

1 Piman-Yor Diffusion Trees

1.1 Modelling of PYDT

$$c \sim G(c|a_c, b_c) \quad (1)$$

$$\frac{1}{\sigma^2} \sim G(\sigma^2|a_{\sigma^2}, b_{\sigma^2}) \quad (2)$$

$$\alpha \sim \text{Beta}(\alpha|a_\alpha, b_\alpha) \quad (3)$$

$$\theta \sim G(\theta|a_\theta, b_\theta) \quad (4)$$

$$p(t_v|c, \mathcal{T}) = c(1 - t_v)^{cJ_{\mathbf{n}_v}^{\theta, \alpha} - 1} \quad (5)$$

$$p(\mathbf{z}_v|\mathbf{z}_u, \sigma^2(t_v - t_u)\mathbf{I}) = (2\pi\sigma^2(t_v - t_u))^{-\frac{D}{2}} \exp\left(-\frac{1}{2} \frac{\|\mathbf{z}_v - \mathbf{z}_u\|^2}{\sigma^2(t_v - t_u)}\right) \quad (6)$$

$$p(\mathbf{z}_k|\mathbf{z}_v, \sigma^2(t_k - t_v)\mathbf{I}) = (2\pi\sigma^2(t_k - t_v))^{-\frac{D}{2}} \exp\left(-\frac{1}{2} \frac{\|\mathbf{z}_k - \mathbf{z}_v\|^2}{\sigma^2(t_k - t_v)}\right) \quad (7)$$

$$(8)$$

1.2 Posterior of $p(t_v|\mathcal{T})$

$$\begin{aligned}
p(t_v, \mathbf{z}_{\{u,v,k\}}, \sigma^2 | c, \mathcal{T}) &= c(2\pi\sigma^2)^{-\frac{D(K+1)}{2}} \exp \left\{ (cJ_{\mathbf{n}_v}^{\theta, \alpha} - 1) \ln(1 - t_v) \right. \\
&\quad - \frac{D}{2} (\ln(t_v - t_u) + \sum_k \ln(t_k - t_v)) \\
&\quad \left. - \frac{\|\mathbf{z}_v - \mathbf{z}_u\|^2}{2\sigma^2} \frac{1}{t_v - t_u} - \sum_k \frac{\|\mathbf{z}_k - \mathbf{z}_v\|^2}{2\sigma^2} \frac{1}{t_k - t_v} \right\} \\
&= C_{\sigma^2} \exp \{u(t_v)\} \tag{9}
\end{aligned}$$

$$\begin{aligned}
\frac{du}{dt_v} &= -\frac{cJ_{\mathbf{n}_v}^{\theta, \alpha} - 1}{1 - t_v} - \frac{D}{2} \left(\frac{1}{t_v - t_u} - \sum_k \frac{1}{t_k - t_v} \right) \\
&\quad + \frac{\|\mathbf{z}_v - \mathbf{z}_u\|^2}{2\sigma^2} \frac{1}{(t_v - t_u)^2} - \sum_k \frac{\|\mathbf{z}_k - \mathbf{z}_v\|^2}{2\sigma^2} \frac{1}{(t_k - t_v)^2} \\
&= -\frac{cJ_{\mathbf{n}_v}^{\theta, \alpha} - 1}{1 - t_v} - \frac{1}{2(t_v - t_u)^2} \left(D(t_v - t_u) - \frac{\|\mathbf{z}_v - \mathbf{z}_u\|^2}{\sigma^2} \right) \\
&\quad + \sum_k \frac{1}{2(t_k - t_v)^2} \left(D(t_k - t_v) - \frac{\|\mathbf{z}_k - \mathbf{z}_v\|^2}{\sigma^2} \right) \\
&= A(t_v) \tag{10}
\end{aligned}$$

$$\begin{aligned}
C_{\sigma^2} A(t_v)^{-1} \int \exp \{u(t_v)\} du &= C_{\sigma^2} A(t_v)^{-1} \exp \{u(t_v)\} \Big|_{t_v=t_u}^{t_v=t_{k'}}, \text{ where } t_{k'} = \min(t_k) \\
&= A(t_{k'})^{-1} p(t_{k'}, \mathbf{z}_{\{u,v,k\}}, \sigma^2 | c, \mathcal{T}) - A(t_u)^{-1} p(t_u, \mathbf{z}_{\{u,v,k\}}, \sigma^2 | c, \mathcal{T}) \\
&= Z \tag{11}
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}_{p(t_v | \mathbf{z}_{\{u,v,k\}}, \sigma^2, c, \mathcal{T})}[t_v] &= \frac{1}{Z} \int t_v p(t_v, \mathbf{z}_{\{u,v,k\}}, \sigma^2 | c, \mathcal{T}) dt_v \\
&= \frac{1}{Z} C_{\sigma^2} A_{\mu}(t_v)^{-1} \int \exp \{u_{\mu}(t_v)\} du \tag{12}
\end{aligned}$$