

# Kyber\_Estimation\_768

April 12, 2024

## 1 Kyber Estimation

Estimate the drop of bit security caused by multiple information leaked from power side channel.

```
[1]: !ls
```

Kyber1024_bikz.pdf	Kyber_Estimation.ipynb	modular_result512.txt
Kyber1024_qbit.pdf	Kyber_Estimation_512.ipynb	modular_result768.txt
Kyber512_bikz.pdf	'Kyber_Estimation_64*3.ipynb'	stdout.txt
Kyber512_qbit.pdf	Kyber_Estimation_768.ipynb	untitled.txt
Kyber768_bikz.pdf	PlotDrops.ipynb	untitled1.txt
Kyber768_qbit.pdf	modular_result1024.txt	

```
[2]: load("../framework/instance_gen.sage")
```

```
[3]: build_centered_binomial_law(3)
```

```
[3]: {-3: 0.015625000000000000,  
      -2: 0.093750000000000000,  
      -1: 0.234375000000000000,  
      0: 0.312500000000000000,  
      1: 0.234375000000000000,  
      2: 0.093750000000000000,  
      3: 0.015625000000000000}
```

```
[4]: ## Lwattackstance initilizaiton

KYBER_K =3

## default as 768

ntt_n = 256

n = KYBER_K *ntt_n
m = n
q = 3329
# D_e = build_centered_binomial?
```

```

bit_security_constant = 0.292

if KYBER_K ==3 or KYBER_K ==4:
    D_e = build_centered_binomial_law(2)
    # D_e = {-2: 0.0625, -1: 0.25, 0: 0.375, 1: 0.25, 2: 0.0625}
elif KYBER_K == 2 or KYBER_K ==1 :
    D_e = build_centered_binomial_law(3)
    # D_e = build_centered_binomial_law(3)
    # D_e = {-2:0.093754,-1:0.2343754,2:0.093754,1:0.2343754, 0:0.31255, -3:
    ↪(1- 0.31255 -0.2343754*2 -0.093754 *2)/2, 3: (1- 0.31255 -0.2343754*2 -0.
    ↪0.093754 *2)/2}
else:
    assert("The input KYBER_N must be in {2,3,4} corresponding to {Kyber_512,
    ↪768, 1024}")

D_s = D_e
A, b, dbdd = initialize_from_LWE_instance(DBDD_predict, n, q, m, D_e, D_s)
# _ = dbdd.integrate_q_vectors(q, report_every=20)
beta, delta = dbdd.estimate_attack()

```

Build DBDD from LWE

n=768      m=768      q=3329

Attack Estimation

dim=1537                  =1.002899                  =624.97

[5]: beta\*0.292

[5]: 182.492197517939

```

[6]: R = IntegerModRing(3329)
V = VectorSpace(R,ntt_n)

import numpy as np

def bit_reverse(x):return 2*int( "0b" + bin(x)[2:].rjust(7,'0')[::-1] ,2)+1

NTT_matrix = []

def add (x,y) : return x +y

for x in range(ntt_n/2):
    NTT_matrix.append(V(reduce(add, [[(17)^(x*bit_reverse(i)),0] for i in
    ↪range(ntt_n/2)])))
    NTT_matrix.append(V(reduce(add, [[0,(17)^(x*bit_reverse(i))] for i in
    ↪range(ntt_n/2)])))

```

```
NTT_matrix = matrix(NTT_matrix)

inv_NTT_matrix = NTT_matrix^-1
```

[ ]:

```
[7]: s = [[0 for i in range(ntt_n)] for j in range(KYBER_K)]

for i in range(KYBER_K):
    for j in range(ntt_n):
        v0 = [0 for i in range(m + n)]
        v0[i*ntt_n+j]=1
        s[i][j] = dbdd.leak(v0)
```

[8]: s?

```
Type:          list
String form:    [[0, -1, 1, 2, 0, 1, 1, 0, -2, 0, 0, 1, 2, -1, -1, -1, 1, 0, 0,
↳0, 2, 0, -1, 2, -1, 2, 1, -1, 0, <...> , 1, -2, -1, 0, -1, -1, 1, 0, 1, 1, 0,
↳0, -1, 1, 0, 0, -1, 0, 1, 1, -1, 0, 1, 1, 1, 0, 0, -1, 0]]
Length:        3
File:
Docstring:
Built-in mutable sequence.
```

If no argument is given, the constructor creates a new empty list. The argument must be an iterable if specified.

**Init docstring:** Initialize self. See help(type(self)) for accurate signature.

```
[9]: s_hat_list = [V(i)*NTT_matrix for i in s]
```

```
[10]: # Hint_list
s_relation_list = [ [R(s_hat[2*i])/R(s_hat[2*i+1]) for i in range(ntt_n/2)]
↳for s_hat in s_hat_list]

v_list = []
v_list_1 = []
for j in range(KYBER_K):
    for i in range(ntt_n/2):
        s_relation = s_relation_list[j]
        k = s_relation[i]
        v = list((NTT_matrix.column(2*i) - k*NTT_matrix.column(2*i+1)))
        v = [0]*int(ntt_n*j) + v + [0]*int(ntt_n*(KYBER_K-j-1))
        v_prime = [int(i) for i in list(v) + [0]*(m)]

        v_prime= vec(v_prime)
        v_list.append(v_prime)
```

```
v_list_1.append(v)
# print(dbdd.leak(v_prime)%3329)
```

```
[11]: from tqdm import tqdm,tqdm_notebook
```

```
[12]: # for i in tqdm(range(10)):
#     sleep(1)
```

```
[13]: # import logging
# import sys
# import datetime

# def init_logger(filename, logger_name):
#     '''
#     @brief:
#         initialize logger that redirect info to a file just in case we lost
#         ↪connection to the notebook
#     @params:
#         filename: to which file should we log all the info
#         logger_name: an alias to the logger
#     '''

#     # get current timestamp
#     timestamp = datetime.datetime.utcnow().strftime('%Y%m%d_%H-%M-%S')

#     logging.basicConfig(
#         level=logging.INFO,
#         format='[%asctime)s] %(name)s {%(filename)s:%(lineno)d}
#         ↪%(levelname)s - %(message)s',
#         handlers=[
#             logging.FileHandler(filename=filename),
#             logging.StreamHandler(sys.stdout)
#         ]
#     )

#     # Test
#     logger = logging.getLogger(logger_name)
#     logger.info('### Init. Logger {} ###'.format(logger_name))
#     return logger

# # Initialize
# my_logger = init_logger("./ml_notebook.log", "ml_logger")
```

```
[14]: # sys.stdout = open('stdout_768.txt', 'w')
```

```

for v0 in tqdm(v_list):
    a=dbdd.leak(v0)%3329
    if a!=0:
        assert("a!=0")
    dbdd.integrate_modular_hint(v0,0,3329)

```

```

0%|
| 0/384 [00:00<?, ?it/s]

integrate modular hint    (smooth)    Worthy hint !
dim=1537, =1.00290198, =624.09

0%|
| 1/384 [00:10<1:04:11, 10.06s/it]

integrate modular hint    (smooth)    Worthy hint !
dim=1537, =1.00290491, =623.21

1%|
| 2/384 [00:20<1:06:04, 10.38s/it]

integrate modular hint    (smooth)    Worthy hint !
dim=1537, =1.00290784, =622.33

1%|
| 3/384 [00:31<1:05:57, 10.39s/it]

integrate modular hint    (smooth)    Worthy hint !
dim=1537, =1.00291077, =621.45

1%|
| 4/384 [00:43<1:10:40, 11.16s/it]

integrate modular hint    (smooth)    Worthy hint !
dim=1537, =1.00291371, =620.57

1%|
| 5/384 [00:59<1:21:53, 12.97s/it]

integrate modular hint    (smooth)    Worthy hint !
dim=1537, =1.00291664, =619.70

2%|
| 6/384 [01:28<1:56:19, 18.46s/it]

integrate modular hint    (smooth)    Worthy hint !
dim=1537, =1.00291957, =618.82

2%|
| 7/384 [02:15<2:53:54, 27.68s/it]

```

```

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00292250, =617.95
    2%|
| 8/384 [02:24<2:16:10, 21.73s/it]

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00292594, =617.08
    2%|
| 9/384 [02:43<2:11:24, 21.02s/it]

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00292887, =616.21
    3%|
| 10/384 [02:55<1:53:22, 18.19s/it]

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00293180, =615.35
    3%|
| 11/384 [03:06<1:38:38, 15.87s/it]

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00293472, =614.49
    3%|
| 12/384 [03:36<2:04:49, 20.13s/it]

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00293765, =613.62
    3%|
| 13/384 [03:46<1:46:54, 17.29s/it]

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00294057, =612.76
    4%|
| 14/384 [03:56<1:32:38, 15.02s/it]

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00294350, =611.91
    4%|
| 15/384 [04:06<1:23:32, 13.58s/it]

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00294694, =611.05

```

```

4%|
| 16/384 [04:16<1:15:23, 12.29s/it]
  integrate modular hint    (smooth)    Worthy hint !
  dim=1537, =1.00294986, =610.20

4%|
| 17/384 [04:25<1:09:28, 11.36s/it]
  integrate modular hint    (smooth)    Worthy hint !
  dim=1537, =1.00295279, =609.34

5%|
| 18/384 [04:34<1:06:00, 10.82s/it]
  integrate modular hint    (smooth)    Worthy hint !
  dim=1537, =1.00295571, =608.49

5%|
| 19/384 [04:44<1:03:09, 10.38s/it]
  integrate modular hint    (smooth)    Worthy hint !
  dim=1537, =1.00295863, =607.64

5%|
| 20/384 [04:53<1:00:03, 9.90s/it]
  integrate modular hint    (smooth)    Worthy hint !
  dim=1537, =1.00296155, =606.80

5%|
| 21/384 [05:04<1:02:28, 10.33s/it]
  integrate modular hint    (smooth)    Worthy hint !
  dim=1537, =1.00296447, =605.95

6%|
| 22/384 [05:14<1:01:48, 10.24s/it]
  integrate modular hint    (smooth)    Worthy hint !
  dim=1537, =1.00296791, =605.11

6%|
| 23/384 [05:24<1:00:33, 10.07s/it]
  integrate modular hint    (smooth)    Worthy hint !
  dim=1537, =1.00297083, =604.27

6%|
| 24/384 [05:33<59:43, 9.96s/it]

```

```

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00297375, =603.43
7%|
| 25/384 [05:44<1:00:28, 10.11s/it]

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00297667, =602.59
7%|
| 26/384 [05:54<1:01:08, 10.25s/it]

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00297958, =601.75
7%|
| 27/384 [06:05<1:00:54, 10.24s/it]

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00298250, =600.92
7%|
| 28/384 [06:15<1:00:37, 10.22s/it]

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00298594, =600.09
8%|
| 29/384 [06:24<59:24, 10.04s/it]

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00298885, =599.26
8%|
| 30/384 [06:35<1:00:21, 10.23s/it]

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00299177, =598.43
8%|
| 31/384 [06:45<59:21, 10.09s/it]

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00299468, =597.60
8%|
| 32/384 [06:56<1:00:42, 10.35s/it]

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00299759, =596.77

```



```

9%|
| 33/384 [07:07<1:01:13, 10.46s/it]
  integrate modular hint    (smooth)    Worthy hint !
  dim=1537, =1.00300050, =595.95

9%|
| 34/384 [07:19<1:04:41, 11.09s/it]
  integrate modular hint    (smooth)    Worthy hint !
  dim=1537, =1.00300395, =595.13

9%|
| 35/384 [07:30<1:03:23, 10.90s/it]
  integrate modular hint    (smooth)    Worthy hint !
  dim=1537, =1.00300686, =594.31

9%|
| 36/384 [07:41<1:03:52, 11.01s/it]
  integrate modular hint    (smooth)    Worthy hint !
  dim=1537, =1.00300977, =593.49

10%|
| 37/384 [07:52<1:03:37, 11.00s/it]
  integrate modular hint    (smooth)    Worthy hint !
  dim=1537, =1.00301267, =592.67

10%|
| 38/384 [08:04<1:05:07, 11.29s/it]
  integrate modular hint    (smooth)    Worthy hint !
  dim=1537, =1.00301558, =591.86

10%|
| 39/384 [08:18<1:10:42, 12.30s/it]
  integrate modular hint    (smooth)    Worthy hint !
  dim=1537, =1.00301902, =591.04

10%|
| 40/384 [08:30<1:09:24, 12.11s/it]
  integrate modular hint    (smooth)    Worthy hint !
  dim=1537, =1.00302193, =590.23

11%|
| 41/384 [08:43<1:10:09, 12.27s/it]

```

```

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00302484, =589.42
11%|
| 42/384 [08:56<1:11:43, 12.58s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00302774, =588.62
11%|
| 43/384 [09:07<1:09:07, 12.16s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00303065, =587.81
11%|
| 44/384 [09:16<1:03:54, 11.28s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00303409, =587.00
12%|
| 45/384 [09:28<1:04:32, 11.42s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00303699, =586.20
12%|
| 46/384 [09:39<1:02:31, 11.10s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00303989, =585.40
12%|
| 47/384 [09:53<1:08:17, 12.16s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00304279, =584.60
12%|
| 48/384 [10:08<1:12:20, 12.92s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00304570, =583.80
13%|
| 49/384 [10:17<1:06:25, 11.90s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00304914, =583.01

```

```

13%|
| 50/384 [10:30<1:08:04, 12.23s/it]
  integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00305204, =582.21

13%|
| 51/384 [10:42<1:06:25, 11.97s/it]
  integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00305494, =581.42

14%|
| 52/384 [11:06<1:26:44, 15.68s/it]
  integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00305783, =580.63

14%|
| 53/384 [11:23<1:28:00, 15.95s/it]
  integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00306073, =579.84

14%|
| 54/384 [11:31<1:15:48, 13.78s/it]
  integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00306417, =579.05

14%|
| 55/384 [11:43<1:12:23, 13.20s/it]
  integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00306707, =578.26

15%|
| 56/384 [12:05<1:26:36, 15.84s/it]
  integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00306997, =577.48

15%|
| 57/384 [12:15<1:16:52, 14.10s/it]
  integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00307286, =576.70

15%|
| 58/384 [12:24<1:08:05, 12.53s/it]

```

```

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00307575, =575.91
15%|
| 59/384 [12:33<1:02:02, 11.45s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00307920, =575.13
16%|
| 60/384 [12:43<59:37, 11.04s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00308209, =574.36
16%|
| 61/384 [12:53<57:37, 10.70s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00308498, =573.58
16%|
| 62/384 [13:10<1:08:03, 12.68s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00308787, =572.81
16%|
| 63/384 [13:22<1:05:32, 12.25s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00309131, =572.03
17%|
| 64/384 [13:42<1:17:45, 14.58s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00309420, =571.26
17%|
| 65/384 [13:52<1:11:14, 13.40s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00309709, =570.49
17%|
| 66/384 [14:13<1:22:11, 15.51s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00309998, =569.72

```

```

17%|
| 67/384 [14:36<1:34:49, 17.95s/it]
  integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00310286, =568.95

18%|
| 68/384 [14:47<1:23:11, 15.80s/it]
  integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00310631, =568.19

18%|
| 69/384 [14:58<1:15:40, 14.42s/it]
  integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00310919, =567.43

18%|
| 70/384 [15:09<1:10:13, 13.42s/it]
  integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00311208, =566.66

18%|
| 71/384 [15:22<1:08:15, 13.08s/it]
  integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00311496, =565.90

19%|
| 72/384 [15:36<1:09:20, 13.34s/it]
  integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00311841, =565.14

19%|
| 73/384 [15:46<1:04:21, 12.42s/it]
  integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00312129, =564.39

19%|
| 74/384 [15:55<59:03, 11.43s/it]
  integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00312417, =563.63

20%|
| 75/384 [16:05<56:21, 10.94s/it]

```

```

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00312705, =562.88
20%|
| 76/384 [16:14<52:50, 10.29s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00313049, =562.12
20%|
| 77/384 [16:30<1:01:31, 12.02s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00313337, =561.37
20%|
| 78/384 [16:39<56:38, 11.11s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00313625, =560.62
21%|
| 79/384 [16:50<57:02, 11.22s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00313913, =559.87
21%|
| 80/384 [17:01<56:18, 11.11s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00314258, =559.13
21%|
| 81/384 [17:10<52:38, 10.43s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00314545, =558.38
21%|
| 82/384 [17:19<50:23, 10.01s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00314833, =557.64
22%|
| 83/384 [17:28<48:31, 9.67s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00315120, =556.90

```

```

22%|
| 84/384 [17:36<46:51, 9.37s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00315465, =556.16

22%|
| 85/384 [17:45<46:09, 9.26s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00315752, =555.42

22%|
| 86/384 [17:54<45:36, 9.18s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00316039, =554.68

23%|
| 87/384 [18:04<46:05, 9.31s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00316327, =553.94

23%|
| 88/384 [18:13<45:41, 9.26s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00316671, =553.21

23%|
| 89/384 [18:24<47:23, 9.64s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00316958, =552.48

23%|
| 90/384 [18:34<47:29, 9.69s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00317245, =551.74

24%|
| 91/384 [18:43<46:29, 9.52s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00317589, =551.01

24%|
| 92/384 [18:51<45:21, 9.32s/it]

```

```

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00317876, =550.29
24%|
| 93/384 [19:01<45:33,  9.39s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00318163, =549.56
24%|
| 94/384 [19:10<44:43,  9.25s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00318450, =548.83
25%|
| 95/384 [19:19<44:53,  9.32s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00318794, =548.11
25%|
| 96/384 [19:28<44:15,  9.22s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00319081, =547.39
25%|
| 97/384 [19:38<43:55,  9.18s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00319367, =546.67
26%|
| 98/384 [19:46<43:08,  9.05s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00319654, =545.95
26%|
| 99/384 [19:55<42:54,  9.03s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00319998, =545.23
26%|
| 100/384 [20:04<42:45,  9.03s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00320284, =544.51

```



```

26%|
| 101/384 [20:13<42:16, 8.96s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00320571, =543.79

27%|
| 102/384 [20:23<43:43, 9.30s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00320915, =543.08

27%|
| 103/384 [20:33<43:40, 9.33s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00321201, =542.37

27%|
| 104/384 [20:42<43:32, 9.33s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00321487, =541.66

27%|
| 105/384 [20:51<42:58, 9.24s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00321773, =540.95

28%|
| 106/384 [21:00<42:22, 9.14s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00322117, =540.24

28%|
| 107/384 [21:10<43:25, 9.41s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00322403, =539.53

28%|
| 108/384 [21:20<43:54, 9.54s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00322689, =538.83

28%|
| 109/384 [21:28<42:05, 9.18s/it]

```

```

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00323033, =538.12
29%|
| 110/384 [21:37<41:13,  9.03s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00323319, =537.42
29%|
| 111/384 [21:45<40:27,  8.89s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00323604, =536.72
29%|
| 112/384 [21:55<40:57,  9.03s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00323949, =536.02
29%|
| 113/384 [22:03<39:56,  8.84s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00324234, =535.32
30%|
| 114/384 [22:11<39:03,  8.68s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00324519, =534.62
30%|
| 115/384 [22:20<38:57,  8.69s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00324804, =533.93
30%|
| 116/384 [22:29<38:33,  8.63s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00325149, =533.23
30%|
| 117/384 [22:37<38:28,  8.65s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00325434, =532.54

```

```

31%|
| 118/384 [22:46<38:02, 8.58s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00325719, =531.85

31%|
| 119/384 [22:54<37:38, 8.52s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00326063, =531.16

31%|
| 120/384 [23:03<37:26, 8.51s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00326348, =530.47

32%|
| 121/384 [23:11<36:38, 8.36s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00326633, =529.78

32%|
| 122/384 [23:20<37:22, 8.56s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00326977, =529.10

32%|
| 123/384 [23:28<37:13, 8.56s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00327262, =528.41

32%|
| 124/384 [23:37<37:17, 8.61s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00327546, =527.73

33%|
| 125/384 [23:45<36:26, 8.44s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00327890, =527.05

33%|
| 126/384 [23:54<37:10, 8.65s/it]

```

```

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00328175, =526.37
33%|
| 127/384 [24:02<36:20, 8.49s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00328459, =525.69
33%|
| 128/384 [24:11<36:58, 8.66s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00328804, =525.01
34%|
| 129/384 [24:20<36:22, 8.56s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00329088, =524.33
34%|
| 130/384 [24:28<36:07, 8.54s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00329372, =523.66
34%|
| 131/384 [24:36<35:53, 8.51s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00329656, =522.98
34%|
| 132/384 [24:45<36:19, 8.65s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00330000, =522.31
35%|
| 133/384 [24:53<35:10, 8.41s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00330284, =521.64
35%|
| 134/384 [25:02<35:15, 8.46s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00330568, =520.97

```

```

35%|
| 135/384 [25:11<36:09, 8.71s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00330912, =520.30

35%|
| 136/384 [25:19<35:09, 8.51s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00331196, =519.64

36%|
| 137/384 [25:28<35:55, 8.73s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00331479, =518.97

36%|
| 138/384 [25:37<35:43, 8.71s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00331824, =518.30

36%|
| 139/384 [25:46<35:14, 8.63s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00332107, =517.64

36%|
| 140/384 [25:55<35:39, 8.77s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00332390, =516.98

37%|
| 141/384 [26:04<35:42, 8.82s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00332735, =516.32

37%|
| 142/384 [26:12<35:19, 8.76s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00333018, =515.66

37%|
| 143/384 [26:20<34:23, 8.56s/it]

```

```

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00333362, =515.00
38%|
| 144/384 [26:30<35:05, 8.77s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00333645, =514.34
38%|
| 145/384 [26:39<36:03, 9.05s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00333928, =513.69
38%|
| 146/384 [26:48<35:01, 8.83s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00334273, =513.03
38%|
| 147/384 [26:56<34:35, 8.76s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00334555, =512.38
39%|
| 148/384 [27:04<33:50, 8.60s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00334838, =511.73
39%|
| 149/384 [27:14<34:41, 8.86s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00335183, =511.08
39%|
| 150/384 [27:23<34:54, 8.95s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00335465, =510.43
39%|
| 151/384 [27:32<34:38, 8.92s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00335747, =509.78

```

```

40%|
| 152/384 [27:41<34:39, 8.96s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00336092, =509.13

40%|
| 153/384 [27:50<34:43, 9.02s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00336374, =508.49

40%|
| 154/384 [27:59<34:15, 8.94s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00336656, =507.84

40%|
| 155/384 [28:07<33:43, 8.84s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00337001, =507.20

41%|
| 156/384 [28:16<32:46, 8.62s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00337283, =506.56

41%|
| 157/384 [28:25<33:19, 8.81s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00337565, =505.92

41%|
| 158/384 [28:34<34:07, 9.06s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00337909, =505.28

41%|
| 159/384 [28:43<33:03, 8.81s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00338191, =504.64

42%|
| 160/384 [28:51<32:33, 8.72s/it]

```

```

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00338536, =504.01
42%|
| 161/384 [29:00<32:39, 8.79s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00338818, =503.37
42%|
| 162/384 [29:09<32:07, 8.68s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00339099, =502.74
42%|
| 163/384 [29:17<32:09, 8.73s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00339444, =502.10
43%|
| 164/384 [29:26<32:15, 8.80s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00339725, =501.47
43%|
| 165/384 [29:35<32:18, 8.85s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00340006, =500.84
43%|
| 166/384 [29:45<33:28, 9.22s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00340351, =500.21
43%|
| 167/384 [29:56<34:32, 9.55s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00340632, =499.58
44%|
| 168/384 [30:06<34:50, 9.68s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00340913, =498.95

```



```

44%|
| 169/384 [30:15<34:23, 9.60s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00341258, =498.33

44%|
| 170/384 [30:24<33:19, 9.34s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00341539, =497.70

45%|
| 171/384 [30:37<36:39, 10.33s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00341883, =497.08

45%|
| 172/384 [30:47<36:29, 10.33s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00342164, =496.46

45%|
| 173/384 [30:56<35:09, 10.00s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00342445, =495.84

45%|
| 174/384 [31:05<33:27, 9.56s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00342789, =495.22

46%|
| 175/384 [31:13<32:24, 9.31s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00343070, =494.60

46%|
| 176/384 [31:23<32:22, 9.34s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00343351, =493.98

46%|
| 177/384 [31:32<31:43, 9.20s/it]

```

```

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00343695, =493.36
46%|
| 178/384 [31:45<35:29, 10.34s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00343975, =492.75
47%|
| 179/384 [31:55<35:06, 10.27s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00344320, =492.14
47%|
| 180/384 [32:05<34:36, 10.18s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00344600, =491.52
47%|
| 181/384 [32:14<33:49, 10.00s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00344880, =490.91
47%|
| 182/384 [32:23<32:41, 9.71s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00345225, =490.30
48%|
| 183/384 [32:32<31:46, 9.49s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00345505, =489.69
48%|
| 184/384 [32:42<31:56, 9.58s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00345849, =489.08
48%|
| 185/384 [32:52<31:59, 9.65s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00346129, =488.48

```

48%|  
| 186/384 [33:01<31:33, 9.57s/it]  
integrate modular hint (smooth) Worthy hint !  
dim=1537, =1.00346409, =487.87

49%|  
| 187/384 [33:13<33:08, 10.09s/it]  
integrate modular hint (smooth) Worthy hint !  
dim=1537, =1.00346753, =487.26

49%|  
| 188/384 [33:35<44:54, 13.75s/it]  
integrate modular hint (smooth) Worthy hint !  
dim=1537, =1.00347033, =486.66

49%|  
| 189/384 [33:45<40:54, 12.59s/it]  
integrate modular hint (smooth) Worthy hint !  
dim=1537, =1.00347378, =486.06

49%|  
| 190/384 [33:54<37:39, 11.65s/it]  
integrate modular hint (smooth) Worthy hint !  
dim=1537, =1.00347657, =485.46

50%|  
| 191/384 [34:03<34:42, 10.79s/it]  
integrate modular hint (smooth) Worthy hint !  
dim=1537, =1.00347936, =484.86

50%|  
| 192/384 [34:14<34:23, 10.75s/it]  
integrate modular hint (smooth) Worthy hint !  
dim=1537, =1.00348281, =484.26

50%|  
| 193/384 [34:23<32:39, 10.26s/it]  
integrate modular hint (smooth) Worthy hint !  
dim=1537, =1.00348560, =483.66

51%|  
| 194/384 [34:32<31:39, 10.00s/it]

```

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00348905, =483.06
51%|
| 195/384 [34:43<32:30, 10.32s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00349184, =482.47
51%|
| 196/384 [34:52<31:04, 9.92s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00349463, =481.87
51%|
| 197/384 [35:03<31:34, 10.13s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00349807, =481.28
52%|
| 198/384 [35:12<30:32, 9.85s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00350086, =480.69
52%|
| 199/384 [35:22<30:52, 10.01s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00350431, =480.09
52%|
| 200/384 [35:32<30:29, 9.94s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00350709, =479.51
52%|
| 201/384 [35:41<29:37, 9.71s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00350988, =478.92
53%|
| 202/384 [35:51<29:15, 9.64s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00351332, =478.33

```

53%|  
| 203/384 [36:00<29:03, 9.63s/it]  
integrate modular hint (smooth) Worthy hint !  
dim=1537, =1.00351611, =477.74

53%|  
| 204/384 [36:10<28:47, 9.60s/it]  
integrate modular hint (smooth) Worthy hint !  
dim=1537, =1.00351955, =477.16

53%|  
| 205/384 [36:19<28:16, 9.48s/it]  
integrate modular hint (smooth) Worthy hint !  
dim=1537, =1.00352234, =476.57

54%|  
| 206/384 [36:29<28:26, 9.58s/it]  
integrate modular hint (smooth) Worthy hint !  
dim=1537, =1.00352512, =475.99

54%|  
| 207/384 [36:37<26:50, 9.10s/it]  
integrate modular hint (smooth) Worthy hint !  
dim=1537, =1.00352856, =475.40

54%|  
| 208/384 [36:47<27:25, 9.35s/it]  
integrate modular hint (smooth) Worthy hint !  
dim=1537, =1.00353134, =474.82

54%|  
| 209/384 [36:56<26:52, 9.22s/it]  
integrate modular hint (smooth) Worthy hint !  
dim=1537, =1.00353479, =474.24

55%|  
| 210/384 [37:05<26:38, 9.19s/it]  
integrate modular hint (smooth) Worthy hint !  
dim=1537, =1.00353757, =473.66

55%|  
| 211/384 [37:14<26:08, 9.07s/it]

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integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00354101, =473.08
55%|
| 212/384 [37:23<26:32, 9.26s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00354379, =472.51
55%|
| 213/384 [37:32<25:55, 9.10s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00354657, =471.93
56%|
| 214/384 [37:42<26:39, 9.41s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00355001, =471.36
56%|
| 215/384 [37:51<26:04, 9.26s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00355279, =470.78
56%|
| 216/384 [38:00<25:38, 9.16s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00355623, =470.21
57%|
| 217/384 [38:09<25:38, 9.21s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00355901, =469.64
57%|
| 218/384 [38:19<25:31, 9.23s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00356245, =469.07
57%|
| 219/384 [38:28<25:05, 9.12s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00356522, =468.50

```

```

57%|
| 220/384 [38:37<24:54, 9.11s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00356799, =467.93

58%|
| 221/384 [38:45<24:09, 8.89s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00357144, =467.36

58%|
| 222/384 [38:55<24:43, 9.16s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00357421, =466.80

58%|
| 223/384 [39:04<24:23, 9.09s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00357766, =466.23

58%|
| 224/384 [39:13<24:37, 9.23s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00358042, =465.67

59%|
| 225/384 [39:22<23:43, 8.95s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00358387, =465.10

59%|
| 226/384 [39:32<24:21, 9.25s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00358664, =464.54

59%|
| 227/384 [39:41<24:06, 9.22s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00358940, =463.98

59%|
| 228/384 [39:50<23:37, 9.09s/it]

```

```

integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00359285, =463.42
60%|
| 229/384 [40:03<26:59, 10.45s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00359561, =462.86
60%|
| 230/384 [40:12<25:16, 9.85s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00359906, =462.30
60%|
| 231/384 [40:21<25:07, 9.85s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00360182, =461.74
60%|
| 232/384 [40:29<23:35, 9.31s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00360526, =461.19
61%|
| 233/384 [40:40<23:58, 9.53s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00360802, =460.63
61%|
| 234/384 [40:48<23:07, 9.25s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00361147, =460.08
61%|
| 235/384 [40:58<23:31, 9.48s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00361423, =459.52
61%|
| 236/384 [41:08<23:30, 9.53s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00361699, =458.97

```



```

62%|
| 237/384 [41:17<22:52, 9.34s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00362043, =458.42

62%|
| 238/384 [41:27<23:20, 9.59s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00362319, =457.87

62%|
| 239/384 [41:36<22:36, 9.36s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00362664, =457.32

62%|
| 240/384 [41:46<23:16, 9.70s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00362939, =456.77

63%|
| 241/384 [41:54<22:07, 9.29s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00363284, =456.23

63%|
| 242/384 [42:04<21:49, 9.22s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00363559, =455.68

63%|
| 243/384 [42:14<22:12, 9.45s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00363904, =455.13

64%|
| 244/384 [42:22<21:30, 9.22s/it]
  integrate modular hint (smooth) Worthy hint !
    dim=1537, =1.00364179, =454.59

64%|
| 245/384 [42:33<22:34, 9.74s/it]

```

integrate modular hint	(smooth)	Worthy hint !
dim=1537,	=1.00364523,	=454.05
64%		246/384 [42:42<21:55, 9.54s/it]
integrate modular hint	(smooth)	Worthy hint !
dim=1537,	=1.00364798,	=453.50
64%		247/384 [42:52<21:56, 9.61s/it]
integrate modular hint	(smooth)	Worthy hint !
dim=1537,	=1.00365073,	=452.96
65%		248/384 [43:01<21:27, 9.46s/it]
integrate modular hint	(smooth)	Worthy hint !
dim=1537,	=1.00365418,	=452.42
65%		249/384 [43:12<21:56, 9.75s/it]
integrate modular hint	(smooth)	Worthy hint !
dim=1537,	=1.00365693,	=451.88
65%		250/384 [43:20<20:45, 9.29s/it]
integrate modular hint	(smooth)	Worthy hint !
dim=1537,	=1.00366037,	=451.35
65%		251/384 [43:30<21:06, 9.52s/it]
integrate modular hint	(smooth)	Worthy hint !
dim=1537,	=1.00366312,	=450.81
66%		252/384 [43:39<20:34, 9.35s/it]
integrate modular hint	(smooth)	Worthy hint !
dim=1537,	=1.00366657,	=450.27
66%		253/384 [43:48<20:21, 9.33s/it]
integrate modular hint	(smooth)	Worthy hint !
dim=1537,	=1.00366931,	=449.74

```

66%|
| 254/384 [43:58<20:36, 9.52s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00367276, =449.20
66%|
| 255/384 [44:08<20:25, 9.50s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00367550, =448.67
67%|
| 256/384 [44:18<20:38, 9.67s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00367894, =448.14
67%|
| 257/384 [44:26<19:31, 9.23s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00368169, =447.60
67%|
| 258/384 [44:35<19:22, 9.23s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00368513, =447.07
67%|
| 259/384 [44:46<20:17, 9.74s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00368787, =446.54
68%|
| 260/384 [44:55<19:51, 9.61s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00369132, =446.02
68%|
| 261/384 [45:04<19:21, 9.45s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00369406, =445.49
68%|
| 262/384 [45:14<19:23, 9.54s/it]

```

integrate modular hint (smooth)	Worthy hint !
dim=1537, =1.00369679, =444.96	
68%	263/384 [45:24<19:35, 9.71s/it]
integrate modular hint (smooth)	Worthy hint !
dim=1537, =1.00370024, =444.44	
69%	264/384 [45:34<19:19, 9.66s/it]
integrate modular hint (smooth)	Worthy hint !
dim=1537, =1.00370298, =443.91	
69%	265/384 [45:42<18:19, 9.24s/it]
integrate modular hint (smooth)	Worthy hint !
dim=1537, =1.00370642, =443.39	
69%	266/384 [45:51<18:00, 9.15s/it]
integrate modular hint (smooth)	Worthy hint !
dim=1537, =1.00370916, =442.86	
70%	267/384 [46:00<17:49, 9.14s/it]
integrate modular hint (smooth)	Worthy hint !
dim=1537, =1.00371260, =442.34	
70%	268/384 [46:09<17:38, 9.12s/it]
integrate modular hint (smooth)	Worthy hint !
dim=1537, =1.00371534, =441.82	
70%	269/384 [46:19<17:58, 9.37s/it]
integrate modular hint (smooth)	Worthy hint !
dim=1537, =1.00371878, =441.30	
70%	270/384 [46:29<18:07, 9.54s/it]
integrate modular hint (smooth)	Worthy hint !
dim=1537, =1.00372151, =440.78	

71%	271/384 [46:39<17:58, 9.54s/it]
<code>integrate modular hint (smooth)</code>	Worthy hint !
<code>dim=1537, =1.00372496, =440.26</code>	
71%	272/384 [46:49<18:14, 9.77s/it]
<code>integrate modular hint (smooth)</code>	Worthy hint !
<code>dim=1537, =1.00372769, =439.74</code>	
71%	273/384 [46:58<18:00, 9.73s/it]
<code>integrate modular hint (smooth)</code>	Worthy hint !
<code>dim=1537, =1.00373113, =439.23</code>	
71%	274/384 [47:07<17:14, 9.41s/it]
<code>integrate modular hint (smooth)</code>	Worthy hint !
<code>dim=1537, =1.00373386, =438.71</code>	
72%	275/384 [47:17<17:20, 9.54s/it]
<code>integrate modular hint (smooth)</code>	Worthy hint !
<code>dim=1537, =1.00373731, =438.20</code>	
72%	276/384 [47:27<17:21, 9.64s/it]
<code>integrate modular hint (smooth)</code>	Worthy hint !
<code>dim=1537, =1.00374003, =437.68</code>	
72%	277/384 [47:36<17:06, 9.60s/it]
<code>integrate modular hint (smooth)</code>	Worthy hint !
<code>dim=1537, =1.00374348, =437.17</code>	
72%	278/384 [47:46<17:03, 9.66s/it]
<code>integrate modular hint (smooth)</code>	Worthy hint !
<code>dim=1537, =1.00374620, =436.66</code>	
73%	279/384 [47:56<16:52, 9.65s/it]

integrate modular hint (smooth)	Worthy hint !
dim=1537, =1.00374965, =436.15	
73%	280/384 [48:05<16:30, 9.53s/it]
integrate modular hint (smooth)	Worthy hint !
dim=1537, =1.00375237, =435.64	
73%	281/384 [48:14<16:14, 9.46s/it]
integrate modular hint (smooth)	Worthy hint !
dim=1537, =1.00375582, =435.13	
73%	282/384 [48:26<17:07, 10.07s/it]
integrate modular hint (smooth)	Worthy hint !
dim=1537, =1.00375854, =434.62	
74%	283/384 [48:36<17:13, 10.24s/it]
integrate modular hint (smooth)	Worthy hint !
dim=1537, =1.00376199, =434.11	
74%	284/384 [48:45<16:09, 9.70s/it]
integrate modular hint (smooth)	Worthy hint !
dim=1537, =1.00376471, =433.61	
74%	285/384 [48:55<16:03, 9.73s/it]
integrate modular hint (smooth)	Worthy hint !
dim=1537, =1.00376815, =433.10	
74%	286/384 [49:04<15:47, 9.66s/it]
integrate modular hint (smooth)	Worthy hint !
dim=1537, =1.00377087, =432.60	
75%	287/384 [49:13<15:02, 9.31s/it]
integrate modular hint (smooth)	Worthy hint !
dim=1537, =1.00377432, =432.09	

75%	288/384 [49:23<15:30, 9.70s/it]
<code>integrate modular hint (smooth)</code>	<code>Worthy hint !</code>
<code>dim=1537, =1.00377704, =431.59</code>	
75%	289/384 [49:36<16:56, 10.70s/it]
<code>integrate modular hint (smooth)</code>	<code>Worthy hint !</code>
<code>dim=1537, =1.00378048, =431.09</code>	
76%	290/384 [49:49<17:54, 11.44s/it]
<code>integrate modular hint (smooth)</code>	<code>Worthy hint !</code>
<code>dim=1537, =1.00378320, =430.58</code>	
76%	291/384 [50:25<28:45, 18.55s/it]
<code>integrate modular hint (smooth)</code>	<code>Worthy hint !</code>
<code>dim=1537, =1.00378664, =430.08</code>	
76%	292/384 [50:38<25:50, 16.86s/it]
<code>integrate modular hint (smooth)</code>	<code>Worthy hint !</code>
<code>dim=1537, =1.00378936, =429.58</code>	
76%	293/384 [50:46<21:54, 14.45s/it]
<code>integrate modular hint (smooth)</code>	<code>Worthy hint !</code>
<code>dim=1537, =1.00379280, =429.09</code>	
77%	294/384 [51:13<27:17, 18.19s/it]
<code>integrate modular hint (smooth)</code>	<code>Worthy hint !</code>
<code>dim=1537, =1.00379551, =428.59</code>	
77%	295/384 [51:35<28:29, 19.21s/it]
<code>integrate modular hint (smooth)</code>	<code>Worthy hint !</code>
<code>dim=1537, =1.00379896, =428.09</code>	
77%	296/384 [51:44<23:37, 16.10s/it]

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integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00380167, =427.60
77%|
| 297/384 [51:53<20:18, 14.00s/it]

integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00380512, =427.10
78%|
| 298/384 [52:01<17:23, 12.13s/it]

integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00380783, =426.61
78%|
| 299/384 [52:10<15:56, 11.26s/it]

integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00381127, =426.11
78%|
| 300/384 [52:20<15:25, 11.02s/it]

integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00381398, =425.62
78%|
| 301/384 [52:31<15:08, 10.95s/it]

integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00381743, =425.13
79%|
| 302/384 [52:41<14:20, 10.50s/it]

integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00382013, =424.64
79%|
| 303/384 [52:49<13:31, 10.02s/it]

integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00382358, =424.15
79%|
| 304/384 [52:59<13:12, 9.91s/it]

integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00382628, =423.66

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79%|
| 305/384 [53:07<12:21, 9.38s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00382973, =423.17
80%|
| 306/384 [53:19<13:13, 10.18s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00383243, =422.68
80%|
| 307/384 [53:28<12:36, 9.83s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00383588, =422.20
80%|
| 308/384 [53:37<12:03, 9.52s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00383858, =421.71
80%|
| 309/384 [53:50<13:19, 10.66s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00384202, =421.23
81%|
| 310/384 [54:11<16:55, 13.72s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00384472, =420.74
81%|
| 311/384 [54:22<15:32, 12.77s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00384817, =420.26
81%|
| 312/384 [54:30<13:48, 11.51s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00385086, =419.78
82%|
| 313/384 [54:40<13:07, 11.10s/it]

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integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00385431, =419.29
82%|
                                     | 314/384 [54:51<12:38, 10.83s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00385700, =418.81
82%|
                                     | 315/384 [55:01<12:11, 10.60s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00386045, =418.33
82%|
                                     | 316/384 [55:10<11:31, 10.17s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00386314, =417.85
83%|
                                     | 317/384 [55:20<11:25, 10.23s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00386659, =417.38
83%|
                                     | 318/384 [55:31<11:27, 10.42s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00386928, =416.90
83%|
                                     | 319/384 [55:40<10:53, 10.06s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00387273, =416.42
83%|
                                     | 320/384 [55:50<10:31, 9.86s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00387542, =415.95
84%|
                                     | 321/384 [55:58<09:50, 9.38s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00387887, =415.47

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84%|
| 322/384 [56:09<10:11, 9.86s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00388155, =415.00
84%|
| 323/384 [56:20<10:25, 10.25s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00388500, =414.52
84%|
| 324/384 [56:29<09:56, 9.95s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00388845, =414.05
85%|
| 325/384 [56:48<12:19, 12.54s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00389113, =413.58
85%|
| 326/384 [56:57<11:05, 11.47s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00389458, =413.11
85%|
| 327/384 [57:18<13:35, 14.30s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00389726, =412.64
85%|
| 328/384 [57:27<11:54, 12.76s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00390071, =412.17
86%|
| 329/384 [57:37<10:58, 11.96s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00390339, =411.70
86%|
| 330/384 [57:46<09:58, 11.09s/it]

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integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00390684, =411.23
86%|
                                | 331/384 [57:57<09:40, 10.95s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00390952, =410.77
86%|
                                | 332/384 [58:07<09:15, 10.69s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00391297, =410.30
87%|
                                | 333/384 [58:19<09:28, 11.16s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00391565, =409.83
87%|
                                | 334/384 [58:27<08:29, 10.20s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00391909, =409.37
87%|
                                | 335/384 [58:39<08:46, 10.75s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00392177, =408.91
88%|
                                | 336/384 [58:49<08:27, 10.57s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00392522, =408.44
88%|
                                | 337/384 [59:29<15:00, 19.16s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00392789, =407.98
88%|
                                | 338/384 [1:00:09<19:37, 25.61s/it]
integrate modular hint    (smooth)    Worthy hint !
    dim=1537, =1.00393134, =407.52

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88%|
                                | 339/384 [1:00:19<15:33, 20.75s/it]
integrate modular hint      (smooth)      Worthy hint !
    dim=1537, =1.00393479, =407.06
89%|
                                | 340/384 [1:00:28<12:38, 17.24s/it]
integrate modular hint      (smooth)      Worthy hint !
    dim=1537, =1.00393746, =406.60
89%|
                                | 341/384 [1:00:37<10:43, 14.97s/it]
integrate modular hint      (smooth)      Worthy hint !
    dim=1537, =1.00394091, =406.14
89%|
                                | 342/384 [1:00:48<09:30, 13.59s/it]
integrate modular hint      (smooth)      Worthy hint !
    dim=1537, =1.00394358, =405.68
89%|
                                | 343/384 [1:00:57<08:29, 12.43s/it]
integrate modular hint      (smooth)      Worthy hint !
    dim=1537, =1.00394703, =405.23
90%|
                                | 344/384 [1:01:11<08:37, 12.93s/it]
integrate modular hint      (smooth)      Worthy hint !
    dim=1537, =1.00394970, =404.77
90%|
                                | 345/384 [1:01:21<07:45, 11.95s/it]
integrate modular hint      (smooth)      Worthy hint !
    dim=1537, =1.00395314, =404.31
90%|
                                | 346/384 [1:01:31<07:06, 11.22s/it]
integrate modular hint      (smooth)      Worthy hint !
    dim=1537, =1.00395581, =403.86
90%|
                                | 347/384 [1:01:40<06:31, 10.57s/it]

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integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00395926, =403.40
91%|
| 348/384 [1:02:06<09:09, 15.28s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00396192, =402.95
91%|
| 349/384 [1:02:32<10:43, 18.39s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00396537, =402.50
91%|
| 350/384 [1:02:41<08:55, 15.74s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00396882, =402.04
91%|
| 351/384 [1:02:51<07:35, 13.81s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00397148, =401.59
92%|
| 352/384 [1:03:00<06:43, 12.62s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00397493, =401.14
92%|
| 353/384 [1:03:22<07:55, 15.34s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00397759, =400.69
92%|
| 354/384 [1:03:42<08:19, 16.66s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00398104, =400.24
92%|
| 355/384 [1:03:52<07:06, 14.72s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00398370, =399.80

```

```

93%|
| 356/384 [1:04:01<06:01, 12.91s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00398715, =399.35
93%|
| 357/384 [1:04:14<05:49, 12.93s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00398981, =398.90
93%|
| 358/384 [1:04:29<05:56, 13.71s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00399325, =398.46
93%|
| 359/384 [1:04:56<07:20, 17.64s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00399670, =398.01
94%|
| 360/384 [1:05:06<06:07, 15.30s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00399936, =397.57
94%|
| 361/384 [1:05:14<05:04, 13.26s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00400280, =397.12
94%|
| 362/384 [1:05:24<04:28, 12.22s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00400546, =396.68
95%|
| 363/384 [1:05:32<03:52, 11.07s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00400891, =396.24
95%|
| 364/384 [1:05:44<03:42, 11.12s/it]

```

```

integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00401156, =395.80
95%|
| 365/384 [1:05:52<03:15, 10.27s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00401501, =395.35
95%|
| 366/384 [1:06:01<02:56, 9.82s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00401766, =394.91
96%|
| 367/384 [1:06:28<04:16, 15.08s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00402110, =394.48
96%|
| 368/384 [1:06:42<03:54, 14.63s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00402455, =394.04
96%|
| 369/384 [1:06:52<03:20, 13.39s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00402720, =393.60
96%|
| 370/384 [1:07:01<02:47, 12.00s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00403065, =393.16
97%|
| 371/384 [1:07:10<02:23, 11.03s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00403329, =392.73
97%|
| 372/384 [1:07:21<02:13, 11.11s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00403674, =392.29

```



```

97%|
| 373/384 [1:07:33<02:04, 11.28s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00403939, =391.85
97%|
| 374/384 [1:07:42<01:48, 10.81s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00404283, =391.42
98%|
| 375/384 [1:07:51<01:32, 10.24s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00404548, =390.99
98%|
| 376/384 [1:08:01<01:20, 10.08s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00404892, =390.55
98%|
| 377/384 [1:08:10<01:08, 9.84s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00405237, =390.12
98%|
| 378/384 [1:08:25<01:07, 11.26s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00405501, =389.69
99%|
| 379/384 [1:08:37<00:57, 11.42s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00405846, =389.26
99%|
| 380/384 [1:08:45<00:42, 10.57s/it]
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00406110, =388.83
99%|
| 381/384 [1:08:59<00:34, 11.66s/it]

```

```
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00406455, =388.40
99%|
| 382/384 [1:09:10<00:22, 11.45s/it]
```

```
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00406718, =387.97
```

```
100%|
| 383/384 [1:09:20<00:10, 10.95s/it]
```

```
integrate modular hint (smooth) Worthy hint !
dim=1537, =1.00407063, =387.54
```

```
100%|
| 384/384 [1:09:31<00:00, 10.86s/it]
```

```
[15]: beta_1, delta_1 = dbdd.estimate_attack()
```

Attack Estimation

```
dim=1537          =1.004071          =387.54
```

```
[20]: beta_1*0.292
```

```
[20]: 113.163023945025
```

```
[16]: # dbdd
```

```
[17]: # dbdd.float_type="qd"
      # # dbdd.float_type="mpfr"
```

```
[18]: # dbdd.float_type="dd"
      # dbdd.float_type="dd"
      # res = dbdd.attack(beta_pre=90)
```

```
[ ]:
```