| 8/256 [00:41<21:25, 5.18s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00400407, =396.86
 4%|
| 9/256 [00:46<20:30, 4.98s/it]
integrate modular hint
                                      Worthy hint!
                         (smooth)
 dim=1025, =1.00401063, =395.92
 4% l
| 10/256 [00:50<20:00, 4.88s/it]
integrate modular hint
                                      Worthy hint!
                         (smooth)
 dim=1025, =1.00401719, =394.98
 4%|
| 11/256 [00:54<19:03, 4.67s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00402494, =394.04
 5%1
| 12/256 [00:59<19:24, 4.77s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00403149, =393.10
 5%|
| 13/256 [01:04<19:18, 4.77s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint !
 dim=1025, =1.00403805, =392.17
 5%1
| 14/256 [01:09<19:06, 4.74s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00404460, =391.24
 6%1
| 15/256 [01:14<19:48, 4.93s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00405114, =390.31
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6%1
| 16/256 [01:19<19:46, 4.94s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00405768, =389.39
 7%|
| 17/256 [01:24<19:44, 4.95s/it]
integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00406422, =388.47
 7%1
| 18/256 [01:31<22:08, 5.58s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00407076, =387.55
 7%|
| 19/256 [01:38<23:27, 5.94s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00407729, =386.64
 8%1
| 20/256 [01:46<25:43, 6.54s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00408383, =385.73
 8%1
| 21/256 [01:58<31:42, 8.09s/it]
integrate modular hint
                        (smooth)
                                      Worthy hint!
 dim=1025, =1.00409035, =384.82
 9%1
| 22/256 [02:07<32:34, 8.35s/it]
integrate modular hint
                                      Worthy hint !
                         (smooth)
 dim=1025, =1.00409688, =383.92
 9%1
| 23/256 [02:14<30:49, 7.94s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint !
 dim=1025, =1.00410463, =383.01
 9%1
| 24/256 [02:19<28:24, 7.35s/it]
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integrate modular hint
                       (smooth)
                                      Worthy hint!
 dim=1025, =1.00411115, =382.12
10%|
| 25/256 [02:24<25:11, 6.54s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00411767, =381.22
10%|
| 26/256 [02:28<22:33, 5.89s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint !
 dim=1025, =1.00412419, =380.33
11%|
| 27/256 [02:33<20:39, 5.41s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00413070, =379.44
11%|
| 28/256 [02:37<18:59, 5.00s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00413721, =378.56
11%|
| 29/256 [02:41<17:48, 4.71s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00414372, =377.67
12%|
| 30/256 [02:45<16:56, 4.50s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint !
 dim=1025, =1.00415022, =376.79
12%|
| 31/256 [02:52<20:10, 5.38s/it]
integrate modular hint
                                      Worthy hint!
                         (smooth)
 dim=1025, =1.00415672, =375.92
12%|
| 32/256 [02:59<21:52, 5.86s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00416447, =375.04
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```
13%|
| 33/256 [03:03<19:55, 5.36s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00417097, =374.17
13%|
| 34/256 [03:08<18:45, 5.07s/it]
integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00417746, =373.31
14%|
| 35/256 [03:12<17:40, 4.80s/it]
integrate modular hint
                        (smooth)
                                      Worthy hint!
 dim=1025, =1.00418395, =372.44
14%|
| 36/256 [03:24<25:35, 6.98s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint !
 dim=1025, =1.00419044, =371.58
14%|
| 37/256 [03:30<24:19, 6.66s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00419692, =370.72
15% l
| 38/256 [03:36<22:55, 6.31s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00420340, =369.86
15%|
| 39/256 [03:40<20:43, 5.73s/it]
integrate modular hint
                                      Worthy hint!
                         (smooth)
 dim=1025, =1.00421116, =369.01
16%|
| 40/256 [03:45<20:26, 5.68s/it]
                         (smooth)
integrate modular hint
                                      Worthy hint !
 dim=1025, =1.00421763, =368.16
16%|
| 41/256 [03:50<18:35, 5.19s/it]
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integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00422411, =367.31
16%|
| 42/256 [03:54<17:54, 5.02s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00423058, =366.47
17%|
| 43/256 [04:04<22:32, 6.35s/it]
                                      Worthy hint!
integrate modular hint
                         (smooth)
 dim=1025, =1.00423705, =365.63
17%|
| 44/256 [04:23<36:03, 10.21s/it]
integrate modular hint
                                      Worthy hint!
                         (smooth)
 dim=1025, =1.00424351, =364.79
18%|
| 45/256 [04:28<30:30, 8.68s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00424997, =363.95
18%|
| 46/256 [04:32<26:03, 7.44s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00425772, =363.12
18%|
| 47/256 [04:37<22:53, 6.57s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint !
 dim=1025, =1.00426418, =362.29
19%|
| 48/256 [04:42<20:44, 5.98s/it]
integrate modular hint
                                      Worthy hint!
                         (smooth)
 dim=1025, =1.00427063, =361.46
19%|
| 49/256 [04:47<19:37, 5.69s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00427708, =360.64
```

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20%1
| 50/256 [04:51<18:05, 5.27s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00428353, =359.82
20%1
| 51/256 [04:56<17:18, 5.07s/it]
integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00428998, =359.00
20%1
| 52/256 [05:00<16:40, 4.90s/it]
integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00429773, =358.18
21%|
| 53/256 [05:04<15:53, 4.70s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00430417, =357.37
21%|
| 54/256 [05:08<15:03, 4.47s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00431061, =356.55
21%|
| 55/256 [05:13<15:19, 4.57s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00431704, =355.75
22%|
| 56/256 [05:17<14:45, 4.43s/it]
integrate modular hint
                                      Worthy hint!
                         (smooth)
 dim=1025, =1.00432347, =354.94
22%1
| 57/256 [05:22<14:48, 4.47s/it]
                         (smooth)
integrate modular hint
                                      Worthy hint !
 dim=1025, =1.00433122, =354.14
23%1
| 58/256 [05:26<14:31, 4.40s/it]
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integrate modular hint
                        (smooth)
                                      Worthy hint!
 dim=1025, =1.00433765, =353.34
23%1
| 59/256 [05:30<13:57, 4.25s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00434407, =352.54
23%|
| 60/256 [05:34<14:09, 4.33s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint !
 dim=1025, =1.00435049, =351.74
24%1
| 61/256 [05:39<13:58, 4.30s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00435690, =350.95
24%|
| 62/256 [05:44<15:02, 4.65s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00436466, =350.16
25%1
| 63/256 [05:49<15:00, 4.67s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00437107, =349.37
25%|
| 64/256 [05:53<14:52, 4.65s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint !
 dim=1025, =1.00437748, =348.59
25%1
| 65/256 [05:57<14:07, 4.44s/it]
integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00438388, =347.81
26%1
| 66/256 [06:02<14:04, 4.45s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00439163, =347.03
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```
26%1
| 67/256 [06:06<13:59, 4.44s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00439804, =346.25
27%|
| 68/256 [06:11<14:03, 4.49s/it]
integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00440443, =345.47
27%1
| 69/256 [06:15<13:36, 4.37s/it]
integrate modular hint (smooth)
                                      Worthy hint !
 dim=1025, =1.00441083, =344.70
27%|
| 70/256 [06:19<13:36, 4.39s/it]
integrate modular hint
                                      Worthy hint !
                         (smooth)
 dim=1025, =1.00441721, =343.93
28%1
| 71/256 [06:25<14:24, 4.67s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00442497, =343.17
28%1
| 72/256 [06:29<14:06, 4.60s/it]
integrate modular hint
                        (smooth)
                                      Worthy hint!
 dim=1025, =1.00443135, =342.40
29%1
| 73/256 [06:34<14:02, 4.60s/it]
integrate modular hint
                                      Worthy hint !
                         (smooth)
 dim=1025, =1.00443774, =341.64
29%1
| 74/256 [06:39<14:15, 4.70s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint !
 dim=1025, =1.00444411, =340.88
29%1
| 75/256 [06:44<14:44, 4.89s/it]
```

```
integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00445187, =340.12
30%1
| 76/256 [06:48<14:10, 4.73s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00445824, =339.37
30%|
| 77/256 [06:53<14:18, 4.79s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00446461, =338.62
30%1
| 78/256 [06:58<14:21, 4.84s/it]
integrate modular hint
                                      Worthy hint!
                         (smooth)
 dim=1025, =1.00447098, =337.86
31%|
| 79/256 [07:03<14:10, 4.81s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00447873, =337.12
31%|
| 80/256 [07:07<13:47, 4.70s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00448510, =336.37
32%|
| 81/256 [07:12<14:04, 4.83s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint !
 dim=1025, =1.00449145, =335.63
32%|
| 82/256 [07:17<13:57, 4.81s/it]
integrate modular hint
                                      Worthy hint!
                         (smooth)
 dim=1025, =1.00449781, =334.89
32%1
| 83/256 [07:22<13:44, 4.77s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00450556, =334.15
```

```
33%1
| 84/256 [07:26<13:30, 4.71s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00451191, =333.42
33%|
| 85/256 [07:31<13:24, 4.70s/it]
                                      Worthy hint!
integrate modular hint (smooth)
 dim=1025, =1.00451826, =332.68
34%1
| 86/256 [07:37<14:11, 5.01s/it]
integrate modular hint
                        (smooth)
                                      Worthy hint!
 dim=1025, =1.00452460, =331.95
34%|
| 87/256 [07:42<13:52, 4.93s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00453236, =331.22
34%|
| 88/256 [07:46<13:30, 4.82s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00453870, =330.50
35%1
| 89/256 [07:51<13:36, 4.89s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00454503, =329.77
35%|
90/256 [07:56<13:31, 4.89s/it]
integrate modular hint
                                      Worthy hint!
                         (smooth)
 dim=1025, =1.00455278, =329.05
36%1
| 91/256 [08:03<15:18, 5.57s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint !
 dim=1025, =1.00455912, =328.33
36%1
| 92/256 [08:08<14:25, 5.28s/it]
```

```
integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00456544, =327.62
36%|
| 93/256 [08:12<13:37, 5.02s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00457176, =326.90
37%|
| 94/256 [08:17<13:32, 5.01s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00457952, =326.19
37%1
| 95/256 [08:23<13:38, 5.08s/it]
integrate modular hint
                                      Worthy hint!
                         (smooth)
 dim=1025, =1.00458584, =325.48
38%|
| 96/256 [08:29<14:19, 5.37s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00459215, =324.77
38%|
| 97/256 [08:34<13:57, 5.27s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00459991, =324.06
38%|
| 98/256 [08:38<13:19, 5.06s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint !
 dim=1025, =1.00460621, =323.36
39%1
| 99/256 [08:43<13:05, 5.00s/it]
integrate modular hint
                                      Worthy hint!
                         (smooth)
 dim=1025, =1.00461252, =322.66
39%1
| 100/256 [08:49<13:49, 5.32s/it]
integrate modular hint
                        (smooth)
                                      Worthy hint!
 dim=1025, =1.00461882, =321.96
```

```
39%1
| 101/256 [08:55<14:15, 5.52s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00462658, =321.26
40%1
| 102/256 [09:04<16:33, 6.45s/it]
integrate modular hint
                        (smooth)
                                      Worthy hint!
 dim=1025, =1.00463287, =320.57
40%1
| 103/256 [09:09<15:33, 6.10s/it]
integrate modular hint
                        (smooth)
                                      Worthy hint!
 dim=1025, =1.00463916, =319.87
41%|
| 104/256 [09:14<14:38, 5.78s/it]
integrate modular hint
                                      Worthy hint !
                         (smooth)
 dim=1025, =1.00464692, =319.18
41%|
| 105/256 [09:20<14:26, 5.74s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00465321, =318.50
41%|
| 106/256 [09:26<14:32, 5.82s/it]
integrate modular hint
                        (smooth)
                                      Worthy hint!
 dim=1025, =1.00465949, =317.81
42%|
| 107/256 [09:31<14:15, 5.74s/it]
integrate modular hint
                                      Worthy hint !
                        (smooth)
 dim=1025, =1.00466725, =317.13
42%1
| 108/256 [09:38<14:59, 6.08s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint !
 dim=1025, =1.00467353, =316.44
43%1
| 109/256 [09:43<13:54, 5.67s/it]
```

```
integrate modular hint
                        (smooth)
                                      Worthy hint!
 dim=1025, =1.00467980, =315.76
43%|
| 110/256 [09:47<12:57, 5.33s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00468756, =315.09
43%|
| 111/256 [09:52<12:27, 5.16s/it]
integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00469383, =314.41
44%1
| 112/256 [09:56<11:47, 4.91s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00470009, =313.74
44%|
| 113/256 [10:01<11:34, 4.86s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00470785, =313.06
45%|
| 114/256 [10:06<11:34, 4.89s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00471411, =312.40
45%1
| 115/256 [10:11<11:38, 4.96s/it]
integrate modular hint
                        (smooth)
                                      Worthy hint !
 dim=1025, =1.00472037, =311.73
45%|
| 116/256 [10:16<11:26, 4.90s/it]
integrate modular hint
                                      Worthy hint!
                         (smooth)
 dim=1025, =1.00472812, =311.06
46%1
| 117/256 [10:21<11:16, 4.87s/it]
integrate modular hint
                        (smooth)
                                      Worthy hint!
 dim=1025, =1.00473437, =310.40
```

```
46%1
| 118/256 [10:27<12:00, 5.22s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00474062, =309.74
46%1
| 119/256 [10:32<11:46, 5.15s/it]
integrate modular hint
                        (smooth)
                                      Worthy hint!
 dim=1025, =1.00474838, =309.08
47%|
| 120/256 [10:39<13:12, 5.83s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00475462, =308.42
47%|
| 121/256 [10:46<13:29, 6.00s/it]
integrate modular hint
                                      Worthy hint!
                         (smooth)
 dim=1025, =1.00476086, =307.77
48%|
| 122/256 [10:50<12:18, 5.51s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00476862, =307.11
48%1
| 123/256 [10:55<11:42, 5.28s/it]
integrate modular hint
                        (smooth)
                                      Worthy hint!
 dim=1025, =1.00477485, =306.46
48%|
| 124/256 [10:59<10:54, 4.96s/it]
integrate modular hint
                                      Worthy hint!
                         (smooth)
 dim=1025, =1.00478108, =305.81
49%1
| 125/256 [11:03<10:20, 4.74s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint !
 dim=1025, =1.00478883, =305.17
49%1
| 126/256 [11:09<10:58, 5.07s/it]
```

```
integrate modular hint
                        (smooth)
                                      Worthy hint!
 dim=1025, =1.00479506, =304.52
50%|
| 127/256 [11:15<11:21, 5.29s/it]
integrate modular hint
                        (smooth)
                                      Worthy hint!
 dim=1025, =1.00480128, =303.88
50%|
| 128/256 [11:20<11:03, 5.18s/it]
integrate modular hint
                       (smooth)
                                      Worthy hint!
 dim=1025, =1.00480903, =303.24
50%1
| 129/256 [11:24<10:36, 5.02s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00481525, =302.60
51%|
| 130/256 [11:33<12:55, 6.15s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00482146, =301.96
51%|
| 131/256 [11:48<18:08, 8.71s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00482922, =301.32
52%|
| 132/256 [11:58<18:56, 9.17s/it]
integrate modular hint
                       (smooth)
                                      Worthy hint !
 dim=1025, =1.00483542, =300.69
52%|
| 133/256 [12:03<16:11, 7.90s/it]
integrate modular hint
                                      Worthy hint!
                         (smooth)
 dim=1025, =1.00484318, =300.06
52%1
| 134/256 [12:08<14:01, 6.89s/it]
integrate modular hint
                        (smooth)
                                      Worthy hint!
 dim=1025, =1.00484938, =299.43
```

```
53%|
| 135/256 [12:11<12:00, 5.95s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00485557, =298.80
53%|
| 136/256 [12:16<11:06, 5.55s/it]
integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00486333, =298.18
54%|
| 137/256 [12:21<10:55, 5.51s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00486952, =297.55
54%|
| 138/256 [12:30<12:48, 6.51s/it]
integrate modular hint
                                      Worthy hint !
                         (smooth)
 dim=1025, =1.00487570, =296.93
54%|
| 139/256 [12:50<20:36, 10.57s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00488346, =296.31
55% l
| 140/256 [12:54<16:36, 8.59s/it]
integrate modular hint
                        (smooth)
                                      Worthy hint!
 dim=1025, =1.00488964, =295.69
55%|
| 141/256 [12:59<14:28, 7.56s/it]
integrate modular hint
                                      Worthy hint!
                         (smooth)
 dim=1025, =1.00489740, =295.07
55%|
| 142/256 [13:03<12:09, 6.40s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint !
 dim=1025, =1.00490357, =294.46
56%1
| 143/256 [13:07<10:40, 5.67s/it]
```

```
integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00490974, =293.85
56%|
| 144/256 [13:12<09:53, 5.30s/it]
integrate modular hint
                       (smooth)
                                      Worthy hint!
 dim=1025, =1.00491750, =293.23
57%|
| 145/256 [13:15<08:53, 4.80s/it]
integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00492367, =292.63
57%1
| 146/256 [13:20<08:35, 4.69s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00493142, =292.02
57%|
| 147/256 [13:25<08:43, 4.80s/it]
integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00493758, =291.41
58%|
| 148/256 [13:30<08:54, 4.95s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00494374, =290.81
58%|
| 149/256 [13:36<09:10, 5.14s/it]
integrate modular hint
                       (smooth)
                                      Worthy hint !
 dim=1025, =1.00495149, =290.21
59%|
| 150/256 [13:41<09:07, 5.16s/it]
integrate modular hint
                                      Worthy hint!
                         (smooth)
 dim=1025, =1.00495764, =289.61
59%1
| 151/256 [13:49<10:25, 5.96s/it]
integrate modular hint
                       (smooth)
                                      Worthy hint!
 dim=1025, =1.00496540, =289.01
```

```
59%|
| 152/256 [13:55<10:31, 6.08s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00497154, =288.41
60%1
| 153/256 [13:59<09:22, 5.46s/it]
integrate modular hint
                        (smooth)
                                      Worthy hint!
 dim=1025, =1.00497768, =287.82
60% l
| 154/256 [14:04<09:17, 5.46s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00498544, =287.22
61%|
| 155/256 [14:09<08:49, 5.24s/it]
integrate modular hint
                                      Worthy hint!
                         (smooth)
 dim=1025, =1.00499157, =286.63
61%|
| 156/256 [14:16<09:47, 5.88s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00499933, =286.04
61%|
| 157/256 [14:27<11:59, 7.27s/it]
integrate modular hint
                        (smooth)
                                      Worthy hint!
 dim=1025, =1.00500546, =285.45
62%|
| 158/256 [14:31<10:26, 6.39s/it]
integrate modular hint
                                      Worthy hint !
                        (smooth)
 dim=1025, =1.00501158, =284.87
62%1
| 159/256 [14:36<09:26, 5.84s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint !
 dim=1025, =1.00501934, =284.28
62%1
| 160/256 [14:41<09:10, 5.73s/it]
```

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integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00502546, =283.70
63%1
| 161/256 [14:52<11:12, 7.08s/it]
integrate modular hint
                         (smooth)
                                      Worthy hint!
 dim=1025, =1.00503322, =283.12
 63%|
                                            | 162/256 [15:00<11:51, 7.57s/it]
 integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00503933, =282.54
 64%|
                                            | 163/256 [15:07<11:32, 7.44s/it]
integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00504543, =281.96
 64%|
                                           | 164/256 [15:12<10:11, 6.65s/it]
integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00505319, =281.38
 64%|
                                           | 165/256 [15:16<08:53, 5.86s/it]
 integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00505929, =280.81
 65%1
                                           | 166/256 [15:21<08:11, 5.46s/it]
 integrate modular hint (smooth)
                                      Worthy hint !
 dim=1025, =1.00506705, =280.24
 65%1
                                           | 167/256 [15:25<07:43, 5.21s/it]
integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00507314, =279.66
 66%1
                                          | 168/256 [15:30<07:20, 5.00s/it]
 integrate modular hint (smooth)
                                      Worthy hint!
 dim=1025, =1.00508090, =279.09
```

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66%1
                                          | 169/256 [15:34<06:46, 4.67s/it]
integrate modular hint (smooth)
                                     Worthy hint!
dim=1025, =1.00508699, =278.53
66%1
                                         | 170/256 [15:38<06:36, 4.61s/it]
integrate modular hint (smooth)
                                     Worthy hint!
dim=1025, =1.00509308, =277.96
67%1
                                         | 171/256 [15:43<06:38, 4.69s/it]
integrate modular hint (smooth)
                                     Worthy hint!
dim=1025, =1.00510083, =277.40
67%1
                                         | 172/256 [15:48<06:26, 4.60s/it]
integrate modular hint (smooth)
                                     Worthy hint!
dim=1025, =1.00510691, =276.83
68%1
                                        | 173/256 [15:52<06:16, 4.54s/it]
integrate modular hint (smooth)
                                     Worthy hint!
dim=1025, =1.00511467, =276.27
68%1
                                        | 174/256 [15:57<06:31, 4.78s/it]
integrate modular hint (smooth)
                                     Worthy hint!
dim=1025, =1.00512074, =275.71
68%1
                                        | 175/256 [16:02<06:30, 4.82s/it]
integrate modular hint (smooth)
                                     Worthy hint!
dim=1025, =1.00512850, =275.15
69%1
                                        | 176/256 [16:07<06:25, 4.82s/it]
integrate modular hint (smooth)
                                     Worthy hint!
dim=1025, =1.00513457, =274.60
69%1
                                        | 177/256 [16:15<07:27, 5.67s/it]
```

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integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00514233, =274.04
70%|
                                       | 178/256 [16:20<07:09, 5.51s/it]
integrate modular hint (smooth)
                                     Worthy hint!
dim=1025, =1.00514839, =273.49
70%|
                                       | 179/256 [16:24<06:39, 5.19s/it]
integrate modular hint (smooth)
                                     Worthy hint!
dim=1025, =1.00515444, =272.94
70%1
                                       | 180/256 [16:29<06:28, 5.11s/it]
integrate modular hint (smooth)
                                     Worthy hint!
dim=1025, =1.00516220, =272.39
71%|
                                       | 181/256 [16:33<05:58, 4.78s/it]
integrate modular hint (smooth)
                                     Worthy hint!
dim=1025, =1.00516825, =271.84
71%|
                                       | 182/256 [16:38<05:44, 4.65s/it]
integrate modular hint (smooth)
                                     Worthy hint!
dim=1025, =1.00517601, =271.29
71%|
                                      | 183/256 [16:42<05:27, 4.48s/it]
integrate modular hint (smooth)
                                     Worthy hint !
dim=1025, =1.00518205, =270.75
72%|
                                      | 184/256 [16:46<05:29, 4.58s/it]
integrate modular hint (smooth)
                                     Worthy hint!
dim=1025, =1.00518981, =270.20
72%1
                                      | 185/256 [16:51<05:19, 4.50s/it]
integrate modular hint (smooth)
                                     Worthy hint!
dim=1025, =1.00519585, =269.66
```

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73%|
                                      | 186/256 [16:55<05:00, 4.30s/it]
integrate modular hint
                      (smooth)
                                     Worthy hint!
dim=1025, =1.00520361, =269.12
73%|
                                     | 187/256 [16:59<04:54, 4.27s/it]
integrate modular hint (smooth)
                                     Worthy hint!
dim=1025, =1.00520964, =268.58
73%1
                                     | 188/256 [17:03<04:39, 4.11s/it]
integrate modular hint (smooth)
                                     Worthy hint!
dim=1025, =1.00521740, =268.04
74%|
                                     | 189/256 [17:12<06:13, 5.58s/it]
integrate modular hint (smooth)
                                     Worthy hint!
dim=1025, =1.00522342, =267.51
74%|
                                     | 190/256 [17:18<06:30, 5.91s/it]
integrate modular hint (smooth)
                                     Worthy hint!
dim=1025, =1.00522944, =266.97
75%1
                                    | 191/256 [17:22<05:44, 5.30s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00523720, =266.44
75%|
                                    | 192/256 [17:26<05:21, 5.02s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00524321, =265.91
75%|
                                    | 193/256 [17:31<05:15, 5.00s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00525097, =265.38
76%1
                                    | 194/256 [17:36<04:53, 4.73s/it]
```

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integrate modular hint (smooth)
                                 Worthy hint!
dim=1025, =1.00525698, =264.85
76%|
                                   | 195/256 [17:41<04:59, 4.90s/it]
integrate modular hint (smooth)
                                   Worthy hint!
dim=1025, =1.00526474, =264.32
77%|
                                   | 196/256 [17:46<04:54, 4.90s/it]
integrate modular hint (smooth)
                                   Worthy hint !
dim=1025, =1.00527073, =263.79
77%1
                                   | 197/256 [17:52<05:10, 5.26s/it]
integrate modular hint (smooth)
                                   Worthy hint!
dim=1025, =1.00527850, =263.27
77%|
                                   | 198/256 [17:56<04:41, 4.85s/it]
                                   Worthy hint!
integrate modular hint (smooth)
dim=1025, =1.00528449, =262.75
78%1
                                  | 199/256 [18:00<04:28, 4.71s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00529225, =262.23
78%|
                                  | 200/256 [18:04<04:08, 4.44s/it]
integrate modular hint (smooth)
                                 Worthy hint!
dim=1025, =1.00529824, =261.71
79%|
                                  | 201/256 [18:08<04:01, 4.40s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00530600, =261.19
79%1
                                  | 202/256 [18:12<03:46, 4.20s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00531198, =260.67
```

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79%1
                                 | 203/256 [18:16<03:43, 4.22s/it]
integrate modular hint (smooth)
                                  Worthy hint!
dim=1025, =1.00531974, =260.15
80%1
                                 | 204/256 [18:20<03:39, 4.22s/it]
integrate modular hint (smooth)
                                  Worthy hint!
dim=1025, =1.00532571, =259.64
80%1
                                 | 205/256 [18:24<03:28, 4.09s/it]
integrate modular hint (smooth)
                                  Worthy hint!
dim=1025, =1.00533347, =259.13
80%1
                                | 206/256 [18:28<03:26, 4.13s/it]
integrate modular hint (smooth)
                                  Worthy hint!
dim=1025, =1.00533944, =258.62
81%|
                                | 207/256 [18:32<03:16, 4.01s/it]
integrate modular hint (smooth)
                                  Worthy hint !
dim=1025, =1.00534720, =258.11
81% l
                                | 208/256 [18:36<03:15, 4.08s/it]
integrate modular hint (smooth)
                                Worthy hint!
dim=1025, =1.00535316, =257.60
82%1
                                | 209/256 [18:40<03:09, 4.03s/it]
integrate modular hint (smooth)
                                  Worthy hint!
dim=1025, =1.00536092, =257.09
82%1
                                | 210/256 [18:45<03:12, 4.20s/it]
integrate modular hint (smooth)
                                  Worthy hint!
dim=1025, =1.00536687, =256.59
82%1
                                | 211/256 [18:49<03:10, 4.23s/it]
```

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integrate modular hint (smooth)
                                Worthy hint !
dim=1025, =1.00537464, =256.08
83%|
                               | 212/256 [18:54<03:08, 4.28s/it]
integrate modular hint (smooth)
                                  Worthy hint!
dim=1025, =1.00538058, =255.58
83%|
                               | 213/256 [18:57<02:56, 4.11s/it]
integrate modular hint (smooth)
                                  Worthy hint!
dim=1025, =1.00538834, =255.08
84%1
                              | 214/256 [19:01<02:47, 4.00s/it]
integrate modular hint (smooth)
                                  Worthy hint!
dim=1025, =1.00539428, =254.58
84%|
                              | 215/256 [19:07<03:08, 4.61s/it]
integrate modular hint (smooth) Worthy hint!
dim=1025, =1.00540205, =254.08
84%1
                              | 216/256 [19:12<03:05, 4.63s/it]
integrate modular hint (smooth)
                                   Worthy hint!
dim=1025, =1.00540798, =253.58
85%|
                              | 217/256 [19:16<02:56, 4.53s/it]
integrate modular hint (smooth) Worthy hint!
dim=1025, =1.00541574, =253.08
85%|
                              | 218/256 [19:20<02:49, 4.47s/it]
integrate modular hint (smooth)
                                Worthy hint!
dim=1025, =1.00542167, =252.59
86%1
                             | 219/256 [19:24<02:37, 4.25s/it]
integrate modular hint (smooth)
                                   Worthy hint!
dim=1025, =1.00542943, =252.10
```

```
86%1
                              | 220/256 [19:28<02:34, 4.28s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00543535, =251.60
86%1
                             | 221/256 [19:32<02:24, 4.13s/it]
integrate modular hint (smooth)
                                  Worthy hint!
dim=1025, =1.00544311, =251.11
87%|
                             | 222/256 [19:36<02:21, 4.17s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00544902, =250.62
87%|
                             | 223/256 [19:40<02:14, 4.06s/it]
integrate modular hint (smooth)
                                  Worthy hint !
dim=1025, =1.00545679, =250.14
88%1
                             | 224/256 [19:44<02:10, 4.07s/it]
integrate modular hint (smooth)
                                 Worthy hint !
dim=1025, =1.00546269, =249.65
88%1
                             | 225/256 [19:49<02:10, 4.20s/it]
integrate modular hint (smooth)
                                 Worthy hint !
dim=1025, =1.00547045, =249.16
88%1
                             | 226/256 [19:53<02:06, 4.23s/it]
integrate modular hint (smooth)
                                 Worthy hint !
dim=1025, =1.00547635, =248.68
89%1
                            | 227/256 [19:58<02:04, 4.30s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00548411, =248.20
89%1
                            | 228/256 [20:02<02:02, 4.37s/it]
```

```
integrate modular hint (smooth)
                                 Worthy hint !
dim=1025, =1.00549001, =247.72
89%|
                            | 229/256 [20:06<01:54, 4.25s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00549777, =247.24
90%|
                            | 230/256 [20:10<01:48, 4.16s/it]
integrate modular hint (smooth)
                                   Worthy hint!
dim=1025, =1.00550365, =246.76
90%1
                           | 231/256 [20:15<01:45, 4.23s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00551142, =246.28
91%|
                           | 232/256 [20:19<01:41, 4.22s/it]
integrate modular hint (smooth)
                                   Worthy hint!
dim=1025, =1.00551729, =245.80
91%|
                           | 233/256 [20:23<01:38, 4.30s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00552506, =245.33
91%|
                           | 234/256 [20:27<01:31, 4.18s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00553093, =244.86
92%1
                           | 235/256 [20:31<01:28, 4.21s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00553869, =244.38
92%1
                          | 236/256 [20:35<01:21, 4.06s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00554455, =243.91
```

```
93%1
                           | 237/256 [20:39<01:15, 3.98s/it]
integrate modular hint
                        (smooth)
                                    Worthy hint!
dim=1025, =1.00555231, =243.44
93%1
                          | 238/256 [20:43<01:13, 4.08s/it]
integrate modular hint (smooth)
                                   Worthy hint!
dim=1025, =1.00555817, =242.97
93%|
                          | 239/256 [20:47<01:07, 3.98s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00556593, =242.51
94%|
                          | 240/256 [20:51<01:05, 4.08s/it]
integrate modular hint (smooth)
                                   Worthy hint!
dim=1025, =1.00557369, =242.04
94%|
                          | 241/256 [20:55<00:59, 3.99s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00557954, =241.58
95%1
                         | 242/256 [21:00<01:01, 4.43s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00558731, =241.11
95%|
                         | 243/256 [21:05<00:58, 4.47s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00559315, =240.65
95%|
                        | 244/256 [21:10<00:54, 4.53s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00560091, =240.19
96%1
                         | 245/256 [21:14<00:47, 4.32s/it]
```

```
integrate modular hint (smooth)
                                 Worthy hint!
dim=1025, =1.00560674, =239.73
96%|
                        | 246/256 [21:18<00:43, 4.36s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00561450, =239.27
96%|
                        | 247/256 [21:22<00:37, 4.20s/it]
integrate modular hint (smooth)
                                   Worthy hint !
dim=1025, =1.00562033, =238.81
97%1
                        | 248/256 [21:26<00:34, 4.26s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00562809, =238.36
97%|
                        | 249/256 [21:31<00:30, 4.37s/it]
integrate modular hint
                       (smooth)
                                   Worthy hint!
dim=1025, =1.00563391, =237.90
98%|
                       | 250/256 [21:35<00:25, 4.20s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00564167, =237.45
98%1
                       | 251/256 [21:38<00:20, 4.07s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00564748, =237.00
98%1
                       | 252/256 [21:43<00:16, 4.21s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00565525, =236.55
99%1
                       | 253/256 [21:47<00:12, 4.20s/it]
integrate modular hint (smooth)
                                    Worthy hint!
dim=1025, =1.00566301, =236.10
```

```
99%1
                             | 254/256 [21:51<00:08, 4.17s/it]
      integrate modular hint (smooth)
                                           Worthy hint!
       dim=1025, =1.00566881, =235.65
     100%|
                             | 255/256 [21:56<00:04, 4.38s/it]
      integrate modular hint (smooth)
                                         Worthy hint !
      dim=1025, =1.00567658, =235.20
     100%|
                             | 256/256 [22:00<00:00, 5.16s/it]
[15]: beta_1, delta_1 = dbdd.estimate_attack()
            Attack Estimation
      dim=1025
               =1.005677
                                    =235.20
[19]: beta_1*0.292
[19]: 68.6780061557604
[16]: # dbdd
[17]: # dbdd.float_type="qd"
     # # dbdd.float_type="mpfr"
[18]: # dbdd.float_type="dd"
     # dbdd.float_type="dd"
     # res = dbdd.attack(beta pre=90)
 []:
```