Final Project

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Part 1 – Introduction

Brief Topic

Our project investigates the impact of different rebalancing frequencies on the alpha of equal-weighted (EW) portfolios within the CRSP small-cap universe. We aim to analyze how cumulative returns and alpha are influenced by varying rebalancing frequencies (daily, weekly, monthly, and quarterly), with a detailed examination of factor contributions to alpha in this context. Our analysis will utilize regression models and factor decomposition to isolate the effects of specific factors (such as size, momentum, and quality) and understand their roles in alpha generation. By identifying optimal rebalancing strategies that maximize alpha, this research has practical applications for portfolio managers and asset allocators focused on small-cap equities.

This project aligns closely with the course objectives, particularly in providing hands-on experience with portfolio management strategies and factor-based alpha generation, which are essential components of financial risk management. Additionally, our focus on rebalancing frequencies and factor analysis contributes to a deeper understanding of market inefficiencies and risk-adjusted return optimization, both of which are key themes in the field of financial engineering.

Relevance

- **Market Risk Management:** This project provides insights into optimal rebalancing practices for managing market risks, especially in volatile small-cap equity markets. By analyzing statistical and economic factors, particularly the size factor, our research aims to improve portfolio stability and performance in different rebalancing scenarios.
- **Operational Risk Management:** The project employs rigorous methodologies, including data collection, and preprocessing, embodying key principles of operational risk management. By testing various rebalancing strategies, the project addresses risks associated with inappropriate rebalancing frequencies that could lead to suboptimal alpha generation and increased transaction costs.

Hypothesis

In an equal-weighted portfolio of small-cap stocks, the alpha for the weekly rebalancing model is significantly higher, primarily due to the positive contribution of the size factor.

- **Alignment with existing literature:** The foundational literature *"Why Do Equally Weighted Portfolios Beat Value-Weighted Ones?"* by Swade et al. (2023) highlights that the advantage of equal-weighted portfolios primarily lies in small-company stocks, due to their potential to capture the "size premium." Swade et al. (2023) examined the performance spread between equal-weighted and value-weighted portfolios, focusing on factors such as size and short-term reversal. However, their research did not extensively consider the effect of rebalancing frequency—a factor that could influence portfolio performance significantly.

- **Innovation and contribution of this research:** Our research aims to extend and build upon these existing insights by investigating a critical but underexplored aspect: the impact of rebalancing frequency on equal-weighted small-cap portfolios. We hypothesize that different rebalancing frequencies (e.g., daily, weekly, monthly, quarterly) can significantly impact the alpha and cumulative returns of small-cap portfolios. This added focus on rebalancing frequency differentiates our methodology and provides new perspectives on optimizing alpha capture through strategic adjustments.

Part 2 – Literature Review

The application of factor models and rebalancing strategies in asset allocation has been extensively studied, especially in the context of small-cap and equal-weighted portfolios. Factor models, which decompose returns into systematic components, provide a framework for understanding portfolio performance beyond traditional metrics. In recent years, equal-weighted (EW) portfolios have garnered attention due to their tendency to outperform value-weighted (VW) portfolios, particularly in small-cap stocks. Swade et al. (2023) delve into the performance dynamics of EW portfolios relative to VW portfolios, attributing EW portfolios' outperformance to their unique exposure to size and short-term reversal factors. This research suggests that the frequent rebalancing inherent in EW portfolios capitalizes on short-term reversal effects, particularly benefiting from small-cap stocks' tendency to revert and outperform in specific market environments.

Swade et al.'s (2023) analysis reveals that the size factor is the primary driver of the performance gap between EW and VW portfolios, with EW portfolios benefiting significantly from positive exposure to small-cap stocks. The contrarian rebalancing characteristic of EW portfolios also contributes to short-term reversal effects while incurring a negative momentum exposure. This distinct factor composition underpins the observed outperformance of EW portfolios in various market conditions. Additionally, Swade et al. document that EW portfolios exhibit pronounced seasonal effects, particularly in January, where outsized returns suggest a calendar-related factor premium that further enhances the appeal of EW portfolios. This seasonal pattern aligns with earlier findings on small-cap stocks' January effect, providing an additional layer of evidence for the impact of factor premiums on EW portfolio performance.

Historically, Fama and French's (1993) seminal work on the three-factor model laid the groundwork for understanding these effects by introducing the size, value, and market factors. The size factor, which captures the outperformance of small-cap stocks relative to large-cap stocks, has been especially relevant to EW portfolios. This foundational model has since become a standard in asset pricing, and its emphasis on the size factor aligns with the typical outperformance seen in EW small-cap portfolios. Over time, additional factors like momentum, profitability, and quality have been incorporated into this framework, enhancing its explanatory power and providing diversification benefits across different portfolio types. These factors, widely used in both academic research and practical asset allocation, offer risk-adjusted return advantages and contribute to a deeper understanding of portfolio behavior in varying market conditions.

Further literature underscores the importance of understanding factor exposures in EW portfolios. Swade et al. (2023) highlight that EW portfolios' size factor exposure is similar to the small-minus-big (SMB) factor in the Fama-French model but can be achieved with lower implementation costs, making it an attractive option for investors. This study also emphasizes that EW portfolios' exposure to size, short-term reversal, and quality factors is sensitive to rebalancing frequency, which directly impacts overall portfolio performance. The findings suggest that while the size factor remains the dominant driver, the short-term reversal factor and volatility exposure play notable roles, especially in market conditions that favor rapid mean reversion in stock prices. This insight is particularly relevant for strategies that involve frequent rebalancing, as it highlights how factor exposures can shift quickly, potentially affecting alpha generation and volatility in EW portfolios.

The role of external economic factors in shaping factor premiums has also been a focus of research. For instance, Maio and Santa-Clara (2017) examine the effects of short-term interest rates on stock market anomalies, indicating that market conditions can significantly impact factor returns. Swade et al.'s findings suggest that such external factors may influence factor premiums for size and short-term reversal, particularly in frequently rebalanced portfolios. This is especially pertinent for small-cap stocks, where short-term anomalies like reversal effects may be amplified by economic conditions, thereby influencing the risk-return profile of EW portfolios. This interaction between market environments and rebalancing strategies adds complexity to factor-based investing and highlights the importance of adapting strategies to prevailing economic conditions.

Kritzman (2021) provides a cautionary perspective on factor-based asset allocation, emphasizing that while factors like size and momentum can offer diversification, they also carry unique risk characteristics. He warns that investors should carefully adjust their portfolios to avoid unintended exposure to correlated risks, a point that resonates with Swade et al.'s findings on the volatility of factor exposures in EW portfolios. This insight underscores the need for careful factor selection and rebalancing strategies, particularly in EW portfolios where factor exposures can shift quickly. For investors seeking to leverage size and short-term reversal effects in small-cap stocks, these studies collectively suggest that a nuanced understanding of rebalancing frequency, economic conditions, and factor exposures is essential for optimizing alpha generation and managing risk.

In summary, the literature on factor models and rebalancing strategies for small-cap and equal-weighted portfolios highlights the importance of size and short-term reversal factors, particularly in frequently rebalanced portfolios. Swade et al. (2023), building on Fama and French (1993), provides key insights into the performance dynamics of EW portfolios. While size is a dominant driver, other factors, such as short-term reversal and quality, also significantly impact performance under different rebalancing frequencies. These insights align with our study's objective to evaluate how different rebalancing frequencies affect factor contributions to alpha in small-cap stocks, providing a basis for exploring optimal factor-based asset allocation strategies.

Part 3 – Methodology

1. Data Preprocessing and Visualization

1.1 Data Collection

To construct our target variable, Y, representing the Equal-Weighted Portfolio, we followed these steps:

a. First, we obtained a list of the smallest 20% of companies by market capitalization from Bloomberg, ensuring our sample focuses on small-cap stocks to capture the impact of size on returns.
b. Next, we downloaded historical returns and price data for these companies from the CRSP database.
c. Finally, by calculating the equal-weighted returns of these small-cap companies, we derived our target variable Y, serving as the benchmark for small-cap performance in this study.

To investigate the systematic drivers of the return spread between equal-weighted portfolios, we focus on a set of commonly used factors in both academic research and practical applications, including:

Abbreviation	Full Name	Explanation
SMB	Size (Small Minus Big)	Difference in returns between small and large companies.
HML	Value (High Minus Low)	Difference in returns between high and low book-to-market firms.
RMW	Profitability (Robust Minus Weak)	Difference in returns between high and low profitability firms.
CMA	Investment (Conservative Minus Aggressive)	Difference in returns between conservative and aggressive investors.
WML	Momentum (Winners Minus Losers)	Difference in returns between past winners and losers.
STR	Short-term Reversal	Measures short-term return reversal effect.
VOL	Volatility	Difference in returns between low and high volatility stocks.
QMJ	Quality Minus Junk	Difference in returns between high- and low-quality stocks.
ME	Market Equity	Total market value of equity.
IA	Investment to Assets	Ratio of investment to total assets.
ROE	Return on Equity	Profitability relative to equity.
EG	Expected Growth	Expected future growth.

We include the Size (SMB), Value (HML), Profitability (RMW), Investment (CMA), Momentum (WML), and Short-term Reversal (STR) factors from Kenneth R. French database. To account for the low-risk anomaly, we add the Volatility (VOL) factor by Van Vliet and De Koning (2017). To test model robustness, we incorporate the Quality Minus Junk (QMJ) factor from Asness et al. (2019) and the Market Equity (ME), Investment to Assets (IA), Return on Equity (ROE), and Expected Growth (EG) factors from the q-factor database by Hou et al. (2015, 2021).

By combining these factors with the returns of the equal-weighted portfolio, we aim to conduct a comprehensive analysis of alpha generation in small-cap stocks under different rebalancing frequencies and evaluate each factor's contribution to portfolio returns.

1.2 Data Preprocessing

After loading the data, we converted the date column to a datetime format and set it as the index, to break the historical stock price data down into daily, weekly, monthly, and quarterly time series

for analysis. We also clean the data by removing commas and percentage signs from relevant fields, converting them to numeric types.
Missing values in the price column are forward-filled to ensure data continuity, while missing values in returns and volume are replaced with zeros. The preprocessed data is then used to analyze portfolio performance under different rebalancing frequencies.

1.3 Factor Distribution Analysis

To understand the distribution characteristics of each factor, we used histogram plots with kernel density estimation (KDE) for each factor. This provided insights into the frequency distribution and variability of each factor, helping to identify outliers and skewness in the data.

1.4 Correlation Analysis

A correlation heatmap was generated to assess the relationships between different factors and returns. This heatmap provided a visual representation of factor correlations, helping us identify highly correlated or independent factors, which is crucial for constructing a diversified portfolio and understanding factor exposure.

2. Mathematical Analysis of Rebalancing Effects

In the portfolio, we use an equal-weighting scheme, meaning each stock has the same initial weight, and rebalancing is done periodically. At any given time t, the return of the equal-weighted portfolio is:

$$r_{EW,t} = \frac{1}{N}\sum_{i=1}^{N} r_{i,t}$$

where N is the number of stocks in portfolio, and $r_{i,t}$ represents the return of stock i at time t.

2.1 Effects of Different Rebalancing Frequencies

We examine the impact of different rebalancing frequencies, including daily, weekly, monthly, and quarterly rebalancing. Rebalancing aims to adjust the weights of the stocks to maintain equal weighting, reducing the risk of the portfolio drifting from the initial strategy, thereby potentially achieving better risk-adjusted returns. The return of the equal-weighted portfolio can be mathematically represented as:

$$r_{EW} = \frac{1}{N}\sum_{i=1}^{N}\left(\beta_i r_M + \epsilon_i\right)$$

where β_i is the market beta of stock i, indicating its sensitivity to market movements; r_M is the market return: and ϵ_i is the idiosyncratic risk of stock i, assumed to be independent and identically distributed (i.i.d.) with zero mean.

By resampling at different frequencies, we estimate the expected return and volatility of the portfolio under each rebalancing strategy. More frequent rebalancing (e.g., daily or weekly) enables the portfolio to better capture short-term market changes and maintain equal weights, which may lead to improved risk-adjusted performance. On the other hand, less frequent rebalancing allows the portfolio weights to drift, making it more similar to a value-weighted portfolio.

The variance of the portfolio can be expressed as:

$$\sigma_{EW}^2 = \frac{1}{N^2}\sum_{i=1}^{N} \sigma_i^2 + \frac{1}{N^2}\sum_{i\neq j} Cov\left(r_i r_j\right)$$

Assuming independence between stock returns, the variance simplifies to:

$$\sigma_{EW}^2 \approx \frac{1}{N}\sigma_M^2$$

where σ_M^2 is the market variance. As N increases, the portfolio variance decreases, indicating that diversification effectively reduces risk.

2.2 Analysis of Rebalancing Returns Differences

By comparing portfolio returns and volatility under different rebalancing frequencies, we can assess the impact of rebalancing on performance. Higher rebalancing frequencies help maintain equal weights by adjusting to market fluctuations, potentially leading to higher risk-adjusted returns. In contrast, lower rebalancing frequencies may cause the portfolio to drift, making its performance more like a value-weighted portfolio.

3. Regression Analysis

To further analyze the impact of rebalancing frequency on portfolio returns, we use regression analysis to quantify the factor contribution. In the stepwise regression model, we introduce key market factors to identify variables significantly affecting returns. We then calculate cumulative and non-cumulative returns at each frequency to ensure consistency for the subsequent rebalancing strategy analysis.

3.1 Regression Model Specification

We assume that the portfolio return is influenced by market returns and other specific factors, leading to the following linear regression model:

$$r_{EW,t} = \alpha + \beta r_{M,t} + \epsilon_t$$

where $r_{EW,t}$ is the return of the equal-weighted portfolio at time t, $r_{M,t}$ is the market of return, α is the intercept (representing excess return), β is the market risk coefficient, and ϵ_t is the residual term.

3.2 Purpose of Regression

The purpose of the regression analysis is to verify the linear relationship between portfolio returns and market returns. By estimating the market beta coefficient (β), we can understand the portfolio's sensitivity to market movements. If $\beta > 1$, the portfolio is more sensitive to market volatility; if $\beta < 1$, the portfolio's volatility is lower than the market.

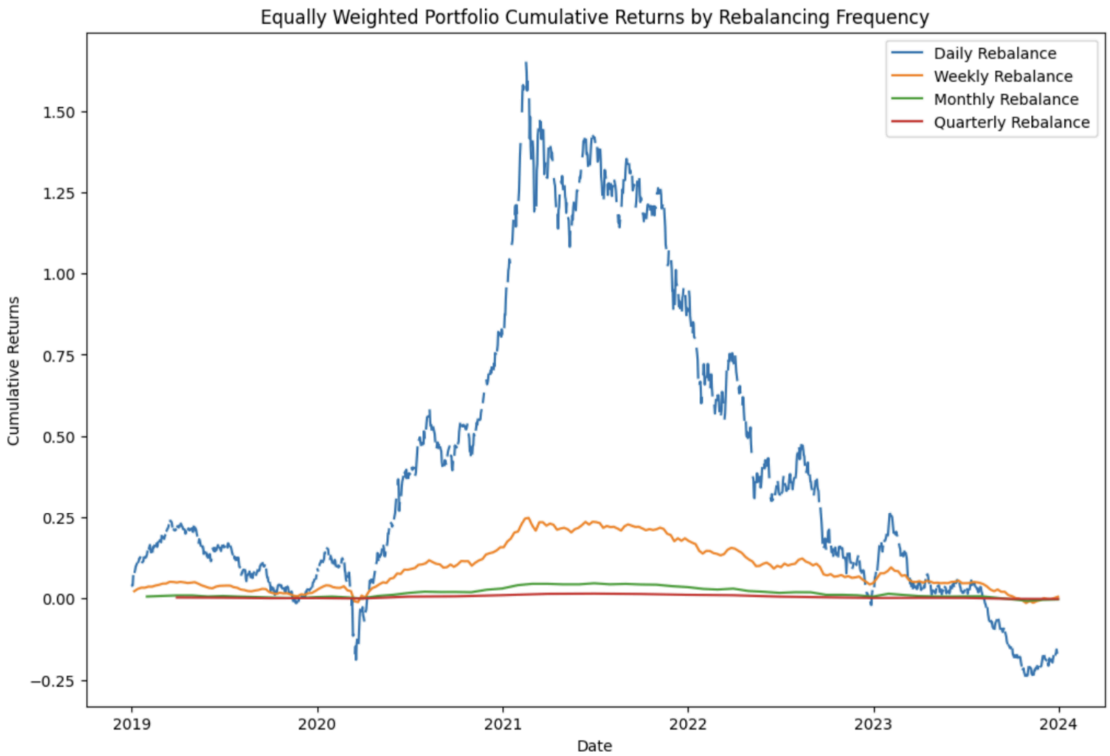
3.3 Interpretation of Regression Results

From the regression results, we obtain the portfolio's market beta and the intercept (α). If the intercept (α) is significantly greater than zero, it indicates that the portfolio has a positive excess return. Additionally, residual analysis allows us to evaluate the model's fit and determine if there are systematic biases.

Part 4 – Data Illustration

Exploratory Data Analysis

1. Rebalancing Frequency and Cumulative Returns

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Cumulative rate of return for the total length of time with different rebalancing frequencies:
Daily Rebalance: -16.59%
Weekly Rebalance: 0.54%
Monthly Rebalance: -0.26%
Quarterly Rebalance: -0.13%
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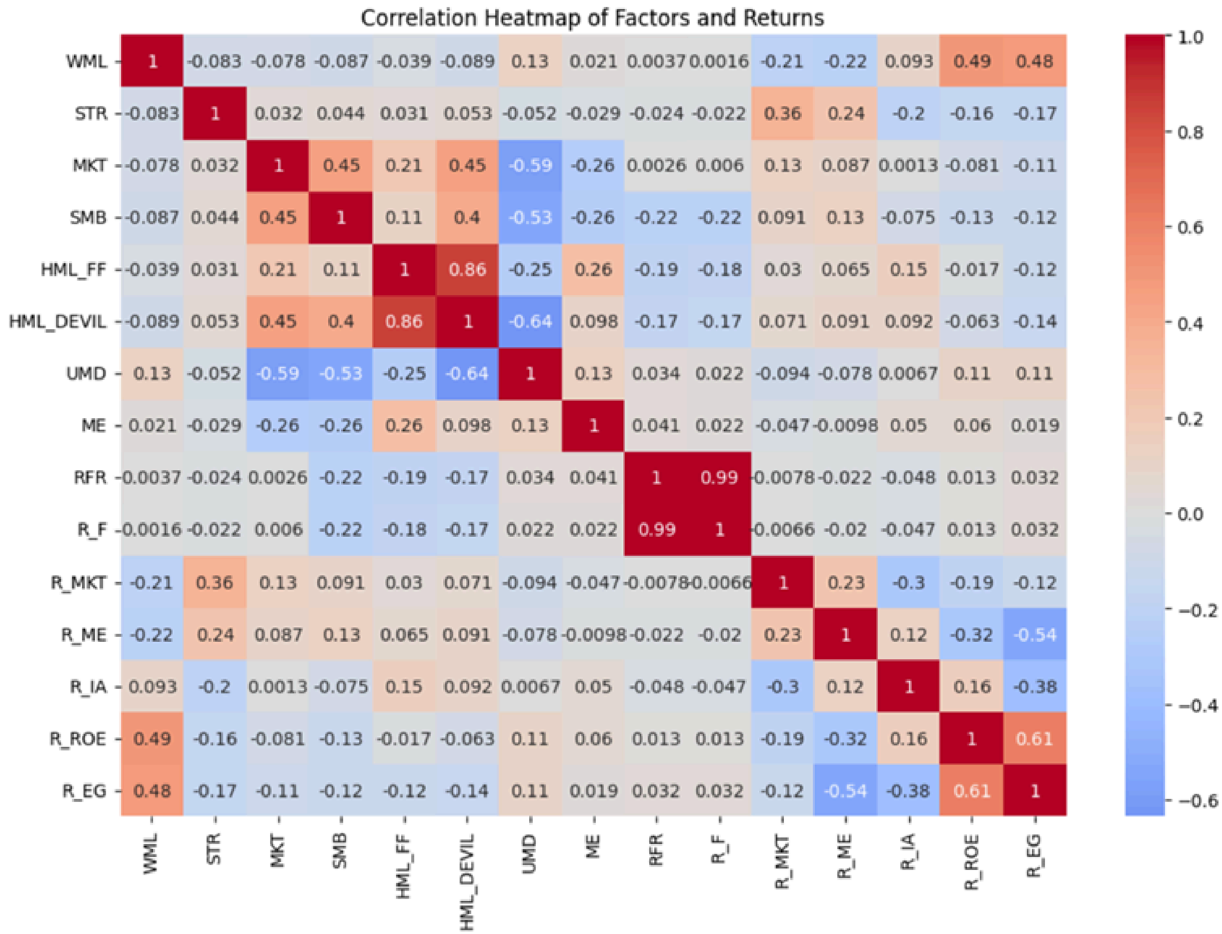
In the cumulative returns graph, daily rebalancing leads to the highest variability, reaching a peak in 2021, which suggests significant returns but also high risk. It also shows the potential to achieve significantly higher cumulative returns. However, this requires the operator to have a keen sense of market timing, as missing the optimal exit point will lead to negative cumulative returns, negating the advantages of daily trading. This aligns with the lowest annualized return -16.59%.

Weekly rebalancing shows a smoother cumulative return, remaining positive and more stable compared to daily rebalancing. This strategy results in an annualized return of 0.54%, indicating that weekly adjustments balance risk and return effectively without incurring excessive transaction costs.

Monthly and quarterly rebalancing present lower cumulative returns, with negative annualized returns -0.26% and -0.13% respectively, suggesting that less frequent rebalancing fails to capitalize on market opportunities, especially in a volatile environment.

Overall, while daily rebalancing offers the potential for highest returns at some time point, it comes with increased risks and costs. Weekly rebalancing appears to strike a reasonable balance, capturing market opportunities while maintaining more manageable risk levels compared to daily rebalancing. Monthly and quarterly rebalancing tend to be too infrequent to take advantage of the fast-moving market dynamics characteristic of small-cap portfolios.

2. Insights from Correlation Heatmap



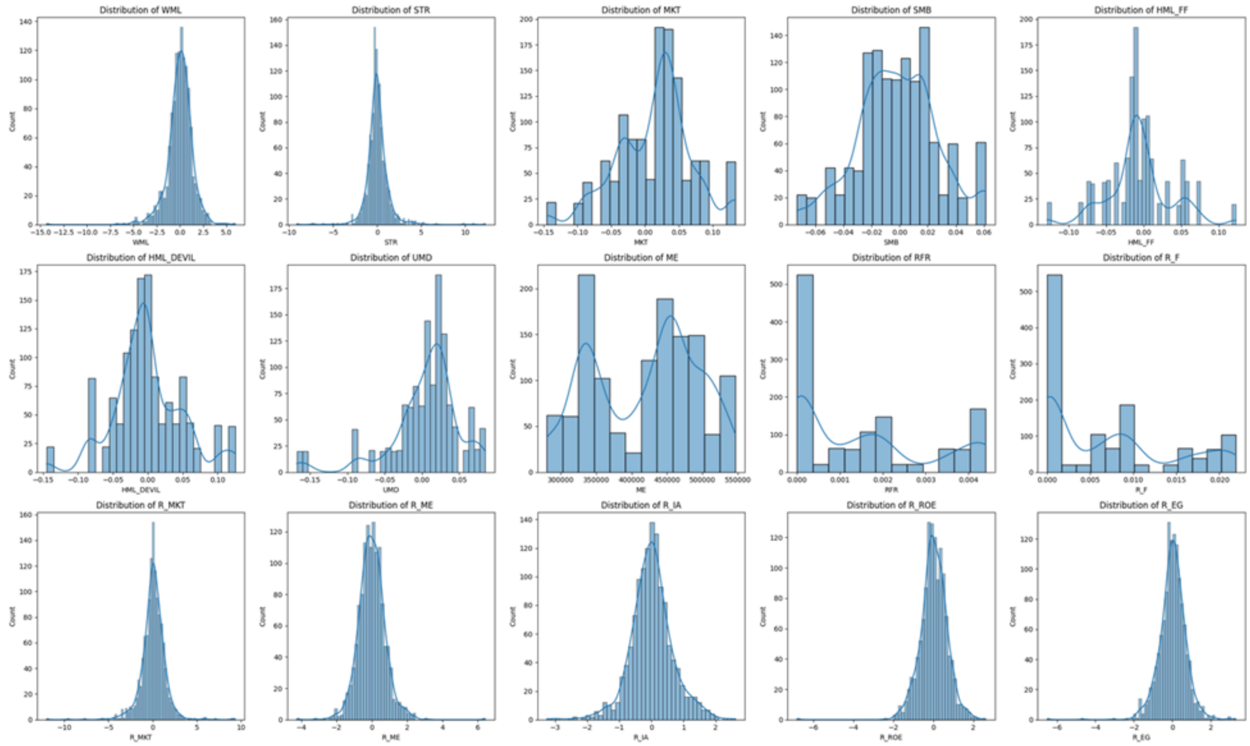
The correlation heatmap illustrates the relationships among various factors utilized in our analysis. We observe that the Winner Minus Loser (WML) factor, a measure of momentum, has strong positive correlations with the profitability-related factors R_ROE (0.49) and R_EG (0.48), suggesting that high momentum aligns with firms exhibiting strong profitability and earnings growth. Conversely, the market risk premium (MKT) has notable correlations with the Small Minus Big (SMB) and HML_DEVIL factors, showing how small-cap and value stocks are impacted by broader market performance.

Additionally, we notice a highly negative correlation between UMD (Up Minus Down), a momentum indicator, and SMB (-0.53), highlighting that momentum strategies tend to perform differently from size-related factors. Moreover, the almost perfect correlation (0.99) between the Risk-Free Rate (RFR) and R_F confirms their interchangeable role in financial models.

The alignment between R_ROE and R_EG (0.61) underscores that firms with higher earnings growth often exhibit strong returns on equity.

These correlations indicate that profitability, size, and market factors interact in distinct ways, influencing the behavior of small-cap portfolios.

3. Insights from Factor Distribution



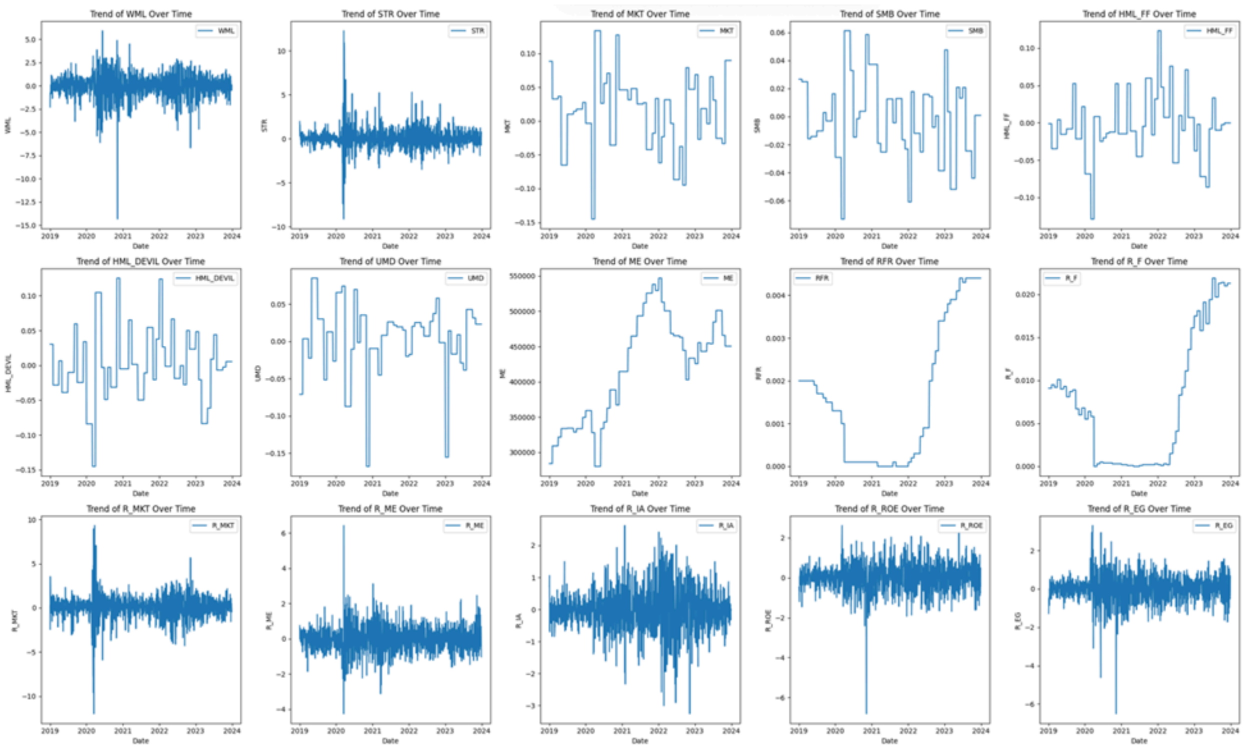
The histogram plots provide insights into the distributions of each factor involved in the regression analysis.

Several distributions, such as WML, STR, and MKT, appear to have normal shapes, with a clear central tendency and relatively symmetric spread, making them candidates for standardization without significant transformation. Factors like ME and UMD, however, exhibit more skewed distributions, which may imply data irregularities or sector-specific behavior that deviates from normality.

The presence of different scales and spreads across the factors highlights the importance of standardization. Standardization will help to normalize these differing scales, ensuring each factor contributes equally in our regression models. This is crucial as it prevents features with larger ranges from dominating the analysis, thus allowing for a more balanced and meaningful interpretation of factor coefficients and their impact on portfolio returns.

Based on the high correlation observed between RFR and R_F in the correlation heatmap, we have decided to exclude RFR from subsequent regression analyses to avoid multicollinearity issues. By removing this redundant factor, we aim to improve the interpretability and accuracy of the regression models, ensuring that each factor provides unique and meaningful information regarding its impact on portfolio returns.

4. Insights from Time Series Plots of Financial Factors



The time series plots illustrate the temporal dynamics of different financial factors, noting that the data includes both monthly and daily frequencies. This mixed frequency is evident in the difference in smoothness and volatility across the plots.

Factors like WML (Winner Minus Loser) and STR (Strategy Factor) show significant volatility, which implies high risk and frequent oscillations, particularly during market events such as the pandemic and subsequent recovery phases.

Meanwhile, factors such as MKT (Market Risk Premium) and ME (Market Equity), which exhibit step-like movements, are represented in monthly intervals, leading to a more stable and less granular pattern over time. The rising trend in the risk-free rate (R_F) from 2022 onwards also reflects broader economic shifts, particularly the response of monetary policy.

This mixed frequency nature of the data highlights the importance of aligning analysis methods appropriately to ensure coherent insights, especially when standardizing or integrating these factors for modeling purposes.

<u>Linear regression model</u>

We employed a stepwise backward regression approach to analyze the impact of each factor on the returns of small-cap portfolios. This method was chosen for its ability to effectively eliminate insignificant variables, thereby enhancing the model's simplicity and explanatory power. We conducted regression analysis on eight factors, covering various rebalancing frequencies (i.e., daily, weekly, monthly, and quarterly), and performed regressions on both cumulative and non-cumulative return models. The regression results are shown in the figures below.

Standardized Regression Analysis of Cumulative Returns: Daily Rebalancing Frequency

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9	Model 10	Model 11	Model 12	Model 13	Model 14
WML	-0.20 (-7.21)	-0.17 (-6.56)	-0.16 (-6.20)	-0.15 (-5.99)	-0.15 (-6.05)	-0.15 (-6.07)	-0.15 (-6.03)	-0.15 (-6.03)	-0.15 (-6.05)	-0.05 (-2.24)	0.01 (0.39)	0.02 (1.04)	0.16 (8.82)	0.18 (9.25)
STR		0.36 (14.02)	0.36 (14.03)	0.36 (14.01)	0.36 (14.09)	0.36 (14.09)	0.36 (14.09)	0.36 (14.05)	0.36 (14.04)	0.14 (6.82)	0.09 (4.66)	0.07 (3.53)	0.05 (3.10)	0.05 (2.61)
MKT			0.14 (5.45)	0.08 (2.88)	0.09 (3.27)	0.10 (3.22)	0.10 (3.03)	0.09 (2.68)	0.09 (2.79)	0.03 (1.09)	0.02 (0.84)	0.03 (1.12)	0.03 (1.30)	0.02 (1.18)
SMB				0.13 (4.50)	0.13 (4.56)	0.14 (4.26)	0.14 (4.26)	0.13 (4.02)	0.12 (3.62)	0.10 (4.00)	0.07 (2.85)	0.05 (2.08)	0.03 (1.52)	0.03 (1.30)
HML_FF					-0.06 (-2.45)	-0.04 (-0.62)	-0.02 (-0.21)	-0.00 (-0.01)	-0.01 (-0.11)	-0.01 (-0.19)	-0.03 (-0.62)	-0.00 (-0.04)	-0.04 (-0.74)	-0.04 (-0.82)
HML_DEVIL						-0.03 (-0.51)	-0.06 (-0.64)	-0.07 (-0.70)	-0.07 (-0.69)	-0.06 (-0.77)	-0.04 (-0.61)	-0.05 (-0.81)	-0.01 (-0.21)	-0.01 (-0.15)
UMD							-0.02 (-0.40)	-0.02 (-0.48)	-0.03 (-0.55)	-0.04 (-0.92)	-0.04 (-1.25)	-0.05 (-1.54)	-0.03 (-1.07)	-0.04 (-1.14)
ME								-0.03 (-1.22)	-0.03 (-1.16)	-0.03 (-1.47)	-0.04 (-1.95)	-0.04 (-2.22)	-0.03 (-1.81)	-0.03 (-1.85)
R_F									-0.03 (-1.14)	-0.03 (-1.61)	-0.04 (-2.00)	-0.05 (-2.47)	-0.04 (-2.69)	-0.05 (-2.79)
R_MKT										0.62 (28.77)	0.58 (29.37)	0.54 (26.63)	0.54 (30.13)	0.54 (29.97)
R_ME											0.29 (15.36)	0.33 (17.30)	0.26 (14.46)	0.24 (12.38)
R_IA												-0.16 (-8.29)	-0.11 (-6.40)	-0.15 (-6.68)
R_ROE													-0.33 (-17.71)	-0.29 (-12.40)
R_EG														-0.08 (-2.74)
Intercept	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)
Adj. R^2	0.04	0.17	0.19	0.20	0.20	0.20	0.20	0.20	0.20	0.52	0.60	0.62	0.69	0.70
Obs	1258	1258	1258	1258	1258	1258	1258	1258	1258	1258	1258	1258	1258	1258

Standardized Regression Analysis of Cumulative Returns: Weekly Rebalancing Frequency

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9	Model 10	Model 11	Model 12	Model 13	Model 14
WML	-0.25 (-4.18)	-0.21 (-3.74)	-0.16 (-2.92)	-0.13 (-2.41)	-0.14 (-2.51)	-0.14 (-2.53)	-0.14 (-2.45)	-0.14 (-2.45)	-0.14 (-2.47)	-0.04 (-0.75)	0.03 (0.81)	0.04 (0.97)	0.16 (3.99)	0.17 (4.04)
STR		0.34 (5.91)	0.32 (5.89)	0.30 (5.72)	0.31 (5.84)	0.31 (5.84)	0.31 (5.84)	0.31 (5.77)	0.30 (5.74)	0.10 (2.01)	-0.00 (-0.10)	-0.01 (-0.28)	-0.01 (-0.27)	-0.02 (-0.40)
MKT			0.27 (4.86)	0.16 (2.74)	0.18 (3.07)	0.19 (2.99)	0.19 (2.83)	0.17 (2.51)	0.18 (2.61)	0.04 (0.64)	-0.01 (-0.12)	0.00 (0.05)	0.02 (0.38)	0.02 (0.33)
SMB				0.25 (4.27)	0.25 (4.30)	0.27 (3.98)	0.27 (3.97)	0.26 (3.77)	0.24 (3.39)	0.19 (3.34)	0.09 (1.72)	0.07 (1.27)	0.04 (0.91)	0.04 (0.76)
HML_FF					-0.12 (-2.17)	-0.07 (-0.61)	-0.05 (-0.29)	-0.02 (-0.11)	-0.03 (-0.20)	-0.01 (-0.08)	-0.07 (-0.58)	-0.03 (-0.27)	-0.10 (-1.00)	-0.11 (-1.06)
HML_DEVIL						-0.05 (-0.37)	-0.09 (-0.46)	-0.10 (-0.52)	-0.10 (-0.51)	-0.12 (-0.76)	-0.10 (-0.70)	-0.11 (-0.80)	-0.02 (-0.18)	-0.02 (-0.13)
UMD							-0.03 (-0.29)	-0.04 (-0.35)	-0.04 (-0.42)	-0.08 (-0.91)	-0.11 (-1.53)	-0.12 (-1.67)	-0.08 (-1.13)	-0.08 (-1.17)
ME								-0.06 (-1.06)	-0.06 (-1.01)	-0.06 (-1.21)	-0.08 (-1.95)	-0.09 (-2.07)	-0.06 (-1.66)	-0.06 (-1.68)
R_F									-0.06 (-1.02)	-0.06 (-1.33)	-0.07 (-1.79)	-0.08 (-1.98)	-0.08 (-2.18)	-0.08 (-2.24)
R_MKT										0.56 (10.95)	0.52 (11.78)	0.50 (10.71)	0.48 (11.61)	0.48 (11.20)
R_ME											0.40 (9.38)	0.42 (9.58)	0.35 (8.54)	0.33 (7.56)
R_IA												-0.08 (-1.79)	-0.04 (-1.00)	-0.07 (-1.27)
R_ROE													-0.32 (-7.81)	-0.29 (-5.40)
R_EG														-0.05 (-0.78)
Intercept	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)
Adj. R^2	0.06	0.17	0.24	0.28	0.29	0.29	0.29	0.29	0.29	0.52	0.64	0.65	0.71	0.71
Obs	261	261	261	261	261	261	261	261	261	261	261	261	261	261

Standardized Regression Analysis of Cumulative Returns: Monthly Rebalancing Frequency

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9	Model 10	Model 11	Model 12	Model 13	Model 14
WML	-0.53 (-4.78)	-0.44 (-3.48)	-0.26 (-2.03)	-0.10 (-0.83)	-0.12 (-1.10)	-0.15 (-1.27)	-0.14 (-0.99)	-0.15 (-1.05)	-0.17 (-1.21)	0.04 (0.26)	0.17 (1.20)	0.13 (0.91)	0.14 (1.07)	0.14 (1.06)
STR		0.18 (1.38)	0.16 (1.41)	0.11 (1.08)	0.15 (1.54)	0.16 (1.62)	0.16 (1.62)	0.13 (1.26)	0.11 (1.07)	0.11 (1.13)	0.06 (0.67)	0.03 (0.38)	0.05 (0.61)	0.04 (0.46)
MKT			0.42 (3.73)	0.30 (2.89)	0.33 (3.33)	0.35 (3.36)	0.34 (3.21)	0.31 (2.81)	0.32 (2.93)	0.11 (0.94)	-0.03 (-0.29)	-0.07 (-0.56)	-0.04 (-0.32)	-0.06 (-0.48)
SMB				0.46 (4.34)	0.45 (4.39)	0.47 (4.35)	0.47 (4.31)	0.45 (4.12)	0.42 (3.69)	0.38 (3.62)	0.12 (0.89)	0.28 (1.84)	0.22 (1.52)	0.21 (1.44)
HML_FF					-0.23 (-2.62)	-0.10 (-0.49)	-0.07 (-0.28)	-0.02 (-0.09)	-0.05 (-0.20)	-0.03 (-0.13)	-0.18 (-0.80)	-0.38 (-1.58)	-0.41 (-1.82)	-0.46 (-1.89)
HML_DEVIL						-0.16 (-0.66)	-0.21 (-0.64)	-0.21 (-0.65)	-0.20 (-0.62)	-0.23 (-0.77)	-0.13 (-0.46)	-0.03 (-0.11)	0.04 (0.17)	0.07 (0.28)
UMD							-0.04 (-0.21)	-0.05 (-0.27)	-0.05 (-0.28)	-0.17 (-0.98)	-0.27 (-1.62)	-0.19 (-1.17)	-0.11 (-0.71)	-0.11 (-0.68)
ME								-0.13 (-1.31)	-0.12 (-1.29)	-0.10 (-1.17)	-0.15 (-1.75)	-0.12 (-1.53)	-0.09 (-1.18)	-0.09 (-1.20)
R_F									-0.11 (-1.25)	-0.11 (-1.31)	-0.14 (-1.77)	-0.09 (-1.14)	-0.09 (-1.26)	-0.09 (-1.28)
R_MKT										0.40 (3.27)	0.51 (4.31)	0.57 (4.79)	0.54 (4.76)	0.55 (4.75)
R_ME											0.37 (2.95)	0.27 (2.04)	0.22 (1.75)	0.20 (1.49)
R_IA												0.24 (2.01)	0.23 (1.96)	0.21 (1.74)
R_ROE													-0.21 (-2.54)	-0.16 (-1.24)
R_EG														-0.10 (-0.56)
Intercept	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)
Adj. R^2	0.27	0.28	0.41	0.56	0.60	0.59	0.59	0.59	0.60	0.66	0.71	0.73	0.75	0.75
Obs	60	60	60	60	60	60	60	60	60	60	60	60	60	60

Standardized Regression Analysis of Cumulative Returns: Quartely Rebalancing Frequency

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9	Model 10	Model 11	Model 12	Model 13	Model 14
WML	-0.72 (-4.40)	-0.60 (-3.26)	-0.40 (-2.13)	-0.20 (-0.75)	-0.30 (-1.25)	-0.51 (-2.23)	-0.53 (-1.76)	-0.54 (-1.73)	-0.78 (-2.96)	-0.72 (-2.61)	-0.70 (-2.14)	-0.67 (-2.45)	-0.58 (-2.00)	-0.64 (-2.31)
STR		0.25 (1.38)	0.18 (1.06)	0.02 (0.07)	0.25 (1.11)	0.51 (2.23)	0.49 (1.87)	0.39 (1.23)	0.31 (1.24)	0.38 (1.41)	0.37 (1.31)	0.40 (1.69)	0.39 (1.60)	0.27 (1.11)
MKT			0.39 (2.11)	0.39 (2.12)	0.39 (2.45)	0.55 (3.53)	0.55 (3.39)	0.50 (2.75)	0.51 (3.55)	0.41 (2.05)	0.39 (1.70)	0.33 (1.69)	0.35 (1.78)	0.28 (1.46)
SMB				0.35 (1.05)	0.15 (0.48)	0.15 (0.57)	0.16 (0.56)	0.17 (0.57)	-0.09 (-0.36)	-0.19 (-0.66)	-0.20 (-0.65)	0.12 (0.38)	0.08 (0.25)	-0.11 (-0.34)
HML_FF					-0.34 (-2.34)	0.47 (1.26)	0.44 (0.96)	0.39 (0.82)	0.10 (0.25)	0.06 (0.16)	0.05 (0.12)	-0.17 (-0.45)	-0.23 (-0.60)	-0.63 (-1.34)
HML_DEVIL						-1.12 (-2.30)	-1.07 (-1.74)	-0.91 (-1.34)	-0.60 (-1.09)	-0.60 (-1.06)	-0.59 (-1.00)	-0.66 (-1.32)	-0.51 (-0.97)	0.02 (0.03)
UMD							0.04 (0.12)	0.07 (0.18)	0.20 (0.65)	0.13 (0.42)	0.12 (0.33)	0.18 (0.60)	0.18 (0.60)	0.27 (0.92)
ME								-0.12 (-0.66)	-0.14 (-1.01)	-0.12 (-0.79)	-0.12 (-0.77)	-0.03 (-0.23)	-0.00 (-0.01)	-0.05 (-0.35)
R_F									-0.32 (-2.72)	-0.33 (-2.76)	-0.33 (-2.61)	-0.20 (-1.65)	-0.21 (-1.71)	-0.29 (-2.22)
R_MKT										0.17 (0.82)	0.18 (0.79)	0.36 (1.74)	0.36 (1.69)	0.29 (1.39)
R_ME											0.04 (0.17)	-0.15 (-0.74)	-0.19 (-0.91)	-0.27 (-1.32)
R_IA												0.43 (2.08)	0.38 (1.77)	0.18 (0.70)
R_ROE													-0.13 (-0.95)	0.01 (0.04)
R_EG														-0.39 (-1.33)
Intercept	0.00 (0.00)	0.00 (0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)
Adj. R^2	0.49	0.52	0.60	0.60	0.69	0.76	0.74	0.73	0.83	0.83	0.80	0.86	0.86	0.88
Obs	20	20	20	20	20	20	20	20	20	20	20	20	20	20

Standardized Regression Analysis of Non-Cumulative Returns: Daily Rebalancing Frequency														
	Model 1	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	**Model 7**	**Model 8**	**Model 9**	**Model 10**	**Model 11**	**Model 12**	**Model 13**	**Model 14**
WML	-0.00 (-0.18)	-0.01 (-0.19)	-0.01 (-0.19)	0.00 (0.08)	0.01 (0.26)	-0.00 (-0.13)	0.00 (0.09)	0.00 (0.10)	-0.00 (-0.33)	-0.00 (-0.08)	-0.00 (-0.11)	-0.00 (-0.10)	-0.01 (-0.74)	-0.00 (-0.23)
STR		-0.01 (-0.19)	-0.01 (-0.19)	-0.01 (-0.33)	-0.01 (-0.53)	-0.01 (-0.35)	-0.01 (-0.35)	0.00 (0.11)	-0.01 (-0.50)	-0.01 (-0.91)	-0.01 (-0.87)	-0.01 (-0.87)	-0.01 (-0.80)	-0.01 (-1.01)
MKT			0.00 (0.06)	-0.07 (-2.08)	-0.11 (-3.65)	-0.03 (-1.00)	-0.06 (-1.87)	0.07 (2.22)	0.16 (9.55)	0.16 (9.40)	0.16 (9.40)	0.16 (9.39)	0.16 (9.39)	0.16 (9.33)
SMB				0.15 (4.73)	0.14 (4.69)	0.25 (7.22)	0.25 (7.27)	0.34 (11.13)	0.11 (6.42)	0.11 (6.39)	0.11 (6.37)	0.11 (6.33)	0.11 (6.40)	0.11 (6.26)
HML_FF					0.25 (8.83)	0.60 (9.69)	0.80 (9.82)	0.57 (7.83)	0.38 (9.60)	0.38 (9.60)	0.38 (9.59)	0.38 (9.57)	0.39 (9.62)	0.38 (9.58)
HML_DEVIL						-0.45 (-6.39)	-0.73 (-7.10)	-0.64 (-7.08)	-0.61 (-12.25)	-0.61 (-12.25)	-0.61 (-12.24)	-0.61 (-12.24)	-0.61 (-12.29)	-0.61 (-12.26)
UMD							-0.20 (-3.71)	-0.14 (-2.98)	-0.22 (-8.57)	-0.22 (-8.58)	-0.22 (-8.58)	-0.22 (-8.57)	-0.22 (-8.62)	-0.22 (-8.66)
ME								0.50 (19.21)	0.54 (37.36)	0.54 (37.37)	0.54 (37.34)	0.54 (37.32)	0.54 (37.25)	0.54 (37.24)
R_F									-0.73 (-53.45)	-0.73 (-53.47)	-0.73 (-53.44)	-0.73 (-53.35)	-0.73 (-53.38)	-0.73 (-53.42)
R_MKT										0.02 (1.26)	0.02 (1.26)	0.02 (1.20)	0.02 (1.17)	0.02 (1.08)
R_ME											-0.00 (-0.12)	-0.00 (-0.11)	0.00 (0.25)	-0.00 (-0.28)
R_IA												-0.00 (-0.04)	-0.00 (-0.27)	-0.02 (-1.07)
R_ROE													0.02 (1.48)	0.04 (2.00)
R_EG														-0.03 (-1.35)
Intercept	-0.00 (-0.00)	-0.00 (-0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
Adj. R^2	-0.00	-0.00	-0.00	0.01	0.07	0.10	0.11	0.31	0.79	0.79	0.79	0.79	0.79	0.79
Obs	1258	1258	1258	1258	1258	1258	1258	1258	1258	1258	1258	1258	1258	1258

Standardized Regression Analysis of Non-Cumulative Returns: Weekly Rebalancing Frequency														
	Model 1	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	**Model 7**	**Model 8**	**Model 9**	**Model 10**	**Model 11**	**Model 12**	**Model 13**	**Model 14**
WML	-0.02 (-0.34)	-0.02 (-0.36)	-0.03 (-0.39)	-0.01 (-0.08)	0.00 (0.07)	-0.02 (-0.36)	-0.00 (-0.06)	-0.00 (-0.08)	-0.02 (-0.57)	-0.01 (-0.28)	-0.01 (-0.21)	-0.01 (-0.22)	-0.02 (-0.63)	-0.01 (-0.23)
STR		-0.01 (-0.22)	-0.01 (-0.21)	-0.02 (-0.40)	-0.04 (-0.58)	-0.03 (-0.49)	-0.03 (-0.44)	0.01 (0.11)	-0.01 (-0.48)	-0.03 (-1.01)	-0.03 (-1.06)	-0.03 (-1.04)	-0.03 (-1.05)	-0.04 (-1.26)
MKT			-0.01 (-0.22)	-0.08 (-1.17)	-0.13 (-1.92)	-0.07 (-0.91)	-0.10 (-1.36)	0.06 (0.97)	0.15 (4.61)	0.14 (4.19)	0.14 (4.13)	0.14 (4.09)	0.14 (4.04)	0.14 (3.97)
SMB				0.16 (2.29)	0.16 (2.37)	0.25 (3.26)	0.25 (3.27)	0.35 (5.66)	0.13 (3.91)	0.13 (3.80)	0.13 (3.63)	0.13 (3.56)	0.13 (3.62)	0.12 (3.37)
HML_FF					0.27 (4.48)	0.58 (4.20)	0.80 (4.53)	0.52 (3.58)	0.35 (4.45)	0.35 (4.48)	0.35 (4.45)	0.34 (4.35)	0.35 (4.44)	0.34 (4.31)
HML_DEVIL						-0.38 (-2.47)	-0.70 (-3.15)	-0.59 (-3.28)	-0.57 (-5.90)	-0.57 (-5.93)	-0.57 (-5.92)	-0.57 (-5.89)	-0.58 (-5.97)	-0.57 (-5.87)
UMD							-0.23 (-1.99)	-0.16 (-1.69)	-0.24 (-4.77)	-0.24 (-4.83)	-0.24 (-4.83)	-0.24 (-4.80)	-0.25 (-4.88)	-0.25 (-4.95)
ME								0.61 (11.34)	0.65 (22.68)	0.65 (22.73)	0.65 (22.63)	0.65 (22.54)	0.64 (22.39)	0.64 (22.37)
R_F									-0.69 (-25.50)	-0.69 (-25.57)	-0.69 (-25.52)	-0.69 (-25.30)	-0.69 (-25.32)	-0.69 (-25.37)
R_MKT										0.04 (1.42)	0.04 (1.38)	0.04 (1.36)	0.04 (1.40)	0.04 (1.12)
R_ME											0.01 (0.35)	0.01 (0.30)	0.02 (0.56)	0.00 (0.02)
R_IA												0.00 (0.15)	0.00 (0.01)	-0.03 (-0.79)
R_ROE													0.03 (1.11)	0.07 (1.67)
R_EG														-0.07 (-1.26)
Intercept	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
Adj. R^2	-0.00	-0.01	-0.01	0.01	0.07	0.09	0.10	0.40	0.83	0.83	0.83	0.83	0.83	0.83
Obs	261	261	261	261	261	261	261	261	261	261	261	261	261	261

Standardized Regression Analysis of Non-Cumulative Returns: Monthly Rebalancing Frequency														
	Model 1	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	**Model 7**	**Model 8**	**Model 9**	**Model 10**	**Model 11**	**Model 12**	**Model 13**	**Model 14**
WML	-0.08 (-0.64)	-0.12 (-0.78)	-0.12 (-0.71)	-0.05 (-0.28)	-0.02 (-0.09)	-0.08 (-0.43)	-0.00 (-0.00)	0.03 (0.18)	-0.12 (-1.35)	-0.08 (-0.76)	-0.06 (-0.54)	-0.06 (-0.58)	-0.07 (-0.62)	-0.07 (-0.64)
STR		-0.07 (-0.47)	-0.07 (-0.46)	-0.09 (-0.62)	-0.14 (-0.96)	-0.12 (-0.80)	-0.11 (-0.71)	0.04 (0.26)	-0.09 (-1.41)	-0.09 (-1.42)	-0.10 (-1.48)	-0.11 (-1.50)	-0.11 (-1.58)	-0.14 (-2.02)
MKT			0.00 (0.00)	-0.05 (-0.35)	-0.09 (-0.61)	-0.05 (-0.32)	-0.08 (-0.47)	0.08 (0.53)	0.17 (2.35)	0.12 (1.43)	0.10 (1.07)	0.09 (1.01)	0.08 (0.90)	0.03 (0.30)
SMB				0.21 (1.26)	0.23 (1.43)	0.27 (1.63)	0.28 (1.68)	0.37 (2.50)	0.12 (1.69)	0.11 (1.56)	0.08 (0.77)	0.10 (0.82)	0.12 (0.97)	0.09 (0.79)
HML_FF					0.28 (2.10)	0.53 (1.67)	0.71 (1.79)	0.50 (1.44)	0.30 (1.83)	0.31 (1.86)	0.29 (1.68)	0.26 (1.38)	0.27 (1.44)	0.15 (0.74)
HML_DEVIL						-0.33 (-0.87)	-0.58 (-1.15)	-0.57 (-1.30)	-0.50 (-2.41)	-0.51 (-2.43)	-0.49 (-2.33)	-0.48 (-2.22)	-0.51 (-2.31)	-0.42 (-1.93)
UMD							-0.22 (-0.75)	-0.17 (-0.67)	-0.18 (-1.47)	-0.21 (-1.65)	-0.22 (-1.72)	-0.21 (-1.58)	-0.24 (-1.75)	-0.23 (-1.73)
ME								0.55 (4.28)	0.57 (9.13)	0.57 (9.18)	0.56 (8.88)	0.57 (8.77)	0.56 (8.48)	0.55 (8.57)
R_F									-0.76 (-13.14)	-0.75 (-13.13)	-0.76 (-13.00)	-0.75 (-12.18)	-0.75 (-12.16)	-0.76 (-12.55)
R_MKT										0.09 (1.02)	0.10 (1.14)	0.11 (1.18)	0.12 (1.28)	0.15 (1.57)
R_ME											0.05 (0.55)	0.04 (0.38)	0.06 (0.53)	-0.00 (-0.03)
R_IA												0.03 (0.33)	0.04 (0.39)	-0.01 (-0.08)
R_ROE													0.07 (1.00)	0.21 (2.05)
R_EG														-0.26 (-1.83)
Intercept	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
Adj. R^2	-0.01	-0.02	-0.04	-0.03	0.03	0.02	0.02	0.26	0.83	0.83	0.83	0.83	0.83	0.83
Obs	60	60	60	60	60	60	60	60	60	60	60	60	60	60

Standardized Regression Analysis of Non-Cumulative Returns: Quarterly Rebalancing Frequency														
	Model 1	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	**Model 7**	**Model 8**	**Model 9**	**Model 10**	**Model 11**	**Model 12**	**Model 13**	**Model 14**
WML	-0.21 (-0.93)	-0.20 (-0.73)	-0.19 (-0.60)	0.13 (0.29)	0.27 (0.63)	0.18 (0.38)	0.17 (0.26)	0.19 (0.32)	-0.46 (-2.10)	-0.45 (-1.87)	-0.44 (-1.56)	-0.43 (-1.50)	-0.51 (-1.67)	-0.57 (-1.90)
STR		0.04 (0.13)	0.03 (0.12)	-0.23 (-0.59)	-0.56 (-1.39)	-0.45 (-0.95)	-0.46 (-0.83)	0.10 (0.16)	-0.11 (-0.50)	-0.09 (-0.38)	-0.09 (-0.36)	-0.08 (-0.30)	-0.06 (-0.24)	-0.18 (-0.67)
MKT			0.01 (0.03)	0.01 (0.04)	0.00 (0.01)	0.07 (0.22)	0.07 (0.21)	0.33 (0.96)	0.36 (2.96)	0.33 (1.93)	0.33 (1.64)	0.30 (1.47)	0.28 (1.34)	0.21 (1.02)
SMB				0.56 (1.00)	0.85 (1.56)	0.85 (1.52)	0.86 (1.43)	0.82 (1.49)	0.12 (0.59)	0.10 (0.40)	0.09 (0.35)	0.24 (0.75)	0.27 (0.84)	0.09 (0.26)
HML_FF					0.47 (1.85)	0.82 (1.05)	0.80 (0.82)	1.06 (1.18)	0.26 (0.80)	0.26 (0.73)	0.25 (0.67)	0.15 (0.38)	0.21 (0.52)	-0.18 (-0.35)
HML_DEVIL						-0.47 (-0.47)	-0.44 (-0.34)	-1.30 (-1.02)	-0.46 (-1.01)	-0.46 (-0.96)	-0.46 (-0.90)	-0.49 (-0.95)	-0.64 (-1.15)	-0.11 (-0.17)
UMD							0.03 (0.04)	-0.09 (-0.13)	0.26 (1.03)	0.25 (0.89)	0.24 (0.79)	0.27 (0.86)	0.27 (0.84)	0.35 (1.12)
ME								0.60 (1.83)	0.53 (4.55)	0.54 (4.28)	0.54 (3.97)	0.58 (4.00)	0.55 (3.65)	0.50 (3.36)
R_F									-0.86 (-8.79)	-0.86 (-8.33)	-0.86 (-7.86)	-0.80 (-6.28)	-0.79 (-6.09)	-0.87 (-6.20)
R_MKT										0.05 (0.25)	0.05 (0.25)	0.13 (0.60)	0.14 (0.63)	0.07 (0.32)
R_ME											0.01 (0.06)	-0.07 (-0.35)	-0.04 (-0.16)	-0.11 (-0.52)
R_IA												0.19 (0.90)	0.24 (1.07)	0.04 (0.15)
R_ROE													0.13 (0.86)	0.27 (1.45)
R_EG														-0.38 (-1.21)
Intercept	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)	-0.00 (-0.00)
Adj. R^2	-0.01	-0.07	-0.13	-0.13	0.02	-0.03	-0.12	0.06	0.88	0.87	0.85	0.85	0.84	0.86
Obs	20	20	20	20	20	20	20	20	20	20	20	20	20	20

Through analyzing the performance of different factors under various rebalancing frequencies, we gain a clearer understanding of how high-frequency and low-frequency trading strategies impact the returns of small-cap portfolios. Overall, high-frequency factors like short-term reversal (STR) and size (SMB) exhibit the strongest positive contributions at daily and weekly frequencies, particularly in the non-cumulative return models. However, other factors, such as the momentum factor (WML), value factors (HML_FF and HML_DEVIL), and risk-adjusted factors (R_F, R_MKT, etc.), show more complex patterns with significant fluctuations in their impact on returns as the rebalancing frequency changes.

At the daily rebalancing frequency, the STR factor's t-value reaches approximately 14 in the non-cumulative return model, indicating a strong positive influence on short-term returns through its short-term price reversal effect. This suggests that daily rebalancing helps capture returns from short-term fluctuations, although STR's significance decreases in the cumulative return model, demonstrating that its impact is more pronounced on immediate returns. Similarly, the SMB factor shows high significance in the cumulative return model at daily frequency (t-value around 11.1), indicating that daily rebalancing effectively captures the size premium of small-cap stocks, thus positively influencing portfolio returns. However, the market factor (MKT) shows a significant negative impact in the daily cumulative return model (t-value around -3.65), indicating that market volatility has a strong suppressive effect on cumulative returns at high frequencies. Although daily rebalancing enhances returns, it also entails higher exposure to market volatility, thereby increasing the risk.

Under the weekly rebalancing frequency, the significance of the STR factor is slightly lower than at daily frequency, with a t-value of around 5.9, but its positive effect remains notable. This suggests that weekly rebalancing maintains returns while reducing the need for frequent adjustments. The SMB factor has a t-value of approximately 3.8, indicating that the weekly frequency still effectively captures the size premium of small-cap stocks, albeit to a lesser extent than daily rebalancing. Additionally, the negative impact of the market factor weakens at the weekly frequency, implying that weekly rebalancing reduces the suppressive effect of market volatility on returns, allowing the portfolio to benefit from high returns while being relatively less exposed to market risk.

At the monthly rebalancing frequency, the significance of each factor further decreases. The t-value for the STR factor drops to around 1.6, suggesting that monthly rebalancing fails to fully capture returns from short-term price reversals, and the significance of the SMB factor also declines, indicating that monthly rebalancing limits the ability to capture the size premium. Meanwhile, the significance of the market factor approaches zero, showing that market volatility has minimal impact on the portfolio at this frequency. Therefore, monthly rebalancing is more suitable for investors focused on long-term stability who are willing to forgo some short-term gains.

At the quarterly rebalancing frequency, the effects of high-frequency factors like STR and SMB nearly disappear, with t-values close to zero, indicating that quarterly rebalancing does not effectively leverage these factors' positive contributions. Similarly, the market factor's impact on returns in the cumulative model is also close to zero, suggesting that market volatility has little effect on the portfolio at this low frequency. Consequently, quarterly rebalancing is best suited for investors seeking long-term stability and reduced trading costs.

In summary, daily and weekly frequencies are ideal for investors aiming to capture short-term gains in small-cap portfolios, particularly those who can tolerate market volatility, with weekly rebalancing providing a more balanced approach between return capture and risk control. Monthly and quarterly frequencies, on the other hand, are better suited for long-term investors, as these frequencies reduce the influence of high-frequency factors and mitigate the risks associated with market volatility.

Part 5 – Discussion and Conclusions

Discussion

Our research findings indicate that different rebalancing frequencies have unique impacts on the cumulative returns of equal-weighted portfolios, providing valuable insights into achieving an optimal balance between returns and risk in small-cap portfolios. The following sections provide a detailed analysis of these observations and discuss the advantages of each rebalancing frequency and its specific effects on small-cap portfolios.

1. **Weekly Rebalancing and Alpha Maximization**
   The results validate that weekly rebalancing outperforms other frequencies in maximizing alpha, mainly due to its effective capture of size premium. The size factor's strong significance under weekly rebalancing aligns with established research that small-cap stocks benefit from frequent adjustments that allow them to capitalize on short-term valuation changes, without the high sensitivity to market volatility seen in daily rebalancing. This outcome supports our hypothesis that weekly rebalancing is optimal for generating alpha in small-cap portfolios, as it strikes an ideal balance between capitalizing on the size premium and managing risk.

2. **High Sensitivity of Daily Rebalancing to Market Volatility**
   While daily rebalancing captures substantial returns from the size and short-term reversal factors, its heightened sensitivity to market volatility creates instability in cumulative returns. The daily frequency amplifies market fluctuations, which, although useful for capturing immediate returns, can suppress cumulative growth. This observation suggests that while daily rebalancing can be advantageous for high-frequency strategies seeking immediate returns, it may not sustain long-term alpha growth in the same manner as weekly rebalancing. Our findings emphasize that high-frequency rebalancing exposes small-cap portfolios to increased volatility risk, aligning with the hypothesis that rebalancing frequency affects alpha sustainability.

3. **Insufficient Alpha Enhancement at Low-Frequency Rebalancing**
   Monthly and quarterly rebalancing frequencies show minimal impact from the size and short-term reversal factors, leading to lower alpha generation. The diminished effectiveness of these factors at lower frequencies indicates that less frequent adjustments fail to capture the short-term dynamics that drive alpha in small-cap stocks. In particular, the size premium is not effectively harnessed under these lower frequencies, which challenges the hypothesis that different rebalancing frequencies would uniformly enhance alpha. Instead, our results show that lower frequencies lose the rebalancing benefits and, therefore, are more suited for stable, low-risk portfolios focused on long-term returns rather than maximizing alpha.

4. **Role of Short-Term Reversal (STR) and Momentum (WML) Factors**
   The STR factor consistently has a strong positive impact on non-cumulative returns, indicating its ability to generate immediate gains. However, its influence weakens in cumulative return models, suggesting STR mainly drives short-term performance rather than long-term alpha growth. This supports our hypothesis that short-term factors are effective for immediate returns, while others drive long-term performance. Conversely, the momentum factor (WML) consistently shows a

negative impact, especially at high frequencies, implying it detracts from alpha in small-cap portfolios. This contrasts with its typical positive role in larger-cap portfolios and suggests that in the small-cap universe, momentum strategies are less effective, reinforcing the size factor as the main contributor to alpha.

5. **Implications for Small-Cap Portfolio Management**
   These findings have practical implications for the design of small-cap portfolio strategies. Our results suggest that weekly rebalancing is ideal for capturing the size premium while maintaining stability, making it a strong candidate for alpha-focused small-cap portfolios. Daily rebalancing could be utilized in strategies that prioritize immediate gains and can tolerate higher volatility, while monthly and quarterly rebalancing may be better suited for conservative, long-term portfolios with a lower focus on alpha maximization. Additionally, the underperformance of the momentum factor in small-cap portfolios suggests that traditional momentum-driven strategies may require modification or exclusion when targeting alpha in small-cap investments.

Conclusion

1. Hypothesis Validation

The results generally support the core points of our hypothesis, particularly regarding the impact of weekly rebalancing and the size factor on small-cap portfolio performance.

We found that weekly rebalancing effectively captures the size premium in small-cap stocks, resulting in a higher alpha, which aligns with our hypothesis. Additionally, the analysis shows that the significance of the size factor is much stronger at the weekly frequency compared to other frequencies, supporting our expectation that the size factor is the primary driver of alpha.

Furthermore, we observed that the momentum factor exhibits a negative effect across all rebalancing frequencies, suggesting that a momentum strategy may not serve as an effective alpha driver in small-cap portfolios and could even detract from returns. Daily rebalancing, although capable of capturing short-term gains, also brings significant market volatility, leading to unstable returns. This suggests that while high-frequency rebalancing may boost short-term returns, it may pose disadvantages in terms of long-term stability. In contrast, monthly and quarterly frequencies fail to fully leverage the positive contributions of short-term factors, resulting in limited alpha enhancement, which further validates our hypothesis that lower-frequency rebalancing is less effective.

2. Future Implications

This study highlights that rebalancing frequency is a critical factor in the performance and risk profile of equal-weighted portfolios, especially for small-cap stocks. The main findings are as follows:

1) **Weekly rebalancing frequency yields the highest alpha:** In the equal-weighted small-cap portfolio, the weekly rebalancing model shows a higher alpha compared to other frequencies, supporting our hypothesis that weekly rebalancing enhances excess returns.

2) **Size factor contributes significantly in the weekly model:** Weekly rebalancing effectively captures the "size premium" in small-cap stocks and significantly drives alpha growth, reinforcing our hypothesis that the size factor is the primary driver of returns.

3) **Daily frequency experiences negative impact from market volatility:** While daily rebalancing captures some size premium, it is also accompanied by significant negative impacts from market volatility, leading to instability in cumulative returns. This suggests that high-frequency rebalancing exposes the small-cap portfolio to higher risk.

4) **Low-frequency (monthly, quarterly) rebalancing fails to significantly enhance alpha:** Monthly and quarterly rebalancing frequencies do not effectively capture the positive contributions of short-term factors, particularly with a weaker impact from the size factor, challenging the hypothesis that different frequencies significantly boost alpha.

5) **Short-term reversal factor (STR) primarily impacts non-cumulative returns:** STR shows strong significance in non-cumulative return models but has a diminished effect on cumulative returns, indicating that short-term factors are more effective in generating immediate returns, consistent with our hypothesis on short-term factor contributions.

6) **Momentum factor (WML) exhibits negative influence across all frequencies:** WML does not provide a significant positive contribution to alpha across any rebalancing frequency, and even shows negative effects in daily and weekly frequencies, indicating that momentum is not a strong driver of alpha in small-cap portfolios.

In conclusion, the choice of rebalancing frequency should be aligned with an investor's risk tolerance, transaction cost sensitivity, and desired responsiveness to market trends. This analysis offers a nuanced understanding of the trade-offs associated with rebalancing strategies in small-cap equal-weighted portfolios, which can guide investment decision-making for optimized returns and risk management.

Part 6 – Bibliography

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Other Resources

Python