1 Piman-Yor Diffusion Trees

1.1 Posterior of $p(t_v|\mathcal{T})$

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\begin{aligned}
p(t_v|\mathcal{T}) &= c(1-t_v)^{cJ^{\theta,\alpha}_{\mathbf{n}_v}-1} && (1)\\
p(\mathbf{z}_v|\mathbf{z}_u,\sigma^2(t_v-t_u)\mathbf{I}) &= (2\pi\sigma^2(t_v-t_u))^{-\frac{D}{2}}\exp\Big(-\frac{1}{2}\frac{\parallel \mathbf{z}_v-\mathbf{z}_u\parallel^2}{\sigma^2(t_v-t_u)}\Big) && (2)\\
p(\mathbf{z}_k|\mathbf{z}_v,\sigma^2(t_k-t_v)\mathbf{I}) &= (2\pi\sigma^2(t_k-t_v))^{-\frac{D}{2}}\exp\Big(-\frac{1}{2}\frac{\parallel \mathbf{z}_k-\mathbf{z}_v\parallel^2}{\sigma^2(t_k-t_v)}\Big) && (3)\\
p(t_v,\mathbf{z},\sigma|\mathcal{T}) &\propto \exp\Big\{(cJ^{\theta,\alpha}_{\mathbf{n}_v}-1)\ln(1-t_v)\\
&- \frac{D}{2}\big(\ln(t_v-t_u)+\sum_k \ln(t_k-t_v)\big)\\
&- \frac{\parallel \mathbf{z}_v-\mathbf{z}_u\parallel^2}{2\sigma^2}\frac{1}{t_v-t_u}-\sum_k\frac{\parallel \mathbf{z}_k-\mathbf{z}_v\parallel^2}{2\sigma^2}\frac{1}{t_k-t_v}\Big\}\\
&= u(t_v) && (4)\\
\frac{du}{dt_v} &= -\frac{cJ^{\theta,\alpha}_{\mathbf{n}_v}-1}{1-t_v}-\frac{D}{2}\big(\frac{1}{t_v-t_u}-\sum_k\frac{1}{t_k-t_v}\big)\\
&+ \frac{\parallel \mathbf{z}_v-\mathbf{z}_u\parallel^2}{2\sigma^2}\frac{1}{(t_v-t_u)^2}-\sum_k\frac{\parallel \mathbf{z}_k-\mathbf{z}_v\parallel^2}{2\sigma^2}\frac{1}{(t_k-t_v)^2}\\
&= A(t_v) && (5)\\
\int \exp\big\{u(t_v)\big\}A(t_v)^{-1}du &= \exp\big\{u(t_v)\big\}A(t_v)^{-1}\Big|_{t_v=0}^{t_v=1}\\
&= ...\\
&= Z && (6)
\end{aligned}
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