Two Methods to Generate Normal Random Variables

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Abstract

This paper introduces two methods not mentioned in our STAT 428 lectures to generate normal random variables with mean equal to 0 and variance equal to 1. Both the methods can be easily implemented in R.

Polar Method

Polar Method is an advanced version of Box-Muller method which was mentioned in our homework. It improves calculation efficiency by making fewer calls to the mathematical library. It is also more numerically robest. The algorithm is as follows:

1. U_1 and U_2 denotes independent Uniform(0,1) random variables.
2. $V_i = 2U_i - 1$ and $W = V_1^2 + V_2^2$
3. if $W < 1$, let $y = \sqrt{\frac{-2lnW}{W}}$, else go back to 1.
4. Then $X_1 = V_1 Y$ and $X_2 = V_2 Y$ are independent N(0,1).

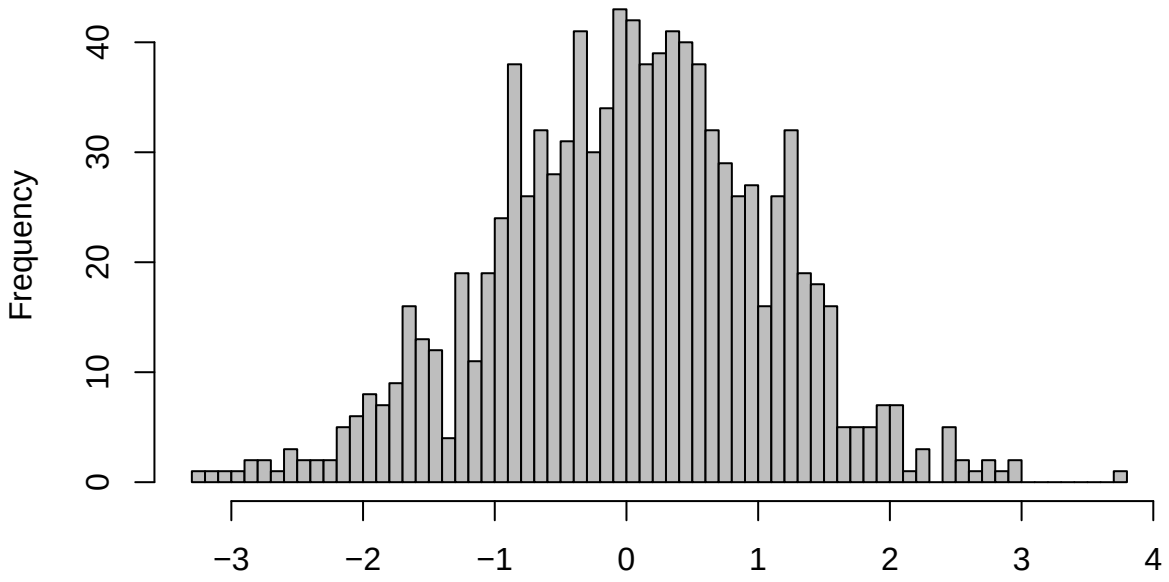

Figure 1

The histgram of random variables generated by polar method is as above, based on 1,000 samples.

Method 2: Sum of Uniform Random Variables

Based on Central Limit Theory, sum of iid random variables are approximately normally distributed. For Uniform(0,1) variables, the normal random variable can be calculated as:

$$Y = \frac{\sum_{i=1}^{n} X_i - \frac{n}{2}}{\sqrt{\frac{n}{12}}}$$

for a big enough value of n.
This method is theoretically feasible but could be computationally expensive.

```
n = 1000
y = numeric(n)
for(i in 1:n){
    u = runif(n)
    y[i] = (sum(u) - n / 2) / sqrt(n / 12)
}
hist(y, breaks = 50, main = " ", col = "grey", xlab = "Figure 2")
```

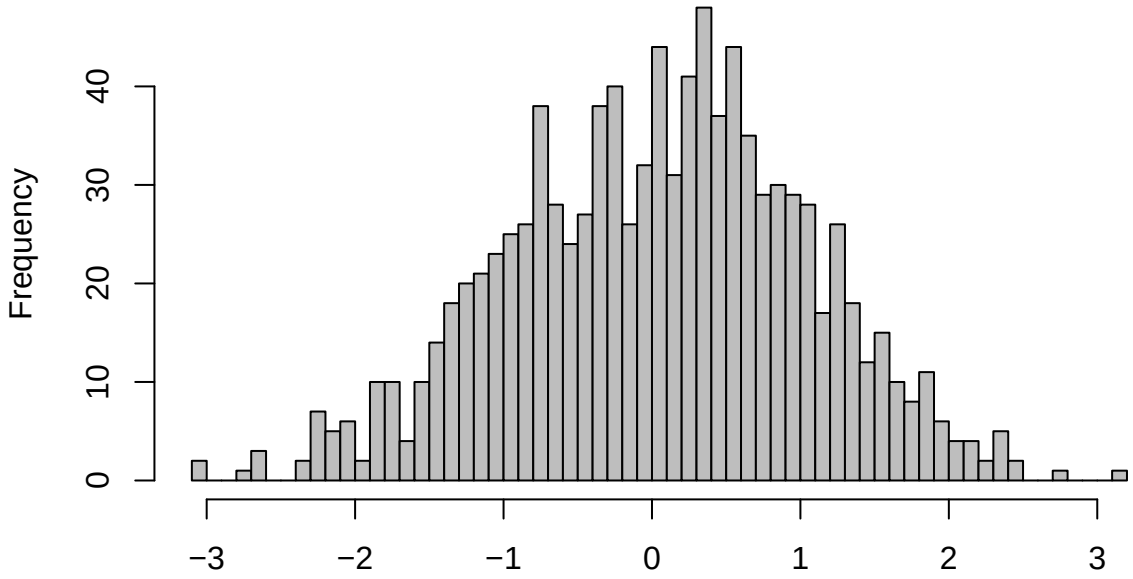

Figure 2 is the histgram of 1,000 gaussian samples. Each sample is generated by 1,000 uniform random variables.

References

[1] Roy R. Comparison of different techniques to generate normal random variables[J]. J. East Central Europe, 2002, 545 5-6.

Appendix

method 1

```
n = 500
count = 0
x = numeric()
while(count < n){
    u1 = runif(1)
    u2 = runif(1)
    v1 = 2*u1 - 1
    v2 = 2*u2 - 1
    w = v1^2 + v2^2
    if(w > 1) next
    y = sqrt(-2*log(w) / w)
    count = count + 1
    x = append(x, c(v1*y, v2*y))
}
hist(x, breaks = 50, main = " ", col = "grey", xlab = "Figure 1")
```

method 2

```
n = 1000
y = numeric(n)
for(i in 1:n){
    u = runif(n)
    y[i] = (sum(u) - n / 2) / sqrt(n / 12)
}
hist(y, breaks = 50, main = " ", col = "grey", xlab = "Figure 2")
```