NTTRU_Estimation

April 12, 2024

1 NTTRU Estimation

Estimate the drop of bit security caused by multiple information leaked from power side channel.

```
[4]: load("../framework/instance_gen.sage")

[ ]: ## NTRU instance initilization

n = 768
m = n
q = 7681

# D_e = {-1: 5/16, 0:6/16, 1:5/16}

# D_s = D_e
# number of one in f and g In NTTRU, the distribution of -1,0,1 is 5/16, 6/
-16,6/16

Dg = int(round(6*n/16))
Df = Dg
bit_security_constant = 0.292

A, b, dbdd = initialize_NTRU_instance(DBDD, n, q, Df, Dg, verbosity=0)
# A, b, dbdd = initialize_from_LWE_instance(DBDD_predict, n, q, m, D_e,D_s)
```

hello

```
[17]: | ## example s_hat
```

```
s_{hat} = [4516, 2276, 4914, 5522, 2472, 5117, 7189, 4818, 5814, 1201, 1768]
  47572, 3018, 5671, 2316, 996, 6907, 6410, 2334, 3437, 7231, 4926, 6416, 1720, u
   →7430, 2082, 1105, 2672, 1596, 3064, 2279, 6081, 695, 5503, 3467, 5420, 3861, □
   45016, 2590, 6879, 3688, 4971, 3467, 7062, 3762, 3535, 7002, 559, 440, 6840, u
  47548, 4458, 4500, 5452, 6788, 7424, 4462, 1650, 4791, 7390, 2845, 6256, u
  →6006, 3723, 7136, 6350, 1561, 6260, 3051, 86, 5942, 721, 6415, 5482, 868, U
   42459, 1561, 3848, 7522, 6214, 2874, 6307, 3801, 16, 4851, 5708, 5874, 688, u
   41082, 6398, 5252, 3344, 5333, 4710, 900, 4981, 7255, 6223, 1488, 5212, 5951, L
   41789, 7208, 1478, 6567, 3085, 7176, 834, 102, 5594, 1980, 6670, 4149, 1332, u
  4888, 2318, 2330, 4196, 3930, 3250, 4589, 108, 2761, 6388, 6578, 2312, 1610, u
  4943, 7320, 7451, 4167, 4161, 2832, 5735, 7086, 7393, 2624, 1840, 7478, 4592, 1
  4280, 464, 6395, 5864, 3626, 2199, 3651, 3268, 6670, 3312, 1430, 5538, 5984, u
  46753, 4129, 727, 4918, 5661, 6042, 2498, 4212, 1973, 2079, 4977, 2387, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 4547, 
   47605, 163, 6628, 6136, 2867, 1456, 769, 4996, 686, 1200, 4761, 1931, 477, u
  4766, 894, 5169, 5875, 3919, 4983, 363, 128, 2287, 4073, 2398, 4901, 4766, u
  →5572, 6981, 5990, 4181, 6995, 893, 5123, 3268, 4434, 3012, 1704, 712, 537, ⊔
   →1003, 2100, 7539, 4758, 1817, 7385, 612, 6885, 2377, 5414, 3853, 6143, 3117, □
   41638, 4684, 607, 4627, 745, 4854, 4869, 183, 586, 1276, 3051, 4086, 4343, ∪
   42905, 1035, 3329, 1936, 5084, 272, 4985, 31, 253, 4550, 907, 1079, 546, u
   45452, 6725, 673, 2905, 1424, 1601, 6047, 2994, 6777, 2771, 1097, 323, 2046, u
   47311, 2540, 7169, 682, 6265, 6599, 3487, 6608, 5150, 7447, 4125, 1270, 2353, L
   4342, 6965, 4290, 1010, 4200, 3542, 3453, 4575, 3976, 6033, 741, 4041, 6034, L
  42553, 4441, 3423, 4379, 7370, 2147, 4243, 5072, 94, 42, 4717, 2101, 3, 6901, u
  41147, 3461, 3229, 1776, 6254, 4576, 7095, 7408, 4630, 3151, 5063, 3518, u
  41765, 764, 341, 6200, 5867, 3430, 7555, 3497, 7305, 5325, 5274, 5679, 472, u
  41501, 5398, 3589, 6722, 7130, 6049, 2782, 5222, 723, 7464, 38, 4328, 6008, U
  46458, 6082, 3102, 3766, 3205, 5245, 5224, 3691, 2179, 2279, 4945, 1991, u
   →6899, 3898, 509, 870, 3606, 4113, 6816, 3293, 3006, 7097, 5440, 2275, 4172, u
  42874, 7586, 583, 2204, 6288, 2087, 6755, 6593, 6592, 588, 189, 7161, 58, L
  →6828, 1545, 7128, 4916, 132, 3461, 2197, 1, 6035, 4164, 636, 2939, 7607, ⊔
   4329, 6173, 1323, 2827, 7285, 4506, 6167, 2331, 3906, 5741, 849, 5454, 6157, u
  →638, 4616, 7358, 3754, 3646, 1220, 251, 6155, 2733, 6515, 5262, 6387, 6227, ⊔
  →3970, 2237, 3621, 2783, 183, 1341, 4858, 7483, 3590, 2305, 6997, 1967, 6664, u
  46463, 2526, 6922, 3706, 4610, 181, 7429, 5752, 2595, 2311, 6751, 5, 1630, U
   ⊶6141, 4774, 3375, 3663, 4277, 447, 1889, 6765, 3660, 5139, 3795, 6880, 317,<sub>⊔</sub>
   45023, 1390, 1337, 4462, 4676, 5781, 725, 2274, 3471, 5600, 2916, 6024, 4213, 5000, 2916, 6024, 4213, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 50000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 50000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000, 5000
   →2895, 5809, 1809, 3599, 6473, 321, 6570, 6988, 3231, 3814, 4947, 4722, 7232, □
   4176, 5668, 3001, 2782, 6479, 6300, 2904, 585, 3376, 4931, 3312, 2139, 6065, u
   45423, 7122, 4999, 67, 1833, 1862, 7387, 1758, 2282, 1978, 4079, 784, 4515, u
   47292, 6006, 923, 5078, 5860, 6711, 3596, 5897, 168, 2935, 3140, 6282, 1184, u
```

```
[19]: ## NTT initialization
      bit_reverse_tree = [1, 128, 64, 320, 32, 224, 160, 352, 16, 208, 112, 304, 80, __
       4272, 176, 368, 8, 200, 104, 296, 56, 248, 152, 344, 40, 232, 136, 328, 88, L
       4280, 184, 376, 4, 196, 100, 292, 52, 244, 148, 340, 28, 220, 124, 316, 76, L
       4268, 172, 364, 20, 212, 116, 308, 68, 260, 164, 356, 44, 236, 140, 332, 92, U
       4284, 188, 380, 2, 194, 98, 290, 50, 242, 146, 338, 26, 218, 122, 314, 74, U
       4266, 170, 362, 14, 206, 110, 302, 62, 254, 158, 350, 38, 230, 134, 326, 86, L
       4278, 182, 374, 10, 202, 106, 298, 58, 250, 154, 346, 34, 226, 130, 322, 82, U
       4274, 178, 370, 22, 214, 118, 310, 70, 262, 166, 358, 46, 238, 142, 334, 94, U
       4286, 190, 382, 1, 193, 97, 289, 49, 241, 145, 337, 25, 217, 121, 313, 73, 11
       4265, 169, 361, 13, 205, 109, 301, 61, 253, 157, 349, 37, 229, 133, 325, 85, u
       4277, 181, 373, 7, 199, 103, 295, 55, 247, 151, 343, 31, 223, 127, 319, 79, U
       4271, 175, 367, 19, 211, 115, 307, 67, 259, 163, 355, 43, 235, 139, 331, 91, L
       4283, 187, 379, 5, 197, 101, 293, 53, 245, 149, 341, 29, 221, 125, 317, 77, I
       4269, 173, 365, 17, 209, 113, 305, 65, 257, 161, 353, 41, 233, 137, 329, 89, L
       4281, 185, 377, 11, 203, 107, 299, 59, 251, 155, 347, 35, 227, 131, 323, 83, U
       4275, 179, 371, 23, 215, 119, 311, 71, 263, 167, 359, 47, 239, 143, 335, 95, L
       →287, 191, 383]
      R = IntegerModRing(q)
      V = VectorSpace(R,n)
      import numpy as np
      # def \ bit \ reverse(x): return \ int("0b" + bin(x)[2:].rjust(8,'0')[::-1],2)+1
      def bit_reverse(x): return bit_reverse_tree[x]
      NTT_matrix = []
      def add (x,y) : return x +y
      for x in range(n/3):
          NTT matrix.append(V(reduce(add, [[20^(x*bit reverse(i)),0,0] for i in_
       \rightarrowrange(n/3)])))
          NTT_matrix.append(V(reduce(add, [[0,20^(x*bit_reverse(i)),0] for i in_
       \rightarrowrange(n/3)])))
          NTT_matrix.append(V(reduce(add, [[0,0,20^(x*bit_reverse(i))] for i in_
       \negrange(n/3)])))
      NTT_matrix = matrix(NTT_matrix)
      # inv_NTT_matrix = NTT_matrix^-1
```

[20]: dbdd.leak?

Signature: . ()
Init docstring:

```
File:
Type:
```

```
[46]: zeta = 20
      from tqdm import trange
      def enumerate_z(tripple_indexes):
          curr_zeta = R(zeta**bit_reverse_tree[tripple_indexes])
          f_hat_0= s_hat[tripple_indexes*3]
          f_hat_1= s_hat[tripple_indexes*3+1]
          f_hat_2= s_hat[tripple_indexes*3+2]
          for z in trange(q):
              for z_prime in range(q):
                  if (z*f_hat_0+f_hat_1+z_prime*f_hat_2*curr_zeta==0 and_
       \(\sigma_prime*f_hat_0 + z*f_hat_1 + f_hat_2 == 0):
                      return (z,z prime)
          print("flase")
          return 0,0
      f_hat_0= s_hat[0]
      f_hat_1= s_hat[1]
      f_hat_2= s_hat[2]
      0*f_hat_0+f_hat_1+0*f_hat_2*20==0
      \# enumerate_z(0)
```

[46]: 2276

```
[32]: v_list = []
index_s = 0
zeta = R(20)

s_relation = [R(-1*s_hat[3*i+1]-s_hat[3*i+2])/R(s_hat[3*i]) for i in range(n/3)]

for i in range(n/3):
    k = s_relation[i]
    v = list((NTT_matrix[3*i+2] + NTT_matrix[3*i+1]+ k *NTT_matrix[3*i]))
    # v = [0]*int(256*index_s) + v + [0]*int(256*(KYBER_N-index_s-1))
    v_prime = [int(i) for i in list(v)] + [0]*768

v_prime= vec(v_prime)
```

```
v_list.append(v_prime)

v = list((NTT_matrix[3*i+2]*k + NTT_matrix[3*i+1]+ *NTT_matrix[3*i]))
# v = [0]*int(256*index_s) + v + [0]*int(256*(KYBER_N-index_s-1))
v_prime = [int(i) for i in list(v)] + [0]*768

v_prime= vec(v_prime)
v_list.append(v_prime)

dbdd.leak(v_prime)
```

[]: dbdd.integrate_modular_hint?


```
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00327435, =527.95
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00327779, =527.27
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00328063, =526.59
integrate modular hint
                                     Worthy hint !
dim=1537, =1.00328348, =525.91
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00328692, =525.23
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00328976, =524.55
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00329260, =523.88
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00329605, =523.20
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00329889, =522.53
integrate modular hint
                                     Worthy hint !
dim=1537, =1.00330173, =521.86
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00330517, =521.19
```

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integrate modular hint
                                     Worthy hint!
 dim=1537, =1.00330801, =520.52
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00331084, =519.85
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00331429, =519.18
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00331712, =518.52
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00331996, =517.86
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00332340, =517.19
integrate modular hint
                                     Worthy hint!
 dim=1537, =1.00332623, =516.53
integrate modular hint
                                     Worthy hint !
dim=1537, =1.00332906, =515.87
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00333251, =515.21
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00333534, =514.56
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00333817, =513.90
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00334161, =513.25
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00334444, =512.59
integrate modular hint
                                     Worthy hint !
dim=1537, =1.00334727, =511.94
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00335071, =511.29
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00335354, =510.64
integrate modular hint
                                     Worthy hint !
dim=1537, =1.00335636, =509.99
integrate modular hint
                                     Worthy hint!
 dim=1537, =1.00335981, =509.34
```

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integrate modular hint
                                     Worthy hint!
 dim=1537, =1.00336263, =508.70
integrate modular hint
                                     Worthy hint!
 dim=1537, =1.00336607, =508.05
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00336890, =507.41
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00337172, =506.77
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00337516, =506.13
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00337798, =505.49
integrate modular hint
                                     Worthy hint!
 dim=1537, =1.00338080, =504.85
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00338424, =504.21
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00338706, =503.58
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00338988, =502.94
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00339332, =502.31
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00339614, =501.67
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00339958, =501.04
integrate modular hint
                                     Worthy hint !
dim=1537, =1.00340240, =500.41
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00340521, =499.78
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00340865, =499.16
integrate modular hint
                                     Worthy hint !
dim=1537, =1.00341146, =498.53
integrate modular hint
                                     Worthy hint!
 dim=1537, =1.00341427, =497.91
```

```
integrate modular hint
                                     Worthy hint!
 dim=1537, =1.00341772, =497.28
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00342053, =496.66
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00342397, =496.04
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00342678, =495.42
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00342959, =494.80
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00343303, =494.18
integrate modular hint
                                     Worthy hint!
 dim=1537, =1.00343584, =493.56
integrate modular hint
                                     Worthy hint !
dim=1537, =1.00343864, =492.95
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00344209, =492.33
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00344489, =491.72
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00344833, =491.11
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00345113, =490.50
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00345393, =489.89
integrate modular hint
                                     Worthy hint !
dim=1537, =1.00345738, =489.28
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00346018, =488.67
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00346362, =488.07
integrate modular hint
                                     Worthy hint !
dim=1537, =1.00346642, =487.46
integrate modular hint
                                     Worthy hint!
 dim=1537, =1.00346922, =486.86
```

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integrate modular hint
                                     Worthy hint!
 dim=1537, =1.00347266, =486.25
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00347546, =485.65
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00347890, =485.05
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00348169, =484.45
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00348449, =483.85
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00348793, =483.26
integrate modular hint
                                     Worthy hint!
 dim=1537, =1.00349072, =482.66
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00349417, =482.06
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00349696, =481.47
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00349975, =480.88
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00350319, =480.29
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00350598, =479.70
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00350942, =479.11
integrate modular hint
                                     Worthy hint !
dim=1537, =1.00351221, =478.52
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00351499, =477.93
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00351844, =477.34
integrate modular hint
                                     Worthy hint !
dim=1537, =1.00352122, =476.76
integrate modular hint
                                     Worthy hint!
 dim=1537, =1.00352467, =476.18
```

```
integrate modular hint
                                     Worthy hint!
 dim=1537, =1.00352745, =475.59
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00353089, =475.01
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00353367, =474.43
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00353645, =473.85
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00353990, =473.27
integrate modular hint
                                     Worthy hint!
dim=1537, =1.00354268, =472.69
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