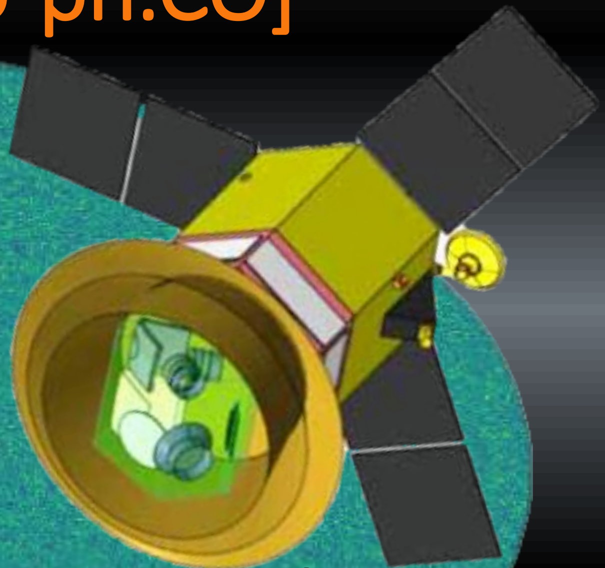
Simultaneous determination of the cosmic birefringence and miscalibrated polarisation angles from CMB experiments
arXiv:1904.12440 [astro-ph.CO]

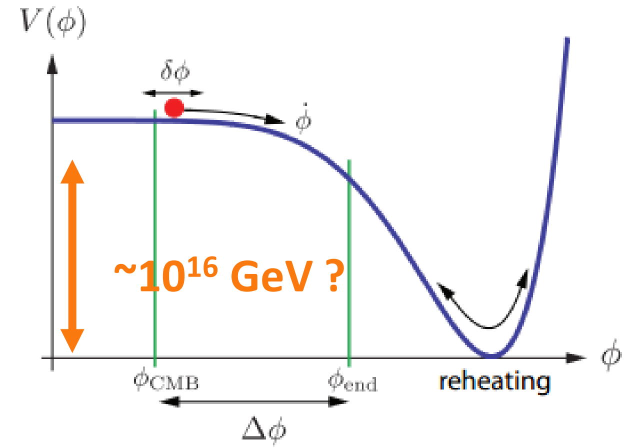


Yuto Minami , Hiroki Ochi, Kiyotomo Ichiki , Nobu Katayama, Eiichiro Komatsu, and Tomotake Matsumura

Before we dive into the content of the paper

Introduction to cosmic microwave background (CMB) measurement

Inflation: Physics of high energy scale



- Exponential expansion of space in the early universe
- Hope to explore GUT-scale physics
- The potential of single field slow-roll model is

$$V^{1/4} \sim 1.04 \cdot \left(\frac{r}{0.01}\right)^{\frac{1}{4}} \times 10^{16}\ \mathrm{GeV}$$

r: tensor-to-scalar ratio
Power-spectrum ratio of the tensor type perturbation to scalar type perturbation

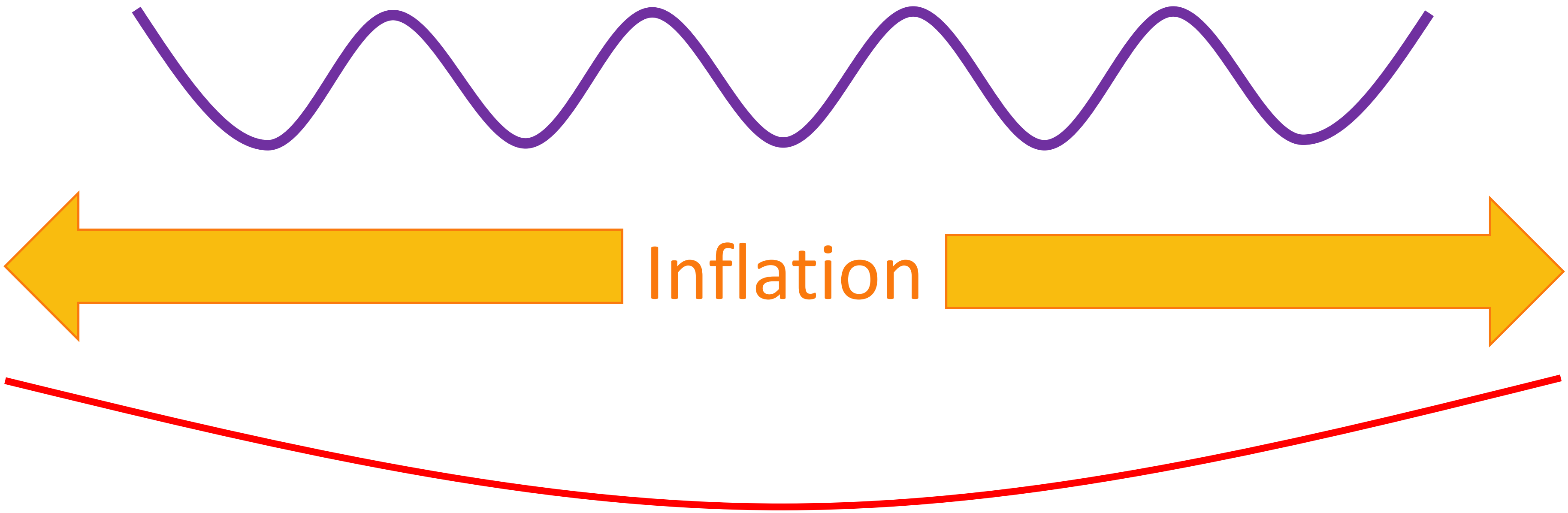
$$r \equiv \frac{\langle h_{ij} h^{ij} \rangle}{\langle \zeta^2 \rangle}$$

- Two types of fluctuation in inflation
  - scalar $\langle \zeta^2 \rangle$: density
  - tensor $\langle |h|^2 \rangle$: primordial gravitational wave (PGW)

$$ds^2 = a^2(t)\,(1 + 2\zeta(x,t))\big(\delta_{ij} + h_{ij}(x,t)\big)dx^i dx^j$$

Inflation and Gravitational Waves (GWs)

Inflation expands the space by $\sim 10^{26}$ times in $\sim 10^{-36}$ sec

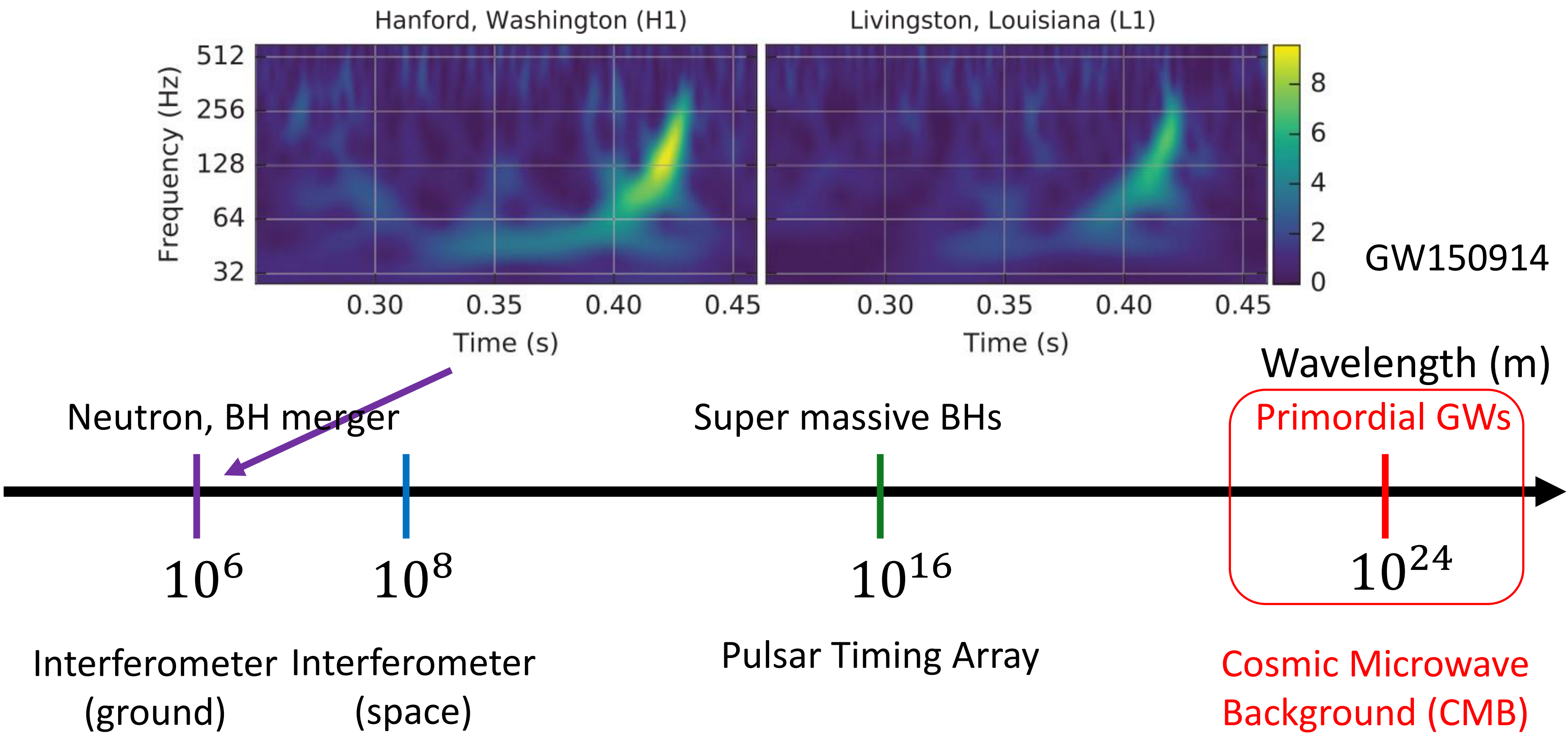


Quantum fluctuation becomes large scale fluctuation

Primordial gravitational waves

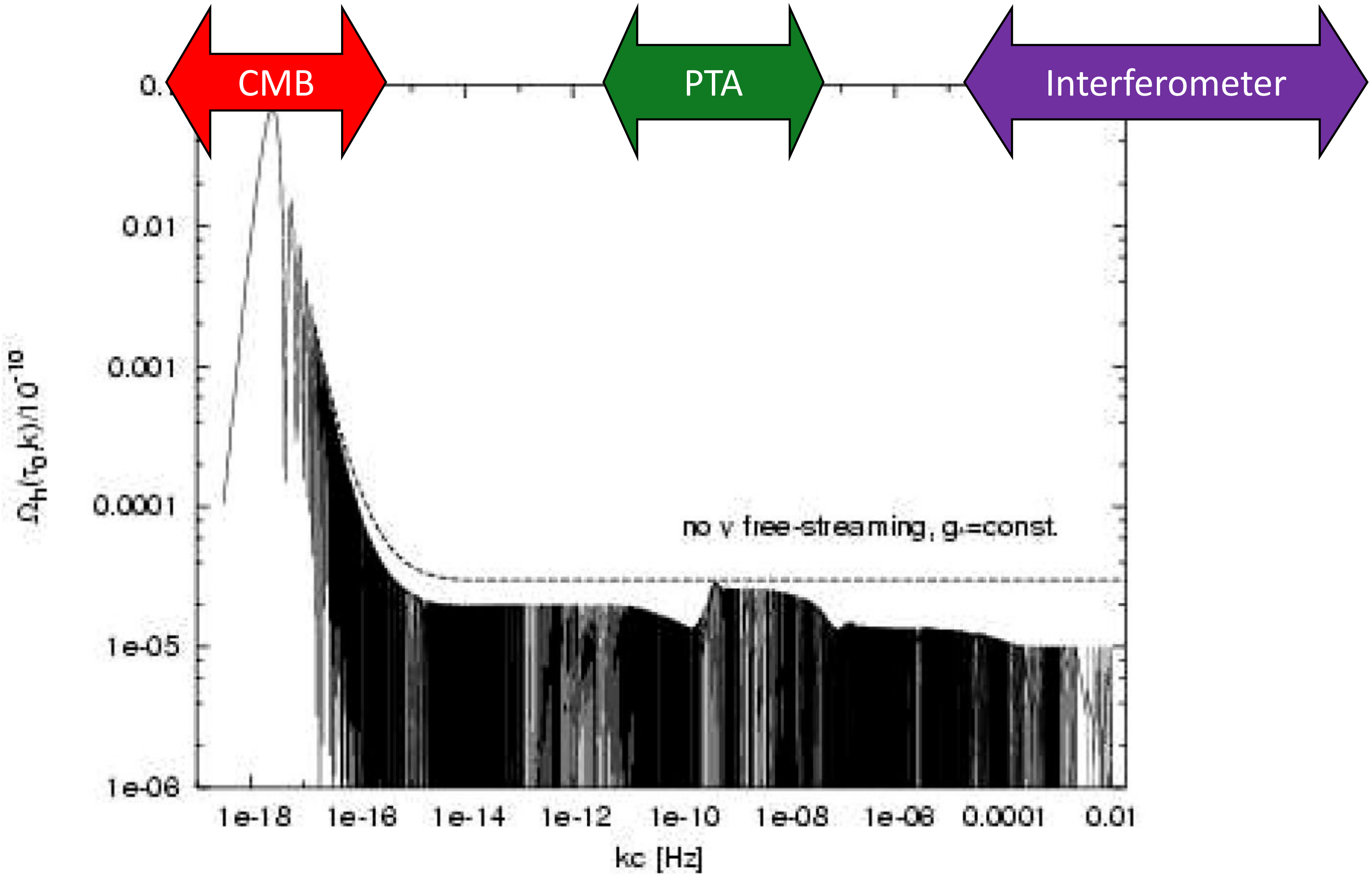
Primordial gravitational wave (PGW)

- LIGO made a first observation of gravitational wave
  - Wavelength is $\sim 10^6$ m



We target the 10 billion light year scale of gravitational waves

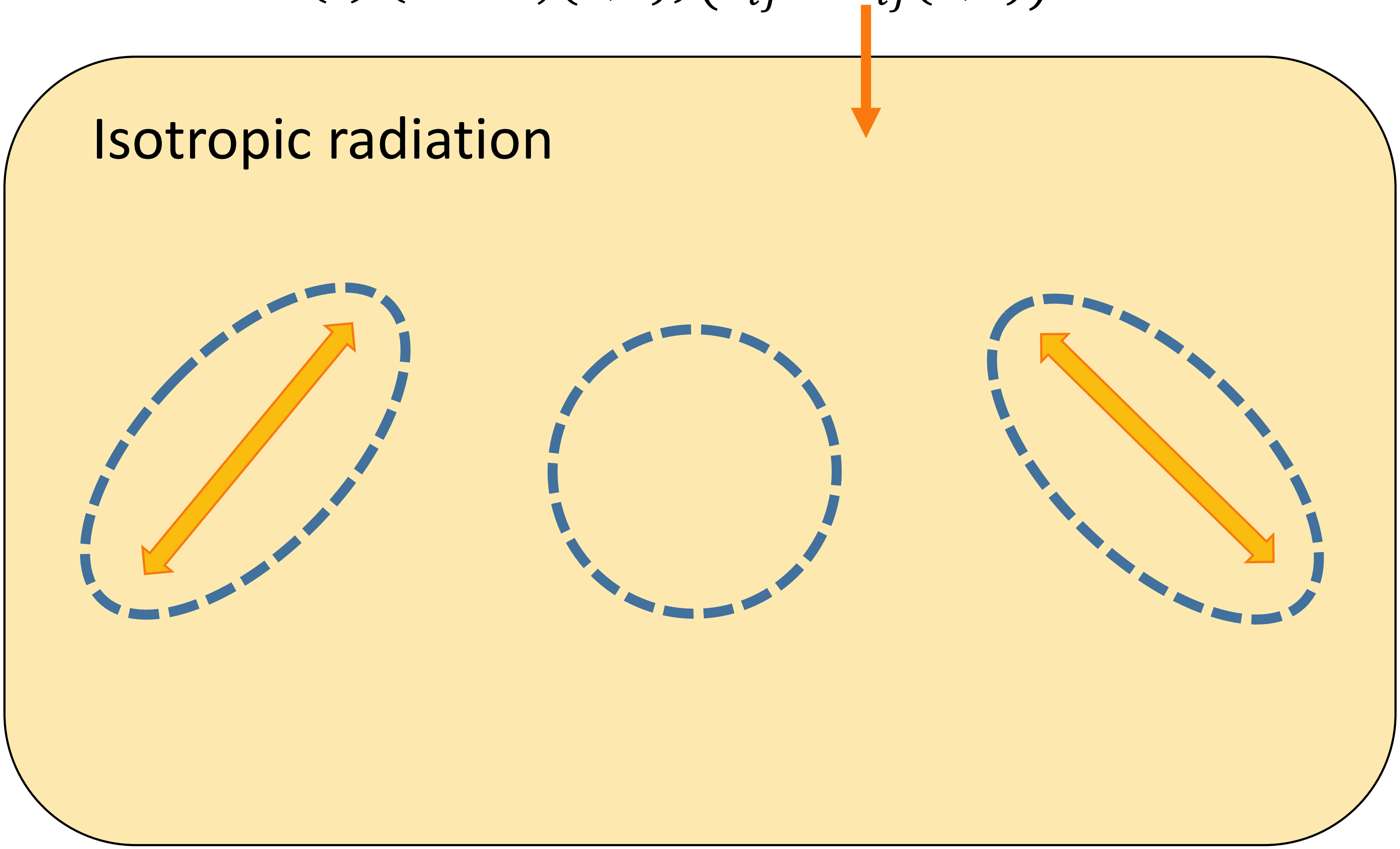
Power spectrum of PGWs



Watanabe and Komatsu (2006)

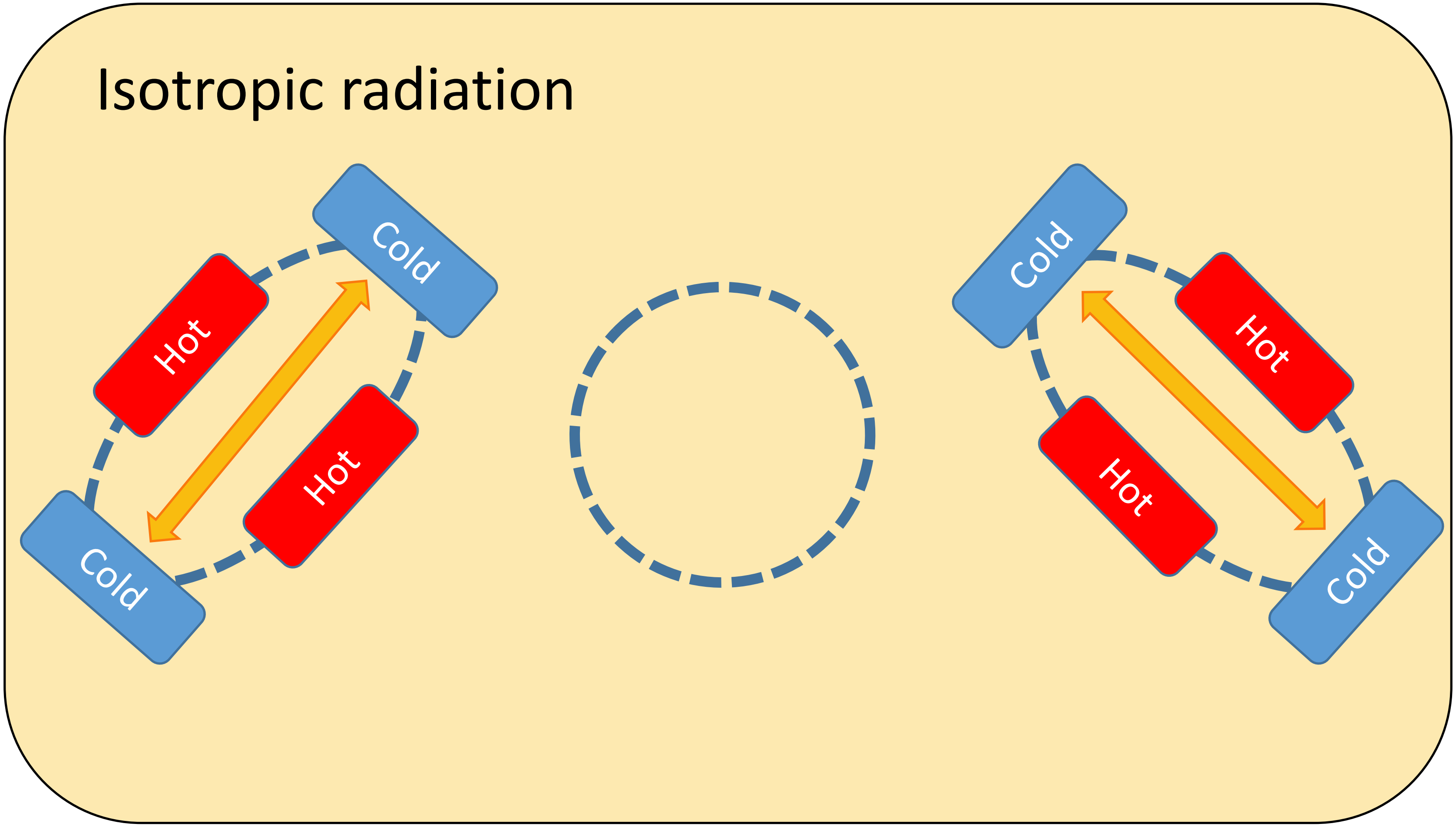
How to measure PGWs with CMB?

$$ds^2 = a^2(t)\,(1 + 2\zeta(x,t))\big(\delta_{ij} + h_{ij}(x,t)\big)dx^i dx^j$$



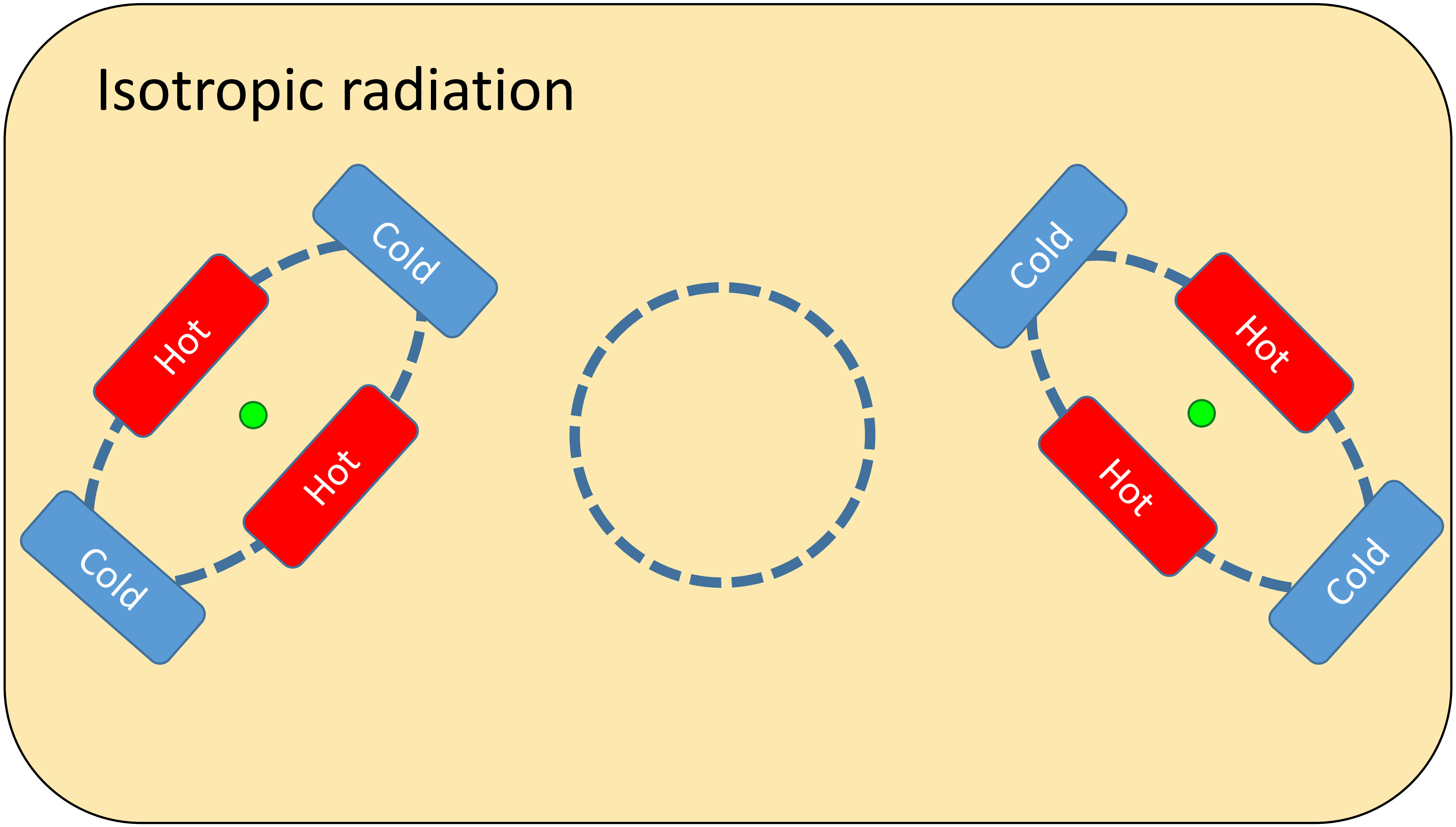
How to measure PGWs with CMB?

Space is distorted by fluctuation

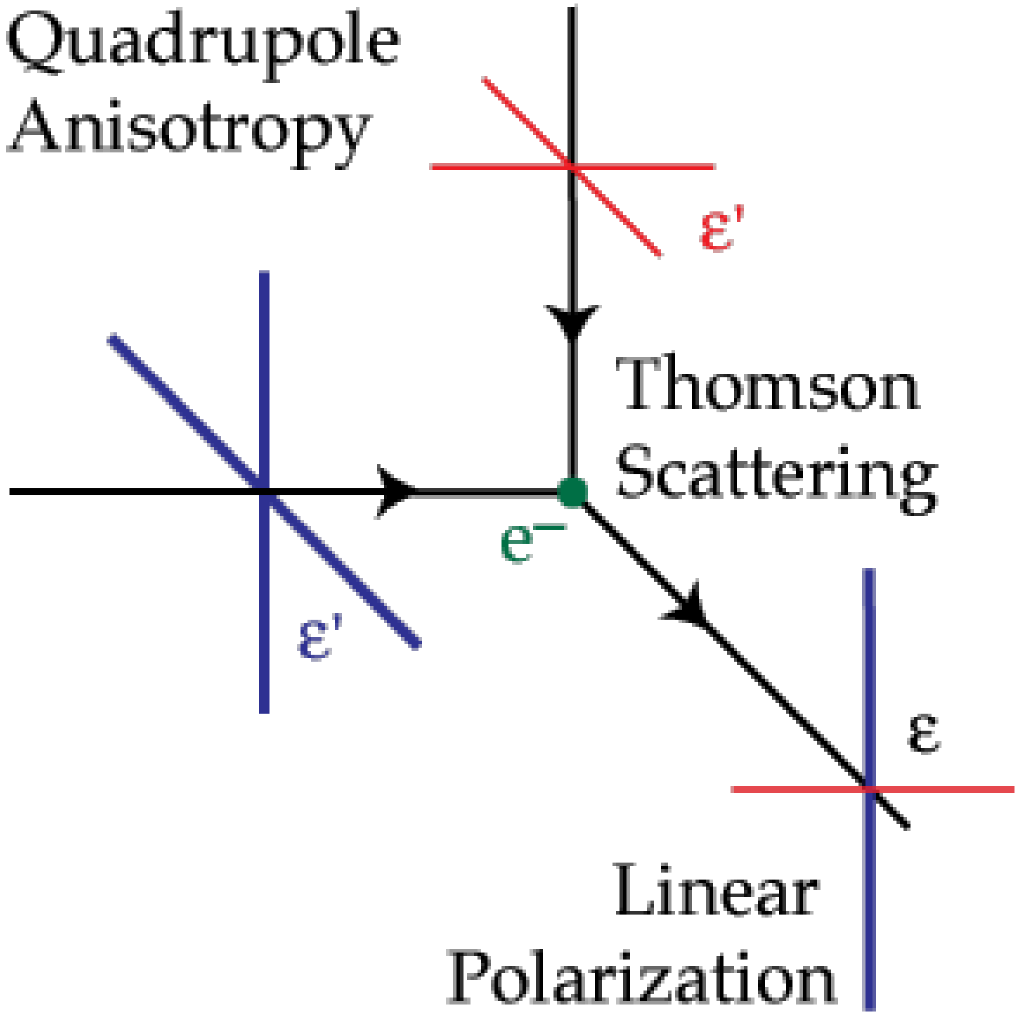


How to measure PGWs with CMB?

What if electrons are surrounded by the quadrupole?



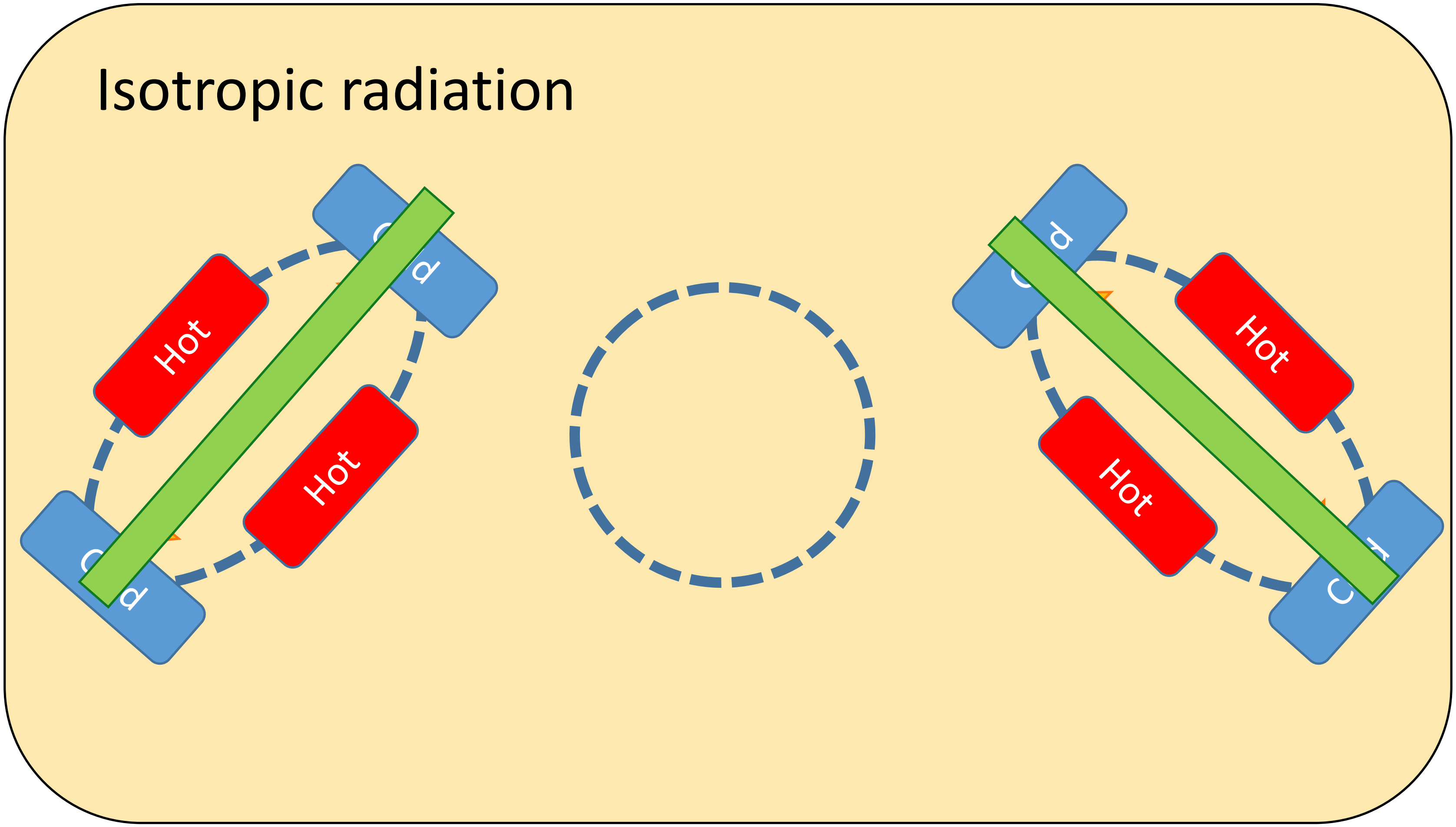
Linear polarization from quadrupole anisotropy



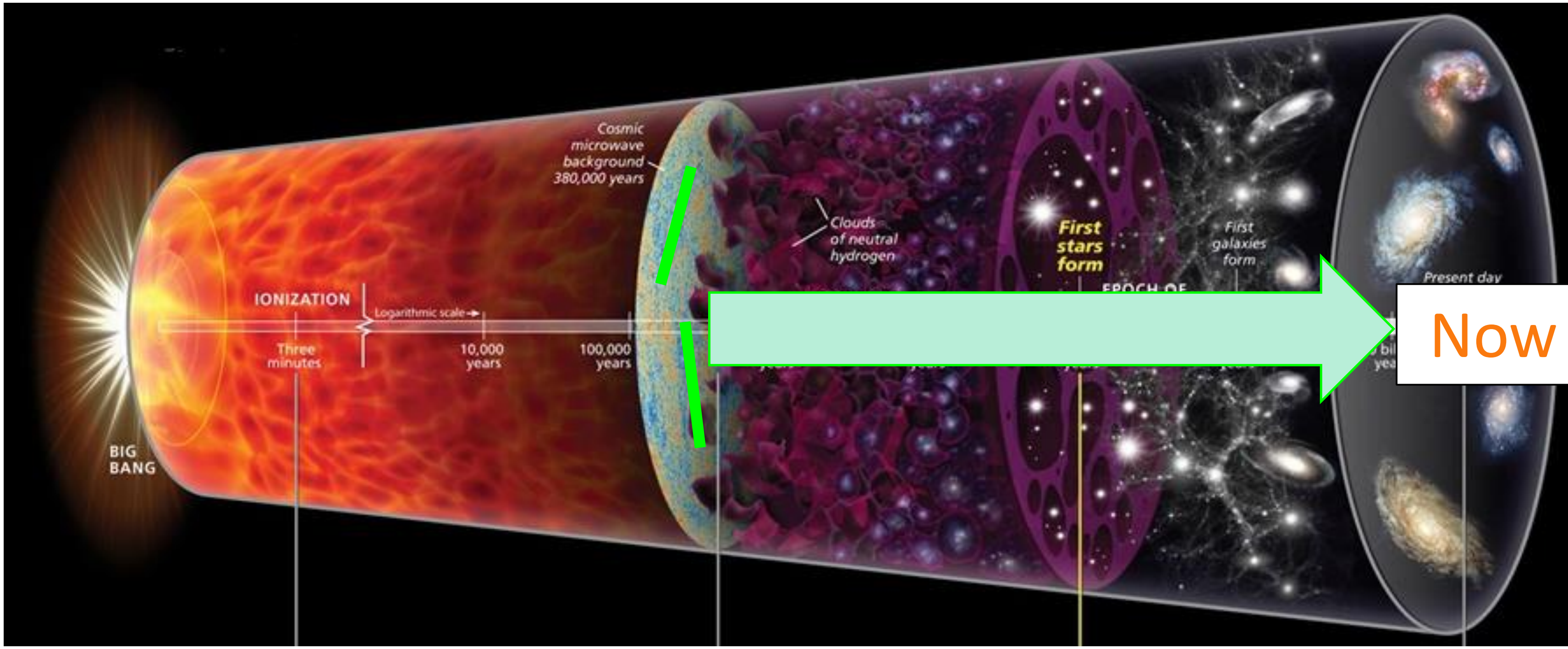
by Wayne Hu

Linear polarization from gravitational waves

Quadrupole creates linear polarization



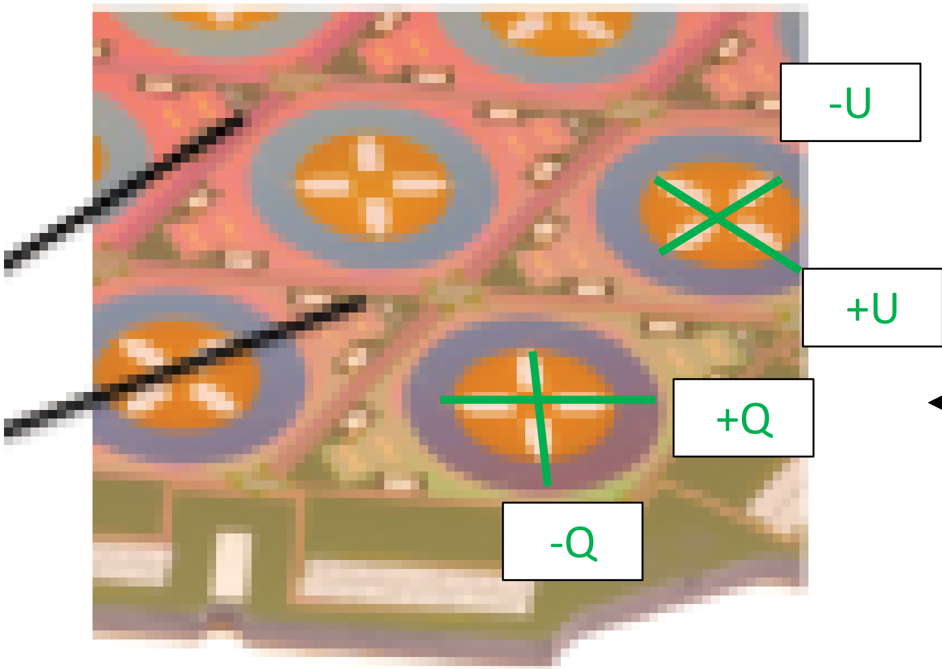
Cosmic microwave background

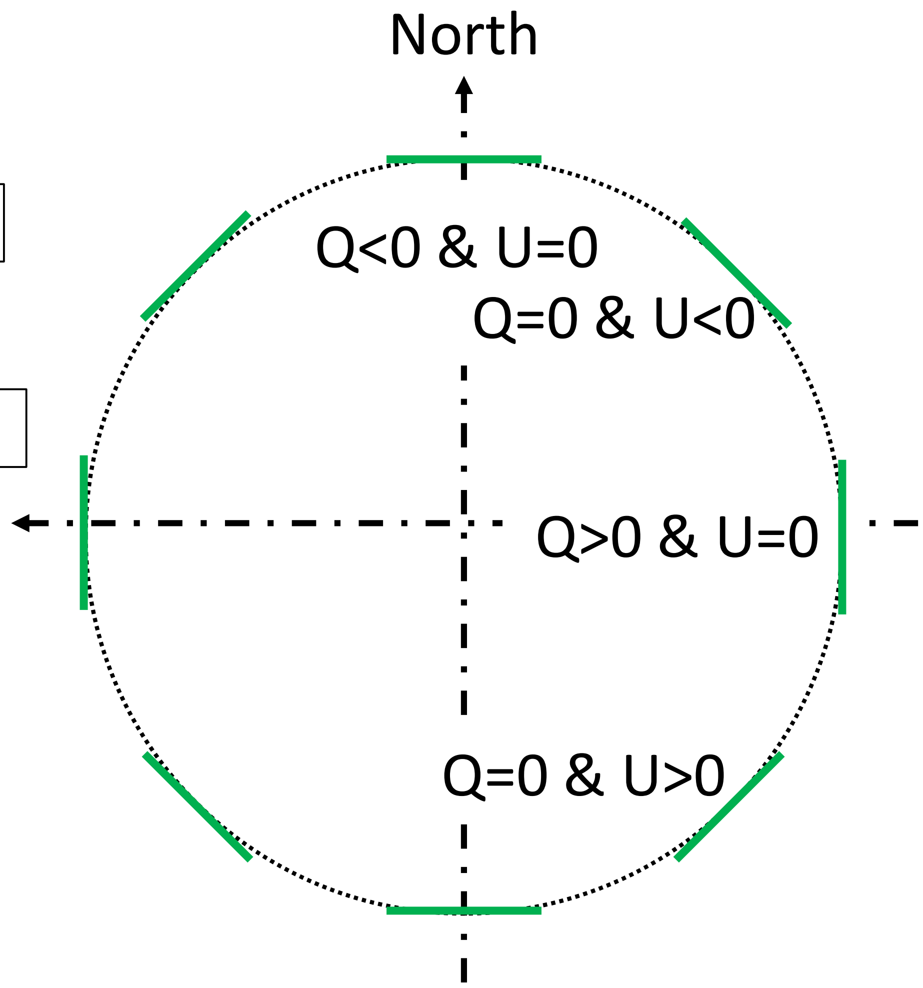


Credit: Roen Kelly, Discovermagazine

Last scatterings at the recombination era imprint the quadrupole anisotropy on CMB

Stokes parameter

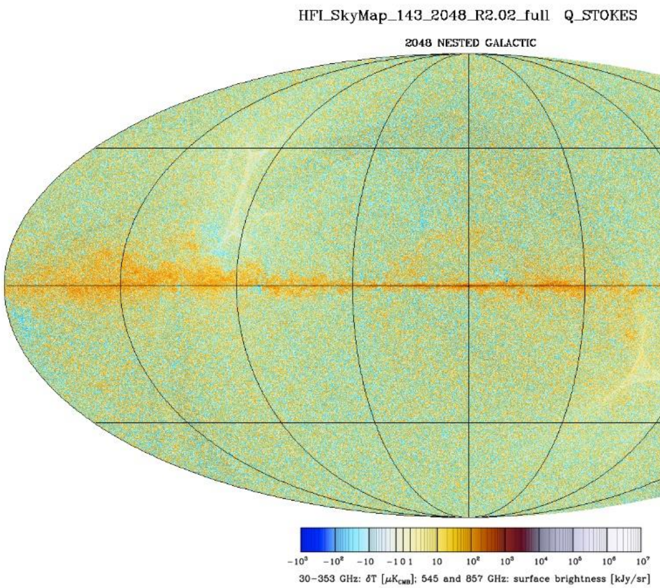




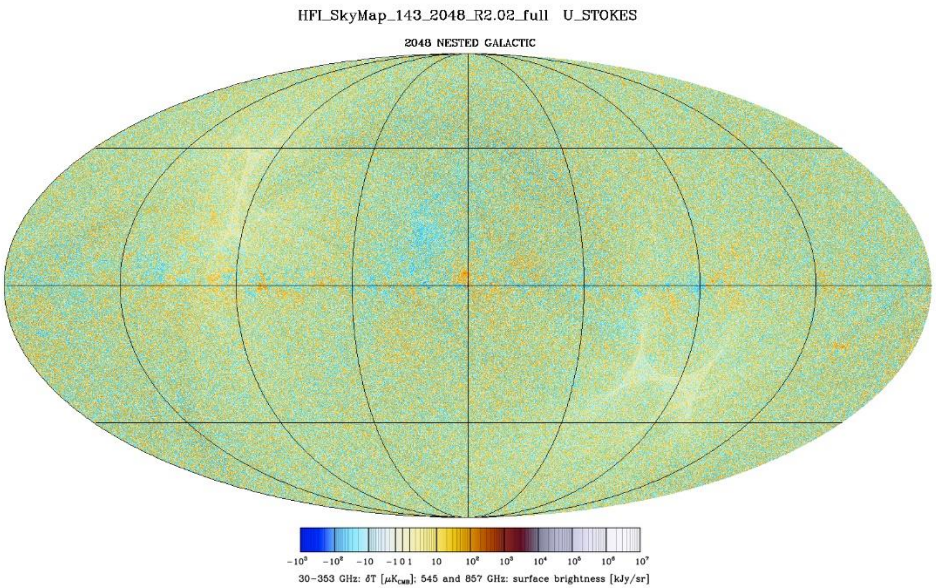
Linear polarization is measured as Stokes parameters

Map

Planck Map of 143 GHz channel



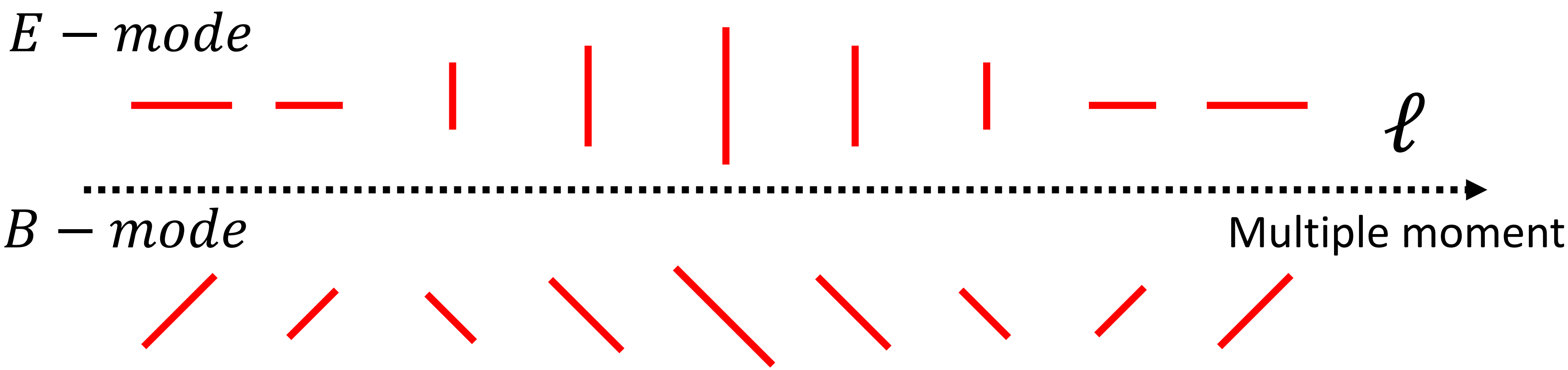
Stokes Q



Stokes U

E-mode and *B*-mode

$$E(\ell) \pm iB(\ell) = e^{\mp 2i\phi_\ell} \int d\hat{\boldsymbol{n}}\,[Q(\hat{\boldsymbol{n}}) + iU(\hat{\boldsymbol{n}})]e^{-i\ell\cdot\hat{\boldsymbol{n}}}$$



Rotation invariant *E*- and *B*- modes are used for the analysis of CMB polarization

E-mode and *B*-mode polarization

Scalar perturbation

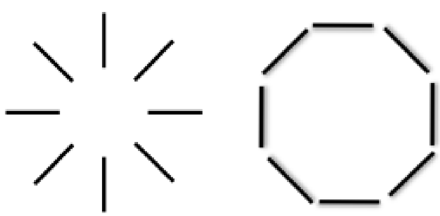
- **density**

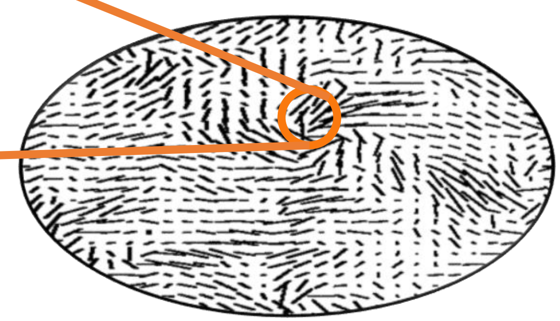
Tensor perturbation

- **Primordial gravitational waves**

E-mode

B-mode





Divergence,
even parity

Curl,
odd parity

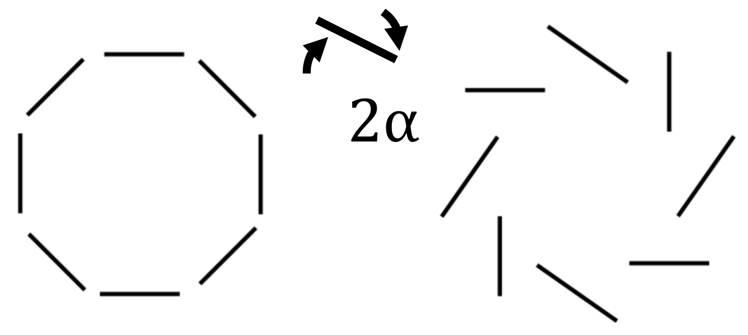
r = (tensor perturbation)/(scalar perturbation)

B-mode search is needed to determine r!

Next: Contents of the paper

Introduction:

- Miscalibration of detector rotation angle (α) creates spurious B mode from E mode



$$C_\ell^{BB,\mathrm{o}} = C_\ell^{EE}\sin^2(2\alpha) + C_\ell^{BB}\cos^2(2\alpha)$$

observed

We need to estimate α and calibrate rotation angle

- In past experiments, this α was calculated assuming EB correlation of CMB is zero

$$C_\ell^{EB,\mathrm{o}} = \frac{1}{2}\left(C_\ell^{EE} - C_\ell^{BB}\right)\sin(4\alpha)$$

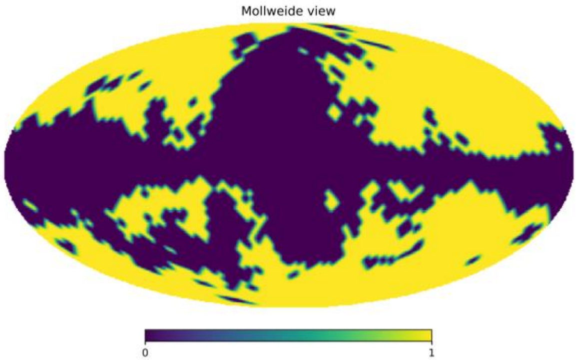
From theory

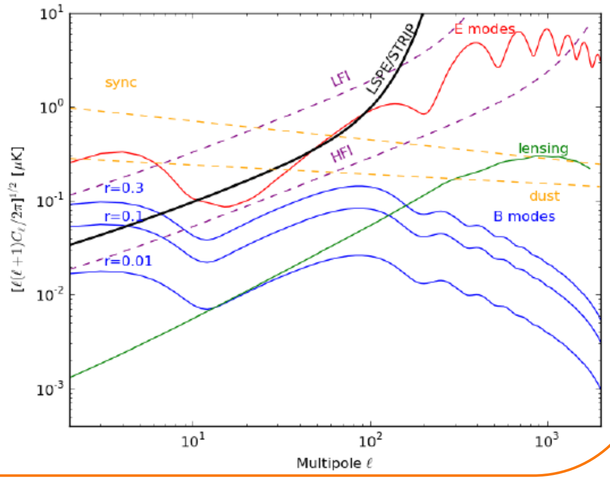
Brian G. Keating et al.（2013）

(Assumption in the method)

CMB is dominant

- Only can be used in CMB channel?
- Need to mask foreground





Cosmic birefringence is negligible

- Create EB correlation
- Explain afterwards

Review the relation from the beginning

We started from the coefficients of spherical harmonics:

$$E^{\mathrm{o}}_{\ell,m} = E_{\ell,m}\cos(2\alpha) - B_{\ell,m}\sin(2\alpha),$$

$$B^{\mathrm{o}}_{\ell,m} = E_{\ell,m}\sin(2\alpha) + B_{\ell,m}\cos(2\alpha),$$

From these equations, we find

$$C_\ell^{EB,\mathrm{o}} = \frac{1}{2}\left(C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}}\right)\tan(4\alpha) + \frac{C_\ell^{EB}}{\cos(4\alpha)}\,.$$

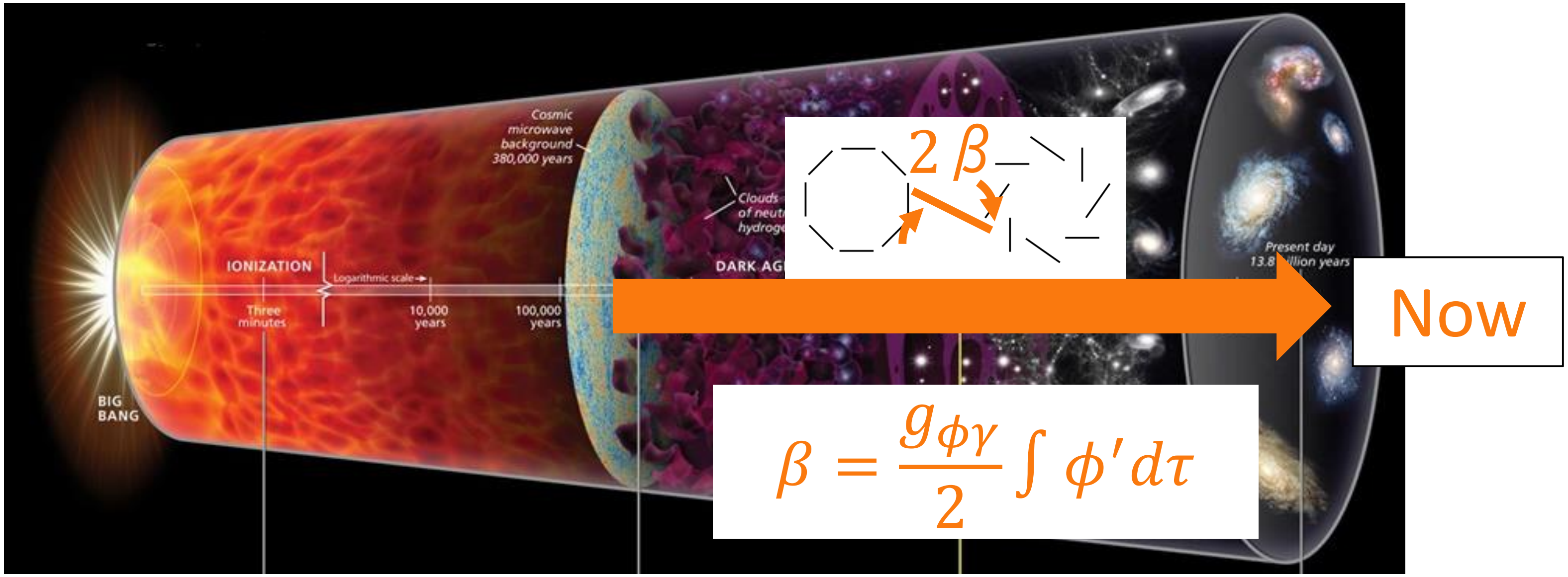
G. B. Zhao et al.（2015）

Our work

- We can estimate α only with observed data
- If we assume theory CMB power spectra, we can estimate an additional angle!

Cosmological birefringence

During the long travel from recombination era, CMB can be rotated by some physics (e.g. axionic fields)



Credit: Roen Kelly, Discovermagazine

In such case,

- Foreground term: rotated only with α
- CMB term: rotated with $\alpha + \beta$

Equations including birefringence rotation:

The coefficients becomes

$$E^{\mathrm{o}}_{\ell,m} = E^{\mathrm{fg}}_{\ell,m}\cos(2\alpha) - B^{\mathrm{fg}}_{\ell,m}\sin(2\alpha) + E^{\mathrm{CMB}}_{\ell,m}\cos(2\alpha+2\beta) - B^{\mathrm{CMB}}_{\ell,m}\sin(2\alpha+2\beta)$$

$$B^{\mathrm{o}}_{\ell,m} = E^{\mathrm{fg}}_{\ell,m}\sin(2\alpha) + B^{\mathrm{fg}}_{\ell,m}\cos(2\alpha) + E^{\mathrm{CMB}}_{\ell,m}\sin(2\alpha+2\beta) + B^{\mathrm{CMB}}_{\ell,m}\cos(2\alpha+2\beta)$$

From them, we derived

$$\begin{aligned}
&\left(C_\ell^{EB,\mathrm{o}} - \frac{\tan(4\alpha)}{2}\left(C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}}\right)\right)\cos(4\alpha) = \\
&\quad (C_\ell^{EE,\mathrm{CMB}} - C_\ell^{BB,\mathrm{CMB}})\sin(4\beta)/2 \\
&\quad - (C_\ell^{EE,\mathrm{N}} - C_\ell^{BB,\mathrm{N}})\sin(4\alpha)/2 \\
&\quad + C_\ell^{EB,\mathrm{fg}} + C_\ell^{EB,\mathrm{N}}\cos(4\alpha) + C_\ell^{EB,\mathrm{CMB}}\cos(4\beta) \\
&\quad + (C_\ell^{E^{\mathrm{fg}}B^{\mathrm{CMB}}} + C_\ell^{E^{\mathrm{CMB}}B^{\mathrm{fg}}})\cos(2\beta) + (C_\ell^{E^{\mathrm{fg}}E^{\mathrm{CMB}}} - C_\ell^{B^{\mathrm{fg}}B^{\mathrm{CMB}}})\sin(2\beta) \\
&\quad + (C_\ell^{E^{\mathrm{fg}}B^{\mathrm{N}}} + C_\ell^{B^{\mathrm{fg}}E^{\mathrm{N}}})\cos(2\alpha) - (C_\ell^{E^{\mathrm{fg}}E^{\mathrm{N}}} - C_\ell^{B^{\mathrm{fg}}B^{\mathrm{N}}})\sin(2\alpha) \\
&\quad + (C_\ell^{E^{\mathrm{CMB}}B^{\mathrm{N}}} + C_\ell^{B^{\mathrm{CMB}}E^{\mathrm{N}}})\cos(2\alpha-2\beta) \\
&\quad - (C_\ell^{E^{\mathrm{CMB}}E^{\mathrm{N}}} - C_\ell^{B^{\mathrm{CMB}}B^{\mathrm{N}}})\sin(2\alpha-2\beta)\,.
\end{aligned}$$

Equations including birefringence rotation:

If we take ensemble average

$$\langle C_\ell^{EB,\mathrm{o}} \rangle = \frac{\tan(4\alpha)}{2}\left(\langle C_\ell^{EE,\mathrm{o}}\rangle - \langle C_\ell^{BB,\mathrm{o}}\rangle\right) + \frac{\sin(4\beta)}{2\cos(4\alpha)}\left(\langle C_\ell^{EE,\mathrm{CMB}}\rangle - \langle C_\ell^{BB,\mathrm{CMB}}\rangle\right)$$

$$\boxed{+\frac{1}{\cos(4\alpha)}\langle C_\ell^{EB,\mathrm{fg}}\rangle + \frac{\cos(4\beta)}{\cos(4\alpha)}\langle C_\ell^{EB,\mathrm{CMB}}\rangle .}$$

Assume these to be zero
Revisit

Therefore, we can determine both miscalibration angle and birefringence-rotation angle simultaneously!

Log-likelihood

If we take ensemble average

$$\langle C_\ell^{EB,\mathrm{o}}\rangle = \frac{\tan(4\alpha)}{2}\left(\langle C_\ell^{EE,\mathrm{o}}\rangle - \langle C_\ell^{BB,\mathrm{o}}\rangle\right) + \frac{\sin(4\beta)}{2\cos(4\alpha)}\left(\langle C_\ell^{EE,\mathrm{CMB}}\rangle - \langle C_\ell^{BB,\mathrm{CMB}}\rangle\right)$$
$$\boxed{+\frac{1}{\cos(4\alpha)}\langle C_\ell^{EB,\mathrm{fg}}\rangle + \frac{\cos(4\beta)}{\cos(4\alpha)}\langle C_\ell^{EB,\mathrm{CMB}}\rangle\,.}$$

Assume these to be zero

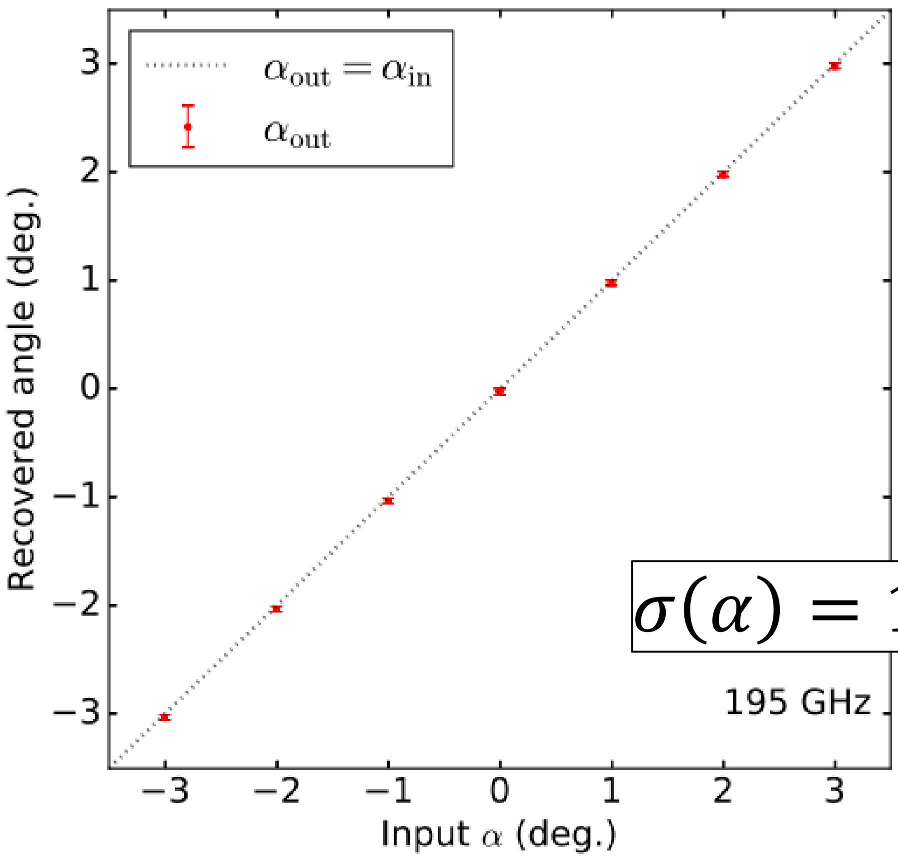
Revisit

$$-2\ln\mathcal{L} = \sum_{\ell=2}^{\ell_{\max}} \frac{\left[C_\ell^{EB,\mathrm{o}} - \frac{\tan(4\alpha)}{2}\left(C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}}\right) - \frac{\sin(4\beta)}{2\cos(4\alpha)}\left(C_\ell^{EE,\mathrm{CMB}} - C_\ell^{BB,\mathrm{CMB}}\right)\right]^2}{\mathrm{Var}\left(C_\ell^{EB,\mathrm{o}} - \frac{\tan(4\alpha)}{2}\left(C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}}\right)\right)}$$

Minimise this log-likelihood to determine α and β

Before the simultaneous determination: α only case

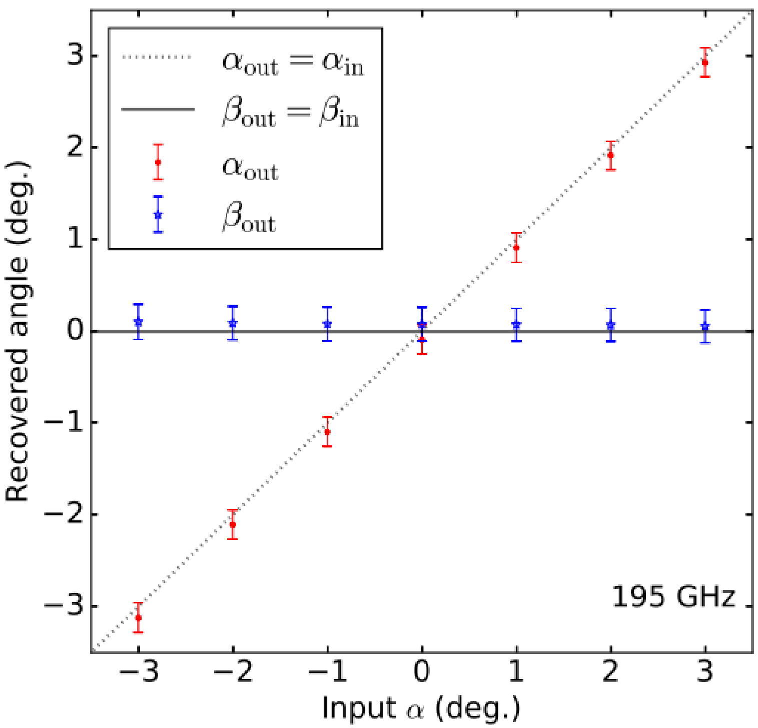
Assuming β=0, we can determine only α as the previous method



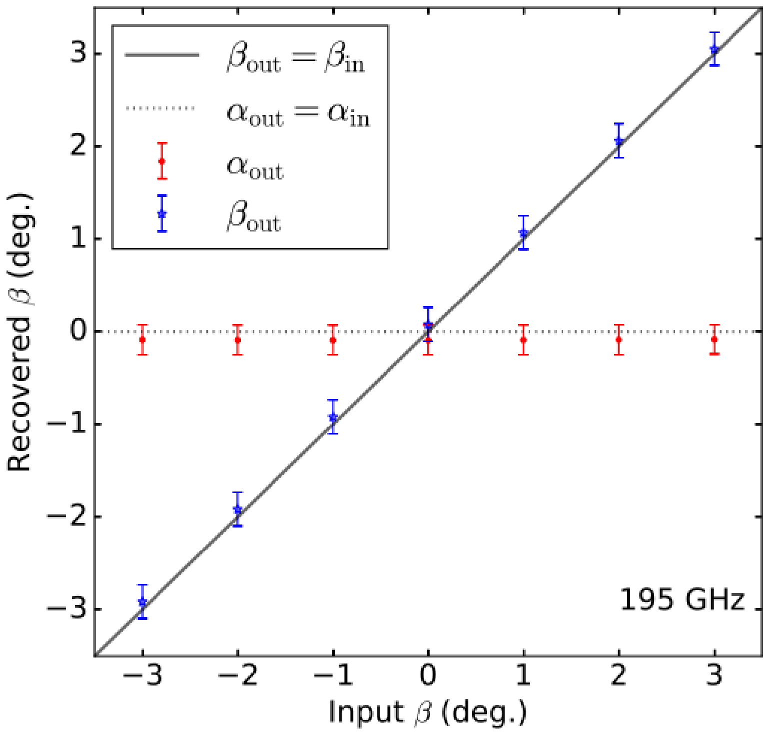
$\sigma(\alpha) = 1.5$ arcmin

Simultaneous determination

input $\beta = 0$ case



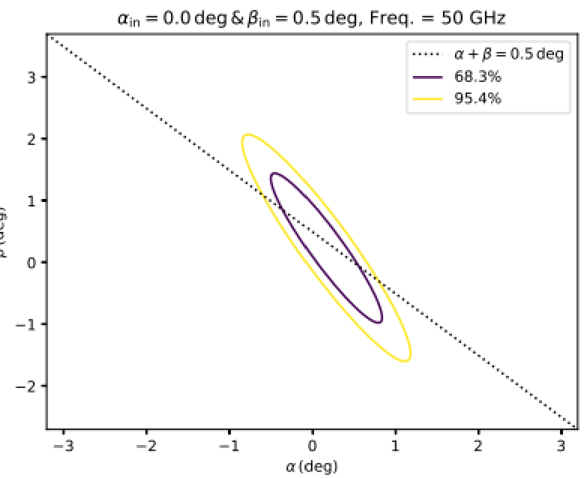
input $\alpha = 0$ case



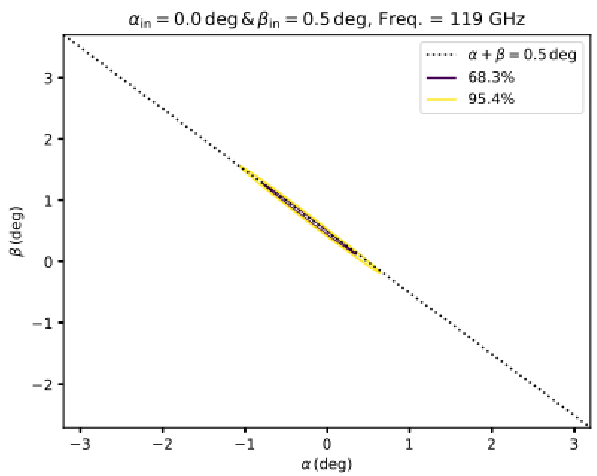
$\sigma(\alpha) = 9.6$ arcmin and $\sigma(\beta) = 11$ arcmin

Correlation between α and β

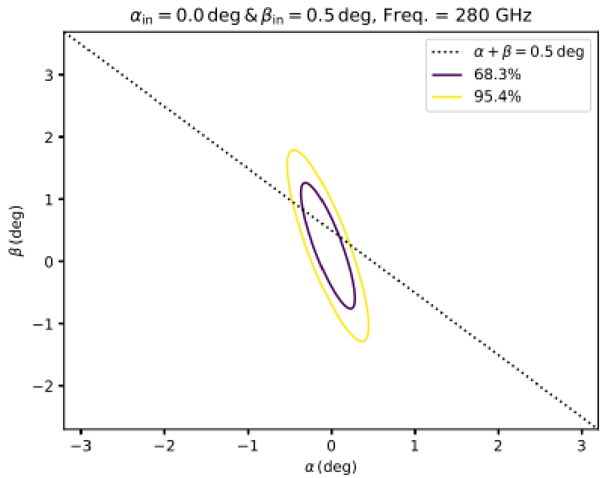
$$E^{\mathrm{o}}_{\ell,m} = E^{\mathrm{fg}}_{\ell,m}\cos(2\alpha) - B^{\mathrm{fg}}_{\ell,m}\sin(2\alpha) + E^{\mathrm{CMB}}_{\ell,m}\cos(2\alpha+2\beta) - B^{\mathrm{CMB}}_{\ell,m}\sin(2\alpha+2\beta)$$



Synchrotron channel



CMB channel



dust channel

- CMB has a power to determine α+β
- FG has a power to determine α

Summary

There was a consensus in the CMB community that the measurement of the cosmic birefringence and the polarization angle calibration cannot be done simultaneously.

We have shown that this is not the case.

We can determine the birefringence angle of order 10 arcmin.

This is a great news!

Assumed specification

Table 1: Polarisation sensitivity and beam size of the LiteBIRD telescopes [27]

Frequency (GHz)	Polarisation Sensitivity (μK$'$)	Beam Size in FWHM (arcmin)
40	37.5	69
50	24.0	56
60	19.9	48
68	16.2	43
78	13.5	39
89	11.7	35
100	9.2	29
119	7.6	25
140	5.9	23
166	6.5	21
195	5.8	20
235	7.7	19
280	13.2	24
337	19.5	20
402	37.5	17

Foreground EB correlation

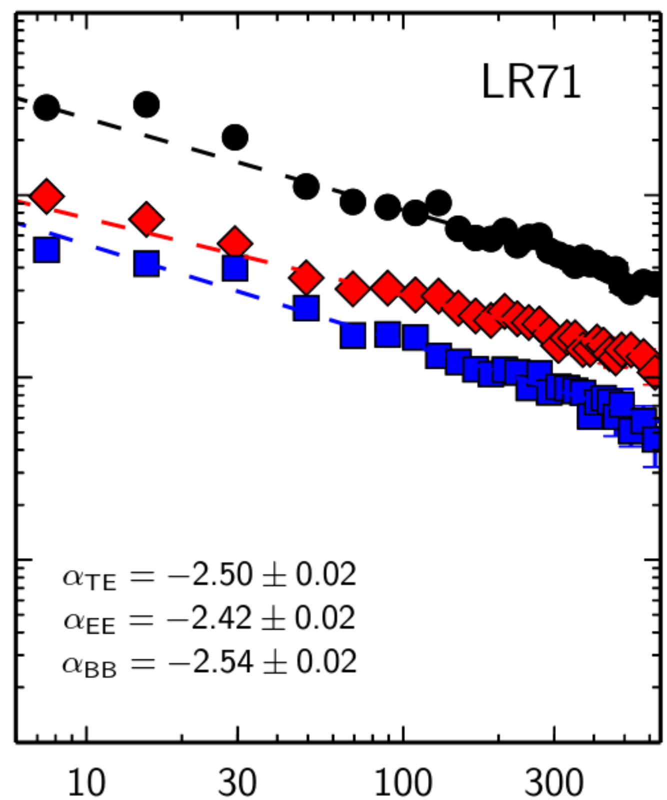
M. H. Abitbol, J. C. Hill, and B. R. Johnson, Mon. Not. Roy. Astron. Soc., 457(2), 1796–1803 (2016),
200 arXiv:1512.06834.

$$C_\ell^{dust,XY} = A^{XY}\left(\frac{\ell}{80}\right)^{-2.42} \quad (1)$$

$$C_{\ell,mult}^{dust,XY} = mC_\ell^{dust,XY} \quad (2)$$

$$C_{\ell,corr}^{dust,ZB} = f_c\sqrt{C_{\ell,mult}^{dust,ZZ}C_{\ell,mult}^{dust,BB}}\,, \quad (3)$$

where A^{XY} is the best-fitting amplitude, m is a multiplicative factor, f_c is a correlation fraction, $X, Y \in \{T, E, B\}$ and $Z \in \{T, E\}$.



Planck Collaboration XI (2018), arXiv:1801.04945.

Foreground

$$\langle C_\ell^{EB,\mathrm{fg}}\rangle = \frac{f_c\sqrt{\xi}}{1-\xi}\left(\langle C_\ell^{EE,\mathrm{fg}}\rangle - \langle C_\ell^{BB,\mathrm{fg}}\rangle\right).$$

$\rightarrow$

$$\frac{\sin(4\gamma)}{2}\left(\left\langle C_\ell^{EE,fg}\right\rangle - \left\langle C_\ell^{BB,fg}\right\rangle\right)$$

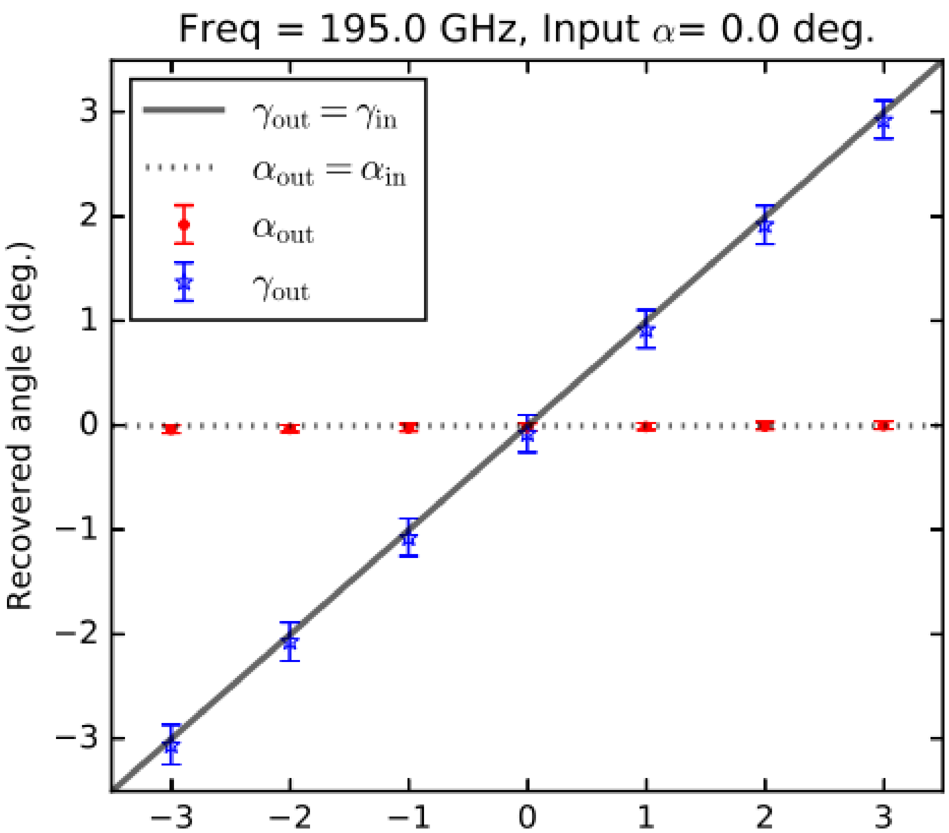
$$E_{\ell,m}^{\mathrm{o,fg}} = E_{\ell,m}^{\mathrm{fg}}\cos(2\gamma) - B_{\ell,m}^{\mathrm{fg}}\sin(2\gamma),$$
$$B_{\ell,m}^{\mathrm{o,fg}} = E_{\ell,m}^{\mathrm{fg}}\sin(2\gamma) + B_{\ell,m}^{\mathrm{fg}}\cos(2\gamma),$$

Replace α-> α+γ
β->-γ
in birefringence
estimation

$$\begin{aligned}
&\left(C_\ell^{EB,\mathrm{o}} - \frac{\tan(4\alpha)}{2}\left(C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}}\right)\right)\cos(4\alpha+4\gamma) = \\
&\quad (C_\ell^{EE,\mathrm{CMB}} - C_\ell^{BB,\mathrm{CMB}})\sin(-4\gamma)/2 \\
&\quad + C_\ell^{EB,\mathrm{fg}} + C_\ell^{EB,\mathrm{N}}\cos(4\alpha+4\gamma) + C_\ell^{EB,\mathrm{CMB}}\cos(-4\gamma) \\
&\quad - (C_\ell^{EE,\mathrm{N}} - C_\ell^{BB,\mathrm{N}})\sin(4\alpha+4\gamma)/2 \\
&\quad + (C_\ell^{E^{\mathrm{fg}}B^{\mathrm{CMB}}} + C_\ell^{E^{\mathrm{CMB}}B^{\mathrm{fg}}})\cos(-2\gamma) + (C_\ell^{E^{\mathrm{fg}}E^{\mathrm{CMB}}} - C_\ell^{B^{\mathrm{fg}}B^{\mathrm{CMB}}})\sin(-2\gamma) \\
&\quad + (C_\ell^{E^{\mathrm{fg}}B^{\mathrm{N}}} + C_\ell^{B^{\mathrm{fg}}E^{\mathrm{N}}})\cos(2\alpha+2\gamma) - (C_\ell^{E^{\mathrm{fg}}E^{\mathrm{N}}} - C_\ell^{B^{\mathrm{fg}}B^{\mathrm{N}}})\sin(2\alpha+2\gamma) \\
&\quad + (C_\ell^{E^{\mathrm{CMB}}B^{\mathrm{N}}} + C_\ell^{B^{\mathrm{CMB}}E^{\mathrm{N}}})\cos(2\alpha+4\gamma) \\
&\quad - (C_\ell^{E^{\mathrm{CMB}}E^{\mathrm{N}}} - C_\ell^{B^{\mathrm{CMB}}B^{\mathrm{N}}})\sin(2\alpha+4\gamma)
\end{aligned}$$

Results of FG estimation

Replace α-> α+γ
β->-γ
in birefringence estimation



Because β is the same in any frequency, we can still determine β even with FG EB correlation.

Variance

$$\begin{aligned}
&\mathrm{Var}\left[C_\ell^{EB,\mathrm{o}} - (C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}})\tan(4\alpha)/2\right] \\
&= \langle\left[C_\ell^{EB,\mathrm{o}} - (C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}})\tan(4\alpha)/2\right]^2\rangle - \langle C_\ell^{EB,\mathrm{o}} - (C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}})\tan(4\alpha)/2\rangle^2 \\
&= \frac{1}{2\ell+1}\langle C_\ell^{EE}\rangle\langle C_\ell^{BB}\rangle + \frac{\tan^2(4\alpha)}{4}\frac{2}{2\ell+1}\left(\langle C_\ell^{EE}\rangle^2 + \langle C_\ell^{BB}\rangle^2\right) \\
&\quad - \tan(4\alpha)\frac{2}{2\ell+1}\langle C_\ell^{EB}\rangle\left(\langle C_\ell^{EE}\rangle - \langle C_\ell^{BB}\rangle\right) + \frac{1}{2\ell+1}\left(1-\tan^2(4\alpha)\right)\langle C_\ell^{EB}\rangle^2. \qquad \text{(A1)}
\end{aligned}$$

$$\begin{aligned}
&\mathrm{Var}\left[C_\ell^{EB,\mathrm{o}} - (C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}})\tan(4\alpha)/2\right] \\
&\approx \frac{1}{2\ell+1}C_\ell^{EE,\mathrm{o}}C_\ell^{BB,\mathrm{o}} + \frac{\tan^2(4\alpha)}{4}\frac{2}{2\ell+1}\left[(C_\ell^{EE,\mathrm{o}})^2 + (C_\ell^{BB,\mathrm{o}})^2\right] \\
&\quad - \tan(4\alpha)\frac{2}{2\ell+1}C_\ell^{EB,\mathrm{o}}\left(C_\ell^{EE,\mathrm{o}} - C_\ell^{BB,\mathrm{o}}\right).
\end{aligned}$$