Searching

Task:

9	6	5	3	2	8	7	2	5

Find an element in an array, if it exists.
Eg: find 2. Or find 13.

Linear search

```
/* returns index of target if found in A, -1 otherwise. */
int linear_search(int target, int A[], int n) {
    for (int i = 0; i < n; i++) {
        if (A[i] == target) return i;
    }
    return NOT_FOUND;  // #define NOT_FOUND -1
}
```

Linear search

Find 2.

5 comparisons.

Linear search

Best case? Target is first item of array.
1 comparison => **O(1)**

(9 found)

Worst case? Target not in the array.
n comparisons => **O(n)**

(1 not found)

Average case? **O(n)**

The element is not in the array (likely) => O(n)
If in the array, on average n/2 comparisons => O(n)

Linear Search: Avg Case

Assuming array is **randomly distributed**, and 5 is **guaranteed to be in the array**:

5 has probability 1/n of being in any position of the array.

If **5** is in array index *i*, it will require *i* comparisons to find it.

Expected (mean) number of comparisons:

= sum(val * prob) (summing over vals)
= sum(i / n) (summing over i = 1 .. n)
= sum (i) / n (n is invariant)

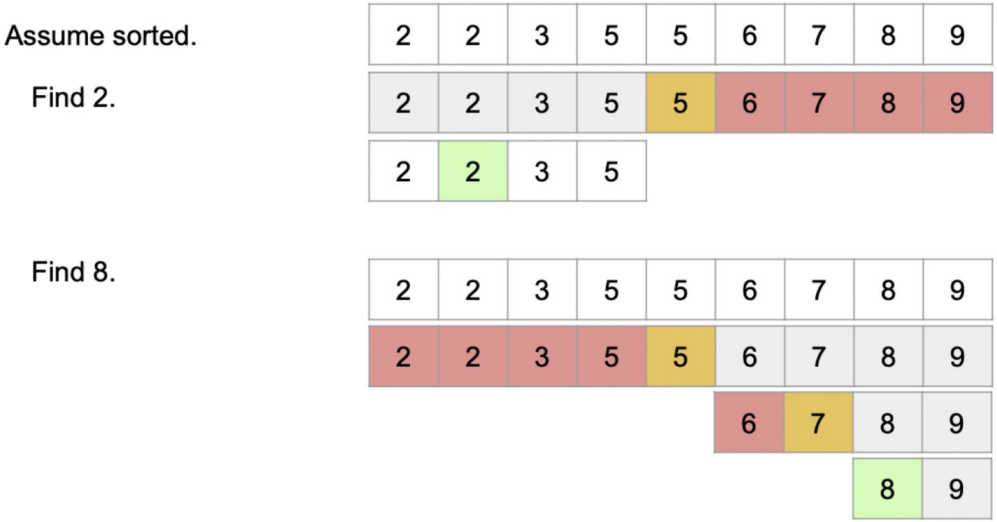
"sum(i)" = $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$, = $n^2/2 + n/2$

= $(n^2/2 + n/2) / n$
= $n/2 + ½$
= O(n)

Binary Search

Search by repeatedly dividing the search interval by half.

Requires **sorted data**, but cost of O(log n) (< O(n) linear search)



Binary Search

```
int binary_search(int A[], int n, int target){
    int lo = 0, hi = n;
    while (lo < hi) {
        int mid = (lo + hi) / 2;
        if (target > A[mid])
            lo = m + 1;
        else if (target < A[mid])
            hi = m;
        else
            return m;
    }
    return -1;
}
```

Binary Search = Time complexity

Best case: Median. => O(1)

Worst case: Not in array. => O(log n)

Average case: O(log n)

$$T(n) = \begin{cases} 1 & \text{if } n \leq 1 \\ 1 + T(\lfloor n/2 \rfloor) & \text{if } n > 1 \end{cases}$$

Recurrence relations often arise in the analysis of algorithms.

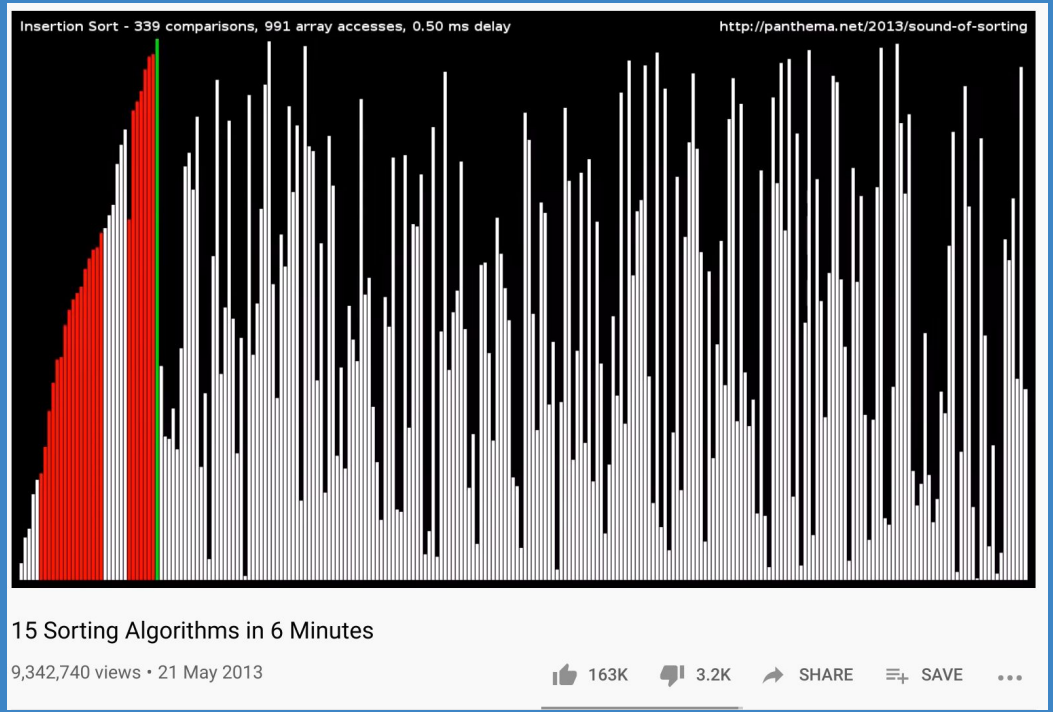
The exact solution of this one is
$T(n) = 1 + \lfloor \log_2 n \rfloor \in O(\log n)$.

Comparing search algorithms

	Linear Search	**Binary Search**
Idea	Sequential scan through array until target is found	Divide the search interval in half, repeating until target is found or have empty interval.
Requirements	*None*	**Sorted** array, items can be ordered (< = >)
Time Complexity	O(1) best (first element) O(n) worst (not found; end of array) O(n) average (not found; n/2 = O(n))	O(1) best (first element) O(log n) average and worst.
Space Complexity	O(1) (auxiliary)	O(1) (auxiliary)

O(1) auxiliary space means we do not need to

Sorting



… in ascending order ;)

https://www.youtube.com/watch?v=kPRA0W1kECg

Insertion Sort

Space Complexity:

2 auxiliary variables (i, j) => O(1) (O(n) total, store the array)

So for small arrays,
Or nearly sorted arrays
Insertion sort is pretty good!

Time Complexity:

Best case: Fully sorted! 1 2 3 4 5 6 7 8 9
=> O(n) comparisons, O(1) swaps.

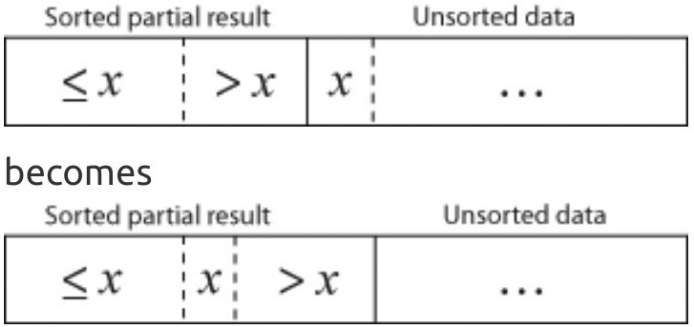
2nd Best case: Nearly sorted 1 2 3 4 5 6 7 9 8
=> approx O(n) comparisons, O(1) swaps (non-rigorous)

Worst case: Reversed array 9 8 7 6 5 4 3 2 1
=> O(n^2) comparisons, O(n^2) swaps.

Average case: O(n^2) comparisons, O(n^2) swaps

Insertion Sort

- Builds sorted array one element at a time.



Steps

1. Pick the first unsorted element, x
2. Shift it towards the left, until it is "in place"
    - "In place": element left <= x
3. Repeat until there are no more unsorted elements

Insertion Sort - Algorithm

```
void insertion_sort(int A[], int n) {
  for (int i = 1; i < n; i++) {     // i : first 'not sorted' index
    for (int j = i; j > 0; j--) {   // j : element being sorted
      if (A[j - 1] <= A[j]) {       // A[j] is in place?
         break;                     // Yep, onto next element.
      }
      int_swap(A + j - 1, A + j);   // No, move A[j] one left:
                                    //     swap A[j] and A[j-1].
    }
  }
}
```

Modified from https://people.eng.unimelb.edu.au/ammoffat/ppsaa/c/insertionsort.c

Insertion Sort - Algorithm

```
void insertion_sort(int A[], int n) {
  for (int i = 1; i < n; i++) {
    for (int j = i; j > 0 && A[j - 1] > A[j]; j--) {
      int_swap(A + j - 1, A + j);
    }
  }
}
```

Insertion Sort

Pros:

- Rather simple to implement
- Easy to interpret - it's the same way one sorts a hand of cards.
- Low memory usage
- Fast insertion for one element
- Very efficient for small datasets.
    - More efficient than mergesort / quicksort there

Cons:

- Slow for larger arrays $O(n^2)$

Selection Sort

- Builds sorted array one element at a time. .. but less efficiently.

Steps

1. Find the maximum value that is unsorted, x
2. Swap x with the first value of the unsorted region.
3. Repeat until there are no more unsorted elements

Selection Sort

Find the max in the (unsorted) array, swap it to the end. Repeat. (Or, find min and swap to the start.)

```
void selection_sort(int A[], int n) {
  for (int i = n; i > 1; i--)
      int_swap(&A[find_max_index(A, i)], &A[i - 1]);
}
int find_max_index(int A[], int n) { // ‘argmax’
  int max_i = 0;
  for (int i = 1; i < n; i++)
      if (A[i] > A[max_i]) max_i = i;
  return max_i;
}
```

Selection Sort

Space Complexity:

3 auxilliary variables (i, j) => O(1) (O(n) total, store the array)

Time Complexity:

All cases: $O(n^2)$ comparisons, O(n) swaps.

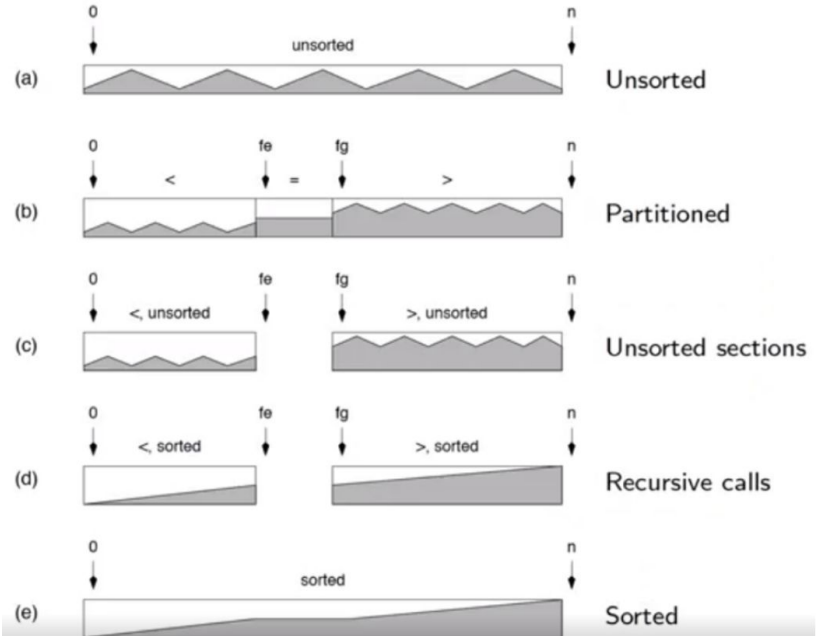
At least it's consistent.

Pros:

- Simple
- Sometimes has performance advantages (low auxilliary memory usage)
- Fewer swaps than insertion sort, minimal memory utilisation. (writes slower than reads?)

Quicksort (Hoare, 1959)

Recursively partition array so that we have ‘smaller’ items on the left of our ‘pivot’ and ‘larger’ items on the right of our pivot.

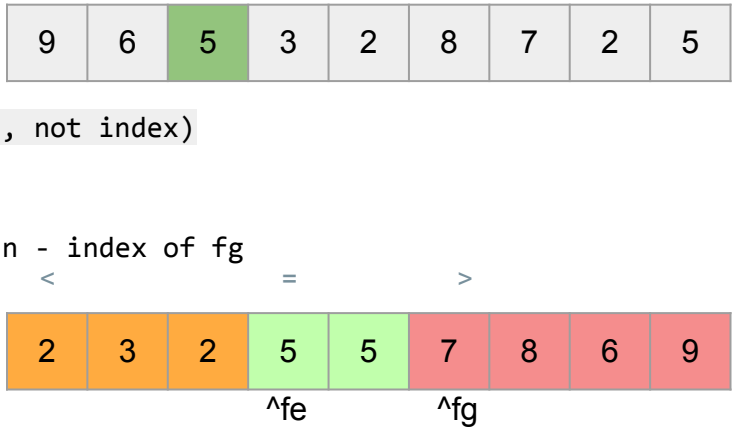


Quicksort Implementation

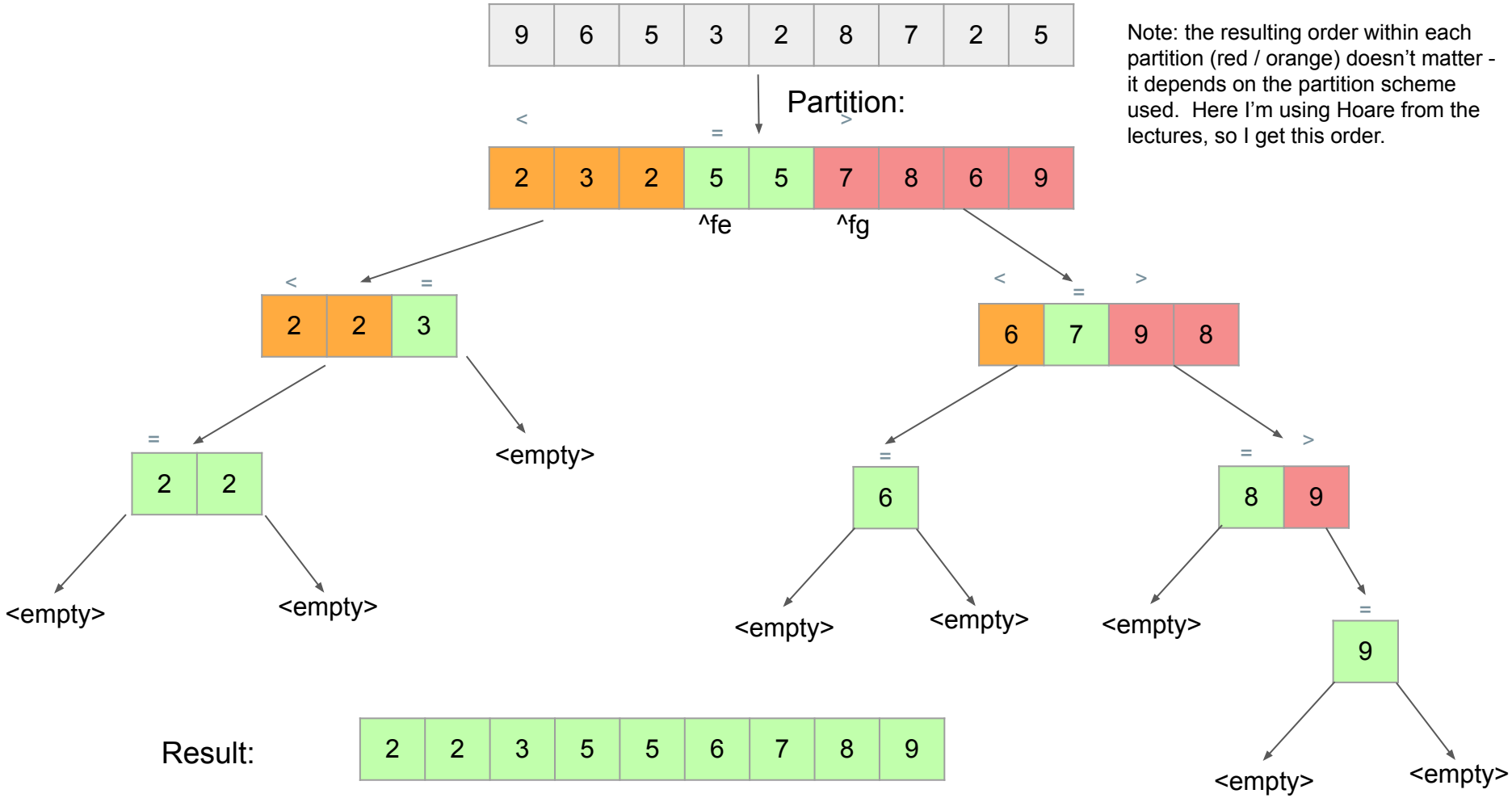
```
void quicksort(int *A, int n){
  if (n <= 1) return; // base case -- sorted
  int fe, fg;                                  // index: first equal=, first greater>
  int pivot = choose_pivot(A);                 // pick a pivot (value)
  partition(A, n, &fe, &fg, pivot);            // partition the array.  (b)
  quicksort(A, fe);                            // length of 1st partition: fe
  quicksort(A + fg, n - fg);                   // length of 2nd partition: (n - fg)
  // equals are in place -- we're done!
}
```

Quicksort

```
void quicksort(int *A, int n){
  if (n <= 1) return; // base case -- sorted
  int fe, fg;  // "first equal =", "first greater >"
  int pivot = choose_pivot(A);  // pick a pivot (value of the pivot, not index)
  partition(A, n, &fe, &fg, pivot); // partition the array.  O(n)
  quicksort(A, fe); // length of first partition is index of fe
  quicksort(A + fg, n - fg); // length of second partition is size n - index of fg
  // equals are in place -- we're done!
}
```



Now run quicksort on both partitions.

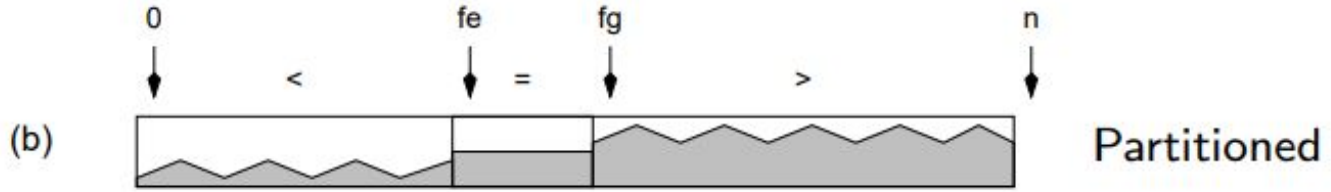


Note: the resulting order within each partition (red / orange) doesn't matter - it depends on the partition scheme used. Here I'm using Hoare from the lectures, so I get this order.

Quicksort (Hoare, 1959)

Recursively partition array so that we have 'smaller' items on the left of our 'pivot' and 'larger' items on the right of our pivot.

0. **Base case**: if the array has 0 or 1 elements, it is sorted.
1. Pick a pivot. (to partition the array with)
2. Partition the array into <smaller>, <equal> and <greater> partitions.
3. Recurse: quicksort the <smaller> partition, and quicksort the <greater> partition.



Quicksort Correctness

Quicksort:

if $n \leq 1$
    return
else
    $p \leftarrow$ any element in $A[0 \ldots n-1]$
    assert: $n > 1$ **and** $p \in A[0 \ldots n-1]$
    $(fe, fg) \leftarrow partition(A, n, p)$
    assert: R
    $quicksort(A[0 \ldots fe-1])$
    $quicksort(A[fg \ldots n-1])$
    assert: $A[0 \ldots fe-1]$ is sorted **and** $A[fg \ldots n-1]$ is sorted
        and $R \implies A[0 \ldots n-1]$ is sorted

p: pivot

Partition:

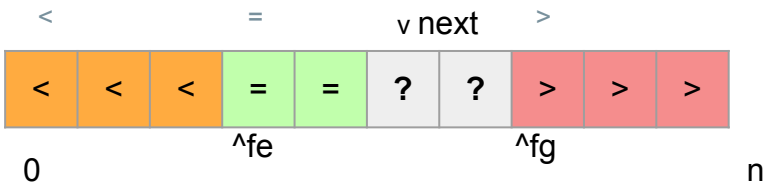
Initialization:
$next, fe, fg \leftarrow 0, 0, n$
assert: P

Loop:
while $next < fg$
    assert: P **and** $next < fg$
    if $A[next] < p$
        swap $A[fe]$ and $A[next]$
        $fe, next \leftarrow fe + 1, next + 1$
        assert: P
    else if $A[next] > p$
        swap $A[next]$ and $A[fg-1]$
        $fg \leftarrow fg - 1$
        assert: P
    else
        $next \leftarrow next + 1$
        assert: P **and** $fe < next$

Hoare Partitioning Implementation

```
void partition(int *A, int n, int* fe, int* fg, int pivot) {
  int next = 0; // next item to partition
  *fe = 0; // first equals,  will go ->
  *fg = n; // first greater, will go <-

  while (next < *fg) {
    if (A[next] < pivot) {
      int_swap(A + *fe, A + next);
      (*fe)++;   next++;
    }
    else if (A[next] > pivot) {
      (*fg)--;
      int_swap(A + *fg, A + next);
    }
    else { // if (A[next] == pivot)
      next++;
    }
  }
}
```



Hoare Partitioning Implementation

```
void partition(int *A, int n, int* fe, int* fg, int pivot) {
  int next = 0; // next item to partition
  *fe = 0; // first equals,  will go ->
  *fg = n; // first greater, will go <-

  while (next < *fg) {
    if (A[next] < pivot) {
      int_swap(A + *fe, A + next);
      (*fe)++;   next++;
    }
    else if (A[next] > pivot) {
      (*fg)--;
      int_swap(A + *fg, A + next);
    }
    else { // if (A[next] == pivot)
      next++;
    }
  }
}
```

Pivot: **5**

Hoare Partitioning Implementation

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void partition(int *A, int n, int* fe, int* fg, int pivot) {
  int next = 0; // next item to partition
  *fe = 0; // first equals,  will go ->
  *fg = n; // first greater, will go <-

  while (next < *fg) {
    if (A[next] < pivot) {
      int_swap(A + *fe, A + next);
      (*fe)++;   next++;
    }
    else if (A[next] > pivot) {
      (*fg)--;
      int_swap(A + *fg, A + next);
    }
    else { // if (A[next] == pivot)
      next++;
    }
  }
}
```

Pivot: **5**

9 > 5

Hoare Partitioning Implementation

```
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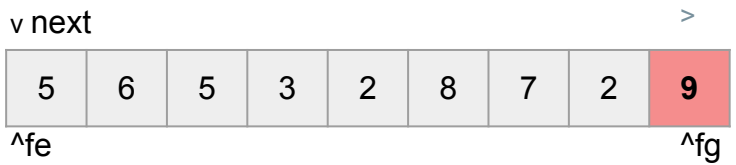
      int_swap(A + *fe, A + next);
      (*fe)++;   next++;
    }
    else if (A[next] > pivot) {
      (*fg)--;
      int_swap(A + *fg, A + next);
    }
    else { // if (A[next] == pivot)
      next++;
    }
  }
}
```

Pivot: **5**

9 > 5



Hoare Partitioning Implementation

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  int next = 0; // next item to partition
  *fe = 0; // first equals,  will go ->
  *fg = n; // first greater, will go <-

  while (next < *fg) {
    if (A[next] < pivot) {
      int_swap(A + *fe, A + next);
      (*fe)++;   next++;
    }
    else if (A[next] > pivot) {
      (*fg)--;
      int_swap(A + *fg, A + next);
    }
    else { // if (A[next] == pivot)
      next++;
    }
  }
}
```
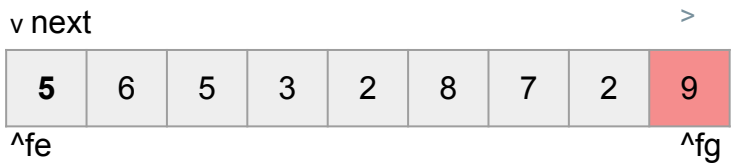

Pivot: **5**

5 = 5



Hoare Partitioning Implementation

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      int_swap(A + *fe, A + next);
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    }
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      (*fg)--;
      int_swap(A + *fg, A + next);
    }
    else { // if (A[next] == pivot)
      next++;
    }
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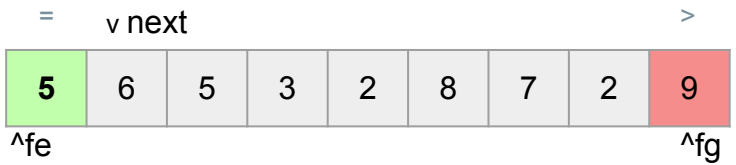
}
```

Pivot: **5**

5 = 5



Hoare Partitioning Implementation

```
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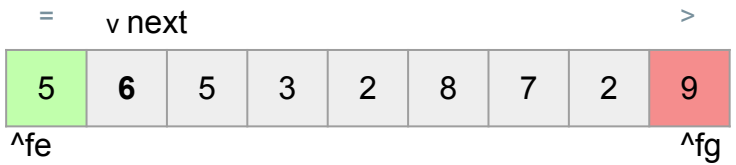
  while (next < *fg) {
    if (A[next] < pivot) {
      int_swap(A + *fe, A + next);
      (*fe)++;   next++;
    }
    else if (A[next] > pivot) {
      (*fg)--;
      int_swap(A + *fg, A + next);
    }
    else { // if (A[next] == pivot)
      next++;
    }
  }
}
```

Pivot: **5**

6 > 5



Hoare Partitioning Implementation

```
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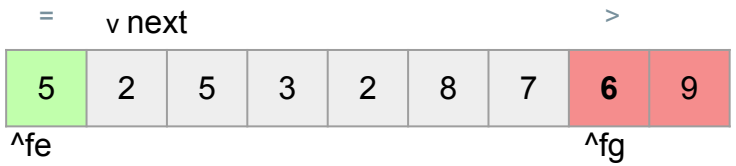
  int next = 0; // next item to partition
  *fe = 0; // first equals,  will go ->
  *fg = n; // first greater, will go <-

  while (next < *fg) {
    if (A[next] < pivot) {
      int_swap(A + *fe, A + next);
      (*fe)++;   next++;
    }
    else if (A[next] > pivot) {
      (*fg)--;
      int_swap(A + *fg, A + next);
    }
    else { // if (A[next] == pivot)
      next++;
    }
  }
}
```

Pivot: **5**

6 > 5



Hoare Partitioning Implementation

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  while (next < *fg) {
    if (A[next] < pivot) {
      int_swap(A + *fe, A + next);
      (*fe)++;    next++;
    }
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      (*fg)--;
      int_swap(A + *fg, A + next);
    }
    else { // if (A[next] == pivot)
      next++;
    }
  }
}
```
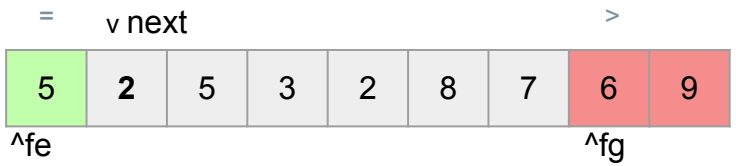

Pivot: **5**

2 < 5



Hoare Partitioning Implementation

```
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  int next = 0; // next item to partition
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  while (next < *fg) {
    if (A[next] < pivot) {
      int_swap(A + *fe, A + next);
      (*fe)++;    next++;
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      int_swap(A + *fg, A + next);
    }
    else { // if (A[next] == pivot)
      next++;
    }
  }
}
```
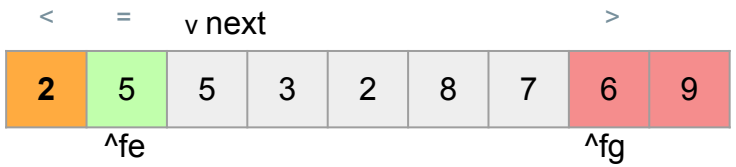

Pivot: **5**

2 < 5



Hoare Partitioning Implementation

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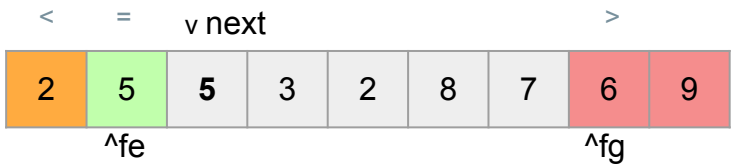
      (*fg)--;
      int_swap(A + *fg, A + next);
    }
    else { // if (A[next] == pivot)
      next++;
    }
  }
}
```

Pivot: **5**

5 = 5



Hoare Partitioning Implementation

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  while (next < *fg) {
    if (A[next] < pivot) {
      int_swap(A + *fe, A + next);
      (*fe)++;   next++;
    }
    else if (A[next] > pivot) {
      (*fg)--;
      int_swap(A + *fg, A + next);
    }
    else { // if (A[next] == pivot)
      next++;
    }
  }
}
```
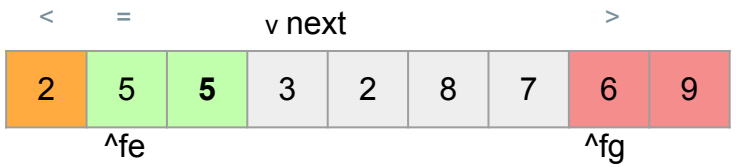

Pivot: **5**

5 = 5



Hoare Partitioning Implementation

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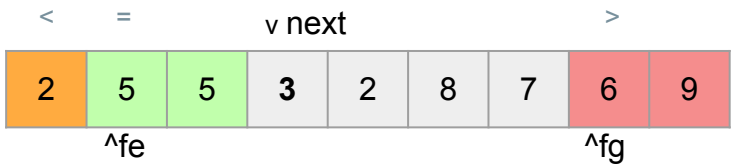
Pivot: **5**

3 **<** 5



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Pivot: **5**

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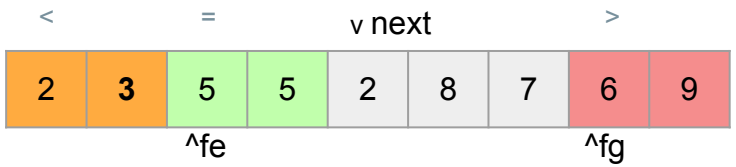

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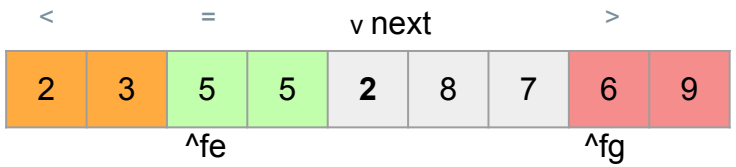
Pivot: **5**

2 < 5



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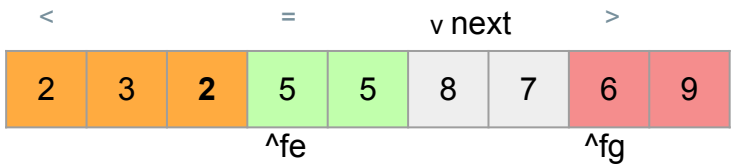
Pivot: **5**

2 < 5



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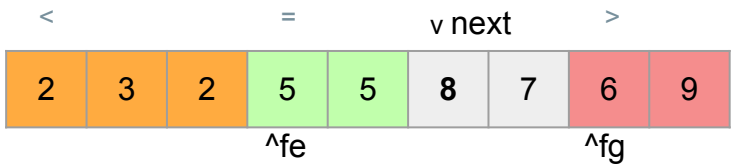
```

Pivot: **5**

8 > 5



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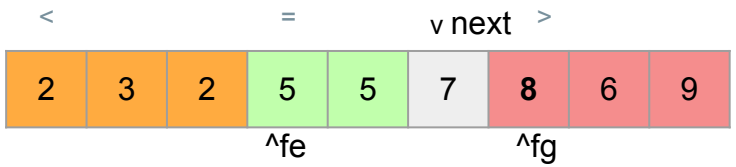

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```
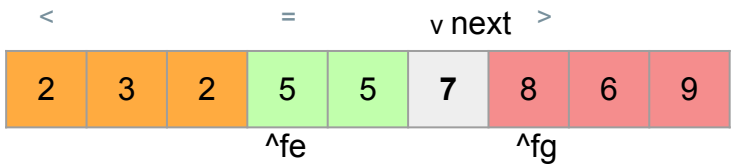

Pivot: **5**

7 > 5



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Pivot: **5**

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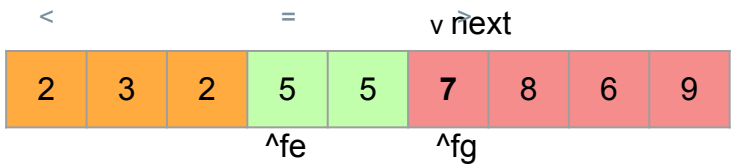

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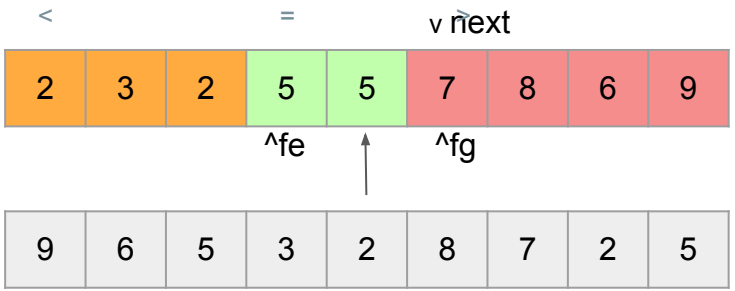

Finished!
How many operations was that?

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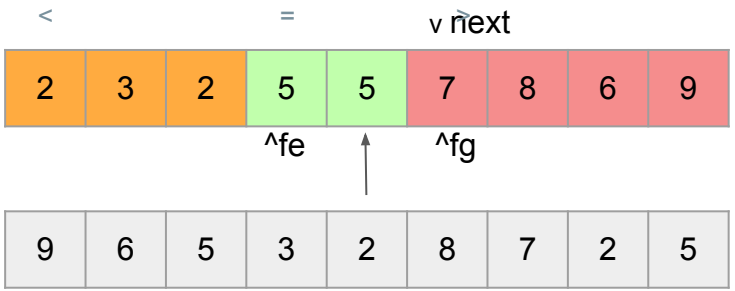
Pivot: **5**



Finished!
How many operations was that?

O(n)

Quicksort Implementation

```
void quicksort(int *A, int n){
  if (n <= 1) return; // base case -- sorted
  int fe, fg;  // "first equal =", "first greater >"
  int pivot = choose_pivot(A);  // pick a pivot (value of the pivot, not index)
  partition(A, n, &fe, &fg, pivot); // partition the array.  (b)
  quicksort(A, fe); // length of first partition is index of fe
  quicksort(A + fg, n - fg); // length of second partition is size n - index of fg
  // equals are in place -- we're done!
}
```

How does choosing a pivot change the complexity of quicksort?
What is the best choice of pivot?

Quicksort Performance

… depends on our choice of pivot!

eg: 2 4 6 8 7 5 3 1

Best case quicksort

sort => (< 2 3 1) (= 4) (> 7 8 6) Partitioning is n operations

sort left => (< 1) (= 2) (> 3) n/2 ops

sort left => (<) (= 1) (>) (done) n/4 ops

sort right => (<) (= 3) (>) (done) n/4 ops

sort right => (< 6) (= 7) (> 8) n/2 ops

sort left => (<) (= 6) (>) (done) n/4 ops

sort right => (<) (= 8) (>) (done) n/4 ops

Total: n + 2*n/2 + 4*n/4 = 3n = **O(n log n)**

(log2(8) = 3)

Quicksort Performance

… depends on our choice of pivot!

eg: 2 4 6 8 7 5 3 1
Worst case quicksort…? Just wait.

Choosing a pivot

1. **First element**

1 2 3 4 5 6 7 8

^ pivot

Any issues?

Choosing a pivot

1. **First element**

1 2 3 4 5 6 7 8

^ pivot

Any issues? Yep.
If the array is already sorted, we will get worst case time complexity.
Partitioning: (recall: Hoare partitioning reverses > elements)

(<)	(=)	(>)
	1	8 7 6 5 4 3 2
7 6 5 4 3 2	8	
6 5 4 3 2	7	
5 4 3 2	6	
4 3 2	5	
3 2	4	
2	3	
	2	

Choosing a pivot

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1 2 3 4 5 6 7 8

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Partitioning:

(<)	(=)	(>)
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6 5 4 3 2	7	
5 4 3 2	6	
4 3 2	5	
3 2	4	
2	3	
	2	

(recall: Hoare partitioning reverses > elements)

n partitions,
partitioning is O(n)

Total time complexity:
n * O(n) = **$O(n^2)$**

Quicksort Complexity - Pivot choice

Pivot: **choose first item**

	1	2	3	4	5	6	7
O(n)	1	2	3	4	5	6	7
O(n)	1	2	3	4	5	6	7
O(n)	1	2	3	4	5	6	7
O(n)	1	2	3	4	5	6	7
			...				
O(n)	1	2	3	4	5	6	7

n * O(n) => $O(n^2)$, worst case

This is O(n) under insertion sort!!

Choosing a pivot

2. **Last element**

Any issues?

Choosing a pivot

2. Last element

Any issues?

8 7 6 5 4 3 2 1
^ pivot

If the array is in reverse order, we will get worst case time complexity.
Partitioning:

(<)	(=) 1	(>) 2 3 4 5 6 7 8
(<) 2 3 4 5 6 7	(=) 8	(>)
(<) 2 3 4 5 6	(=) 7	(>)
(<) 2 3 4 5	(=) 6	(>)
(<) 2 3 4	(=) 5	(>)
(<) 2 3	(=) 4	(>)
(<) 2	(=) 3	(>)
(<)	(=) 2	(>)

n partitions,
partitioning is O(n)

n * O(n) = **$O(n^2)$**

Choosing a pivot

3. Middle element

eg: 5 7 2 9 3 2 5 7

Any issues?

Choosing a pivot

3. Middle element

eg: 5 7 2 9 3 2 5 7

Any issues? Not with that one, but consider: 2 4 6 8 7 5 3 1

Partitioning:

(<)	2 4 6 7 5 3 1	(=) 8	(>)
(<)	2 4 6 5 3 1	(=) 7	(>)
(<)	2 4 5 3 1	(=) 6	(>)
(<)	2 4 3 1	(=) 5	(>)
(<)	2 3 1	(=) 4	(>)
(<)	2 1	(=) 3	(>)
(<)	1	(=) 2	(>)
(<)		(=) 1	(>)

n partitions,
partitioning is O(n)

n * O(n) = **$O(n^2)$**

Choosing a pivot

Seems like wherever we pick a pivot, we're always getting $O(n^2)$ some way.

Can we do better?

Choosing a pivot

Seems like wherever we pick a pivot, we're always getting $O(n^2)$ some way.

Can we do better?

4. Random pivot

- On each partition, choose a random index in the array as the pivot.

eg: 2 4 6 8 7 5 3 1

- There's a 2/n chance of choosing the **worst** case. (1 or 8)
- There's a 2/n chance of choosing the **best** case. (4 or 5)
- And we can have everything in between.

Turns out, this ensures O(n log n) as long as there isn't some prior distribution of the array.

Choosing a pivot

```
int choose_pivot(int *A, int n) {
  return A[rand() % n];    // Random pivot  (best option, why?)
  // return A[0];          // first item
  // return A[n-1];        // last item
  // return A[n/2];        // middle item
}
```

Quicksort Complexity

Space Complexity:
O(n) - 2 partition; O(log n) - 3 partitions.

Time Complexity - depends on how the array is partitioned.

Best case:	Pivot always splits the array in half. => O(n log n)	9 8 7 6 5 4 3 2 1 -> choose 5.
Worst case:	Pick pivot that is largest / smallest => O(n^2)	9 8 7 6 5 4 3 2 1 -> choose 1 or 9.
Average case:	Pivot could be anywhere in between. => O(n log n)	Proof in DoA / ADS

Partition is **O(n)**. We partition O(log n) times if our pivot choice is good => O(n log n).

Comparing sorting algorithms

	Insertion sort	**Quicksort**
Idea	Builds sorted array sequentially, swapping elements towards the end	Recursively partition array into two parts, to sort independently (divide and conquer)
Notes	*Easy to implement / understand. Efficient for nearly sorted arrays, and small arrays.*	Complexity depends on pivot choice. Choose random pivot for best results. Hoare Partitioning is O(n)
Time Complexity	O(n) best (already sorted!) $O(n^2)$ average and worst	$O(n^2)$ worst (picking worst pivot) O(n log n) average and best.
Space Complexity	O(1) (auxiliary)	O(n) (2 partition) O(log n) (3 partition)