Application of Multilevel Demand Subscription Pricing for Mobilizing Residential Demand Response in Belgium - Appendix

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Appendix

Following [1], we assume that demand is linear with a constant slope $P(D,\gamma) = a(\gamma) - bD$, where $a(\gamma) = a_0 - a_1 e^{-\lambda_2\gamma}$ and the PDF of γ is given by $f(\gamma) = \lambda_1 e^{-\lambda_1\gamma}$. This PDF corresponds to the exponential distribution, whose CDF is given by $F(\gamma) = 1 - e^{-\lambda_1\gamma}$. In the formulas, D denotes demand and P denotes price, while the 'state of the world' is represented by γ.

Consider the LDC under constant price p_0 and denote $G(L)$ as the probability that demand is lower than L. $G(L)$ can then be expressed as

$$\begin{aligned}
G(L,p_0) &= \Pr\Big(\frac{a(\gamma)-p_0}{b} \le L\Big) \\
&= \Pr\Big(a(\gamma) \le p_0 + bL\Big) \\
&= \Pr\Big(a_0 - a_1 e^{-\lambda_2\gamma} \le p_0 + bL\Big) \\
&= \Pr\Big(e^{-\lambda_2\gamma} \ge \frac{a_0 - (p_0+bL)}{a_1}\Big) \\
&= \Pr\Big(\gamma \le \frac{\ln\frac{a_0-(p_0+bL)}{a_1}}{-\lambda_2}\Big). \qquad (1)
\end{aligned}$$

Using the CDF of γ and letting $\lambda = \frac{\lambda_1}{\lambda_2}$ yield

$$\begin{aligned}
G(L,p_0) &= 1 - \exp\Big[\frac{\lambda_1}{\lambda_2}\ln\frac{a_0-(p_0+bL)}{a_1}\Big] \\
&= 1 - \Big[\frac{a_0-(p_0+bL)}{a_1}\Big]^{\lambda} \qquad (2)
\end{aligned}$$

Then $1 - G(L,p_0) = \Pr(\text{load} \ge L) = \Big[\frac{a_0-(p_0+bL)}{a_1}\Big]^{\lambda}$ can be estimated using the actual load duration curve. The function $L(p,t)$ is then calculated using

$$\Big[\frac{a_0-(p_0+bL)}{a_1}\Big]^{\lambda} = \frac{t}{T} \qquad (3)$$

$$\Rightarrow L(p_0,t) = -\frac{a_1}{b}\Big(\frac{t}{T}\Big)^{\frac{1}{\lambda}} - \frac{p_0}{b} + \frac{a_0}{b} \qquad (4)$$

where T is the horizon.

The next step is to estimate the parameters a_0, a_1, b and λ. Using regression, we can find the optimal values of λ, $m = -\frac{a_1}{b}$ and $n = -\frac{p_0}{b} + \frac{a_0}{b}$.

The parameter b is calculated using elasticity as follows. Since $P(D,\gamma) = a(\gamma) - bD$, the elasticity of demand $e(\pi,\gamma)$ for a given price π in state γ is

$$\begin{aligned}
e(\pi,\gamma) &= \frac{\Delta D/D}{\Delta\pi/\pi} = \frac{\Delta D/D}{(\pi(D+\Delta D)-\pi(D))/\pi} \\
&= \frac{\Delta D/D}{-b\Delta D/\pi} = \frac{\pi}{-bD} \\
&= \frac{\pi}{\pi(D,\gamma)-a(\gamma)} = -\frac{\pi}{a(\gamma)-\pi}. \qquad (5)
\end{aligned}$$

Thus

$$\begin{aligned}
\mathbb{E}[e(\pi,\gamma)] &= \mathbb{E}\Big[-\frac{\pi}{a(\gamma)-\pi}\Big] \\
&= -\pi\int_0^{+\infty}\frac{\lambda_1 e^{-\lambda_1\gamma}}{a_0 - a_1 e^{-\lambda_2\gamma} - \pi}d\gamma. \qquad (6)
\end{aligned}$$

Substituting $e^{-\lambda_1\gamma}$ with x yields

$$\begin{aligned}
\mathbb{E}[e(\pi,\gamma)] &= -\pi\int_0^1\frac{dx}{a_0 - a_1 x^{\frac{1}{\lambda}} - \pi} \qquad (7) \\
&= -\pi\int_0^1\frac{dx}{p_0 + bn + bmx^{\frac{1}{\lambda}} - \pi}. \qquad (8)
\end{aligned}$$

At $\pi = p_0$, assume $\bar{e} = \mathbb{E}[e(\pi,\gamma)] = -0.13$, as observed in [2] using Dutch data, then b can be calculated.

References

[1] T.-O. Leautier, "Is mandating" smart meters" smart?" *Energy Journal*, vol. 35, no. 4, 2014.

[2] P. G. Boonekamp, "Price elasticities, policy measures and actual developments in household energy consumption–a bottom up analysis for the netherlands," *Energy Economics*, vol. 29, no. 2, pp. 133–157, 2007.